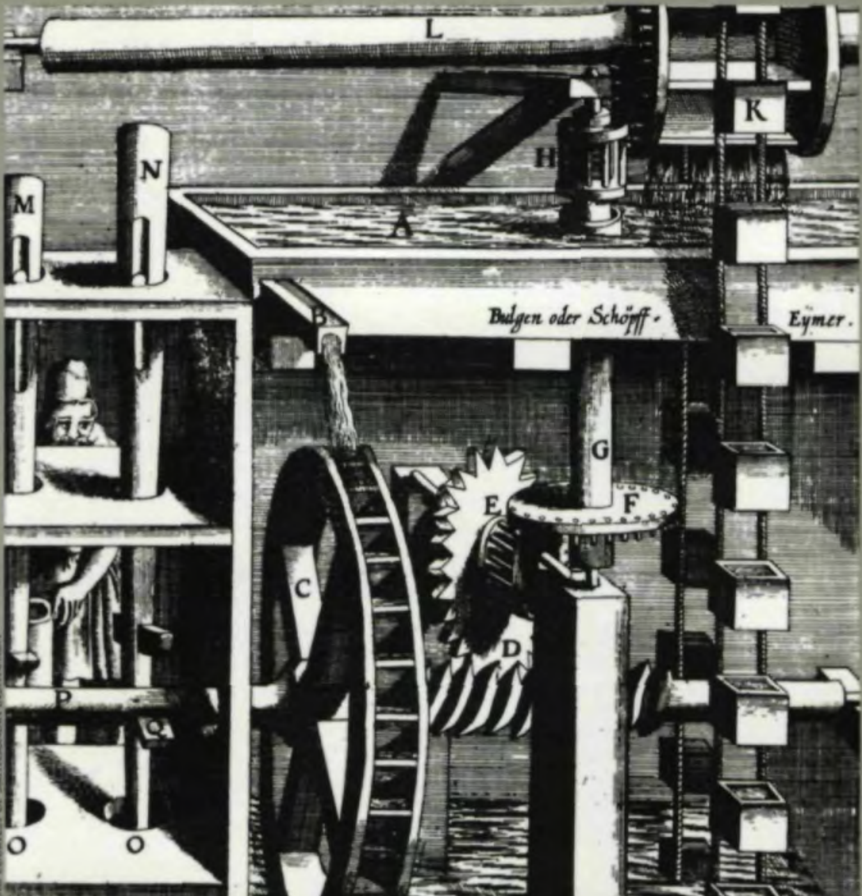
PRESHOW IMTS '88

GEAR TECHNOLOGY

The Journal of Gear Manufacturing

JULY/AUGUST 1988



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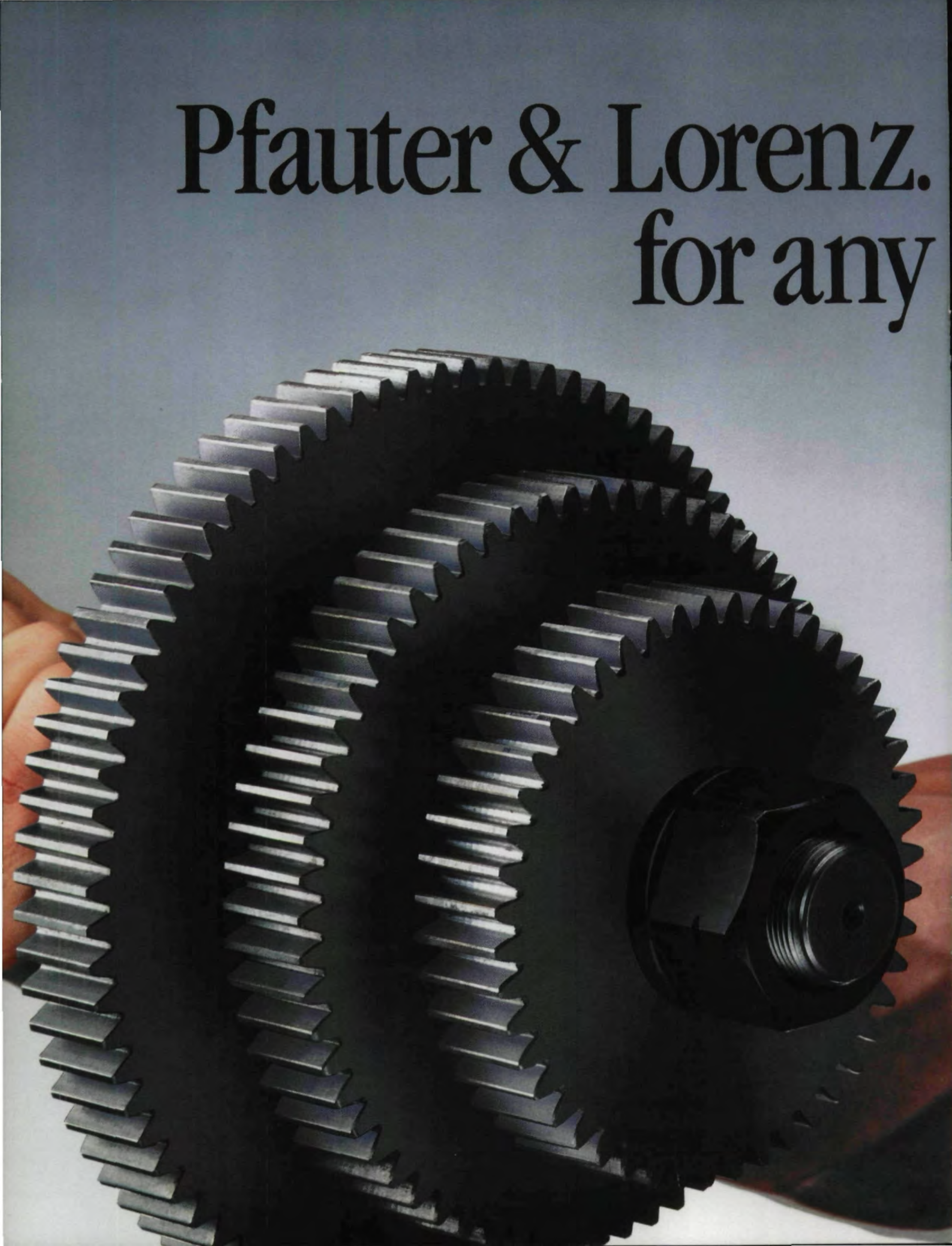
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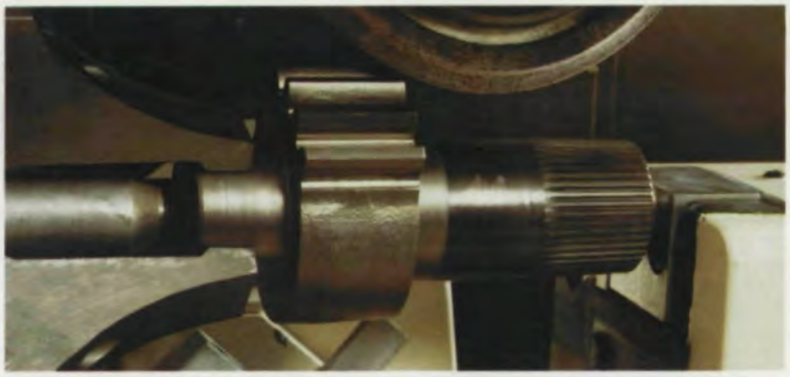

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GEAR TECHNOLOGY

The Journal of Gear Manufacturing

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OUR COVER

A medieval perpetual motion machine. Note the worm gear drive and the pounding machine for pulverizing material. Water drives the large wheel and then is lifted in the buckets to the upper trough where it can be reused. The machine, of course, does not work. Medieval engineers, lacking an understanding of entropy, kept redesigning the perpetual motion machine in the hope of finding a truly practical one. The drawing is taken from a book by George Andream Bocklern published in Numberg in 1661. Photo courtesy of the Smithsonian Institution.

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DEPARTMENTS

July/August 1988 — Vol. 5, No. 4

GEAR TECHNOLOGY, The Journal of Gear Manufacturing (ISSN 0743-6858) is published bimonthly by Randall Publishing Co., Inc., 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007. GEAR TECHNOLOGY, The Journal of Gear Manufacturing is distributed free of charge to qualified individuals and firms in the gear manufacturing industry. Subscription rates for non-qualified individuals and firms are: $35.00 in the United States, $55.00 for foreign countries. Second-Class postage paid at Arlington Heights, IL and at additional mailing office.

Postmaster: Send address changes to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, 1425 Lunt Avenue, P. O. Box 1426, Elk Grove Village, IL 60007.

MANUSCRIPTS: We are requesting technical papers with an educational emphasis for anyone having anything to do with the design, manufacture, testing or processing of gears. Subjects sought are solutions to specific problems, explanations of new: technology, techniques, designs, processes, and alternative manufacturing methods. These can range from the "How to . . ." of gear cutting (BACK TO BASICS) to the most advanced technology. All manuscripts submitted will be carefully considered. However, the Publisher assumes no responsibility for the safety or return of manuscripts. Manuscripts must be accompanied by a self-addressed, self-stamped envelope, and be sent to GEAR TECHNOLOGY, The Journal of Gear Manufacturing, P.O. Box 1426, Elk Grove, IL 60007, (312) 437-6604.

EDITORIAL

UNFAIR FARES

"It's not so much the rocks in the road that wear you down as the bit of gravel in your shoe," says the old maxim. Little annoyances over which we seem to have no control are the ones that take their greatest toll and raise our frustration level to the highest point. I feel fortunate to be the editor of a magazine, so I at least have some means to vent my frustration.

One of the pieces of gravel in my shoes is the way the airlines are taking advantage of business customers. If we all treated our best customers the way airlines treat their business passengers, none of us would have businesses very long. Deregulation has hit the airlines, and the competition among domestic carriers is fierce, but it is the business customer that is being taken for a long and and mostly very unpleasant ride.

More and more often, customers who visit me at my machinery business are complaining about the exhorbitant cost of the airline ticket that they found necessary to buy a day or two prior to the trip. From Chicago, $500 to Dayton; $655 to Houston or LaGuardia, New York; $700 to Boston and $800 to $1000 to Los Angeles or Seattle, and these are *coach* fares!

And what does the business traveler get for these princely sums? Crowded lounges, narrow seats with less leg room than the back seat of a Yugo, jammed overhead racks, filled hanging luggage space, terrible food, delayed and cancelled flights, and, of course, a bag of peanuts and a free airline magazine, because, after all, we are told, flying is fun.

Travelling coach class on the airlines today reminds me a good bit of pictures I've seen of "steerage" class on the ocean liners of another era. At least our immigrant grandparents could comfort themselves with visions of better life at the end of the voyage. All the business traveler has to look forward to is an interminable wait for luggage which may or may not be there and an identical return trip in a day or two.

If this kind of service were the best the airlines could do and if everyone were in the same boat — or airline cabin — it would be unpleasant, but at least it would be fair. As it is, most often the person sitting next to you, who stuffed the overhead rack with all his worldly possessions, and brought along his wriggly, moist and sticky two-year-old, has quite probably paid 20% to 50% of the price of your ticket. He planned his trip six months ago and took advantage of all the cut-rate prices.

Let's get this straight. I'm not opposed to grannys and two-year-olds flying at bargain prices, nor am I opposed to airlines marketing their product the best way they can in a tough competitive environment. The budget traveler, of which I am one for my planned travel, gets exactly what he pays for. He grits his teeth and bears several hours of discomfort in exchange for very low travel prices. But the regular business flyer, who is the backbone of the U.S. domestic airline market and who cannot plan every trip six months in advance, is being ripped off.

The airlines, through their multi-level pricing structure, are, selling virtually the same product at vastly different prices, and the business traveler, who has least control over the circumstances under which he must make his travel arrangements, has to pay the most. Worse yet, as airline competition becomes even fiercer, it is the business traveler's rates that are being jacked up to support the low-end fares.

Frequent mileage programs, which were originally started to reward the airlines' best customers, the business travelers, are now being used as an excuse to raise the cost of our part of the fare structure. And now, perhaps because the frequent flyer programs were too successful for the airlines' own good, even they are being taken away. United recently announced that it has increased the requirement for its top bonuses from 75,000 miles to 250,000 miles. Nice guys!

It's time to bring fairness back into the airline ticket prices. If businesspeople have to pay premium prices for the necessity of travelling at a moment's notice, then there ought to be some additional value received. A business class, available now on some international carriers, is one idea. Such a class should cost present full-fare coach rates, not some additional premium. Some recognition that the business traveler is *at work* when he or she is flying would be nice. Little amenities like additional leg room and work space would help a lot. Better food and drink would be appreciated. Conveniences like free land telephone lines or expedited baggage pick-up and special parking and land transportation services would help too.

(Continued on page 33)

GUEST EDITORIAL

A REAL TEST

It is often easy for those outside of the gear industry to get the impression that nothing is changing in our business. After all, as illustrated bimonthly by the covers of this very journal, the making of gears has been with us for centuries. However, nothing could be further from the truth.

As this year's AGMA president, and for many years a chairman of a technical committee, I've seen firsthand the rapid changes that are occurring. Technical advances, as reported either in this publication or through the publications and meetings of AGMA, have driven the marketplace. The demand for better gears at lower costs has tested our abilities in design, manufacture and application. It has been a real test of our engineering capabilities.

In recent years, there have been other tests of our abilities. External forces keep poking their noses into the engineering tent, testing our ability to compete in an economic environment that often times is neither stable nor fair. Life would be much simpler if the test was just to design and make the best gears.

Some domestic markets have faded as a result of advances in technology. An example can be seen on our own shop floors, where gear generating machine tools have far fewer, if any, gears in them. What was once a part of our industry's market has all but disappeared. Other domestic markets have fallen prey to changing international economies. Gear manufacturers once had a healthy business supplying the American steel industry, and global oil prices have severely reduced our market in the U.S. "oil patch."

Add to this the fact that the U.S. government, regardless of good or bad intentions, has hindered our own efforts to compete. Tax codes, product liability laws, government procurement policies and a long list of other issues have added to our challenge of doing business in today's environment. Some of the forces from outside our plants are ones that we can do little about. For others, there is a chance of ultimately correcting the problem, particularly through our Association. But in either case, this changing environment places greater emphasis on our ability to stay abreast of the latest technology. To compete both domestically and internationally, regardless of the myriad of other constraints, we have to be on the leading edge of our technology.

That's something we can control. We can attend conferences and technical meetings. In AGMA, we hold about 50 sessions per year, described by some as "the best continuing education program available." Most of these are the standards writing committee meetings. Although such committee meetings are working sessions to develop new national standards, they also provide one of the best forums to define state-of-the-art technology. We can also read journals and technical papers. Papers and journals from AGMA, ASME and other societies and publications like *Gear Technology* are only a few of the resources we have commonly available to us.

The point is that we have to make the effort. We have to recognize that as our world changes, it places greater emphasis on our ability to apply the best engineering and technical ideas available. Only in this way can we meet the challenges from our domestic and foreign competitors, as well as those from our economy and government.

That will be the real test of our engineering abilities.

Daniel E. Bailey

Dan Bailey

President
AGMA

DANIEL E. BAILEY *is American Gear Manufacturers' Association president for 1987-88. He has also served on the AGMA Board of Directors for four years and as Chairman of the Fine Pitch Gearing Committee. Mr. Bailey attended LeMoyne College, Syracuse, NY, and earned a degree in manufacturing engineering from Rochester Institute of Technology. He is the president of Rochester Gear, Inc., Rochester, NY.*

1988 International Mac

ADVERTISERS' INDEX

hine Tool Show

For nine days in September, Chicago, Illinois, will become the manufacturing capital of the world. Sept. 7-15, the 1988 International Machine Tool Show will occupy almost all of the 1.8 million square feet of the McCormick Place exposition complex. Systems, processes and equipment from 1,000 exhibitors and covering some 2,000,000 sq. ft. will be on display. These exhibits will showcase state-of-the-art manufacturing technology such as, lasers, robotics, computer-aided design and flexible machining systems and fully automated factories.

A new computerized, touch-screen locator system, with stations throughout the exhibition hall, will help attendees easily locate specific exhibitors and products. In the east and north buildings of the complex, product types will also be grouped together for ease of location.

Running concurrently with the IMTS is the Fourth Biennial International Manufacturing Technology Conference. Forty-eight sessions and more than 200 papers on a wide variety of manufacturing-related subjects will present the latest developments in manufacturing technology.

Admission to the IMTS is by registration only. Those registering before August 5 will receive a 50¢ discount. Early registrants for the Technical Conference will save one-third on conference fees.

Show hours are 9-6 daily and Saturday and 9-4 on Sunday at McCormick Place, just south of the Chicago Loop on Lake Shore Drive.

The exhibition hall is easily accessible by both automobile and public transportation and is near major hotels, restaurants, shopping, entertainment and major museums. There is also both direct public transportation and major highway access to McCormick Place from O'Hare International Airport. For further information and registration forms for the IMTS contact NMTBA, 7901 Westpark Drive, McClean, VA 22102-4269. For more information on hotels, entertainment and transportation in Chicago, call the Chicago Convention and Tourism Bureau, (312) 225-5000 or the Illinois Office of Tourism, (800) 223-0121.

INDUSTRY FORUM

"INDUSTRY FORUM" provides an opportunity for readers to discuss problems and questions facing our industry.

Please address your questions and answers to: INDUSTRY FORUM, GEAR TECHNOLOGY, P.O. BOX 1426, Elk Grove Village, IL 60007.

Dear Editor:

I am involved in a project to investigate alternative shaft/hub connection methods. I would like to request any information on these alternative methods to supplement the investigation.

Ward Machinery manufactures finishing equipment for the corrugated industry (die cutters and flexo/folder/gluers), employing traditional keyed connections between gears and shafts. In certain locations on the machine we have experienced keyway failures, resulting in customer dissatisfaction and expensive service calls.

One option that looks particularly attractive is the use of internal locking devices and shrink disks instead of keys/ keyways. We already use several of these devices in non-critical locations, but there is skepticism that they will not perform in critical locations in the main gear train. Specific information on internal locking devices and shrink disks would be very helpful.

Kevin W. Erbe, Ward Machinery Co.

Could you direct us to a source where we could get information on the "gear planer or shaper"? We would like to get information on this machine for instructional purposes.

C.F. Fitz, Consultant

Reference is made to your November/ December 1987 issue, wherein you published a paper entitled "Finishing of Gears by Ausforming" by M. F. Amateau and R. A. Cellitti. Unfortunately, the authors failed to perform a comprehensive literature search which, if performed, would have discovered that a considerable amount of work had been performed with the ausforming process on bearings and gears. The results reported in this literature do indeed show an improvement in life for many applications. However, the benefit in many instances is offset by increased cost. Further, in large size bearings and gears, the forging capacity of available equipment is insufficient to ausform the components.

A 1976 ASME paper entitled "A Life Study of Ausformed, Standard Forged and Standard Machined AISI Spur Gears" by D.P. Townsend, E.N. Bamberger, and E.V. Zaretsky was published in the *Journal of Lubrication Technology*, Vol. 98, No. 2, 1976, pp. 267-276. The bottom line for this paper is that there was no distinct difference in life between the ausformed and standard forged gears. The reasons are given in the paper.

Erwin V. Zaretsky
NASA Lewis Research Center

High Speed Hobbing of Gears With Shifted Profiles

Kisaburo Nagano
Kurume College of Technology
Kurume, Japan
Masato Ainoura
Kumamoto Institute of Technology
Kumamoto, Japan

Abstract:

Most reduction gears currently manufactured for automobiles and other vehicles are profile-shifted gears.[1]

Therefore, the authors have examined the damage on hobs for profile-shifted gears (spur and helical) with larger outside diameter (long addendum) or smaller outside diameter (short addendum) to improve the design and lengthen the life of a hob.

Introduction

The newer profile-shifted (long and short addendum) gears are often used as small size reduction gears for automobiles or motorcycles. The authors have investigated the damage to each cutting edge when small size mass-produced gears with shifted profiles are used at high speeds.

Table 1 shows the dimensions of hobs and profile-shifted gears in an experiment. The hob has a module m=3 and an outside diameter of 110mm. It is made of high speed M34 steel, triple-threaded, built-up and TiN-coated.[2] In this experiment, three kinds of profile-shifted gears were prepared: (A) a short addendum gear with a coefficient of addendum modification of $a=0.5$ and a smaller outside diameter of $2(1-a)$m; (B) a standard gear ($a=1.0$) without shift; (C) a long addendum gear where ($a=1.5$) and with a larger outside diameter of $2(a-1)$m. The whole depth of cut is 6.75mm. Fig. 1 shows the hobbing method. Each cutting blade is numbered to show the wear of each cutting edge.

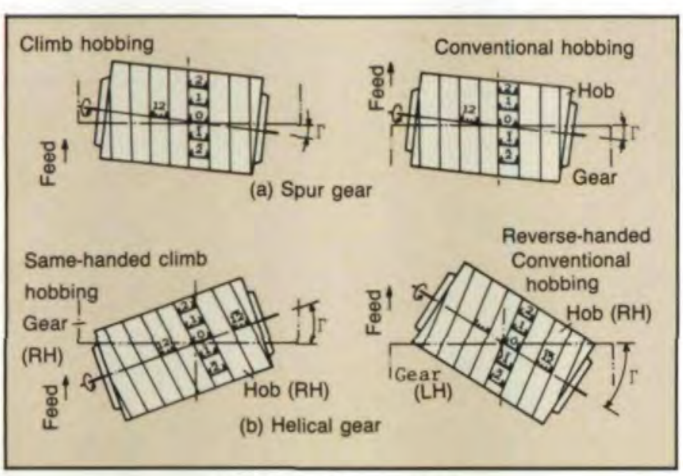


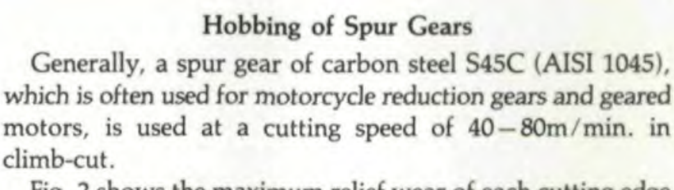
Fig. 1—Hobbing method.

Table 1 Dimensions of hobs & profile shifted gears

Dimensions of tested hobs:			
Normal module	m	3	3
Normal pressure angle	PA	20°	20°
Number of threads		3.RH	3.RH
Outside diameter	D	110mm	80mm
Lead angle	LA	5°04′	7°06′
Rake angle		0°	0°
Number of flutes	N	12	10
Material & Coating		M34+TiN	M34+TiN
Normal tooth profile		Standard	Standard

Dimensions of profile shifted gears:				
Normal module	m	3	3	3
Normal pressure angle	PA	20°	20°	20°
Helix angle	HA	0°	15°	30°
Number of teeth	Z	47	47	57
Coefficient addendum	a	0.5~1.5	0.5~1.5	0.5~1.5
Cutting length/1 gear	l	2.5m	2.8m	2.0m
Material	AISI	1045	1045	4118
Hardness	BHN	180	180	140

Hobbing machine: KS-14, Cutting oil: HS-4M



Hobbing of Spur Gears

Generally, a spur gear of carbon steel S45C (AISI 1045), which is often used for motorcycle reduction gears and geared motors, is used at a cutting speed of 40–80m/min. in climb-cut.

Fig. 2 shows the maximum relief wear of each cutting edge after 13 spur gears of carbon steel with various addendum modifications (cutting length: l = 32.4m) were hobbed at a

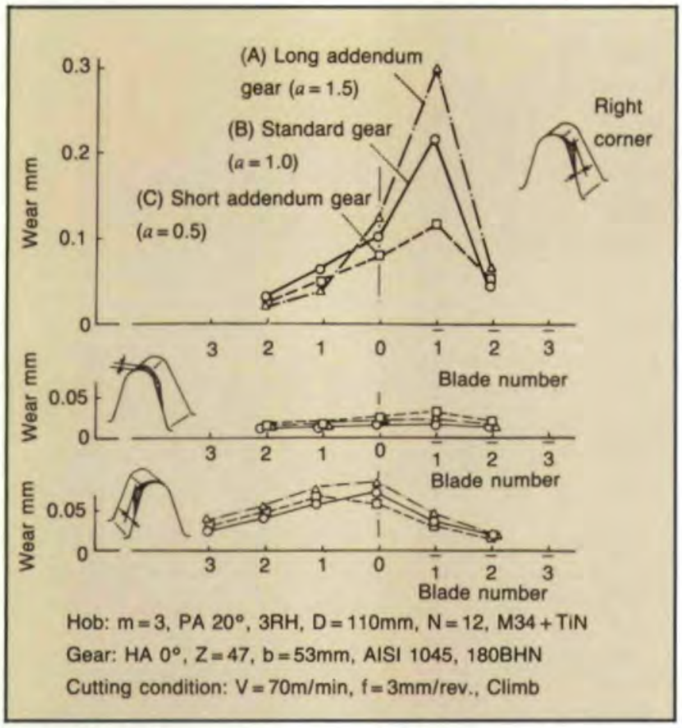


Fig. 2 – The relief wear on each cutting edge of spur gear with various addendum modifications (l = 32.4m).

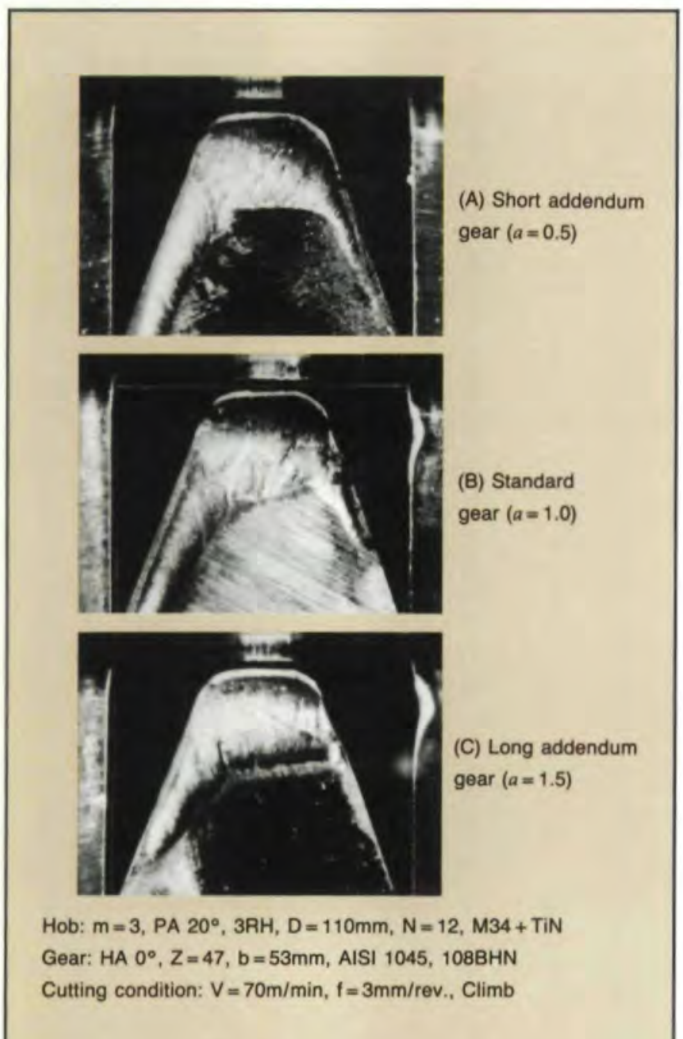


Fig. 3 – Damage to the blade No. $\bar{1}$ where the maximum relief wear occurred (l = 32.4m).

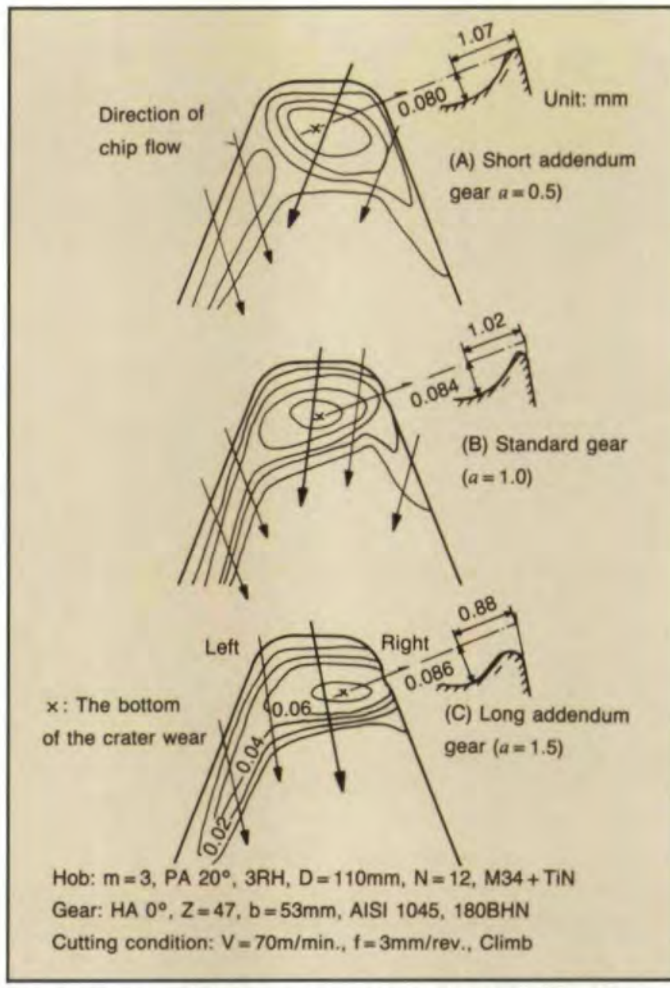
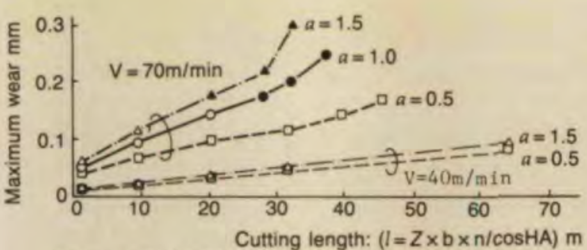


Fig. 4 – Direction of chip flow on each crater (Blade No. $\bar{1}$, l = 32.4m).

cutting speed of 70m/min. and a feed of 3mm/rev. There is little difference in wear at the top cutting edge and the left corner, but the cutting edge No. 1 at the right corner, where the maximum relief wear occurred, is greatly influenced by the addendum modification. The amount of corner wear for long addendum gears is the largest. The standard gear (B) has less wear than (C), and short addendum gear (A) has the least wear. Fig. 3 shows the damage to the blade No. 1 where the maximum relief wear occurred. In the case of hobbing short addendum gears, crater wear occurs on each cutting edge at the top, right corner and left corner, but nothing is yet broken down. In the standard and long addendum gears, the cutting edge at the right corner is broken down because of crater wear, causing extraordinary amounts of relief wear.

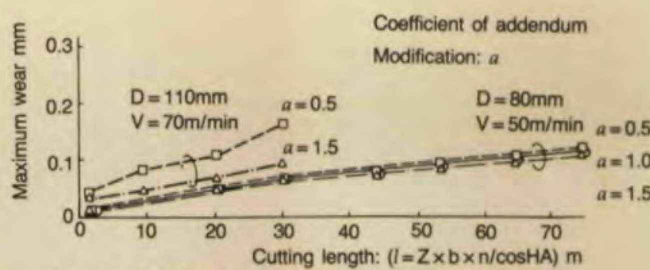
Fig. 4 shows the depth of crater wear with contour lines every 0.02mm and the direction of chip flow on each crater as observed using a microscope. In (A), a short addendum gear, cutting chips that come out from the top cutting edge and the right corner flow to the left cutting edge, but the chip flow from the left corner keeps them away from the left corner. Both cutting edges at the right and left corner are hardly broken down because crater wear is distant from each cutting edge, decreasing corner wear.

On the other hand, in the case of (C), a long addendum gear, cutting chips flow to the right cutting edge without obstruction because the right edge does not cut. Accordingly,



Hob: m=3, PA 20°, 3RH, D=110mm, N=12, M34+TiN
Gear: HA 0°, Z=47, b=53mm, AISI 1045, 180BHN
Cutting condition: F=3mm/rev., Climb hobbing

Fig. 5—Progress of the maximum relief wear in case of climb hobbing (HA 0°).



Hob: m=3, PA 20°, 3RH, M34+TiN coated
Gear: HA 0°, Z=47, b=53mm, AISI 1045, 180BHN
Cutting condition: n=205rpm, f=3mm/rev., Conventional

Fig. 7—Progress of the maximum relief wear in case of conventional hobbing (HA 0°).

Hob: m=3, PA 20°, 3RH, D=110mm, N=12, M34+TiN
Gear: HA 0°, Z=47, b=53mm, AISI 1045, 180BHN
Cutting condition: V=40m/min., f=3mm/rev., Climb hobbing

Fig. 6—Influence of cutting speed (Long addendum gear, a=1.5, Blade No. $\bar{1}$, l=65m).

Hob: m=3, PA 20°, 3RH, D=110mm, N=12, M34+TiN
Gear: HA 0°, Z=47, b=53mm, AISI 1045, 180BHN
Cutting condition: V=70mm/min., f=3mm/rev. Conventional

Fig. 8—Influence of hobbing method (Long addendum gear, a=1.5, Blade No. $\bar{2}$, l=32.4m).

the crater wear along the chip flow grows and extends to the right corner cutting edge. Then the cutting edge is broken down as the corner wear increases, and the bottom of crater wear moves along the chip flow as the number of gears cut increases.[3] In case (C), the bottom of crater wear comes close to the cutting edge at the right corner; that is, the area of highest temperature is much closer to the cutting edge at the right corner, therefore, the cutting edge is broken down more easily.[4]

Fig. 5 shows the amount of maximum relief wear when gears with various addendum modifications are cut. Each solid symbol indicates the relief wear caused by the growth and breakdown of the crater wear. In the case of high speed climb hobbing of spur gears, the life of the hob is shortened when long addendum gears with larger outside diameters are cut. It can be lengthened for short addendum gears with smaller outside diameters. The thin line describes the progress of the maximum relief wear when the cutting speed is 40m/min. Addendum modification has little influence on the relief wear when the cutting speed is slow. Fig. 6 shows the damage on a blade No. $\bar{1}$ with the maximum relief wear when a long addendum gear (C) is climb hobbed at a cutting speed of 40m/min. The crater wear grows slowly when the cutting speed is reduced. The cutting edge at the right corner is not broken down, even with the 65m cutting length of long addendum gears. But the cutting edge at the right corner is nearly broken down, and the relief wear will increase soon after the cutting length increases.

Fig. 7 shows the progress of the maximum relief wear in the case of conventional hobbing at a speed of 70m/min. Other cutting conditions remain the same. In this figure, addendum modification is less influenced than in the results of climb hobbing shown in Fig. 5. The thin line describes the maximum relief wear in case of a hob 80mm outside diameter.[5-6] Fig. 8 shows the damage to a blade No. $\bar{2}$ with the maximum relief wear when long addendum gears are conventionally hobbed using a hob of 110mm outside diameter.

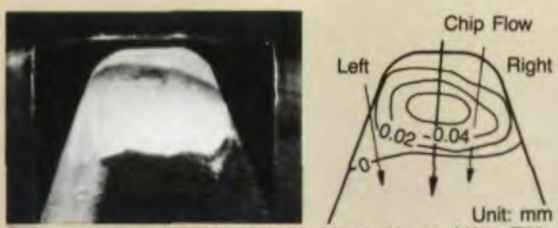
AUTHORS:

DR. KISABURO NAGANO *is Associate Professor of Mechanical Engineering at Kurume National Technical College, Kurume, Japan. A graduate of Saga National University, Dr. Nagano is currently involved in research on high speed hobbing with HSS hobs and the cutting performance of TiN-coated HSS hobs.*

DR. MASATO AINOURA *is Professor of Mechanical Engineering at Kurume National Technical College, Kurume Japan. He has been doing research in gear hobbing for nearly forty years. He is responsible for the development of carbide hobs and hobbers, carbide skiving hobs and shaving hobs and screw-shaped hones. In 1966 he was awarded the Japan Society of Mechanical Engineering Paper Prize for research on hobbing and again in 1979 for research on the development of gear honing machines with screw-shaped hones.*

As shown, cutting chips flow straight to avoid breaking down the right corner of the cutting edge.

Fig. 9 shows the damage to blade No. $\bar{3}$ with the maxmum relief wear when long addendum gears are conventionally hobbed (cutting length: $l=32.4$m) using a smaller outside diameter hob of 80mm. The crater wear is not so deep as in Fig. 8 because the cutting speed is lowered (hobbing time is constant), and so the addendum modification coefficient has little influence on it. It is possible to lessen the addendum modification influence and the damage without lowering the gear production efficiency in the case of conventional hobbing with a small outside diameter hob.

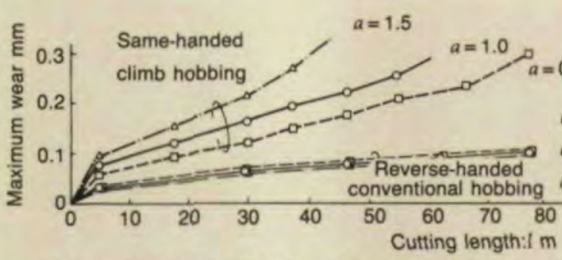


Hob: m=3, PA 20°, 3RH, D=80mm, N=10, M34+TiN
Gear: HA 0°, Z=47, b=53mm, AISI 1045, 180BHN
Cutting condition: V=50m/min., f=3mm/rev., Conventional

Fig. 9—Effect of a small outside diameter hob in case of spur gears (Long addendum gear, $a=1.5$, Blade No. $\bar{3}$, $l=32.4$m).

Hobbing of Helical Gears

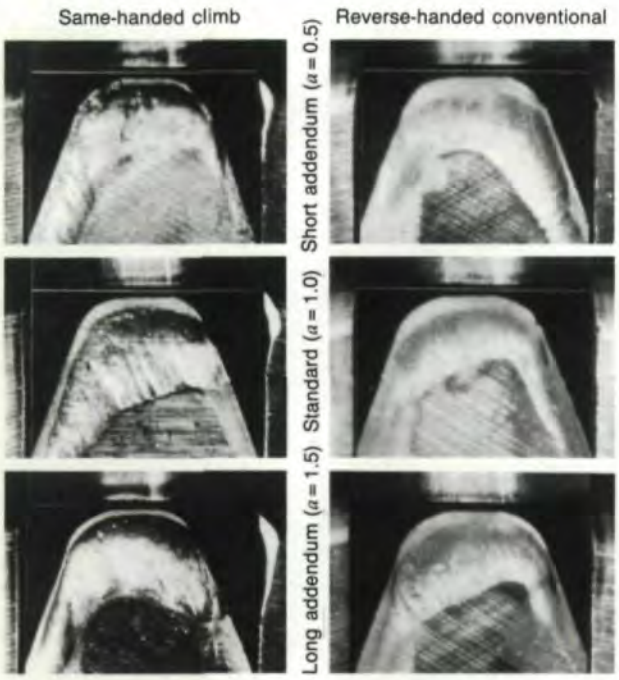
Fig. 10 shows the progress of the maximum relief wear on each hob when profile-shifted gears of a 15° helix angle are cut. The hob is TiN-coated, triple-threaded and of 110mm outside diameter as shown in Table 1. Gear blanks are carbon steel (AISI 1045, 180BHN), cutting speed is 70m/min. and hob feed is 3mm/rev. Judging from this experiment, in the life of a hob used in same-handed climb hobbing, which is now popular, much depends on the addendum modification. On the other hand, it had little influence in reverse-handed conventional hobbing.[7-8]



Hob: m=3, PA 20°, 3RH, D=110mm, N=12, M34+TiN
Gear: HA 15°, Z=47, b=58mm, AISI 4118, 140BHN
Cutting condition: V=70m/min., f=3mm/rev.

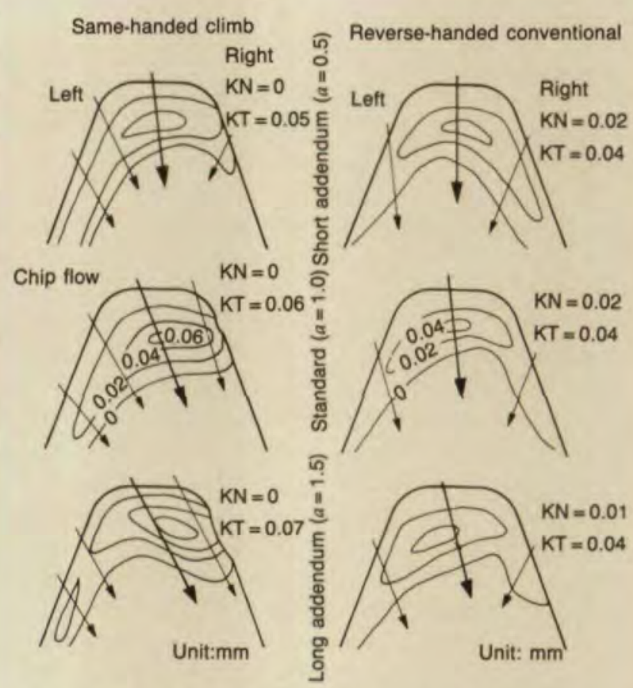
Fig. 10—Progress of the maximum relief wear (HA 15°).

Fig. 11 shows the damage to a blade No. $\bar{6}$ with the maximum relief wear after 45m cutting length. In the case of the same-handed climb hobbing, the amount of the right corner wear on long addendum gears (C) is the largest. A standard



Hob: m=3, PA 20°, 3RH, D=110mm, N=12, M34+TiN
Gear: HA 15°, Z=47, b=58mm, AISI 4118, 140BHN
Cutting condition: V=70m/min., f=3mm/rev.

Fig. 11—Damage to a blade with maximum wear occurred (HA 15°, Blade No. $\bar{6}$, $l=45$m).



Hob: m=3, PA 20°, 3RH, D=110mm, N=12, M34+TiN
Gear: HA 15°, Z=47, b=58mm, AISI 4118, 140BHN
Cutting condition: V=70m/min., f=3mm./rev.

Fig. 12—Direction of chip flow on each crater (HA 15°, Blade No. $\bar{6}$, $l=45$m).

gear (B) comes next, and short addendum gears show the least wear. In the case of the reverse-handed conventional hobbing, the amount of corner wear is very small, and it depends little on the addendum modification.

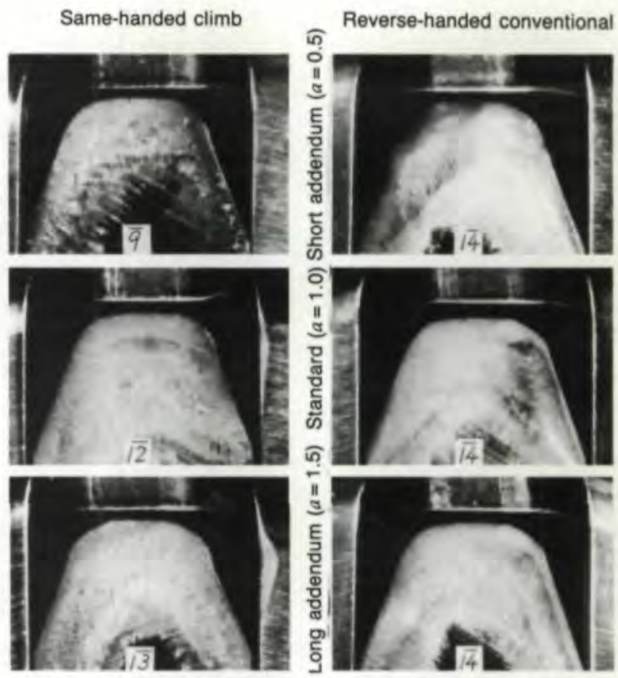
Fig. 12 describes the depth of crater wear and the flow of cutting chips. The crater wear in same-handed climb hobbing grows more rapidly than that in the reverse-handed conventional hobbing, and it extends to the corner of the cutting edge that is broken down. Furthermore, the right side of the cutting edge works little, so that the right corner of the cutting edge is easily broken down by the flow of cutting chips. On the other hand, in the case of the reverse-handed conventional hobbing, the crater wear hardly extends to the right corner of the cutting edge, because the crater wear is not as deep, and the right-side cutting edge works sufficiently to keep the cutting chips away from the cutting edge.

Fig. 13 shows the progress of the maximum relief wear when helical gears (HA 30°, SCM415, AISI 4118, 140BHN) with various addendum modifications are cut at a speed of 90m/min. and a feed of 4mm/rev. using a TiN-coated hob with triple-thread and 110mm outside diameter as shown in Table 1. In the case of helical gears (HA 30°), the addendum modification has much influence on the same-handed climb hobbing, and it has little influence on the reverse-handed conventional hobbing.

Fig. 14 shows the damage of each cutting edge with the maximum relief wear after a cutting length of 30m. The depth of crater wear in helical gears (HA 15°, AISI 1045, 180BHN) is smaller than in Fig. 11, but the right corner of cutting edges are already scooped out, and the cutting edge has retreated because of the crater wear caused by same-handed climb hobbing.

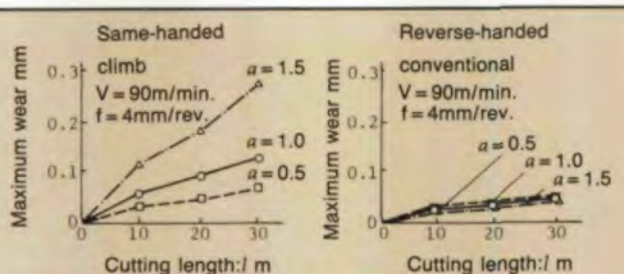
In same-handed climb hobbing, the crater wear is caused by the flow of cutting chips from the top and the left corner of the cutting edge to the right corner, resulting in the retreat of the right corner of the cutting edge. The influence of the addendum modification becomes apparent at the beginning of hobbing, where the cutting edge is breaking down. In this case, the wear of the right corner increases in the following order: short addendum gears (A), standard gears (B) and long addendum gears (C). But in the case of the reverse-handed hobbing, the addendum modification has little influence on the corner wear.

Accordingly, the authors recommend reverse-handed con-



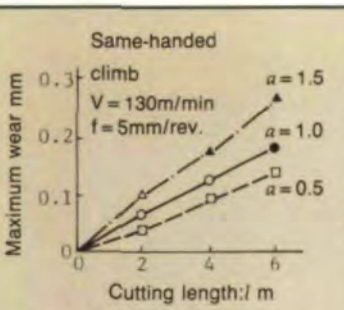
Hob: m = 3, PA 20°, 3RH, D = 110mm, N = 12, M34 + TiN
Gear: HA 30°, Z = 57, b = 30mm, AISI 4118, 140BHN
Cutting condition: V = 90m/min., f = 4mm/rev.

Fig. 14—Damage to a blade where maximum wear occurred (HA 30°, V=90m/min, f=4mm/rev., l=30m).



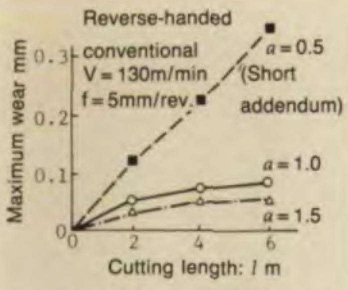
Hob: m = 3, PA 20°, 3RH, D = 110mm, N = 12, M34 + TiN
Gear: HA 30°, Z = 57, b = 30mm, AISI 4118, 140BHN
Cutting condition: V = 90m/min., f = 4mm/rev.

Fig. 13—Progress of the maximum relief wear (HA 30°).



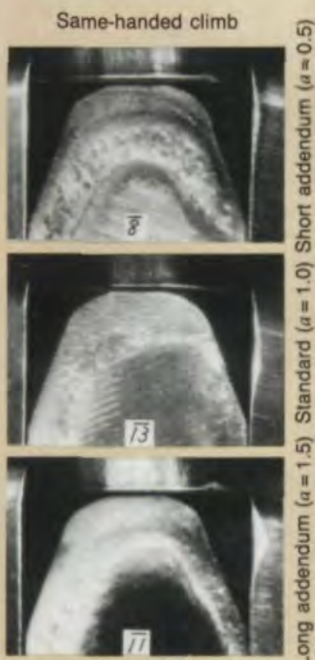
m = 3, PA 20°, 3RH, D = 110mm, M34 + TiN, AISI 4118, 160BHN

Fig. 15 – Progress of the maximum relief wear (HA 30°).



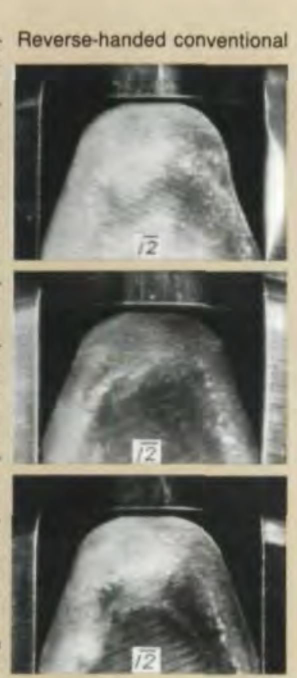
m = 3, PA 20°, 3RH, D = 110mm, M34 + TiN, AISI 4118, 160BHN

Fig. 17 – Progress of the maximum relief wear (HA 30°).



m = 3, PA 20°, 3RH, D = 110mm, M34 + TiN, AISI 4118, 160BHN

Fig. 16 – Damage to a blade where maximum wear occurred (HA 30°, V = 130, f = 5, l = 6m).



m = 3, PA 20°, 3RH, D = 110mm, M34 + TiN, AISI 4118, 160BHN

Fig. 18 – Damage to a blade where maximum wear occurred (HA 30°, V = 130, f = 5, l = 6m).

ventional hobbing for shifted helical gears. The depth of crater wear is decreased, and the flow of cutting chips very effectively lessens the corner wear and the influence of the addendum modification.

Hobbing of Helical Gears at High Speed and Feed

Hobbing at a high speed and feed using a multiple-thread hob increases production efficiency, however, the damage to the hob increases.

Fig. 15 shows the progress of the maximum relief wear on helical gears (HA 30°) with various addendum modifications, cut at a speed of 130m/min. and feed of 5mm/rev. in same-handed climb hobbing. Each solid symbol indicates the relief wear caused by the growth and breakdown of the crater wear. When hobbing at a high speed and feed, the amount of wear is great. Especially in the case of profile-shifted helical gears, the maximum relief wear is nearly 0.3mm when only three gears (φ = 6m) were cut.

Fig. 16 shows a comparison of damage to each cutting edge with the maximum relief wear after 6m cutting length. The wear of the right corner increased more rapidly in comparison to Fig. 14 where a cutting speed of 90m/min. and a feed of 4mm/rev is used. In the case of the same-handed climb hobbing for profile shifted helical gears, the cutting conditions, like a cutting speed of 130m/min. and a feed of 5mm/rev., are impossible for practical use.

Fig. 17 shows the progress of the maximum relief wear in reverse-handed conventional hobbing. Under such conditions, there has been little influence of addendum modification. But when the cutting condition is very severe, like a cutting speed of 130m/min. and a feed of 5mm/rev. for short addendum gears, a large amount of wear at the side cutting edge occurred when only three gears (φ = 6m) were cut.

Fig. 18 shows the damage to each blade No. $\overline{12}$ with the maximum relief wear when the cutting length is 6m. Little crater wear occurs in the case of short addendum gears, but facing the cutting surface, the side cutting edge is much constricted, and a large amount of semicircular relief wear occurs on the relief side. However, the side wear has not occurred in the case of long addendum gears. Therefore, the authors calculated the predicted change of each cutting chip for gears with various addendum modifications to find out the cause of wear.

Fig. 19 shows the shape of cutting chips from the right side

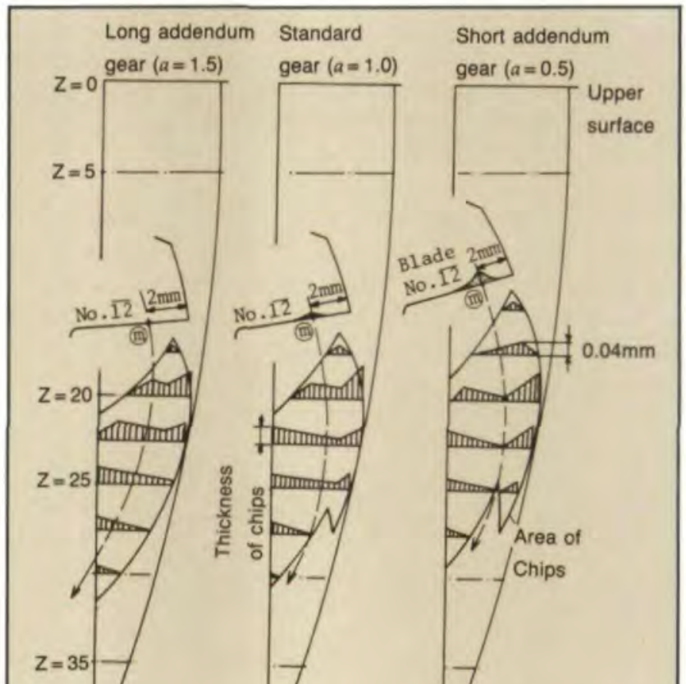
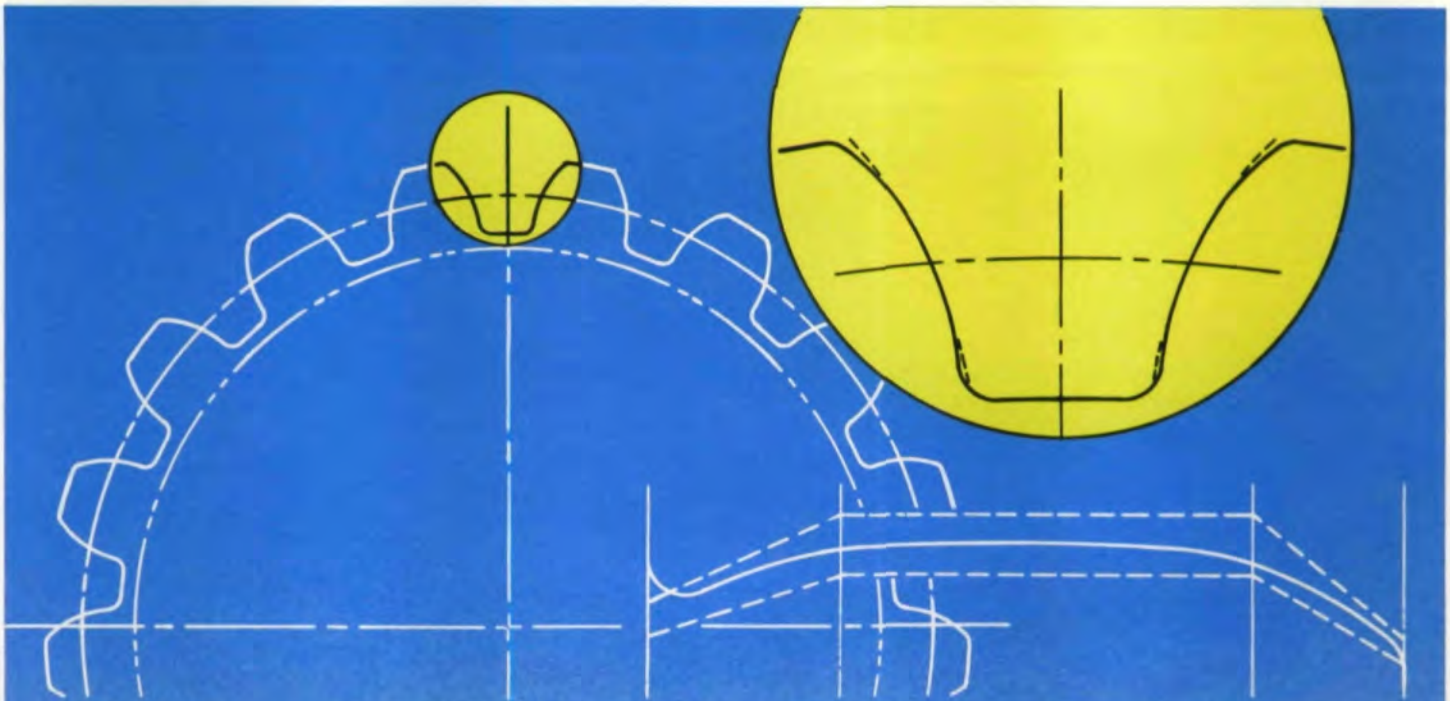


Fig. 19 – Shape of cutting chips from the right side cutting edge in case of reverse-handed conventional hobbing.



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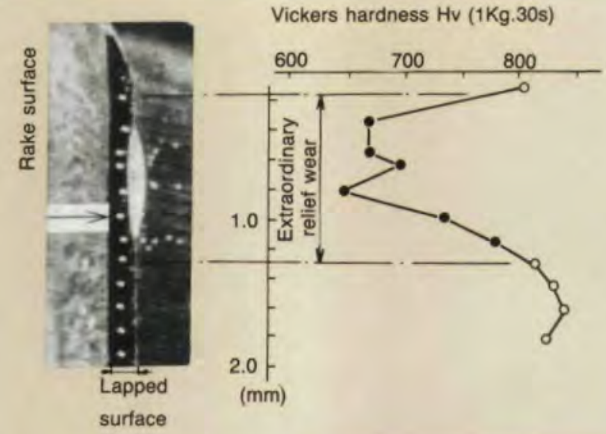
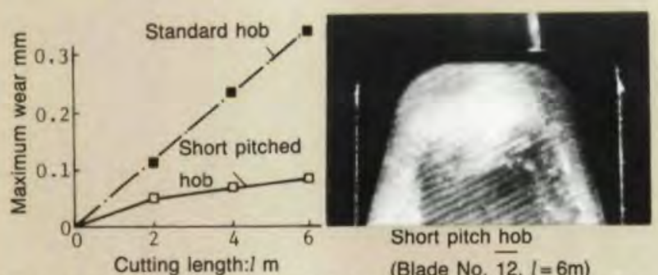
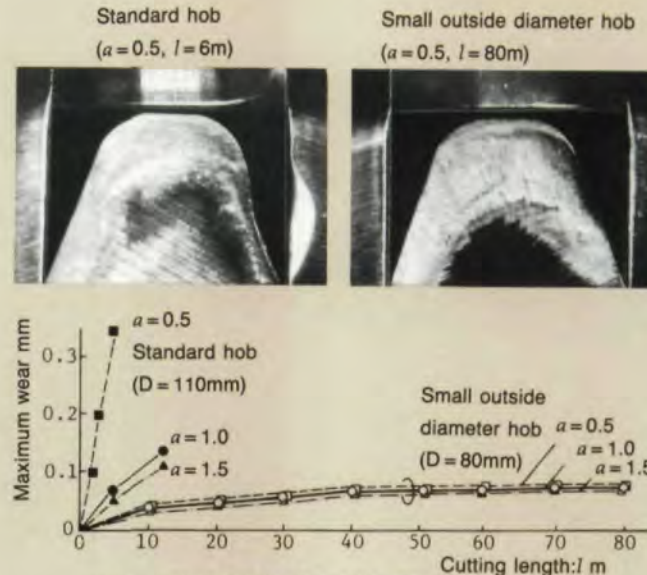


Fig. 20 – (left) Hardness of the right side wear of a blade (Short addendum gear, a=0.5, Blade No. $\overline{12}$).



Short pitched hob: m=2.9118, PA 14.5°, 3RH, D=110mm, M34+TiN
Short addendum gear: m=3, PA 20°, HA 30°, Z=57, AISI 4118, 160BHN
Cutting condition: V=130m/min., f=5mm/rev., Reverse-handed

Fig. 21 – Effect of modified pitched (Short-pitched) hob.



Hob: m=3, PA 20°, 3RH, M34+TiN
Gear: HA 30°LH, Z=57, AISI 4118, 160BHN
Cutting condition: n=375rpm, f=5mm/rev.
n: Revolution of hob axis

Fig. 22 – Effect of small outside diameter hob.

cutting edge of blade No. $\overline{12}$ under cutting conditions as shown in Fig. 18, and the trace of the cutting edge ⓜ with the side wear on the gear tooth surface. The encircled portion is the cutting zone for a cutting blade No. $\overline{12}$. The portion of oblique lines shows the thickness of each cutting chip in axial section at 2.5mm intervals. It is clear that the cutting edge ⓜ with the extraordinary side wear is producing the thinnest cutting chip in the reverse-handed conventional hobbing of short addendum gears.

Fig. 20 shows the hardness of the right side wear of blade No. $\overline{12}$ where it is ground to make it smooth using a micro-Vickers hardness tester. As shown here, the area of wear is remarkably softened.

This may be because the cutting edge around ⓜ produces thin chips at a high speed and feed in the reverse-handed hobbing of short addendum gears and then hardly cuts in. It is important to avoid this rapidly growing damage. Therefore, the authors prepared a modified pitched hob and a smaller outside diameter hob for the next experiment to find out the effect.

Modified pitched hob. As Fig. 18 showed, the side wear does not occur in the reverse-handed conventional hobbing of long addendum gears. Accordingly, the authors prepared a short pitched hob for short addendum gears like those for long addendum gears. Fig. 21 shows the progress of the maximum relief wear and the damage on both hobs. A standard hob such as is shown in Table 1 and a short pitched hob for cutting short addendum helical gears were cut at a speed of 130m/min. and a feed of 5mm/rev. using reverse-handed conventional hobbing. In the case of a short pitched hob, corner wear occurs, but there is little side wear.

Small outside diameter hob. Fig. 22 illustrates the progress of the maximum relief wear when profile-shifted gears are cut under the same conditions using hobs of outside diameter 110mm and 80mm in reverse-handed conventional hobbing. In the case of a smaller outside diameter hob, little wear occurs, and there is little influence of addendum modification. It is proven that a short pitched hob and a small outside diameter hob are both effective to lessen the extraordinary side wear which often occurs when profile-shifted gears are cut at a high speed and feed in the reverse-handed conventional hobbing.

Conclusion

The authors have made experiments on hobbing to find out how to lessen the damage to the cutting edge of profile-shifted gears. As a result:

(1) The distribution of crater wear and the direction of chip flow depend on the addendum modification of gears, and they have an influence on each cutting edge which is broken down.

(2) In the case of climb hobbing for spur gears and same-handed climb hobbing for helical gears, addendum modifica-

(continued on page 47)

A Microcomputer Program To Calculate Spur Gears

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Abstract:

A set of equations to calculate the tooth form factor Y following the procedure of *AGMA Standard 226.01, Information Sheet — Geometry Factors for Determining the Strength of Spur, Helical, Herringbone and Bevel Gear Teeth*, has been developed and programmed in BASIC on an IBM-PC. The program also calculates the tooth deformation, the load transfer points and the load sharing for a large variety of gear pair geometries and material combinations. Sample results are given.

Introduction

The American Gear Manufacturers Association (AGMA) gives the following formula to calculate the bending stress S_t at the root of a spur gear tooth:[1]

$$S_t = \frac{W_t\, P_d}{F\, J}\,\frac{K_a\, K_s\, K_m}{K_v} \qquad (1)$$

where W_t is the transmitted tangential load calculated at the operating pitch circle, P_d is the diametral pitch, F is the face width of the narrowest of the mating gears, J is a geometry factor for bending strength, K_a, K_s, K_m and K_v are application, size, load distribution (across the face) and dynamic factors for bending strength.

Factors K_a, K_s, K_m and K_v depend on the accuracy of the mounting. The nature of the application and values for those factors are suggested in Reference 1. The difficulty related to those factors does not consist in the calculation of their value, but in the decision of which value to use. The decision is often dictated by previous field experience and, since the factors K_a, K_s, K_m and K_v are not the object of this article, they will not be discussed here.

The geometry factor J in Equation 1 evaluates the effect of the gear pair geometry on the bending stress. Before the advent of microcomputers, values for the factor J were most easily obtained from graphs prepared for the most common geometries. Examples of these graphs are those given in AGMA 226.01. For other geometries, a tooth layout has to be prepared, and the geometry factor J calculated by hand. The step by step procedure, described in AGMA 226.01, is tedious and must be repeated each time one of the many components of the gear pair geometry is varied.

The first part of this article gives a set of mathematical equations that permits the calculation of the geometry factor J with a microcomputer. The second part introduces the computer program that has been written to solve this set of equations and that also performs other calculations on spur gears such as the root bending stress, the contact pressure, the load sharing and others. The third part presents some case analyses that were done with the aid of the program to show its capability of solving complex geometry problems in gears. The results presented emphasize the effect of the teeth deformation on the calculated root bending stress, especially with gears made of materials that have a low modulus of elasticity.

The application of this article is limited to the calculation of the root bending stress in spur gear pairs following the procedure mentioned in AGMA 226.01.

Equations to Calculate the Geometry Factor J

For spur gears, the geometry factor J is expressed by the following formula:[1]

$$J = \frac{Y}{K_f\, m_N} \qquad (2)$$

where Y is tooth form factor, K_f is a stress concentration fac-

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QUANG DAN NGUYEN *completed his masters degree at the Ecole Polytechnique de Montreal, where he researched plastic gearing. He is presently a doctoral candidate in the Mechanical Engineering Department at Laval University, Quebec, Canada, where he is studying the kinematic behavior of bevel gears under load.*

tor and m_N is a load sharing ratio ($m_N = 1.0$ for spur gears).

The form factor Y which considers both the tangential (bending) and radial (compressive) components of the normal load W_n (Fig. 1) is calculated by:

$$Y = \cfrac{1}{\cfrac{\cos(\phi_L)}{\cos(\phi_o)}\left(\cfrac{1.5}{X} - \cfrac{\tan(\phi_L)}{t}\right)} \qquad (3)$$

where ϕ_o is the operating pressure angle, ϕ_L is the angle measured between the direction of the normal load and a line perpendicular to the tooth center line, and X and t are dimensions (Fig. 3) that have to be determined. ϕ_L, X and t are defined by the gear pair geometry and the distance S between the pitch point P and the point of application of the load. In this article, the length S is assumed negative when

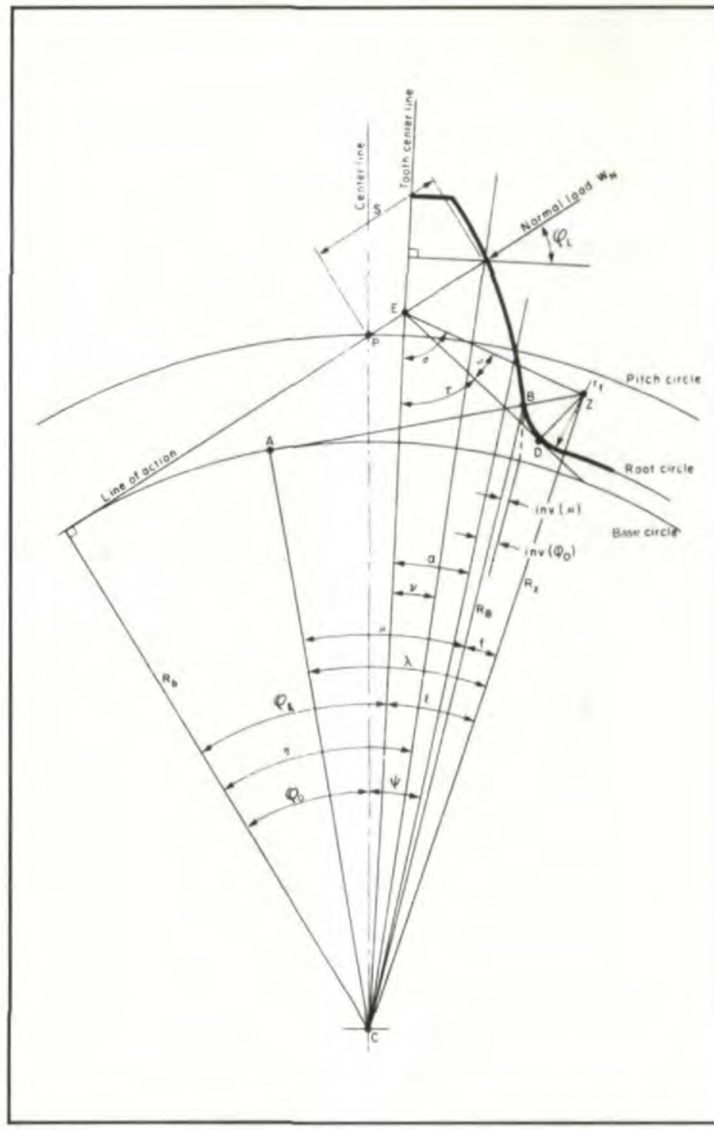

Fig. 1 – Geometrical relations in an involute spur gear tooth when the center of the root fillet radius is above the base circle

the point of application of the load is on the right of the pitch point P, and S is assumed positive when it is on the left.

In deriving the equations to calculate X and t, the junction between the involute tooth profile and the root fillet is assumed to be an arc of a circle of radius r_f calculated as:(2)

$$r_f = \frac{2R}{N}\left(r_T + \frac{(b - r_T)^2}{N/2 + b - r_T}\right) \qquad (4)$$

where R, b and N are, respectively, the pitch circle radius, the dedendum constant and the number of teeth of the gear for which the form factor is calculated and r_T is the edge radius constant of the tool used to cut that gear. In Equation 4, b and r_T are the dimensionless constants that, when divided by the diametral pitch, give the dedendum and the edge radius values in inches, respectively.

In Fig. 1, the radius R_z is calculated by:

$$R_z = R_R + r_f$$

Where R_R is the root radius which allows calculation of R_z

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by:

$$R_z = R\left(1 - \frac{2b}{N}\right) + r_f \qquad (5)$$

At this point, it is necessary to distinguish between two cases:

$$1-\ R_z \geqslant R_b \text{ (Fig. 1)}$$
$$2-\ R_z < R_b \text{ (Fig. 2)}$$

where R_b is the base circle radius.

When $R_z \geqslant R_b$

From Fig. 1, in the right angle triangle CZA:

$$ZA = \sqrt{R_z^2 - R_b^2} \qquad (6)$$

and

$$AB = ZA - r_f \qquad (7)$$

where point B is at the junction of the involute profile with the root fillet.

In triangle CBA, the radius R_B is expressed as:

$$R_B = \sqrt{R_b^2 + AB^2} \qquad (8)$$

and the angle μ

$$\mu = \cos^{-1}[R_b/R_B] \qquad (9)$$

In the triangle CZB, by the cosine law, the angle f is calculated by:

$$f = \cos^{-1}\left(\frac{R_B^2 + R_z^2 - r_f^2}{2R_B\, R_z}\right) \qquad (10)$$

The angle α (Fig. 1) is an angle measured between the tooth center line and a line that joins the gear center to the operating pitch point on the tooth flank. The angle α is a constant for a gear pair and is calculated (in radians) by:

$$\alpha = \frac{1}{N}\left(2e\tan(\phi) + \frac{\pi}{2}\right) + \mathrm{inv}(\phi) - \mathrm{inv}(\phi_o) \qquad (11)$$

where e is the amount the cutting tool (a rack) has been fed in (−) or out (+) from the blank relative to its theoretical position, ϕ is the tool pressure angle and inv is the involute function:

$$\mathrm{inv}(x) = \tan(x) - x \qquad (12)$$

The operating pressure angle ϕ_o is evaluated by:

$$\mathrm{inv}(\phi_o) = \mathrm{inv}(\phi) + \frac{1}{N_p + N_G}\left(2(e_p + e_G)\tan(\phi) + \frac{B\cos(\phi_o)}{\cos(\phi)}\right) \qquad (13)$$

where subscripts P and G refer to the pinion and the gear, respectively, and B is the backlash expressed as:

$$\text{backlash (inch)} = B/P$$
$$\text{backlash (mm)} = B\,m \qquad (14)$$
$$\text{(m is the metric module)}$$

The angles α and ϕ_o known, the angle ξ is calculated by:

$$\xi = \alpha + \mathrm{inv}(\phi_o) - \mathrm{inv}(\mu) + f \qquad (15)$$

The dimension CE in triangle CKE, (Fig. 1) is calculated by:

$$CE = R_b/\cos(\phi_o + \psi - \alpha) = R_b/\cos(\phi_L) \qquad (16)$$

where ψ is related to the distance S by Reference 3:

$$\psi = S/R\cos(\phi) \qquad (17)$$

CE known, it is now possible to calculate the length EZ in triangle CEZ as:

$$ZE = \sqrt{R_z^2 + CE^2 - 2R_z\, CE\cos(\xi)} \qquad (18)$$

Since the line segment ED is tangent to the root fillet at point D, then, in triangle EZD:

$$\varrho = \sin^{-1}[r_f/ZE] \qquad (19)$$

and, from the sine law in tringle CEZ:

$$\sigma = \sin^{-1}\,[R_z \sin(\xi)/ZE] \tag{20}$$

and, finally:

$$\gamma = \sigma - \varrho \tag{21}$$

The angle γ is the starting angle of the procedure that calculates dimensions X and t in Equation 3 (Fig. 3). Before we write the equations to calculate X and t, we should handle the case when $R_Z < R_b$.

When $R_Z < R_b$

When $R_z < R_b$, the portion of tooth profile that extends below the base circle is assumed to follow a radial line (Fig. 2). On the base circle, the circular tooth thickness t_b is expressed as:

$$t_b = 2R\cos(\phi)\left(\frac{2e\tan(\phi)}{N} + \frac{\pi}{2} + \mathrm{inv}(\phi)\right) \tag{22}$$

The angle ζ is given by:

$$\zeta = t_b/2R_b = \frac{2e\tan(\phi)}{N} + \frac{\pi}{2} + \mathrm{inv}(\phi) \tag{23}$$

and the angle ω by:

$$\omega = \tan^{-1}\,[r_f/R_z] \tag{24}$$

From Fig. 2, then ξ is calculated by:

$$\xi = \zeta + \omega \tag{25}$$

When the center of the root fillet radius lies below the base circle, the equations to calculate the angle ξ have to be replaced by Equations 22 to 25.

Dimensions X and t

To satisfy the procedure outlined in AGMA 226.01, starting from the angle γ between the tooth center line and the line ED, the triangle ZDE has to be rotated clockwise about point Z (Fig. 1) to a new position ZD'E' until the line E'D' makes an angle γ' with the tooth center line such that (Fig. 3):

$$E'J = J\,D' \tag{26}$$

In triangle CZF in Fig. 3:

$$ZF = R_z \sin(\xi) \tag{27}$$

In triangle ZGD':

$$D'G = r_f \sin(\gamma') \tag{28}$$

$$ZG = r_f \cos(\gamma') \tag{29}$$

(continued on page 26)

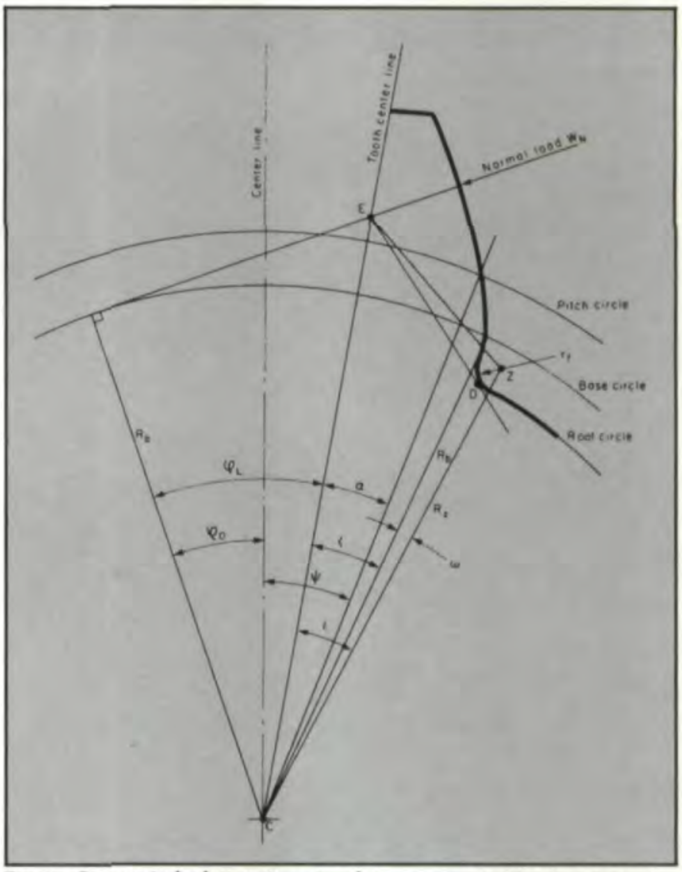

Fig. 2 – Geometrical relations in an involute spur gear tooth when the center of the root fillet radius is below the base circle

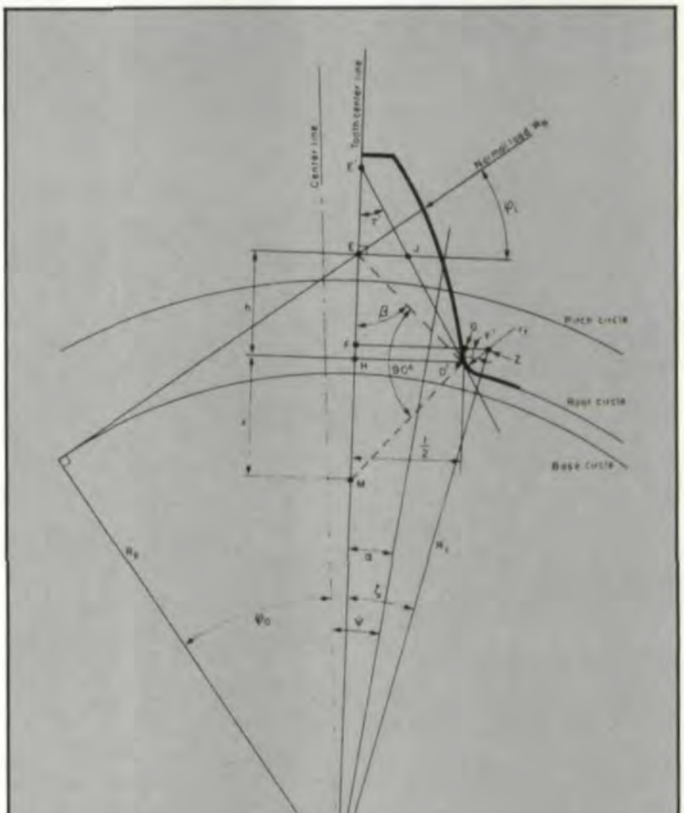

Fig. 3 – Geometrical construction to calculate the tooth form factor Y of an involute spur gear (AGMA 226.01)

From concept to COMPLETION

Gear Manufacturers have access to the followin

Star Cutting Tools

Hobs
Form-Relieved Milling Cutters
Gun Drills
Pressure-Coolant Reamers
Keyseat Cutters
Polycrystalline Diamond Tools

Star Hob Sharpeners

10 x 12 CNC Horizontal
10 x 12 Universal
6 x 6 Vertical

Star Shaper Cutter Sharpener

Borazon Sharpener for helical and spur type cutters with step or dish type grinds.

Star CNC Precision Tool and Cutter Grinders

5-axis model
6-axis model

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(continued from page 23)

and

$$HD' = ZF - ZG = t/2 \tag{30}$$

In triangle E'HD':

$$HE' = HD'/\tan(\gamma') \tag{31}$$

and

$$E'D' = \sqrt{HD'^2 + HE'^2} \tag{32}$$

and

$$HE = CE - [R_z \cos(\xi) - D'G] - h \tag{33}$$

Equations 31 and 33 give:

$$EE' = HE' - HE \tag{34}$$

In triangle E'EJ:

$$E'J = EE'/\cos(\gamma') \tag{35}$$

and, with Equation 32:

$$JD' = E'D' - E'J \tag{36}$$

An iterative process can easily adjust the angle γ' to satisfy Equation 26 within a given precision. Having found that value for γ', then one calculates in triangle EHD':

$$ED' = \sqrt{HE^2 + HD'^2} \tag{37}$$

and

$$\beta = \cos^{-1}[HE/ED'] - \tan^{-1}[HD'/HE] \tag{38}$$

In triangle ED'M:

$$EM = ED'/\cos(\beta) \tag{39}$$

Finally:

$$X = EM - HE \tag{40}$$

Where HE is given by Equation 33 or, in triangle EHD', by:

$$HE = ED' \cos(\beta) - h \tag{41}$$

Placing Equations 39 and 41 in 40 gives:

$$X = ED' \left(\frac{1}{\cos(\beta)} - \cos(\beta) \right) \tag{42}$$

Equation to calculate K_f

The stress concentration factor K_f in Equation 2 is calculated by Reference 1.

$$K_f = H + \left(\frac{t}{r_f}\right)^L \left(\frac{t}{h}\right)^M \tag{43}$$

where the constants H, L and M are given in AGMA 226.01 and reproduced in Table 1. The values of t, r_f and h are the same as calculated before.

Table 1 — Constants H, L and M to use in Equation 44 (from AGMA 226.01)

Pressure angle deg.	H	L	M
14½	0.22	0.20	0.40
20	0.18	0.15	0.45
25	0.14	0.11	0.50

Critical positions of the load

As mentioned before, dimension X, h and t and angle ϕ_L vary as the point of application of the normal load moves on the tooth profile. In AGMA 226.01, the two critical positions of the load are at the tip of the tooth and at the highest point of single tooth contact.

For the load applied at the tip of the tooth, the value of S to use in Equation 17 is calculated by:

$$S = R\left[\sqrt{\left(1+\frac{2a}{N}\right)^2 - \cos^2(\phi)} - \cos(\phi)\tan(\phi_o)\right] \tag{44}$$

where a is the dimensionless addendum constant of the gear for which we calculate the angle ψ.

For the load applied at the highest point of single tooth contact, the value of S to use in Equation 17 is calculated by:

$$S = R_p\left[\sqrt{\left(1+\frac{2a_p}{N_p}\right)^2 - \cos^2(\phi)} - \cos(\phi)\tan(\phi_o)\right] - \frac{2\pi R_p \cos(\phi)}{N_p} \tag{45}$$

where subscript p signifies that the values of R, a and N to use are those of the gear mating with the one for which we calculate the angle.

Equations 2 to 45 completely define the geometry factor J for any gear pair geometry, assuming that:

- the root fillet is tangent to the involute profile,
- the root fillet is a segment of a circle,
- the tooth profile follows a radial line when it extends below the base circle,
- the gear has been cut with a hob.

The equations are valid for any proportions of the hob.

The Computer Program

The set of equations presented in Section 2 to calculate the geometry factor J is complex. While it would take quite a long time to solve those equations by hand, a computer would do it in a fraction of a second. The use of a computer is, therefore, very attractive because in just a few minutes and with little effort, one can obtain a dozen different solutions to the same problem by simply varying design parameters such as the pitch, the number of teeth, the pressure angle, the root fillet radius, the hob proportions or others. A graphical solution as described in AGMA 226.01 would require much more effort and time.

A program that runs on an IBM PC has been written in BASIC to calculate Y, K_f and J by Equations 2 to 42 and S_t by Equation 1 (assuming $K_a = K_s = K_m = K_v = 1.0$) for the load applied at the tip of the tooth, at the highest point of single tooth contact or at any other position. The calculations are made for spur gears made of any material. Although the program does not include strength calculations, it gives a value for the most probable life of gears made from unreinforced polyoxymethylene (acetal) thermoplastic resin. This estimate is based on a statistical analysis that has been made with a relatively large experimental data set (over six hundred points).[4]

Beside the geometric, stress and life calculations, the program also evaluates the probable running temperature of

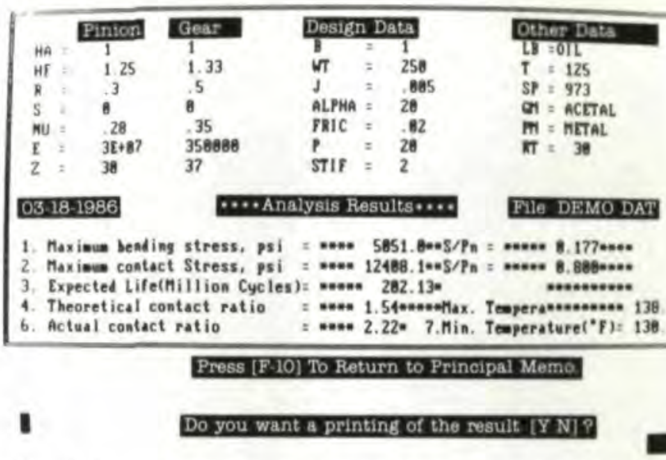

Fig. 4 – Computer input/output data as they appear on the video screen

gears made from three thermoplastic materials: polyamide (nylon), acetal and ultra high molecular weight polyethylene (UHMWPE). The temperature calculations are based on experimental measurements made by the authors in References 5 and 6. The data available permits one to calculate the thermoplastic gear temperature only when it is running against a steel pinion and without lubrication.

To carry those calculations, the program needs a certain number of data. Fig. 4 is a photograph of the screen that shows all the variables that have to be given to the program and those of the results that appear on the screen. Table 2 gives a complete list of the symbols that appear in Fig. 4 with their meaning. This information is entered by choices in menus and answers to questions. Reference 7 discusses the menus available and the questions the computer asks.

Case Analyses

Equation 1 that is used to calculate the root bending stress S_t makes two assumptions. One is that only one tooth pair transmits all of the load ($m_N = 1.0$) Equation 2 at the critical positions. For highly rigid materials, such as steel, this assumption is usually true. However, when one of the gears in the pair is made of a material such as a thermoplastic, that assumption is less well justified.

The second assumption is that the teeth are rigid and that the most severe loading condition occurs exactly at the critical positions calculated by Equations 44 and 45. Because of elastic deformation of the teeth under the load, the positions of the most severe loading condition calculated are shifted. The computer program presented here accounts for the deformation of the teeth by replacing them with springs and then by calculating the load carried by each tooth pair at any position in the meshing zone.[3]

Table 3 has been prepared to illustrate the effect ot the teeth deformation on the positions of load transfer. The gear pair illustrated in Table 3 is a 20-tooth steel pinion driving an 80-tooth steel gear. Geometric and operating conditions given

Table 2 — Variables of the program

Variable symbol	Meaning* (*Values between [ ] are default)
HA	Addendum constant HA, PINION [1] = GEAR [1] =
HF	Dedendum constant HF, PINION [1.25] = GEAR [1.25] =
R	Hob tip radius constant, R, PINION [.3] = GEAR [.3] =
S	Hob shifting constant S, PINION [0] = GEAR [0] =
NU	Poisson's coefficient NU, PINION material [.28] = GEAR material [.28] =
E	Young's modulus E, PINION material (psi) [30E6] = GEAR material (psi) [350E3] =
Z	Number of teeth Z, PINION = GEAR =
B	Gear face width B (inch) =
WT	Tangential load at pitch circle WT (lb) =
J	Backlash J (inch) =
ALPHA	Cutting hob pressure angle ALPHA (degree) [20] =
FRIC	Coefficient of friction FRIC =
P	Diametral pitch P (1/inch) =
STIF	Rim stiffness of gear STIF: Solid rim then STIF = 1.0; Web rim STIF = 2-5
LB	Lubrication mode, LB, OIL [1], GREASE [0], DRY [−1] =
T	Ambient or oil temperature [•F], T=
SP	Gear rotating speed, SP (RPM) =
GM	Gear material, GM: METAL [1], NYLON [3], ACETAL [5], UHMWPE [4] =
PM	Pinion material, PM: METAL [1], NYLON [3], ACETAL [5], UHMWPE [4] =
RT	Metal gear or pinion surface roughness, RT (μ inch). If gear and pinion are of plastic, then RT = 0.

Table 3 — Effect of the tooth deformation on the position of the load transfer

S* inch	W/W_t %	J_P	S_{tP} Kpsi	J_G	S_{tG} Kpsi
0.295	0	?	0	?	0
0.267(1)	34.9	.420	2.49	.297	3.53
0.266	36.7	.420	2.62	.297	3.71
0.236	41.7	.427	2.93	.311	4.02
0.207	47.7	.430	3.33	.327	4.38
0.177	50.9	.428	3.57	.345	4.43
0.148	54.1	.423	3.84	.365	4.45
0.118	57.6	.414	4.17	.389	4.44
0.089	79.9	.403	5.95	.415	5.78
0.066(2)	95.4	.391	7.32	.439	6.52
0.059	100.0	.388	7.73	.446	6.73
0.030	100.0	.372	8.06	.483	6.21
0	100.0	.355	8.45	.526	5.70
−0.028(3)	65.1	.339	5.76	.576	3.39
−0.030	63.3	.338	5.62	.579	3.28
−0.059	58.3	.320	5.47	.644	2.72
−0.089	52.3	.303	5.18	.727	2.16
−0.118	49.1	.287	5.13	.837	1.76
−0.148	45.9	.272	5.06	.989	1.39
−0.177	42.4	.258	4.93	1.216	1.05
−0.207	20.1	.245	2.46	1.601	0.38
−0.229(4)	4.6	.245	0.56	1.601	0.09
−0.236	0	?	0	?	0

*Figures between brackets identify the critical positions.

	PINION	GEAR		
HA =	1.00	1.00	B	= 0.500
HF =	1.25	1.25	WT	= 150
R =	0.35	0.35	J	= 0.005
S =	− 0.03	− 0.03	ALPHA	= 20.000
NU =	0.28	0.28	FRIC	= 0.001
E =	3E+07	3E+07	P	= 10.000
Z =	20	80	STIF	= 1

(continued on page 30)

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at the bottom of the table should be interpreted with the aid of Table 2. The first column represents the length S (inch) measured along the line of action (Fig. 1), assuming that the tooth shown is on the 80-tooth gear. Figures between brackets at the right of certain values of S in Table 3 are to identify particular positions referred to hereafter. Details of the data used to prepare Table 3 are given at the bottom.

For this particular gear pair geometry, it can be calculated (Equation 44) that the 80-tooth gear make first contact by the tip with the flank of the 20-tooth pinion when $S_{(1)} = 0.267''$. Also, it can be calculated that the last contact between the flank of the tooth on the gear and the tip of the tooth on the pinion is at $S_{(4)} = -0.229''$. Since the tooth pairs are spaced by one normal base pitch p_n ($p_n = 0.295''$ in the present case) on the line of action, it appears that one tooth pair should carry all of the load between $S_{(2)} = S_{(4)} + p_n$ and $S_{(3)} = S_{(1)} - p_n$; i.e., between $S_{(2)} = 0.066''$ and $S_{(3)} = -0.028''$, respectively.

The second column of Table 3 is the percentage, calculated by the programs, of the total load W_t transferred by one tooth pair as it goes from the first contact $S_{(1)}$ to the last contact $S_{(4)}$. Theoretically, that column should read 0% at $S_{(1)}$ and $S_{(4)}$ and 100% between $S_{(2)}$ and $S_{(3)}$. As can be seen, the transfer points are not exactly where they should be theoretically. At $S_{(1)}$ and $S_{(4)}$, the tooth pair appears to carry some load, while it should not. On the other hand, at $S_{(2)}$ and $S_{(3)}$, the tooth pair does not carry 100 per cent of the load as it should.

The third column in Table 3 lists the geometry factor of the pinion, Jp, calculated at each value of S. At the highest point of single tooth contact on the pinion ($S_{(3)}$), the calculated value for Jp is 0.339, which is very close to the value given by AGMA 226.01. With the aid of the graph given in that reference, the value of J for a 20-tooth pinion driving an 80-tooth gear cannot be determined with enough precision to say that it is different from the value calculated. The fourth column in Table 3 is the bending stress calculated at the root of the pinion by:

$$S_{tP} = \frac{W_t\ P}{F\ J_P}\ \frac{W/W_t}{100} \qquad (47)$$

The table shows that the maximum bending stress S_t = 8 450 psi (58.3 MPa) at $S = 0''$. Calculations done at finer intervals show that the distribution curve for S_tP goes through a maximum of 8,800 psi (60.7 MPa) at $S = -0.025''$. Using Equation 1 and a geometry factor $J_P = 0.339$, one calculates S_tP = 8,850 psi (61.0 MPa). The discrepancy between the stresses calculated by the two methods can be attributed to the location for the highest point of single tooth contact used. Because of the teeth deformation, the program localizes that point at $S = -0.025''$ compared to the theoretical value $S_{(3)} = -0.028''$ implicitly assumed by Equation 1. For the pinion, the coordinate $S = -0.025''$ corresponds to a lower point on the tooth profile than $S = -0.028''$ and, consequently, to a higher value of J_P. (See Fig. 1.)

Columns five and six in Table 3 are the geometry factor J_G and the root bending stress S_{tG} for the gear. The program calculated a maximum stress S_{tG}=6,730 psi (46.4 MPa) at $S = 0.058''$. The maximum stress calculated by Equation 1, using a geometry factor $J_G = 0.439$, is 6, 830 psi (47.1 MPa) at the theoretical distance $S_{(2)} = 0.066''$. For the gear, the position $S = 0.058''$ corresponds to a lower point of the tooth profile than $S_{(2)} = 0.066''$ and explains why the program calculates a lower value for S_{tG} than Equation 1.

For both cases, the pinion and the gear, the bending stress value calculated by Equation 1 can be compared with the value calculated by the program by the use of a proper value for the load sharing ratio m_N in Equation 2. For the pinion, that value for m_N would be 8,850/8,800 = 1.006 and for the gear, m_N = 6,830/6,730 =1.015. These values for m_N are very close to 1.0 because the load on the gear pair is low. The load sharing ratio would increase at larger load because of increased teeth deformation, but would cause more interference. If the gears are made of highly rigid materials such as steel, it then becomes necessary to cut the teeth with some tip and root reliefs. Since the purpose of tip and root reliefs is to keep the load transfer points close to their theoretical value, calculations such as those reported in Table 3 are quite useless except to give an indication on how much relief should be cut.

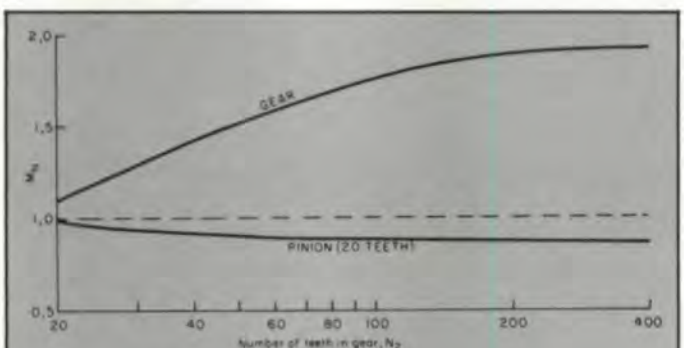
However, when one of the gears is made of a low modulus of elasticity material, the load sharing ratio becomes much more important. For example, suppose that the 80-tooth gear in the example of Table 3 is made of a thermoplastic material with a typical modulus of elasticity of 3.5×10^5 psi (2.4 GPa) instead of steel. Then, the program calculates a maximum bending stress of 4,100 psi (28.3 MPa) compared to the 6,830 psi (47.1 MPa) value calculated before, leading to a load sharing ratio value $m_N = 1.67$ (6,830/4,100). On the gear, the maximum bending stress is located at $S = -0.014''$ which is much lower on the profile than the theoretical position $S_{(2)} = 0.066''$. On the other hand, the maximum bending stress calculated at the root of the 20-tooth steel pinion is 9,800 psi (67.6 MPa) compared to 8,850 psi (61.0 MPa) as stated previously. This means that, for the pinion, the load sharing ratio $m_N = 0.9$ (8,850/9,800). For the pinion, the maximum bending stress is located at $S = 0.210''$, which is higher on the profile than the theoretical position $S = -0.087''$. Fig. 5 has been prepared to show the effect of a low modulus of elasticity gear on the load sharing ratio m_N. The value of m_N for the gear increases with the number of teeth to an asymptotic value of about 1.9. For the pinion, the value of m_N decreases with the increasing number of teeth in the gear to a minimum value of about 0.85. Therefore, in a plastic/steel gear pair, the steel pinion is always more stressed than it would be if the gear was of steel. For the gear, the inverse is true. The geometric and operational parameters used to prepare Fig. 5 are shown in the figure.

The effect that the Young's modulus of the gear has on the load sharing ratio is shown in Fig. 6. For the gear, m_N appears to vary between 1.05 and 2. The load sharing ratio is less than 1.10 for gears made with materials having a modulus of elasticity larger than 10^7 psi (68.9 GPa); i.e., all the metals such as steel, aluminum, brass and cast iron. With thermoplastic materials, however, most of which have a modulus of elasticity below 10^6 psi (6.9 MPa), m_N appears to vary between 1.25 and 2.0. For the pinion, the factor m_N varies between a value slightly above 1.0 when meshing with a steel gear to about 0.8 for a very soft gear material.

Of the parameters that affect the load sharing ratio m_N, the group (W_t P/F) is one of the most important as shown in Fig. 7 for an 80-tooth thermoplastic gear with a Young's modulus of 3.5×10^5 psi (2.4 GPa) driven by a 20-tooth steel pinion. The effect of this group on the value of m_N for the gear is highly nonlinear and more important for (W_t P/F) $> 10^3$ psi (6.9 MPa). For the pinion, $m_N < 1$ due to the soft gear material.

Conclusion

The list of case analyses that can be made with the program is endless. The few examples done above demonstrate that the program reproduces accurately the AGMA method of calculating the geometry factor J in spur gears, but the work and the time necessary to do those calculations are greatly reduced compared to the hand layout method: the computer needs less than ten seconds to analyze a gear pair under one load condition. This part of the program can be



	PINION	GEAR		
HA =	1.00	1.00	B =	0.500
HF =	1.25	1.25	WT =	150
R =	0.35	0.35	J =	0.005
S =	−0.03	−0.03	ALPHA =	20.000
NU =	0.28	0.28	FRIC =	0.001
E =	3E+07	350000	P =	10.000
Z =	20		STIF =	1

Fig. 5 – Effect of the number of teeth of the gear, N_2, on the load sharing ratio, m_N

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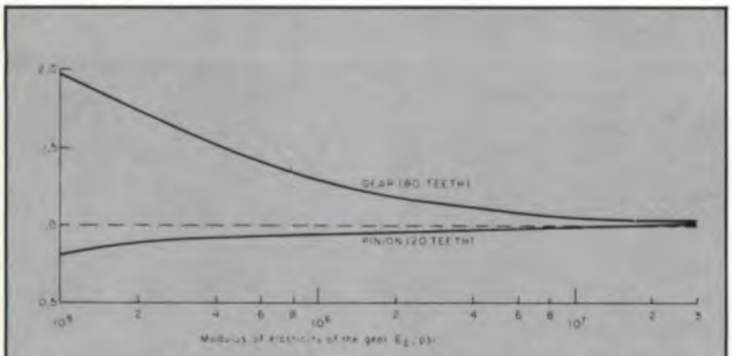
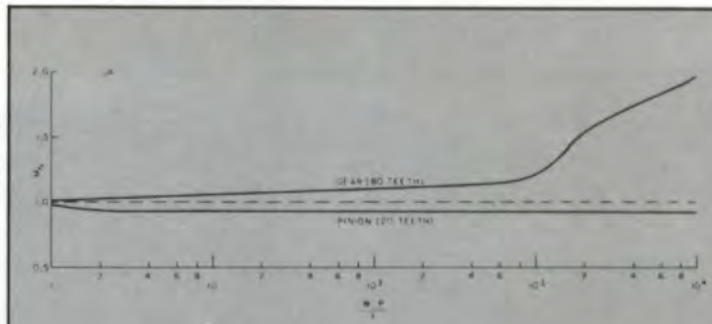


		PINION	GEAR			
HA	=	1.00	1.00	B	=	0.500
HF	=	1.25	1.25	WT	=	150
R	=	0.35	0.35	J	=	0.005
S	=	−0.03	−0.03	ALPHA	=	20.000
NU	=	0.28	0.28	FRIC	=	0.001
E	=	3E+07		P	=	10.000
Z	=	20	80	STIF	=	1

Fig. 6 – Effect of the Young's modulus of the gear, E_2, on the load sharing ratio, m_N



		PINION	GEAR			
HA	=	1.00	1.00			
HF	=	1.25	1.25			
R	=	0.35	0.35	J	=	0.005
S	=	−0.03	−0.03	ALPHA	=	20.000
NU	=	0.28	0.28	FRIC	=	0.001
E	=	3E+07	350000			
Z	=	20	80	STIF	=	1

Fig. 7 – Effect of the group of variables (W_t P/F) on the load sharing ratio, m_N

useful to analyze metal gear pairs. The load sharing portion of the program can also be useful during the design process of a metal gear pair, but it becomes almost mandatory for the analysis of plastic gears or pinions. Also, the freedom it offers in determining the gear geometry makes that program perfectly adapted to plastic gearing design, which is not as rigidly limited by the standardization as is metal gearing.

The program can be improved in several aspects. In particular, it would be interesting if it had graphic capabilities. Actually, we are working on an addition to animate the gear tooth action. In another field, it is intended to add a dynamic analysis module using the spring model that calculates the tooth deformation. That model will also be used to study tooth errors and wear effects.

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Editorial

(Continued from page 5)

The bottom line is that the airlines' best customers are being treated shabbily, given K-Mart services at Gucci prices. The only reason the airlines have gotten away with this for so long is that they are, in a sense, the only game in town. But that will not last forever. Surely someone in the airline industry will have entreprenurial sense enough to see the vast well of resentment filling among its busienss flyers and take advantage of it. Some alternative that acknowledges the needs of business travelers and meets them at a fair price will be found, and when that happens, the airlines that have continued to ignore these important customers will be left with a lot of empty seats. It can't happen too soon for me.

Michael Goldstein
Editor/Publisher

The Uses and Limitations Of Transmission Error

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Abstract:

Transmission error (TE) is described and its relevance for accuracy, noise and vibration investigations and production control is discussed. Difficulties with attempts to predict TE by computer are explained and guidelines suggested for the conditions when testing at zero load can be used.

Limitations of speed, accuracy and vibration frequency are quoted for two of the commonly used test rig encoders currently available.

Introduction

The concept of "transmission error" is relatively new and stems from research work in the late 1950s by Gregory, Harris and Munro,[1] together with the need to check the accuracy of gear cutting machines. The corresponding commercial "single flank" testing equipment became available in the 1960s, but it was not until about ten years ago that it became generally used, and only recently has it been possible to test reliably at full load and full speed.

The ideas involved are new and the equipment is electronics-dominated. Furthermore, the results do not tie up directly with the traditional pitch, involute and helix measurements; therefore, there is much confusion and, understandably, suspicion about the test, especially as there are not as yet useful industry guidelines on permissible values for test results.

Despite the initial resistance, some industries have found that transmission error is so important a measurement that they must inspect all gears, yet some branches of industry do not need to use it. The difference lies in the varied relative importance of noise, strength and drive position accuracy in the branches of industry. As a rough guide, if drive accuracy is important or if noise and vibration are important, it is absolutely essential to control TE for high quality, but if strength is the only factor and speeds are low enough to avoid large dynamic stresses, then TE is not worth measuring. Sometimes it is only necessary to use TE at the development stage as a research tool, but it may be necessary for full production checking.

Definition of Transmission Error

Transmission Error is defined as the difference between the theoretical position of the output gear if the drive were both perfectly accurate and rigid and the actual output gear position.[1]

In a simple case, such as a 3:1 reduction box, we would expect a 3° movement at input to give exactly 1° at output, so if the measured output were 1°2′, the TE change would be +2′ of arc. The equipment typically checks the correctness of output position every minute of arc, making it convenient to give the output as a continuous trace record rather than a series of numbers. However, when subsequent digital analysis is required (though such analysis is often deceptive), sampling of the numbers is used to give perhaps 1,024 samples per revolution of the output gear.

A typical output from a spur drive is given in Fig. 1 with the short wavelength repetition at once per tooth superposed on a long wavelength at once per revolution. Full scale would be 32′ of arc for a small gear or 10″ arc for a large gear.

It is not very convenient to work in angular measure for gearing purposes so, although equipment measures angular error, we normally multiply by a pitch (reference) circle radius to get error in linear terms, usually μm (0.04 thousandths of an inch). This approach is slightly debatable academically, but has the major advantage that we now get the same "error" figure regardless of which end is input and output of the drive. It has the further advantage that the practical levels of error that can be achieved for "good", industrial or "poor" gears are independent of size, in contradiction to the figures in the various gear accuracy definitions. It may seem ridiculous that the same drive accuracies (in μm) are achieved on a gear 10 mm, 100 mm or 4 m (13 ft) diameter, but this happens.

Relevance of Transmission Error

TE simply and solely measures accuracy and, hence, smoothness of drive; therefore, if accuracy or smoothness are important, TE is relevant. There is, however, no connection fundamentally between strength and smoothness in a drive, so TE cannot give information about strength, and it is very foolish to attempt to use it as the sole production control for low speed gears which are highly stressed.

The only reliable check on strength is a full-torque bedding check to verify that contact is occurring over most of the

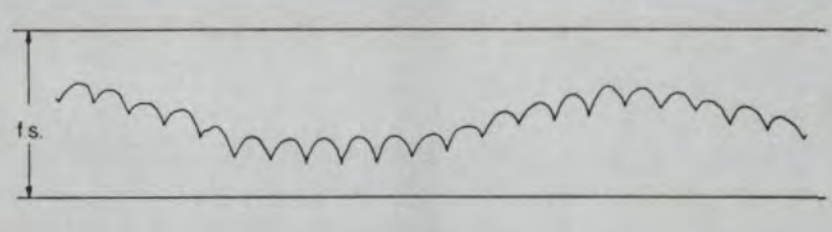

Fig. 1 – Typical transmission error trace; full scale (fs) is 50 μ (2 mil) of error.

tooth face, but this is a time consuming and expensive exercise. Ideally, in critical applications, both TE and bedding must be used.

Using TE for accuracy is an obvious application in industries such as printing, where register between printing cylinders must be accurately maintained or the blue dots may miss the yellow dots and give reject pictures. Accuracy is typically 50 μm (2 thou) maintained through several gear meshes, so that each mesh may have to be accurate to 15 μm. Here the important measurement on the TE check is the peak-to-peak value of the error, since the printing machine is only concerned with the worst relative position. Usually the once-per-revolution components of error are much larger than the once-per-tooth and so dominate the peak-to-peak.

As far as strength is concerned, there is no direct connection with TE so it is possible to design gears which have a very low TE at the expense of gear strength and vice versa. Despite this fundamental lack of correlation with strength, it is preferable to use TE to identify pitch errors, since it is much quicker and cheaper to measure pitch by using TE equipment than to use conventional pitch checkers. A low value of TE (under the correct operating conditions) at the frequency corresponding to once-per-tooth on a helical gear pair infers that either the profile is correct or that the helix matching is good, but does not prove that both are satisfactory. Conversely, a high value of TE infers a poor profile match on the gears *and* a poor helix match.

Vibration and noise are caused primarily by TE, so it is very difficult to investigate or control them without measuring TE. Although relatively few papers have been published on the correlation between noise and TE,[2,3] many checks within industry have confirmed a direct link between them. Gears generate noise for other reasons,[4] but for involute gears of normal accuracy ranges, the other mechanisms may be ignored.

Noise and Vibration Investigation

When a gear drive is too noisy or gives excessive vibration, the initial development work is usually directed towards checking whether there are local resonances which can be eliminated. The presence of such resonances can be checked relatively easily by variation of the operating speed[4] and measurement of natural frequency and mode shape is standard. If there are no "easy" resonances to tackle, or if the excitation, often at once-per-tooth frequency and harmonics, is not near a resonance, then more detailed investigation is necessary.

One approach is to take several complete systems, such as cars or ships, and to instrument each of them thoroughly with as many as 50 accelerometers. Operation under a range of conditions of speed and load with each system will produce a frightening amount of information which can be frequency analysed, correlated, checked for coherence, decomposed into modes, etc.

An alternative approach, which is very popular for work for the state, is to set up a computer model. A very large computer with an extremely large and complex program will compute the output noise and vibration levels from the basic gear shapes. Large numbers of variants on the gear profiles, helices and pitch can be analyzed and the effects on the final vibration levels predicted.

These two approaches both involve massive (and expensive) computation and produce very large and impressive reports which are very good for reassuring customers that the highest possible levels of technological expertise have been applied to the problem. Neither approach, however, provides detailed solutions to the problem, especially when the predictions are that noise levels will be extremely low, but the gear drives do not agree and are not consistent. As this disagreement exists, it is worthwhile looking at the system in more detail to see why the predictive computer approach encounters difficulties.

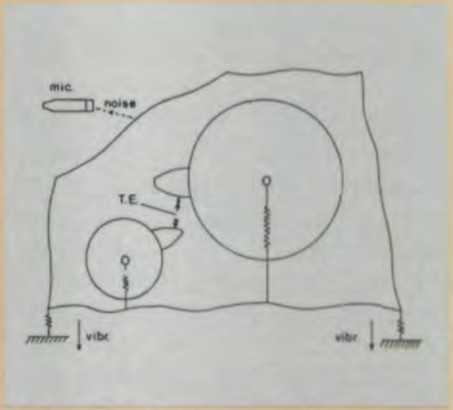

Fig. 2—Sketch of the mechanism of transmission error vibrating the supporting structure and generating noise via internal and external dynamic responses.

The Mechanism of Vibration Generation

Fig. 2 is a sketch of the route by which TE produces noise; this assumes that only TE is relevant (not true for Wildhaber-Novikov gears). The relative

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displacement between the gears (the TE) excites the gears themselves, vibrating mainly on the elasticities of shafts and bearings. Forces are transmitted through the bearings to the gearcase at the bearing housings, the levels of force depending on the size of the original TE and the dynamic response of the gears and their supports. The forces at the bearing housings excite the gearcase, and the vibration from gearcase to the supporting structure may be attenuated by vibration isolators at the gearcase support points.

The two strategic points in this progression from gear shapes to gearcase support vibration are the TE at the gear mesh and the forces which act through the bearings to excite the gearcase. If we put down all the factors which control the stages, the result is as shown in Fig. 3.

A large number of factors contribute to generate the TE, and a relatively small number of factors control the internal dynamic responses of the gears on their shafts. The final response of the gearcase to the bearing forces acting at the bearing depends on a fairly complicated gearcase shape, but relatively few factors, since only stiffness, mass and damping are involved. It is convenient to break down the problem into three stages:

- Gear shapes and design to TE,
- TE to forces through bearings onto gearcase,
- Bearing forces via gearcase to vibration of support.

Of these three stages, the second is relatively simple to predict with reasonable accuracy. The estimates of mass will be very accurate and the estimates of stiffnesses reasonably accurate, since only hydrodynamic bearings will present problems. Estimates of damping will be subject to large errors, but some reasonable guesses can be made based on experimental results from previous designs or, alternatively, variation of speed on a test rig will give useful information about resonances and damping in the internals.

The third stage can be fairly complex if there is a large supporting raft with many resonances, and concepts of statistical energy transfer can be useful. This stage is subject to the same problem that limits the second one in that prediction of the all-important damping is unreliable. The difference between the second and third stages is that it is relatively easy to check the third stage transfer functions experimentally either by running the gearbox at constant torque, but variable speed, or by using electromagnetic vibrators to excite the system to check the force versus displacement transfer function directly.

It is the first stage which presents the really difficult problems due to the accuracy limitations if computer prediction is attempted. Even for the most expensive areas of gearing, such as marine drives, it is difficult to measure to sufficient accuracy, however much care is taken. A rough average rule of thumb for gears of all sizes is that the level of uncertainty in a measurement is of the order of ± 1 μm (0.04 mil). Under typical "precision" conditions it is rather higher at ± 2.5 μm (a tenth of a mil), despite the claims of the manufacturers of the various measuring machines; it is known that some of the measuring machines, though agreeing with others from the same firm, will not agree with machines from rival, equally renowned, firms.

This "uncertainty" of, say, ± 1 μm applies to every one of the many factors contributing to the final TE, so the uncertainty in prediction of TE under operating conditions is likely to be greater than ± 5 μm, even when the averaging effects of the elastic deformations of the stem have been taken into account. This level of uncertainty would not matter too much in a normal industrial gearbox, where the typical level of TE at once-per-tooth frequency might be ± 5 μm. It would matter very much in a very accurate or quiet gearbox, where the requirement could be for an error of not more than ± 1 μm.

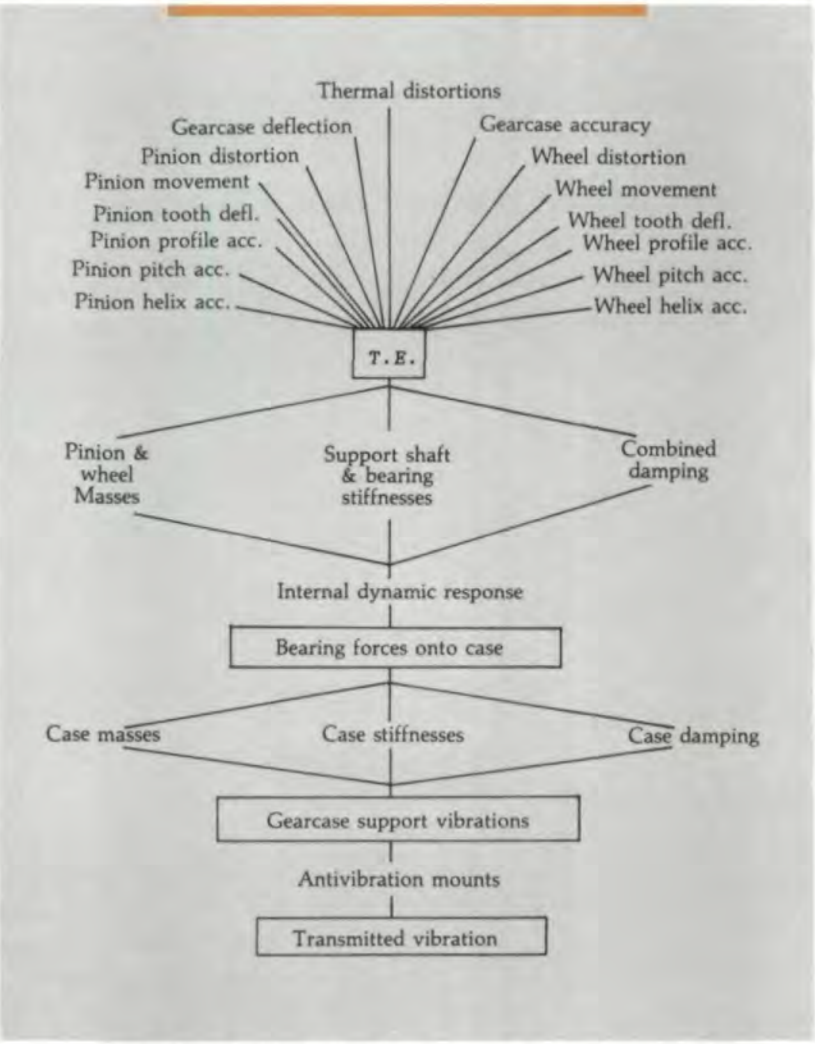

Fig. 3—Factors affecting the vibration generation and transmission.

The practical deduction is that the second and third stages may be predicted with some uncertainty or may be measured rather more reliably. Prediction of the first stages is subject to such high levels of uncertainty that it is difficult to avoid the practical measurement of TE for high quality work on noise or accuracy.

Measurement of Transmission Error

Theoretically, very many possible ways of measuring TE exist, but in practice there are very few which will give a reliable result for accurate gears. The main methods attempted have been:

(a) Magnetic signal methods which "write" a series of pulses onto a magnetic track and decode the resulting "read" signal. These do not appear to work under industrial conditions despite development.
(b) Strain gauges on drive shafts. This approach works for inaccurate drives with known torsional characteristics, but sensitivity is not good enough for precision work.
(c) Torsional vibration transducers of the seismic type. Satisfactory for low frequency work, but not sufficiently sensitive for high frequencies.
(d) Tachometer type devices. Useful for inaccurate work, but not capable of the resolution to one part in 100,000 needed for precision gears.
(e) Tangential accelerometer devices. Cannot be used at once-per-revolution frequencies, but might be suitable for high frequency work.
(f) Grating (rotary encoder) systems. So far this approach has dominated TE testing. A disadvantage is that, like seismic (c) testing, it needs a free shaft end for a coaxial drive to keep torsional frequencies high enough for the system to be adaptable.

To date, the majority of TE testing has been carried out as a production control tool in the inspection shop, either in addition to, or as a replacement for conventional measurement. This type of testing is carried out at very low speeds (10 rpm) and effectively zero torque. The equipment for this is expensive, like all gear measurement equipment, but the testing is so rapid that 100% inspection is possible, and test times of less than a minute can be achieved, so that the cost per gear is low. The gears are mounted on and supported by the spindles of the encoders and driven by an accurate servo-controlled drive which gives a vibration-free input speed. Output can be displayed on a chart recorder or analyzed digitally.

A single TE test on a gear pair is fast and gives a rough check on the combination of helix, profile and pitch errors. In this sense it is similar to the double flank rolling test, though much better in that it gives more reliable information and detects faults which escape the rolling check. A good production compromise is to check 100% with TE checking, since this is so fast and cheap, and to carry out occasional helix and profile checks on a routine basis, or when the TE results show that something is wrong.

In contrast, for development purposes it is sometimes necessary to check at full torque, and an exactly constant speed drive is not realistic. Furthermore, it may be necessary to check the performance of the drive when the gears are mounted in their gearcase, since misalignment errors may be important. For testing under torque it is advisable to run at speeds of above 100 rpm to ensure that an oil film is maintained and to prevent scuffing damage to the teeth surfaces. Provided that free shaft ends are available to connect the rotary encoders, they can be attached to the test rig and give the essential information. Care must be taken that the drive and load are at constant torque or the TE measured will be that of the test rig rather than the gear drive. Costs are relatively low, since the basic encoders are not expensive, and the electronics is much simpler than that used for production testing.

Limitations of Transmission Error Testing

The dominant limitation of TE testing is that it is a vibration measurement and does not give information on strength. Critical applications can overcome this by checking bedding by copper plating and running under load, but, as this requires skill and is time consuming, it is unrealistic for mass production.

Inspection testing is effectively at zero load, whereas, the information is required for operating torques, but at sufficiently low speeds that the system dynamics or resonances do not intrude. There is no general rule and so each case must be assessed separately, but some examples can be quoted.

A vehicle final drive has to be strong enough to withstand full engine torque in bottom gear, yet it is normally only required to be really quiet at 30% torque in top gear. As the torque at cruise is only about 5% of the permissible torque, the cruise deflections are negligible and zero torque testing in inspection is a very good predictor of in-car noise performance. Any other gears, whose noise performance is only relevant at, say, less than 10% of maximum torque, will have small deflections at this torque and so may be tested at zero torque; printing machine gears usually fall into this class because accuracy alone is relevant.

In contrast, a helicopter gearbox is always operating at full torque and will contain gears which have been heavily corrected for wind-up and deflection. Testing at zero torque gives virtually no useful information since the characteristics will be so different at full load.

Many gear drives operate over a wide range of torque, and there is no substitute for testing at low, medium and high load since TE and, hence, gearbox noise may increase, decrease or be independent of load or have a minimum at part loads.

A major limitation of TE testing is seen when shaft free ends are inaccessable, thus ruling out the use of direct coupled rotary encoders. It is possible to use a plain belt drive to a separate bearing block with a rotary encoder at input and /or output. This approach gives rise to two problems: the drive is not exactly synchronous, so much more complicated and temperamental electronics must be used; and, because of the elasticities and masses of the plain belt drive, the maximum reliable frequency for information is limited to about 200 Hz. As an alternative, the tangential accelerometer method can be used, but this needs slip rings or telemetry to transmit the information and does not work for once-per-revolution problems.

Speed and Accuracy of Encoders

Rotary encoders are remarkably cheap considering the accuracy to which they perform. The market is dominated by the firm of Heidenhain, who make two sizes of reasonably accurate encoders which are suitable for gear test rigs.

The smaller ROD 220/260 size is about 100 mm diameter and 50 mm deep and can operate mechanically up to 6,000 rpm without difficulty. Electrically, assuming 9,000 lines, the operating speed is limited to about 1,500 rpm for the 220 and about 6,000 rpm for the faster 260. It is possible to run faster, but the width of the output TTL pulse is then below 0.5 microseconds.

Accuracy is $\pm 5''$ of arc which is satisfactory up to diameters of 80 mm if $\pm 1\ \mu m$ accuracy p-p is required. Noise investigations are not interested in peak-to-peak accuracy, but usually are concerned with once-per-tooth frequencies. The relevant encoder accuracy is, say, that at 20 cycles per revolution and at this frequency the encoders will be accurate to better than $0.2''$ of arc. These relatively small encoders can have their peak-to-peak accuracy greatly improved by using a computer correction on the results, although this requires precalibration and a cumbersome and slow correction of the results.

Although the mechanical and electronic limitation is of the order of 6,000 rpm, the main restriction on the use of the encoder approach is the torsional resonance of the rotary inertia of the disc on the twist stiffness of the 10 mm drive shaft. This imposes a limit of about 1,500 Hz with a well designed coupling and so a practical operating limit for tooth frequency of 1,200 Hz or 400 Hz if third harmonic information is necessary. A 24-tooth pinion at 1,200 Hz allows 3,000 rpm for basic tooth frequency information, or 1,000 rpm for third harmonic, so that the majority of normal industrial drives can be tested.

The larger 800 size of encoder is about 150 mm (6 inches) diameter and 50 mm deep with a lower mechanical limit of the order of 600 rpm for a moderate life. Electronically it is similar to the smaller version, but tends to have higher line numbers of 18,000 or 36,000. It is unlikely to be limited electronically by pulse rate, since it is not used at high speeds.

Torsional frequency limitations apply rather low down the frequency range, since stiffnesses in twist are only about a factor of 4 higher than the 220/260 size, but the inertia is up by a factor of 16, so the torsional resonance will limit the information frequency to about 600 Hz. This would allow only 1,500 rpm on a 24-tooth pinion for tooth frequency; this, of course, exceeds the mechanical limit, but a 200 size encoder could be used on input and the 800 size on the 300 rpm wheel (assuming a 5 to 1 reduction). Checking a hobbing machine drive or a worm and wheel will usually be most satisfactory with a small encoder at input where accuracy is not important.

The accuracy of an 800 size encoder is said to be $\pm 1''$ of arc, and this may be easily verified by cross-checking a pair of encoders. A peak-to-peak measurement of $\pm 1''$ of arc is certainly adequate for most work up to 0.4 m (16 inches) diameter. If greater accuracy is required, 0.1" of arc can be achieved without much difficulty either by selecting a higher accuracy encoder or by correcting a standard encoder.

Discussion of accuracy should also involve the accuracy of the critical coupling which connects the encoder to the shaft which is vibrating torsionally. The coupling must accomodate small alignments and be extremely accurate, yet must be very rigid torsionally. This combination can be achieved, but very careful design and manufacture is needed.

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Helical Gear Mathematics Formulas and Examples

Part II

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The following excerpt is from the *Revised Manual of Gear Design, Section III,* covering helical and spiral gears. This section on helical gear mathematics shows the detailed solutions to many general helical gearing problems. In each case, a definite example has been worked out to illustrate the solution. All equations are arranged in their most effective form for use on a computer or calculating machine.

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Given the proportions of a pair of helical gears in the plane of rotation, to determine the contact ratio:

When,

R_1 = Pitch Radius of First Gear

R_2 = Pitch Radius of Second Gear

R_{o1} = Outside Radius of First Gear

R_{o2} = Outside Radius of Second Gear

R_{b1} = Base Radius of First Gear

R_{b2} = Base Radius of Second Gear

ϕ = Pressure Angle at R_1 and R_2

p = Circular Pitch

C = Center Distance

m_p = Contact Ratio

Then,

$$R_{b1} = R_1 \text{ COS } \phi \qquad R_{b2} = R_2 \text{ COS } \phi$$

$$m_p = \frac{\sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C \text{ SIN } \phi}{p \text{ COS } \phi}$$

Example:

$R_1 = 1.000 \qquad R_2 = 2.250 \qquad R_{o1} = 1.125 \qquad R_{o2} = 2.375$

$\phi = 20° \qquad C = 3.250 \qquad p = .3927 \qquad \text{SIN } \phi = .34202$

$R_{b1} = 1.000 \times .93969 = .93969 \qquad R_{b2} = 2.250 \times .93969 = 2.11430$

$$m_p = \frac{\sqrt{(1.125)^2 - (.93969)^2} + \sqrt{(2.375)^2 - (2.11430)^2} - 3.250 \times .34202}{.3927 \times .93969} = 1.59$$

(Continued on page 42)

Helical Gear Mathematics . . .
(Continued from page 40)

Given the arc tooth thickness and pressure angle in the plane of rotation of an internal helical gear, to determine its tooth thickness at any other radius:

When, r_1 = Given Radius
r_2 = Radius Where Tooth Thickness is to be Determined
ϕ_1 = Pressure Angle at r_1
ϕ_2 = Pressure Angle at r_2
T_1 = Arc Tooth Thickness at r_2
T_2 = Arc Tooth Thickness at r_2

Then,

$$\text{COS } \phi_2 = \frac{r_1 \text{ COS } \phi_1}{r_2}$$

$$T_2 = 2r_2 \left[\frac{T_1}{2r_1} - \text{INV } \phi_1 + \text{INV } \phi_2\right]$$

Example: $r_1 = 5.000$ $\phi = 20°$ $T_1 = .2618$ $r_2 = 5.100$

$\text{COS } \phi_1 = .93969$ $\text{INV } \phi_1 = .014904$

$$\text{COS } \phi_2 = \frac{5.000 \times .93969}{5.100} = .92126$$

$\phi_2 = 22.889°$ $\text{INV } \phi_2 = .022702$

$$T_2 = (2 \times 5.100) \left[\frac{.2618}{2 \times 5.000} - .014904 + .022702\right] = .34657$$

Given the arc tooth thickness and pressure angle in plane of rotation of mating internal helical gear and pinion at given radii, to determine the center distance at which they will mesh tightly:

When, r_1 = Given Radius of Pinion
r_2 = Given Radius of Internal Gear
T_1 = Arc Tooth Thickness at r_1
T_2 = Arc Tooth Thickness at r_2
N_1 = Number of Teeth in Pinion
N_2 = Number of Teeth in Gear
ϕ_1 = Pressure Angle at r_1 and r_2
ϕ_2 = Pressure Angle at Meshing Position
C_1 = Center Distance for ϕ_1
C_2 = Center Distance for ϕ_2

Then,

$$\text{INV } \phi_2 = \frac{2\,\pi r_1 - N(T_1 + T_2)}{2r_1\,(N_2 - N_1)} + \text{INV } \phi_1 \qquad C = r_2 - r_1 \qquad C_2 = \frac{C_1 \text{ COS } \phi_1}{\text{COS } \phi_2}$$

Example: $r_1 = 1.500$ $N_1 = 18$ $T_1 = .2618$ $\phi_1 = 20°$ $\text{COS } \phi_1 = .93969$

$r_2 = 5.000$ $N_2 = 60$ $T_2 = .2500$ $\text{INV } \phi_1 = .014904$

$$\text{INV } \phi_2 = \frac{2\pi\,(1.500) - 18(.2618 + .2500)}{(2 \times 1.500)\,(60 - 18)} + .014904 = .016589$$

$\phi_2 = 20.702°$ $\text{COS } \phi_2 = .93543$

$$C_1 = 5.000 - 1.500 = 3.500 \qquad C_2 = \frac{3.500 \times .93969}{.93543} = 3.5159$$

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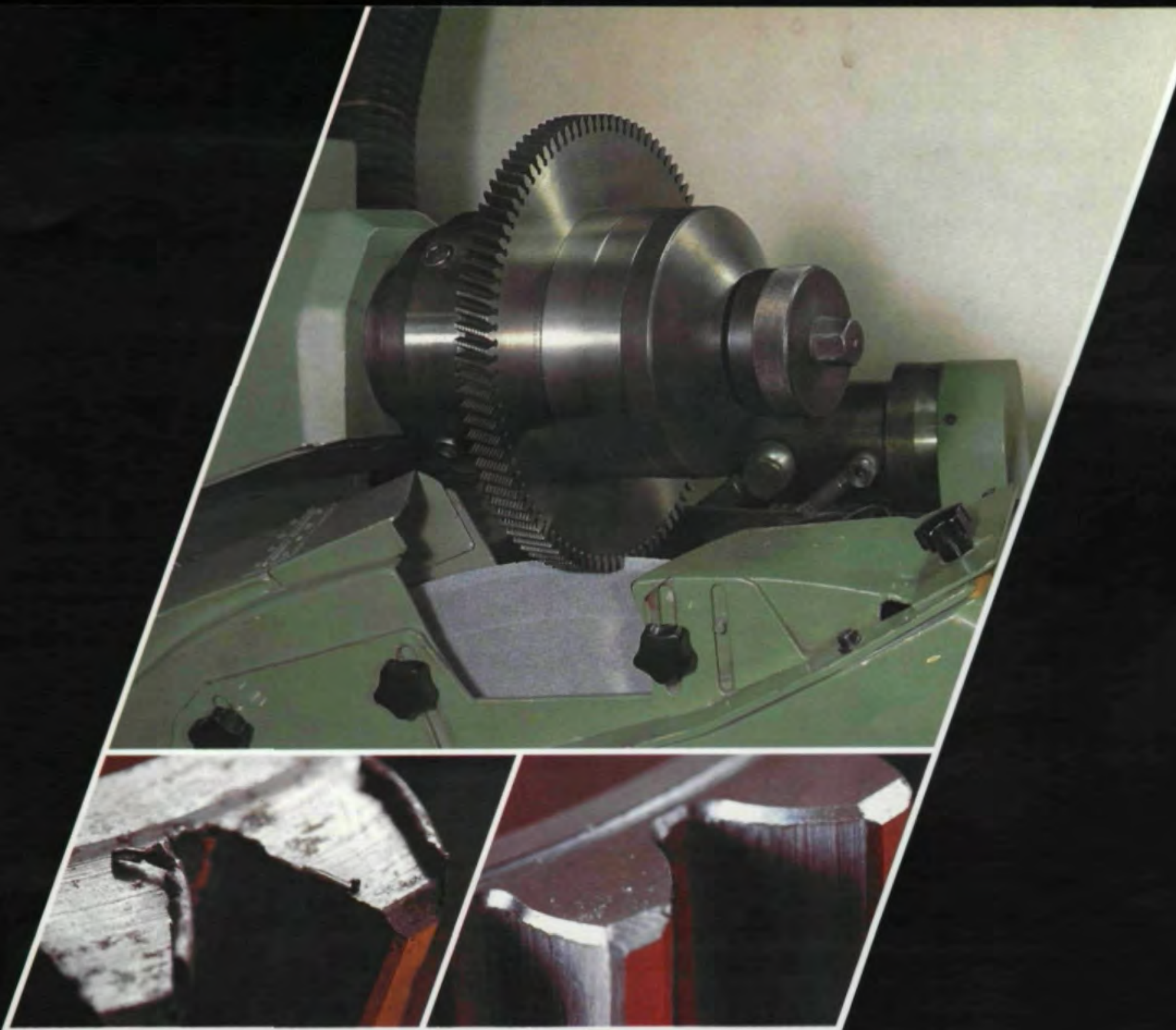

Helical Gear Mathematics . . .
(Continued from page 42)

Given the tooth proportions of a helical pinion and Fellows cutter for an internal gear drive and the center distance, to determine the shaping data for the internal helical gear:

When, C_1 = Center Distance of Operation
ϕ_1 = Pressure Angle of Cutter in Plane of Rotation
R_{b1} = Base Radius of Pinion
R_1 = Pitch Radius of Pinion Where Pressure Angle is ϕ_1
T_1 = Arc Tooth Thickness at R_1
R_{o1} = Outside Radius of Pinion
R_{r1} = Root Radius of Pinion
ϕ_2 = Pressure Angle of Operation in Plane of Rotation
R_{b2} = Base Radius of Internal Gear
R_2 = Pitch Radius of Internal Gear Where Pressure Angle is ϕ_1
T_2 = Arc Tooth Thickness at R_2
R_i = Inside Radius of Internal Gear
R_{r2} = Root Radius of Internal Gear
R_3 = Pitch Radius of Cutter Where Pressure Angle is ϕ_1
C_3 = Center Distance for Cutting
R_{b3} = Base Radius of Cutter
T_3 = Arc Tooth Thickness at R_3
ϕ_3 = Generating Pressure Angle in Plane of Rotation
R_{o3} = Outside Radius of Cutter
p = Circular Pitch at R_3
C = Clearance

Then,

$$R_{b1} = R_1 \cos\phi_1 \qquad R_{b2} = R_2 \cos\phi_1 \qquad R_{b3} = R_3 \cos\phi_1$$

$$\cos\phi_2 = \frac{R_{b2} - R_{b1}}{C_1}$$ Note: C_1 must be greater than $(R_{b2} - R_{b1})$

$$T_2 = p - T_1 - 2(R_2 - R_1)(\text{INV}\,\phi_2 - \text{INV}\,\phi_1) \qquad R_i = C_1 + R_{r1} + C$$

$$\text{INV}\,\phi_3 = \frac{p - (T_2 + T_3)}{2(R_2 - R_3)} + \text{INV}\,\phi_1 \qquad C_3 = \frac{R_{b2} - R_{b3}}{\cos\phi_3}$$

$$R_{r2} = C_3 + R_{o3} \qquad \text{Maximum } R_{o1} = R_{r2} - C_1 - C$$

Example:
8 DP
7-35 Teeth

$\phi_1 = 20°$ $R_1 = .4375$ $R_2 = 2.1875$ $R_3 = 1.750$
$C_1 = 1.750$ $T_1 = .2000$ $p = .3927$ $R_{r1} = .3930$
$C = .0250$ $T_3 = .19635$ $R_{o3} = 1.875$ $R_{o1} = .6180$

$$R_{b1} = .4375 \times .93969 = .41111 \qquad R_{b2} = 2.1875 \times .93969 = 2.05557 \qquad R_{b3} = 1.75 \times .93969 = 1.64446$$

$$\cos\phi_2 = \frac{2.05557 - .41111}{1.750} = .93969 \qquad \phi_2 = 20° \qquad \text{INV}\,\phi_2 = .014904$$

$$T_2 = .3927 - .2000 - 2(2.1875 - .4375)(.014904 - .014904) = .1927$$

$$R_i = 1.750 + .393 + .025 = 2.1680 \qquad \text{INV}\,\phi_3 = \frac{.3927 - (.1927 + .19635)}{2(2.1875 - 1.750)} + .014904 = .019075$$

$$\phi_3 = 21.650° \qquad \cos\phi_3 = .92945$$

$$C_3 = \frac{2.05557 - 1.64446}{.92945} = .4423 \qquad R_{r2} = .4423 + 1.875 = 2.3173$$

$$\text{Maximum } R_{o1} = 2.3173 - 1.750 - .025 = .5423$$

Therefore R_{o1} being greater than maximum, it must be reduced to .542

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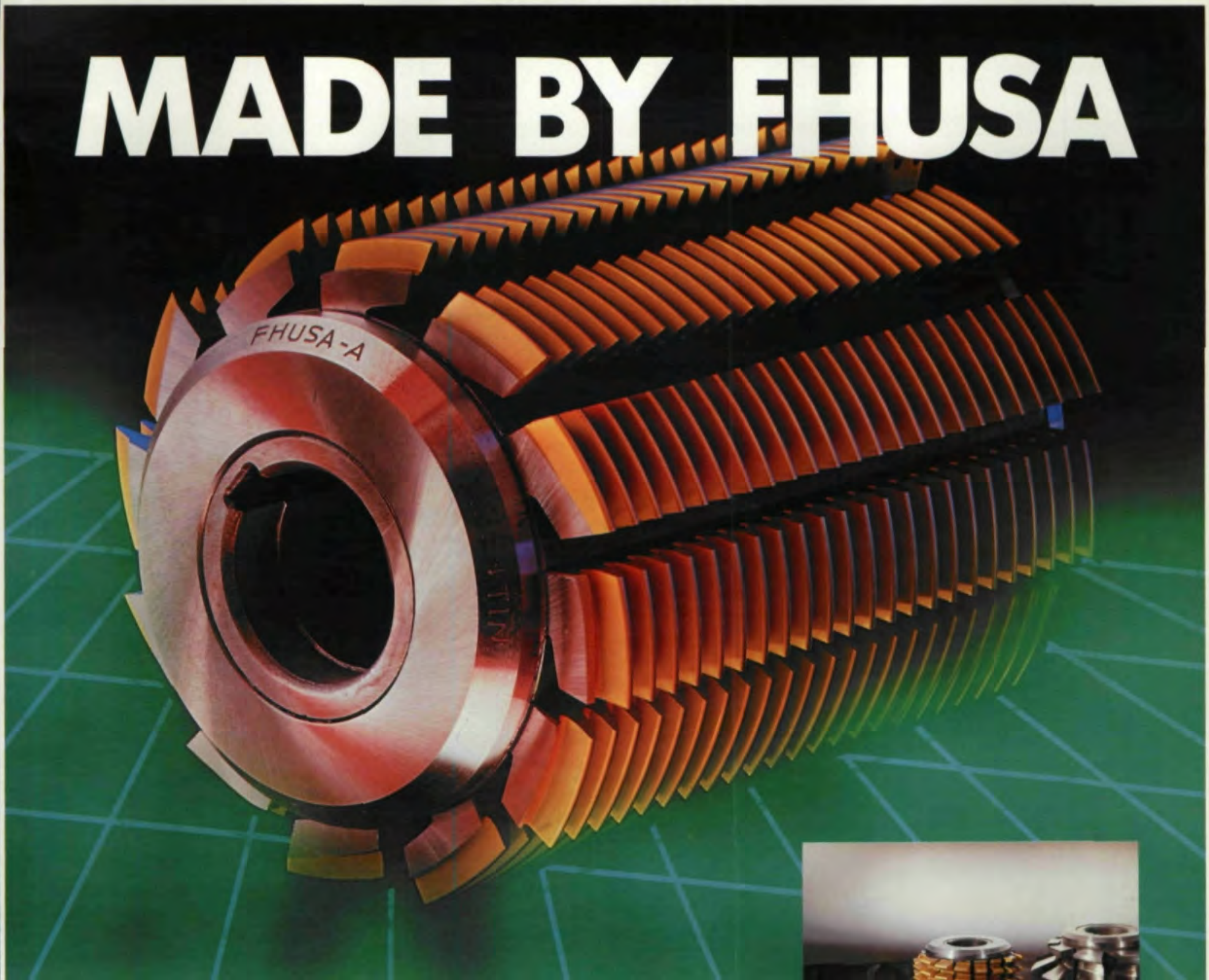

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HIGH SPEED HOBBING . . .
(continued from page 18)

tion has great influence, and the corner wear becomes greater when long addendum gears are cut.

(3) In the case of conventional hobbing for spur gears and reverse-handed conventional hobbing, the influence of addendum modification is comparatively small.

(4) Addendum modification has more influence on crater wear as the cutting speed increases.

(5) Therefore, it is true that conventional hobbing with a small outside diameter hob for spur gears, and reverse-handed hobbing with a small outside diameter hob for helical gears, are very effective for lessening hob wear, even when both short addendum gears and long addendum gears are cut. Addendum modification has no influence on it.

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TECHNICAL CALENDAR

AUGUST 10-12. Ohio State University, Gear Noise Course. Material covered includes noise measurement and analysis, causes, reduction techniques, modeling and modal analysis of gear boxes. For further information, contact: Mr. Richard D. Frasher, College of Engineering, OSU, 2070 Neil Ave., Columbus, OH 43210. (614) 292-8143.

SEPTEMBER 7-15. International Machine Tool Show & Technical Conference, McCormick Place, Chicago, IL. World's largest machine tool show. Technical conference with 48 sessions and over 200 papers on a variety of manufacturing technology subjects. Contact IMTBA, 7901 Westpark Drive, McLean, VA 22102-4269. See also page 8.

SEPTEMBER 27-29. American Society for Metals 11th Annual Heat Treating Conference, McCormick Place, Chicago, IL. Presentations on subjects including heat treating, statistical process control, new energy applications, quenching and cooling improvements. For further information, contact: ASM International, Metals Park, OH 44073. (216) 338-5151.

NOVEMBER 5-10. International Conference on Gearing, Zhengzhou, China. ASME-GRI and several international gear organizations are sponsoring this meeting. For more information contact: Inter—Gear '88 Secretariat, Zhengzhou Research Institute of Mechanical Engineering, Zhongyuan Rd, Zhengzhou, Henan, China. Tel: 47102. Cable 3000. Telex 46033 HSTEC CN.

NOVEMBER 8-10. American Society for Metals Near Net Shape Manufacturing Conference, Hyatt Regency, Columbus, OH. Program will cover precision casting, powder metallurgy, design of dies and molds, forging technology and inspection of precision parts. For further information contact: Technical Department Marketing, ASM International, Metals Park, OH 44073.

CALL FOR PAPERS — Tennessee Technological University for its **1st Internat'l Applied Mechanical Systems Design Conference, March 19-22, 1989, Nashville, TN.** Papers are invited on general mechanical systems subjects including strength, fatigue life, kinematics, vibration, robotics, CAD/CAM, and tribology. Deadline for first drafts is Oct. 1, 1988. For further information, contact: Dr. Cemil Bagci, Dept. of Mech. Eng., TTU, Cookeville, TN 38505. (615) 372-3265.

CALL FOR PAPERS for ASME 5th Annual Power Transmission & Gearing Conference, April 25-27, 1989, Chicago, IL. Papers are invited on emerging technologies for gears, couplings, belts, chains and other power transmission devices — gear geometry, noise, manufacture, inspection, scoring, lubrication, materials, applications, efficiency, dynamics. For more information, contact Donald Borden, P.O. Box 502, Elm Grove, WI 53122.

CHANGE OF DATE: SME's 1988 Gear Processing & Manufacturing Clinic will be held **Oct. 25-27 in Indianapolis, IN.** For more information, contact Dominic Ahearn, SME, One SME Drive, P.O. Box 930, Dearborn, MI 48121. (313) 271-1500 X384.

Helical Gear Mathematics . . .
(Continued from page 44)

Given the proportions of a helical gear and rack, to determine the contact ratio between a helical gear and rack:

When,
R = Pitch Radius of Gear
R_o = Outside Radius of Gear
R_b = Base Radius of Gear
p = Circular Pitch of Rack in Plane of Rotation
ϕ = Pressure Angle of Rack in Plane of Rotation
a = Addendum of Rack
m_p = Contact Ratio

Then,

$$m_p = \frac{a + \text{SIN}\,\phi\,(\sqrt{R_o^2 - R_b^2} + R\,\text{SIN}\,\phi)}{p\,\text{SIN}\,\phi\,\text{COS}\,\phi}$$

Example: $R = 2.250$ $R_o = 2.375$ $R_b = 2.11430$ $p = .3927$ $a = .125$

$\text{SIN}\,\phi = .34202$ $\text{COS}\,\phi = .93969$ $\phi = 20°$

$$m_p = \frac{.125 + .34202\,(\sqrt{(2.375)^2 - (2.11430)^2} - 2.250 \times .34202)}{.3927 \times .34202 \times .93969} = 1.84$$