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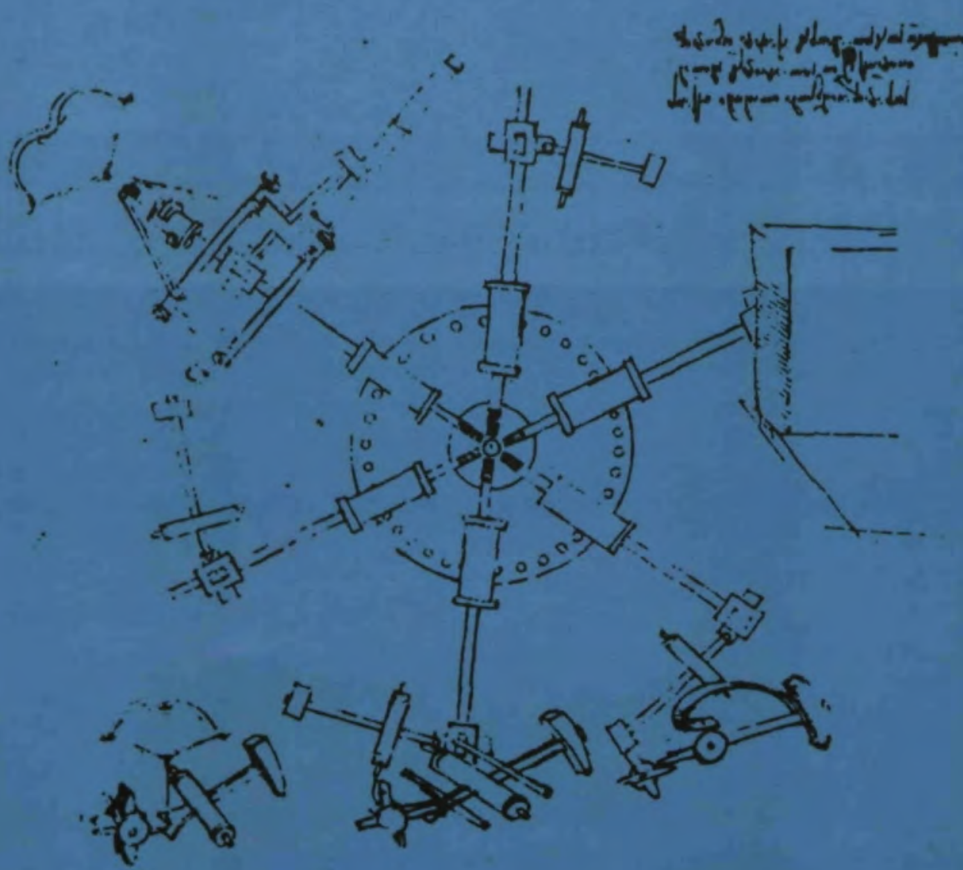
GEAR TECHNOLOGY

The Journal of Gear Manufacturing

JULY/AUGUST 1990



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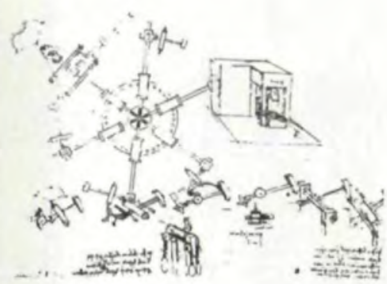

The Advanced Technology of Leonardo da Vinci 1452-1519

COVER

A stamping unit for gold foil—assembly view. Six units are shown, each one activitated by a master crown gear powered by a source, such as an animal or a water mill. The sketch at the right shows how the foil is drawn into the machine, and the little sketches at the bottom of the drawing show various ways of loading the hammer for greater impact. Stamped gold foil was used in Leonardo's time for decorative purposes.

GEAR TECHNOLOGY

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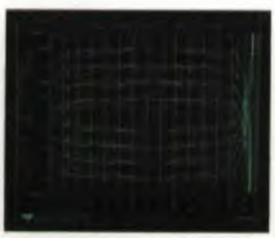

Modified K-Chart Analysis

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CIRCLE A-4 ON READER REPLY CARD

Editorial

OF TREES, WHALES & PRACTICALITY

One of the hot items on the public agenda these days is "The Environment." Suddenly everyone wants to save the whales and the rain forests. Politicians, rock stars, and big business have all discovered that you can't get anything but good press for saying that you're in favor of trees and marine mammals.

Unfortunately, like the environment itself, the business of living in it responsibly is a complicated one. There are no clear right or wrong answers, and almost everything one does is apt to have some negative as well as positive effects.

"A Journey of a thousand miles begins with a single step."

Chinese Proverb

And business people, as usual, are stuck in the middle. We get labeled among the chief villians. (Shame on us for all the toxic waste and poison smoke and industrial waste in the air!) We're supposed to clean up our acts, abide by all the clean air legislaion, fight rising costs, invest for the future, and at the same time, remain competitive.

It's enough to make a person want to sell out and run away to some quiet, unspoiled desert island where they never heard of pollution. Unfortunately, there are no such places left any more.

But despair is not the answer. Neither is paralysis. There are things businesses of any size can do to make a difference – things that don't have to cost a lot of money or disrupt work schedules.

Below are a few suggestions – all of which are relatively inexpensive, quick, and easy to implement. Not every one will work for every office or plant. The list is certainly not complete. I've included them here just to start the ball rolling. This is the direction we need to be walking in. Along with the larger solutions of legislation, funding, and international cooperation, these are the things businesses might do to make a difference in the environment.

- Investigate the possibility of bundling up your waste paper and selling it to recyclers.
- Reuse the boxes and packing material that come to your office.
- Investigate the company pop machine. If it's dispensing aluminum cans, have you made it easy for your employees to recycle the empties? If it's using paper cups, maybe a switch to recyclable cans is the answer.
- Examine your letterhead. Next time you reorder, discuss recycled paper with your supplier. Many paper companies now have some attractive lines of competitively priced, recycled business papers for all uses.
- Examine your operation to see how many disposables you're using just because it's "convenient".
- Cut down on styrofoam or disposable cups for the coffee machine. Encourage employees to bring their own mugs from home.
- Check out the use of disposables in the lunch room. Is it possible to use reusable dishes, trays, silverware?
- Use all the supplies you buy. Make sure things are properly stored, closed, or put away so they are not damaged.
- Make sure the residue from toxic materials is disposed of promptly and safely. The law requires this anyway, and the faster and more efficiently you do so, the smaller the chance of pollution.
- Watch the copier, the fax machine, and the computer printer. How many of those copies are absolutely necessary?
- Check the lights and the thermostat. Are unnecessary lights burning? Does the building *have* to be at a constant 68°? Are you lucky enough to have a facility where the windows open? Do you ever open them when the weather's warm instead of automatically turning on the a.c.?
- Work smarter. Careful work eliminates errors and, therefore, waste. Jobs that don't have to be done over save time, money, and natural resources.
- Look around your grounds. Who maintains the lawn? What kind of fertilizers and pest control methods do they use? How much watering is done? Is it all necessary?
- Plant trees. Trees make your grounds more attractive. They're good for the environment. Planted close to your building, they create shade and cut down on your cooling bills.
- Watch water consumption. How often is the water left running between operations or during clean-up just because it's easier than turning it off?
- Make sure the cleaners and solvents around your operation are biodegradable. Can those that are be substituted for those that aren't?

Okay, so none of these ideas is terribly original. None will, by itself, save a single whale or an acre of the rain forest. But the journey of a thousand miles does begin with a single step. If we all did what we could about the small, controllable aspects of our environments, the cumulative effects would be startling.

The environment is everybody's home and everybody's business. And none of us is powerless to control what happens in our own backyard.

Michael Goldstein,
Editor/Publisher


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IMTS 1990

COMING TO CHICAGO

IMTS-90 — the Western Hemisphere's largest trade show — is coming to Chicago September 5-13, 1990. The event, sponsored by NMTBA - The Association For Manufacturing Technology — will be held at the McCormick Place Complex. Over 1200 exhibitors will display their products at this event.

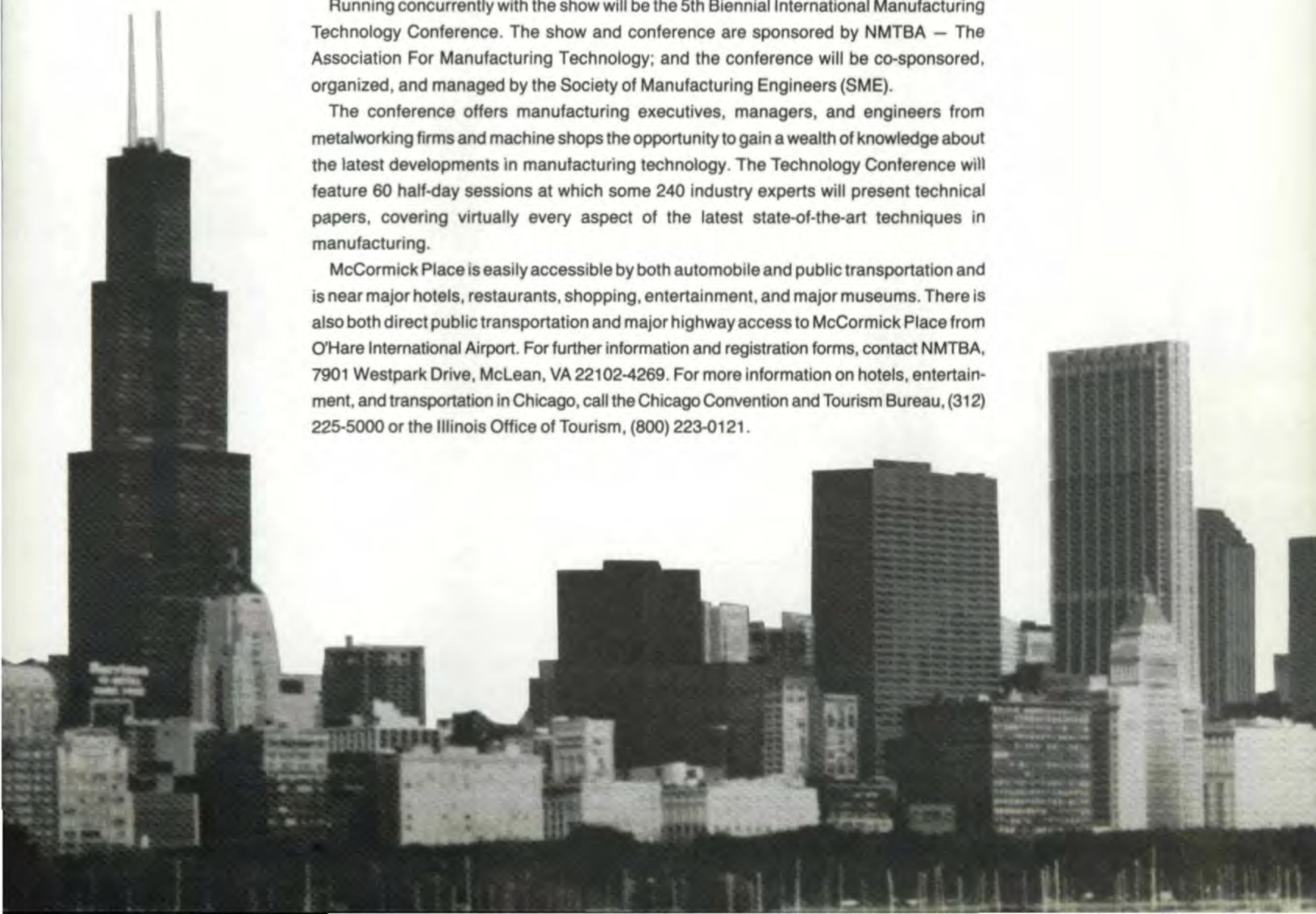
The huge show is held every two years and transforms Chicago's McCormick Place into an international marketplace with more than $315 million worth of exhibits featuring state-of-the-art manufacturing technology. IMTS-88 drew exhibitors from 30 nations and visitors from over 75 different countries.

An added feature of this year's show will be the Product Locator Service. The Product Locator Service will generate custom printed itineraries listing every exhibitor and showing specific products of interest. Signs will be posted in the exhibit halls giving the locations of the terminals.

Running concurrently with the show will be the 5th Biennial International Manufacturing Technology Conference. The show and conference are sponsored by NMTBA — The Association For Manufacturing Technology; and the conference will be co-sponsored, organized, and managed by the Society of Manufacturing Engineers (SME).

The conference offers manufacturing executives, managers, and engineers from metalworking firms and machine shops the opportunity to gain a wealth of knowledge about the latest developments in manufacturing technology. The Technology Conference will feature 60 half-day sessions at which some 240 industry experts will present technical papers, covering virtually every aspect of the latest state-of-the-art techniques in manufacturing.

McCormick Place is easily accessible by both automobile and public transportation and is near major hotels, restaurants, shopping, entertainment, and major museums. There is also both direct public transportation and major highway access to McCormick Place from O'Hare International Airport. For further information and registration forms, contact NMTBA, 7901 Westpark Drive, McLean, VA 22102-4269. For more information on hotels, entertainment, and transportation in Chicago, call the Chicago Convention and Tourism Bureau, (312) 225-5000 or the Illinois Office of Tourism, (800) 223-0121.



Transmission Errors and Bearing Contact of Spur, Helical, and Spiral Bevel Gears

F.L. Litvin, J. Zhang, H.-T. Lee, University of Illinois, Chicago, IL
R.F. Handschuh, NASA Lewis Research Center, Cleveland, OH

Abstract:

An investigation of transmission errors and bearing contact of spur, helical, and spiral bevel gears was performed. Modified tooth surfaces for these gears have been proposed in order to absorb linear transmission errors caused by gear misalignment and to localize the bearing contact. Numerical examples for spur, helical, and spiral bevel gears are presented to illustrate the behavior of the modified gear surfaces with respect to misalignment and errors of assembly. The numerical results indicate that the modified surfaces will perform with a low level of transmission error in non-ideal operating environments.

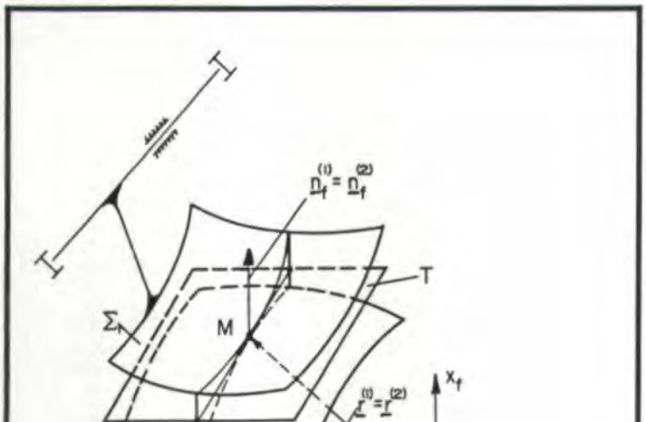


Fig. 1 – Contacting tooth surfaces.

Introduction

The most important criteria of gears are the level of their noise and bearing contact. A main source of gear noise is transmission errors. Traditional methods of gear synthesis that provide conjugate gear tooth surfaces with zero transmission errors and an instantaneous line of contact are not acceptable for the gearing industry due to errors of manufacturing and assembly. Taking into account such errors, the bearing contact is shifted to the edge of the tooth, and transmission errors of an unfavorable shape occur. The new trend of gear synthesis is to localize bearing contact and absorb the transmission errors. These goals may be achieved by a new topology of gear tooth surfaces, and this is the subject of this article.

Simulation of Meshing

The investigation of gear misalignment requires a numerical solution for the simulation of meshing and contact of gear tooth surfaces. The basic ideas of this method[2-3] are as follows:

(1) The meshing of gear tooth surfaces is considered in a fixed coordinate system, S_{F1}.

(2) Due to the continuous tangency of gear tooth surfaces, the position-vectors, $\underset{\sim}{r}^{(1)}$ and $\underset{\sim}{r}^{(2)}$, and the surface unit normals, $\underset{\sim}{n}^{(1)}$ and $\underset{\sim}{n}^{(2)}$, must be equal at every instant. (Fig. 1).

(3) Simulating the errors of manufacturing and assembly and considering that the gear tooth surfaces are in point contact, we may determine function $\phi_2\ (\phi_1)$ that relates the angles of rotation of the gears.

Then, the transmission errors will be determined from the equation

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \qquad (1)$$

where $\phi_2(\phi_1)$ is the function obtained numerically, with the aid of the developed computer program, and N_1 and N_2 are the numbers of teeth.

Simulation of Bearing Contact

The requirement that

$$\underset{\sim}{r}^{(1)} = \underset{\sim}{r}^{(2)} \text{ and } \underset{\sim}{n}^{(1)} = \underset{\sim}{n}^{(2)} \tag{2}$$

provides the information about the path of contact on the tooth surface. Due to the elastic approach of the gear tooth surfaces, their contact is spread over an elliptical area. It is assumed that the magnitude of the elastic approach is known from experiments or may be computed. Knowing in addition the principle curvatures and directions for two contacting surfaces at their point of contact, we may determine the dimensions and orientation of the contact ellipse.[2-3] The instantaneous point of contact is the center of symmetry of the contact ellipse. The set of contact ellipses represents the bearing contact. Simulating the errors of manufacturing and assembly, we are able to determine the real path of contact on the gear tooth surface and the real bearing contact.

Partial Compensation of Transmission Errors

The investigation of the effect of errors of assembly and machine tool settings shows that the real transmission function is a piece-wise linear or almost linear function (Fig. 2a). The transmission errors that are represented by Equation 1 are shown in Fig. 2b. Transmission errors of this type cause an instantaneous jump of the angular velocity of the driven gear at the transfer points, and vibration becomes inevitable. (At transfer points one pair of teeth in mesh is changed for next pair.) The proposed approach allows absorption of the transmission errors by modifying the gear tooth surfaces. The modification is directed to provide a predesigned parabolic function of transmission errors that will be able to absorb linear functions of transmission errors. The predesigned parabolic function of transmission errors will exist even for aligned gears. However, when the gears are misaligned, piece-wise linear transmission errors (Fig. 2b) will not occur. This is based on the possibility of a linear function being absorbed by a parabolic one.

Consider the interaction of a parabolic function given by (Fig. 3a)

$$\Delta\phi_2^{(1)} = -a\phi_1^2 \tag{3}$$

with a linear function represented by

$$\Delta\phi_2^{(2)} = b\phi_1 \tag{4}$$

The resulting function

$$\Delta\phi_2 = b\phi_1 - a\phi_1^2 \tag{5}$$

may be represented in a new coordinate system by (Fig. 3b):

$$\psi_2 = -a\psi_1^2 \tag{6}$$

where

$$\psi_2 = \Delta\phi_2 - \frac{b^2}{4a},\ \psi_1 = \phi_1 - \frac{b}{2a} \tag{7}$$

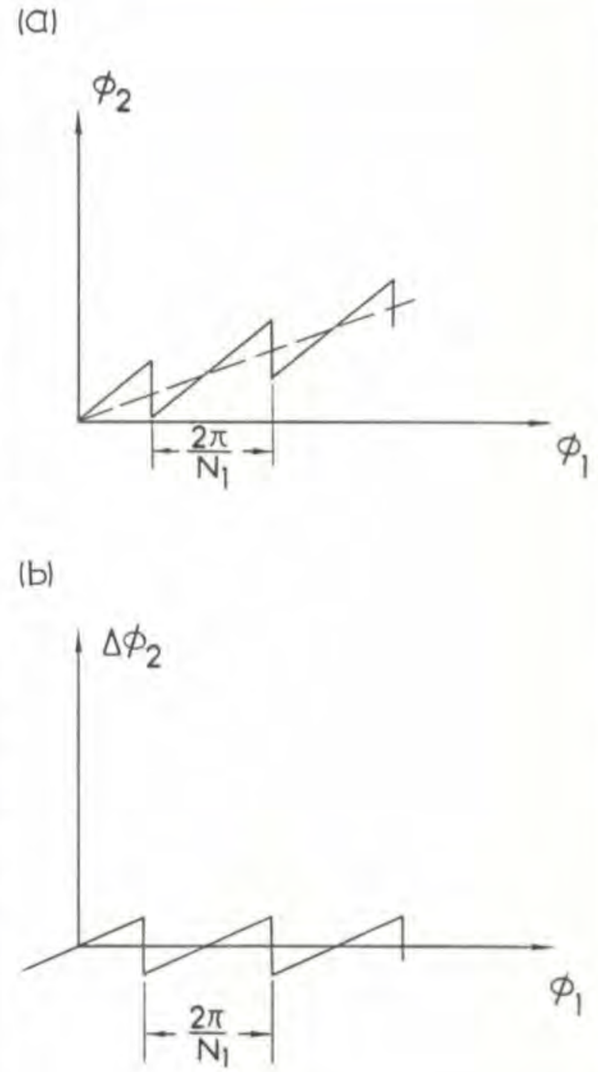


Fig. 2 – Transmission error caused by gear misalignment.

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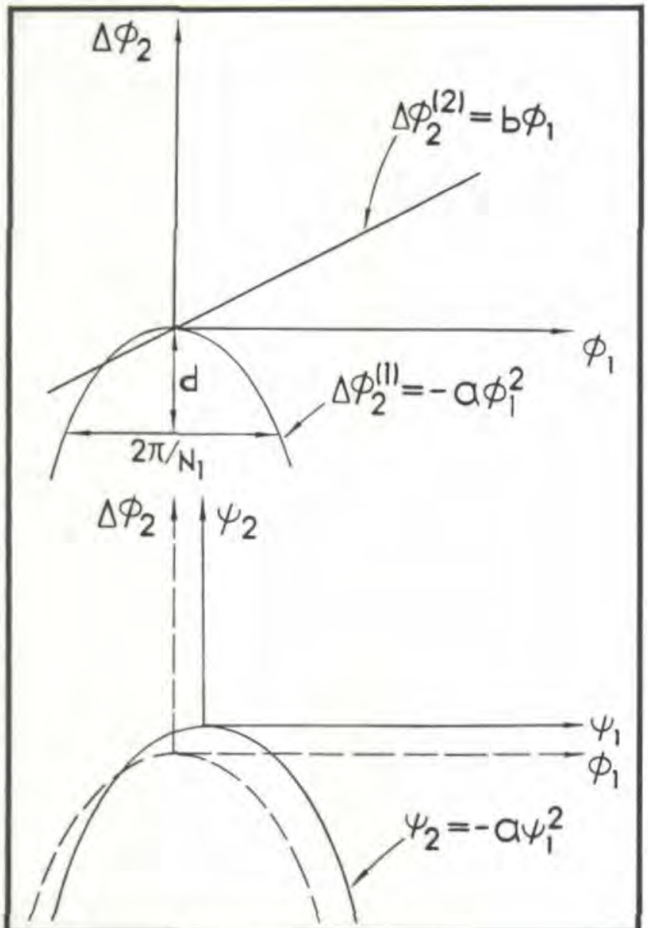

Fig. 3 – Interaction of parabolic and linear functions.

We consider that $\Delta\phi_2^{(1)} = -a\phi_1^2$ is the predesigned function that exists even if misalignments do not appear. The absorption of function $\Delta\phi_2^{(2)} = b\phi_1$ by the parabolic function $\Delta\phi_2^{(1)} = -a\phi_1^2$ means that gear misalignment does not change the predesigned parabolic function of transmission errors. Thus the resulting functions of transmission errors $\Delta\phi_2 = \Delta\phi_2^{(1)} + \Delta\phi_2^{(2)}$ will keep its shape as a parabolic function, although the gears are misaligned. The resulting function of transmission errors $\phi_2(\phi_1)$ may be obtained by translation of the parabolic function $\phi_2^{(1)}$.

The absorption of a linear function of transmission errors by a parabolic function is accompanied by the change of transfer points. The transfer points determine the position of the gears where one pair of teeth is going out of mesh and the next pair is coming into mesh. It is necessary to emphasize that theoretically the contact ratio of gears with transmission errors is one. The real contact ratio can be larger than one only due to elastic deformations. The change of transfer points is determined with $\Delta\phi_1 = \left|\frac{b}{2a}\right|$ and $\Delta\phi_2 = \frac{b^2}{4a}$. The cycle of meshing of one pair of teeth is $\phi_1 = \frac{2\pi}{N}$. It may happen that the absorption of a linear function by a parabolic function is accompanied by a change of transfer points that is too large. If this occurs, the transfer points and the resulting parabolic function of transmission errors, $\psi_2(\psi_1)$, will be represented as a discontinous function for one cycle of meshing (Fig. 4.) To avoid this, it is necessary to limit the tolerances for gear misalignment or increase the level of the predesigned parabolic function.

Modification of Pinion Tooth Surface for Spur Gears

The gear is provided with a regular involute surface. The pinion is provided with a crowned tooth surface, and two types of the crowned surface are considered. The surface of

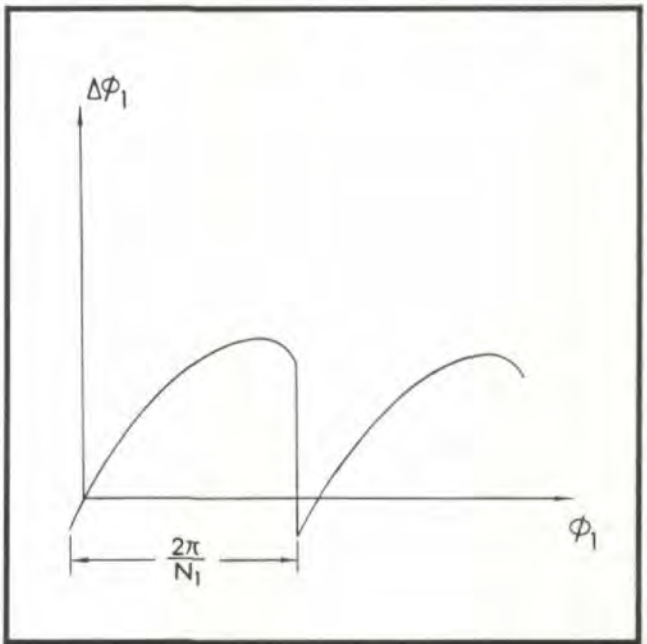

Fig. 4 – Discontinued parabolic function of transmission errors.

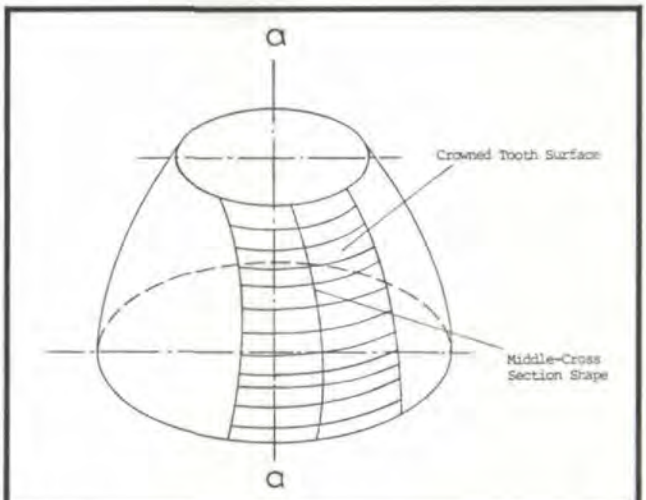

Fig. 5 – Crowned pinion tooth surface.

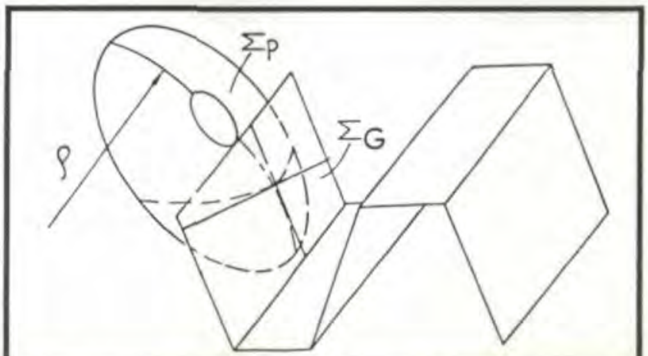

Fig 6 – Generating tool surface for helical pinion and gear.

the first type is considered as a surface of revolution (Fig. 5) whose axial sections slightly deviate from a regular involute curve. The surface of the second type is based on the method of generation that is presented in Fig. 6.

Pinion Tooth Surface as a Surface of Revolution. Consider first that the axial section of the pinion tooth surface is a regular involute curve. The pinion tooth surface, which is a surface of revolution, may be generated by rotation of the involute curve about the a-a axis (Fig. 5.) It is evident that the pinion and gear tooth surfaces will contact each other in the middle cross-section and transform rotation with a *constant* gear ratio if the gears are not misaligned. However, it is necessary to compensate for the transmission errors of misaligned gears. For this reason the shape of the pinion is to be synthesised as a curve that deviates from an involute curve to provide a predesigned parabolic function of transmission errors.

The sections of the pinion tooth surface that are perpendicular to axis a-a (Fig. 5) are circles. The radius of such a circle for the mean contact point may be determined from the requirements to the dimensions of the contact ellipse.

The proposed pinion tooth surface may be generated by form-grinding or by a computer controlled grinding or cutting machine.

Example 1. Given numbers of teeth $N_1 = 20$, $N_2 = 40$, diametral pitch $P = 10\,\frac{1}{in}$, then, pressure angle $\psi_c = 20°$. The pinion tooth surface is crowned as described above, and a parabolic function of transmission errors with the level of 2 arc seconds has been predesigned.

The influence of gear misalignment has been investigated with the developed TCA program for the following cases:

Table 1. Function of Transmission Errors.

$\phi_1^{(°)}$	-3	0	3	6	9	12	15
$\Delta\phi_2^{('')}$	-0.71	0.00	0.40	0.50	0.33	-0.09	-0.74

Table 2. Function of Transmission Errors.

$\phi_1^{(°)}$	-10	-7	-4	-1	0	2	5	8
$\Delta\phi_2^{('')}$	-2.02	-0.92	-0.24	0.01	0.00	-0.16	-0.75	-1.78

Case 1. The change of the center distance is $\frac{\Delta c}{c} = 1\%$; the gear axes are not parallel, but crossed, and the twist angle is 5 arc minutes. The resulting function of transmission errors is still a parabolic one with the maximum value of 1.2 arc seconds as shown in Table 1.

Case 2. The axes of the same gears are intersected and form an angle of 5 arc minutes. The resulting function of transmission errors is again a parabolic one with the maximal value of 2 arc seconds as shown in Table 2.

Generation of the Pinion Tooth Surface by Tool With a Surface of Revolution. Consider that two generating surfaces — a plane and a cone — are rigidly connected with each other and generate the gear tooth surface and the pinion crowned tooth surface, respectively. (Fig. 6.) The generating plane is the surface of a regular rack cutter. In the process of generation the rack cutter and the cone perform a translational motion, while the pinion and the gear rotate about their axes. (Fig. 7.) The rotation of the cone about its axis c-c is not related to other motions for the tooth surface generation, and the angular velocity of the cone depends only on the desired velocity of cutting. The tool for the crowning of the pinion can be designed as a grinding wheel or as a shaver. The opposite side of the pinion tooth is to be generated separately.

The described process of crowning of the pinion by a regular cone provides an involute shape of the pinion tooth surface in its middle section. The crowned pinion and the involute gear, if they are not aligned, can transform rotation without transmission errors. Also, their bearing contact is localized. But in reality, due to the misalignment of gears, they will transform rotation with a piece-wise linear function of transmission errors. (Fig. 2.) To absorb such errors it is necessary to predesign a parabolic function of transmission errors. This can be achieved if a tool surface of revolution that slightly deviates from the cone surface is used. (Fig. 8.) The curvature radius ϱ and the level of predesigned transmission errors of parabolic type are related. The deter-

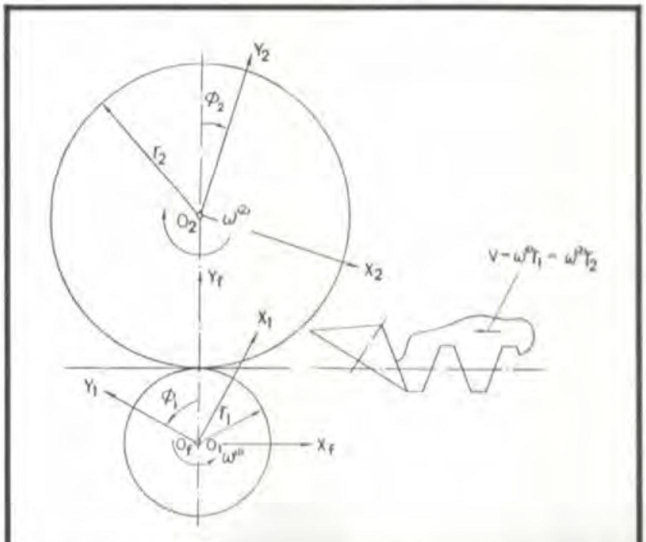

Fig 7 – Generation of gears.

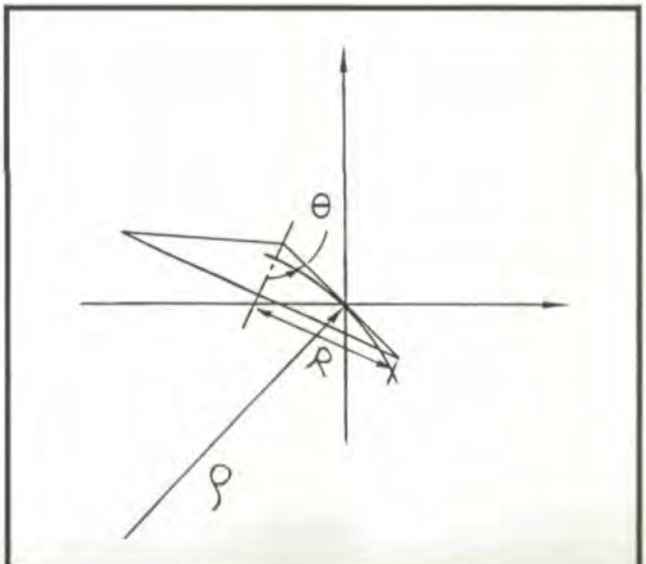

Fig 8 – Surface of revolution and its parameters.

mination of ϱ is the subject of synthesis of the pair of gears with the crowned pinion.

Example 2. The input data are the same as in Example 1. The parameters of the tool surface (Fig. 8) are: $\theta = 20°$, $\varrho = 500$ in., R = 10.6". The influence of misalignment has been investigated by the developed computer programs for two cases.

Case 1. The change of center distance is $\frac{\Delta C}{C} = 1\%$, and the twist angle of crossed axes is 10 arc minutes. The resulting function of transmission errors is of a parabolic type with the maximum value of 0.28 arc seconds as shown in Table 3.

Case 2. The gear axes are interesected and form an angle of 10 arc minutes. The resulting function of transmission errors is of a parabolic type with the maximum value of 0.34 arc seconds as shown in Table 4.

Table 3. Function of Transmission Errors.

$\phi_1^{(°)}$	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2^{('')}$	−0.26	−0.10	0.00	0.02	0.00	−0.02	−0.14	−0.33

Table 4. Function of Transmission Errors.

$\phi_1^{(°)}$	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2^{('')}$	−0.33	−0.15	−0.03	0.01	0.00	−0.02	−0.11	−0.20

Modification of Pinion Tooth Surface for Helical Gears with Parallel Axes

The crowning of the pinion of helical gears with a parallel axis is directed at the achievement of two goals: (i) to localize the bearing contact, and (ii) to reduce the level of transmission errors. These goals are contradictive, and in some cases a compromise solution has to be found. The following methods for crowning are based on: (i) generation of the pinion tooth surface by a tool with a surface of revolution, (Fig. 8.) (ii) generation of the pinion with circular arc teeth, and (iii) the change of the lead angle.

Generation of Helical Pinion Tooth Surface by a Surface of Revolution. The method of generation is based on the same ideas that have been developed for the generation of spur pinions. The only difference is the installment of the tool. Considering again two generating surfaces, (Fig. 6.) we have to require that the generating plane will be installed as a skew rack cutter and form with the gear axis an angle that is equal to or approximately equal to the helix angle on the pitch cylinder. By varying the installment angle, a slight change in the orientation of the bearing contact can be made. It is necessary to emphasize, however, that the path of contact will be in the middle section of the gear tooth or near this direction. (Fig. 9.) The results of investigation of helical gears with crowned pinion tooth surfaces are represented in the following example.

Example 3. Given: $N_1 = 20$, $N_2 = 40$, diametral pitch in the normal section $= P_n = 10$ in^{-1}, the pressure angle in the normal section is $\psi = 20°$, the helix angle on the pitch cylinder is $\beta = 15°$. Parameters of the surface of revolution (Fig. 8.) are $\theta = 20°$, $\varrho = 30$ in., R = 10.6 in. Helical gears with modified pinion tooth surface provide a parabolic type

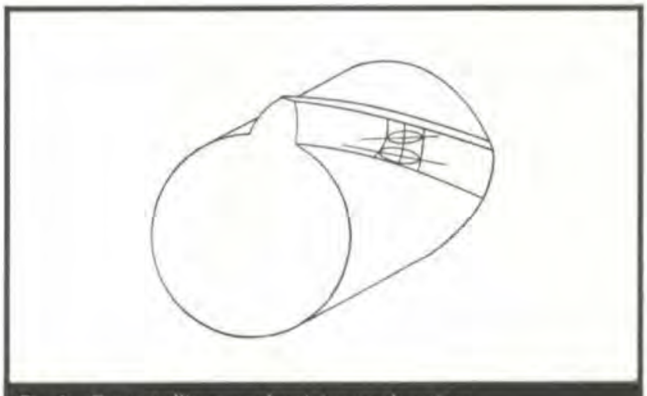

Fig. 9—Contact ellipses on the pinion tooth surface.

of predesigned transmission error with d = 6 arc seconds, (Fig. 3a.) and a path contact that is directed across the tooth surface. (Fig. 9.)

The influence of gear misalignment has been investigated by computer program, and the results of computation are represented in Tables 5 and 6 for crossed and intersected gear axes, respectively. The misalignment of gear axes is 5 arc minutes.

Table 5. Transmisson Errors of Crossed Helical Gears.

$\phi_1^{(°)}$	−14	−11	−8	−5	−2	1	4
$\Delta\phi_2^{('')}$	−4.99	−1.51	0.65	1.51	1.05	−1.75	−3.87

Table 6. Transmission Errors of Intersected Helical Gears.

$\phi_1^{(°)}$3	−11	−8	−5	−2	1	4	7
$\Delta\phi_2^{('')}$	−6.15	−2.72	−0.60	0.20	−0.32	−2.19	−5.40

The results of computation show that the resulting function of transmission errors is a parabolic one. Thus the linear function of transmission errors that was caused by gear misalignment has been absorbed by the predesigned parabolic function.

Change of the Pinion Lead. Helical gears in this case are designed as helical gears with crossed axes. The crossing angle γ is chosen with respect to the expected tolerances of the gear misalignment. (γ is in the range of 10 to 15 arc minutes.) The gear ratio for helical gears with crossed axes may be represented.[2]

$$m_{12} = \frac{\omega^{(1)}}{\omega^{(2)}} = \frac{r_{b2} \sin \lambda_{b2}}{r_{b1} \sin \lambda_{b1}} \qquad (8)$$

where r_{bi} and λ_{bi} are the radius of the base cylinder, and the lead angle on this cylinder; i.e., 1,2. $|\lambda_{p2} - \lambda_{p1}| = \gamma$. Here: λ_{pi} is the lead angle on the pitch cylinder. The advantage of application of crossed helical gears is that the gear ratio is not changed by the misalignment (by the change of γ). The tooth surfaces contact each other at a point during meshing. The disadvantage of this type of surface deviation is that location of the bearing contact of the gears is very sensitive to gear misalignment. A slight change of the crossing angle causes shifting of the contact to the edge of the tooth. (Fig. 10.)

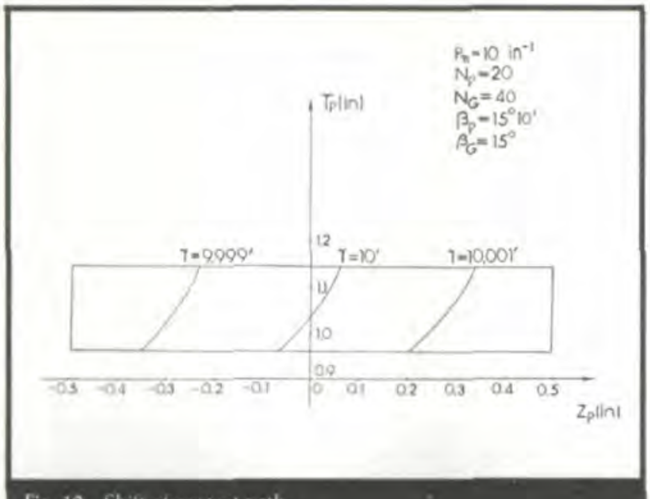

Fig. 10 – Shift of contact path.

The discussed type of surface deviation is reasonable to apply for manufacturing of expensive reducers of large dimensions when the lead of the pinion can be adjusted by regrinding. While changing by regrinding the parameters r_{b1} and λ_{b1}, it is required that the product $r_{b1} \sin \lambda_{b1}$ must be kept constant. Then, the gear ratio m_{21} will be of the prescribed value, and transmission errors caused by the crossing of axes will be zero.

Theoretically, transmission errors are inevitable if the axes of crossed helical gears become intersected. Actually, if gear misalignment is of the range of 5 to 10 arc minutes, the transmission errors are very small and may be neglected. The main problem for this type of misalignment is again the shift of the bearing contact to the edge. (Fig. 10.)

Crowned Helical Pinion with Longitudinal Path Contact. A longitudinal path of contact means that the gear tooth surfaces are in contact at a point at every instant, and the instantaneous contact ellipse moves *along* and not *across* the surface. (Fig. 11a.) This path can be achieved by crowning gear surfaces. It can be expected that this type of contact provides improved conditions of lubrication. Until now only the Novikov-Wildhaber gears[1] could provide a longitudinal path of contact. A disadvantage of this type of gearing is the sensitivity to the change of the center distance and the axis misalignment. The sensitivity to non-ideal orientation of the meshing gears causes a higher level of gear noise in comparison with regular involute helical gears. Litvin and Tsay[4] proposed a compromise type of nonconformal helical gears that may be placed between regular helical gears and Novikov-Wildhaber helical gears. The gears of the proposed gear train are the combination of a regular involute helical gear and a specially crowned helical pinion. The investigation of transmission errors for helical gears with a longitudinal path of contact shows that their good bearing contact is accompanied by an undesirable increased level of linear transmission error. To compensate for this disadvantage, a predesigned parabolic function of transmission errors that will absorb the linear function of transmission errors is proposed.

The generation of gears is based on the following idea:

Consider that two rigidly connected generating surfaces, Σ_g and Σ_p, are used for the generation of the gear and the pinion, respectively. (Fig. 11b.) Surface Σ_g is a plane and

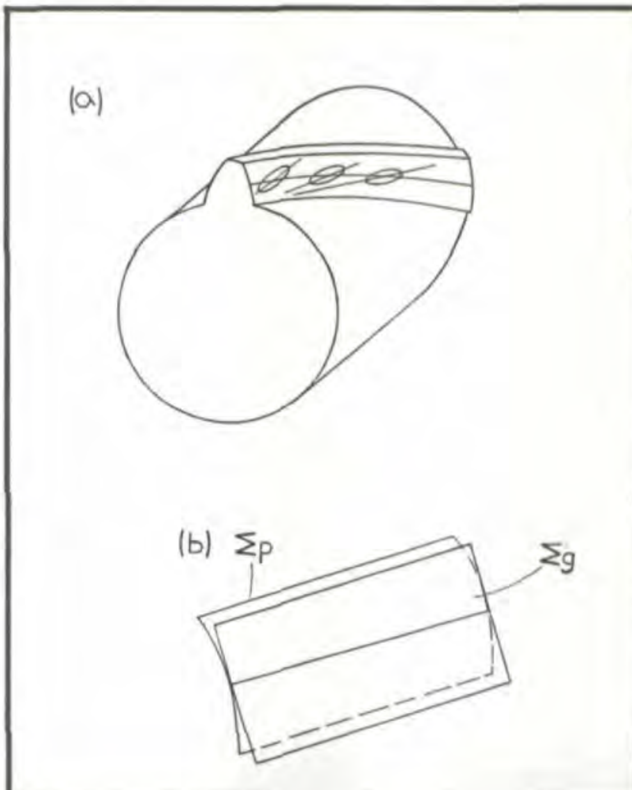

Fig. 11 – Helical gear and generating tool surface.

represents the surface of a regular rack cutter; surface Σ_p is a cylindrical surface whose cross-section is a circular arc. We may imagine that while surfaces Σ_g and Σ_p translate the pinion and the gear rotate about their axes. To provide the predesigned parabolic function of transmission errors, it is necessary to observe the following transmission functions by generation:

$$\frac{V}{\omega^{(2)}} = r_2 = \text{const}, \frac{V}{\omega^{(1)}} = r_2\frac{N_1}{N_2} - 2a\phi_1 = f(\phi_1) \qquad (9)$$

Here $\omega^{(1)}$ and $\omega^{(2)}$ are the angular velocities of pinion and gear by cutting; V is the velocity of the rack cutter in translational motion; N_1 and N_2 are the gear and pinion tooth numbers; ϕ_1^1 is the angle of rotation of the pinion by cutting. The generated gears will be in point contact at every instant and transform rotation with the function

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 - a\phi_1^2 \; 0 < \phi_1 < \frac{2\pi}{N_1} \qquad (10)$$

This function relates the angles of rotation of the pinion and the gear, ϕ_1 and ϕ_2, respectively, for one cycle of meshing. The predesigned function of transmission errors is

$$\Delta\phi_2 = -a\phi_1^2 \qquad (11)$$

It is evident that after differentiation of Function (10), the gear ratio $\omega^{(2)}/\omega^{(1)}$ satisfies Equation (9).

To apply this method of generation in practice it is necessary to vary the angular velocity of the pinion in the

(continued on page 44)

Efficient Methods for the Synthesis of Compound Planetary Differential Gear Trains for Multiple Speed Ratio Generation

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Abstract:

This article presents an efficient and direct method for the synthesis of compound planetary differential gear trains for the generation of specified multiple speed ratios. It is a train-value method that utilizes the train values of the integrated train components of the systems to form design equations which are solved for the tooth numbers of the gears, the number of mating gear sets and the number of external contacts in the system. Application examples, including vehicle differential transmission units, rear-end differentials with unit and fractional speed ratios, multi-input function generators and robot wrist joints are given.

Introduction

In a simple planetary gear train each planet gear shaft carries one gear; in a compound planetary gear train each planet gear shaft carries two or more gears connected to each other. (See the simple and compound planetary gear trains in Figs. 1 and 3, respectively.) Planet gears in a compound planetary gear train cause speed reduction and change in direction of rotation; planet gears in simple planetary gear trains merely cause change in direction of rotation. Synthesis of compound planetary gear trains can be done by the tabulation method, which utilizes relative motions of gears with respect to the planet arm. In general, it is an iterative process.

Using the equations of motion of planetary gear trains instead of the tabulation method yields a very simple, direct method for analysis as well as for synthesis of both simple and compound planetary gear trains. The equations of motion for a simple planetary gear train, such as shown in Fig. 1, are formed by writing the velocity loop-closure equations for contact points A and C and solving them simultaneously.

Given the following criteria:

$\bar{\omega}_i = \omega_i \bar{k}$ is angular velocity;

r_i is the pitch circle radius of the i^{th} gear or member; member 3 is the planet arm;

$\bar{\omega}3 \equiv \bar{\omega}_a$; $\bar{V}_A = -V_A \bar{i}$; $\bar{V}_c = -V_c \bar{i}$;

$\bar{V}_B = -V_B \bar{i}$; $\bar{V}_{AB} = V_{AB} \bar{i}$; and $\bar{V}_{CB} = -V_{CB} \bar{i}$

where

$V_A = r_2 \omega_2$; $V_c = r_5 \omega_5$; $V_{CB} = r_4 \omega_4 = V_{AB}$ and

$V_B = (r_2 + r_4)\, \omega_a$

The velocity loop-closure equations are

$$\bar{V}_A\big|_2 = (\bar{V}_B + \bar{V}_{AB})\big|_4 \tag{1}$$

$$\bar{V}_C\big|_5 = (\bar{V}_B + \bar{V}_{CB})\big|_4 \tag{2}$$

where the subscripts 2, 4, 5 designate the contact points on members 2, 4, 5, respectively. Equations 1 and 2, in terms of r_i and ω_i, become

$$-r_2\omega_2 = -\omega_a (r_2 + r_4) + r_4\omega_4 \tag{1a}$$

$$-r_5\omega_5 = -\omega_a (r_2 + r_4) - r_4\omega_4 \tag{2a}$$

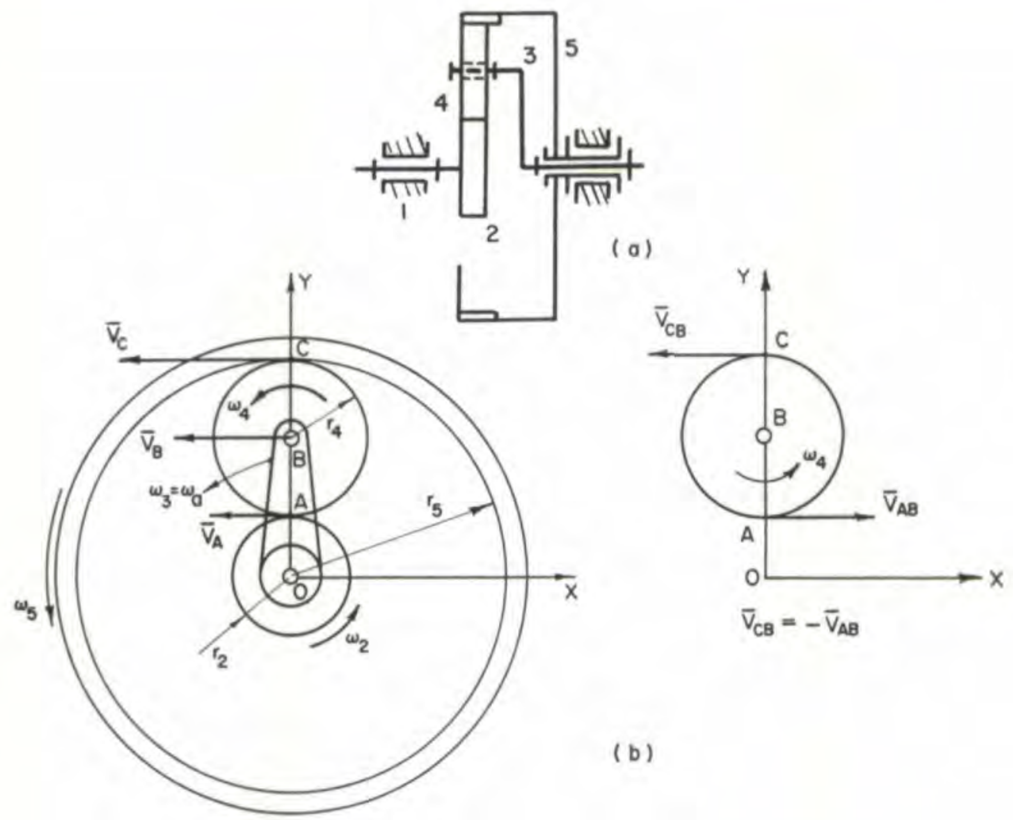

Fig. 1 – (a) Simple planetary gear train, (b) velocity vectors.

Eliminating ω_4 in Equations 1a and 2a we obtain the equations of motion relating speeds of gears 2, 5 and the planet arm as

$$r_2\omega_2 + r_5\omega_5 = 2\omega a\,(r_2+r_4) \tag{3}$$

with the constraint that

$$r_5 = r_2 + 2r_4 \tag{4}$$

Substitute r_5 from (4) into (3) and note that $r_i = N_i/2P$ and $\omega_i = \pi n_i/30$; P is the diametral pitch; n_i is the speed of the i^{th} gear in rpm; n_a is the speed of the planet arm. Then the equation of motion in terms of tooth numbers of gears becomes

$$N_2\,(n_2-n_a) + N_5\,(n_5-n_a) = 0 \tag{5}$$

which is written in the form of train value as

$$e = -\frac{N_2}{N_5} = -\frac{1}{K} = \frac{n_5 - n_a}{n_2 - n_a} \tag{6}$$

In this equation e is the train value formed as the ratio of the products of tooth numbers of driving gears starting with the input gear, ΣPDVER, to the products of tooth numbers of the driven gears, and ΣPDVEN, when the planet arm is considered stationary. The sign of e, however, is very important in order that no error is introduced during the analysis and synthesis. Thus, e is defined as

$$e = (-1)^q \frac{\Sigma PDVER}{\Sigma PDVEN} \tag{7}$$

where q is the number of external contacts of the mating gears. For the gear train in Fig. 1, e is defined as

$$e = -\frac{N_2N_4}{N_4N_5} = -\frac{N_2}{N_5} \tag{8}$$

with $q=1$ for contact between gears 2 and 4. For the gear train of gears 2 to 7 in Fig. 2, e is defined with $q=3$ as

$$e = (-1)^3 \frac{N_2N_4N_5N_6}{N_4N_5N_6N_7} = -\frac{N_2}{N_7} \tag{9}$$

where gear 6 is a double length gear to mate with gears 7 and

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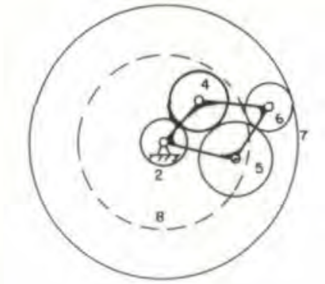

Fig. 2 – Simple planetary gear train with idler planet gears.

8. For the gear train of gears 2 to 8, e is defined with $q=4$,

$$e = \frac{N_2}{N_8} \tag{10}$$

As observed in Equations 8-10, idler gears do not affect the speed ratio, but their number defines the sign of e.

Subtract Equation 1 from Equation 2 to have

$$\overline{V}_A - \overline{V}_C = 2\,\overline{V}_{AB} \tag{11}$$

or

$$-r_2\omega_2 + r_5\omega_5 = 2r_4\omega_4 \tag{12}$$

which, upon substituting r_i and ω_i in terms of N_i and n_i, gives the speed of the planet gear 4 as

$$n_4 = \frac{n_5N_5 - n_2N_2}{2N_4} \tag{13}$$

Either substituting e and n_5 from Equation 6 and

$$N_5 = 2N_4 + N_2 \tag{14}$$

from Equation 4 into Equation 14, or considering Equation 1a, we obtain

$$n_4 = n_a - \frac{N_2}{N_4}(n_2 - n_a) \tag{15}$$

defining n_4 in terms of n_2. Also, either substituting e and n_2 from Equation 6 and N_2 from Equation 14 into Equation 13, *or considering Equation 2a, we obtain*

$$n_4 = n_a + \frac{N_2}{N_5}(n_5 - n_a) \tag{16}$$

defining n_4 in terms of n_5.

The train value defined by Equation 6 is written in general form as

$$e = \frac{n_L - n_a}{n_f - n_a} \tag{17}$$

where n_F and n_L are the speeds of the first and last gears in the train considered, and e is written correspondingly. The first gear tooth number in ΣPDVER is the tooth number of the first gear of the train, whichever end of the train one starts with, and the last gear tooth number in the train is the last number in ΣPDVEN.

Similarly, Equations 15 and 16 can also be written in the following general forms: Where n_p is the designated speed of the planet gear, e_{Fp} and e_{Lp} are the train values between the first and planet gear and last and planet gear, respectively.

$$n_p = n_a + e_{Fp}(n_F - n_a) \tag{18}$$

$$n_p = n_a + e_{Lp}(n_L - n_a) \tag{19}$$

Note that the coefficient $[(-1)^1 n_2/n_4]$ of $(n_2 - n_a)$ in Equation 15 is the train value e_{24} between gears 2 and 4; the coefficient $[(-1)^\circ N_5/N_4]$ of $(n_5 - n_a)$ in Equation 16 is the train value e_{54} between gears 5 and 4.

Equations 18 and 19 form bases for the tabulation method to define the speeds of planet gears. This method simply assumes all moving members are fixed to the planet arm rotating with the arm speed. Then it adds to that speed their individual speeds relative to the planet arm, as if they were ordinary gear trains whose frames were fixed to the planet arm.

As shown in the following section, Equations 17-19 are also the equations of motion for all compound gear trains, bevel gear planetary gear trains and differentials.

Equations of Motion for Compound Planetary Gear Trains

Two compound planetary gear trains are shown in Figs. 3(a) and (b). In (a) the last gear is an internal gear; in (b) it is an external gear. Equations of motion for the compound gear train are written in the same manner as for the simple planetary gear train. Thus, as shown in Fig. 4, velocity loop-closure equations for A and C are

$$\overline{V}_A|2 = (\overline{V}_B + \overline{V}_{AB})|4 \tag{20}$$

$$\overline{V}_C|6 = (\overline{V}_B + \overline{V}_{CB})|5 \tag{21}$$

which reduce to

$$-r_2\omega_2 = -\omega_a(r_2 + r_4) + r_4\omega_4 \tag{22}$$

$$-r_6\omega_6 = -\omega_a(r_2 + r_4) - r_5\omega_5 \tag{23}$$

Noting that $\omega_5 = \omega_4$ and eliminating them in Equations 22 and 23, we obtain

$$r_5r_2(\omega_2 - \omega_a) + r_4r_6\omega_6 = \omega_a[r_4^2 + r_4(r_2 + r_5)] \tag{24}$$

Noting the following equation of constraint

$$r_2 + r_4 = r_6 - r_5 \tag{25}$$

or P_{ij} defining diametral pitch between gears i and j,

$$\frac{N_2 + N_4}{2P_{24}} = \frac{N_6 - N_5}{2P_{56}} \tag{26}$$

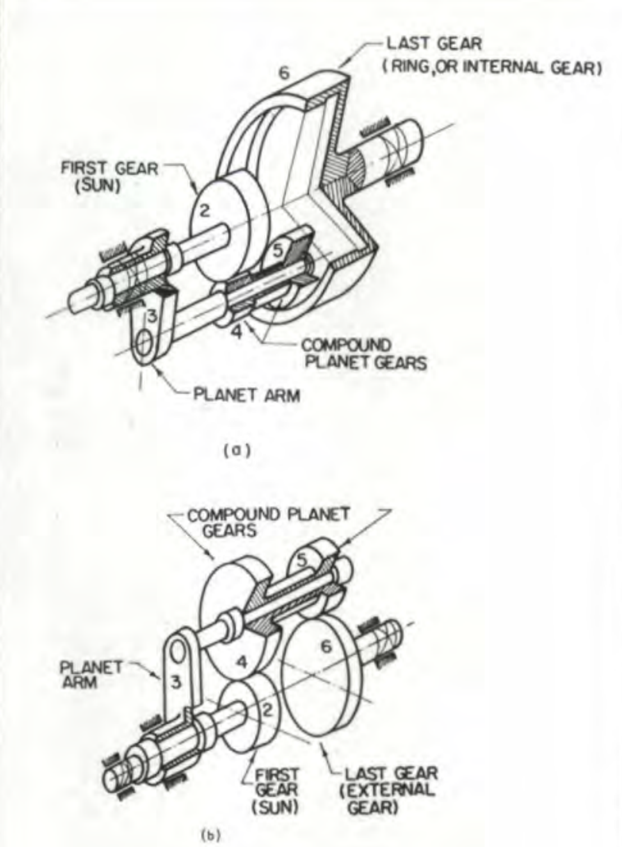

Fig. 3 – Compound planetary gear trains, (a) with internal gear, (b) with all external gears.

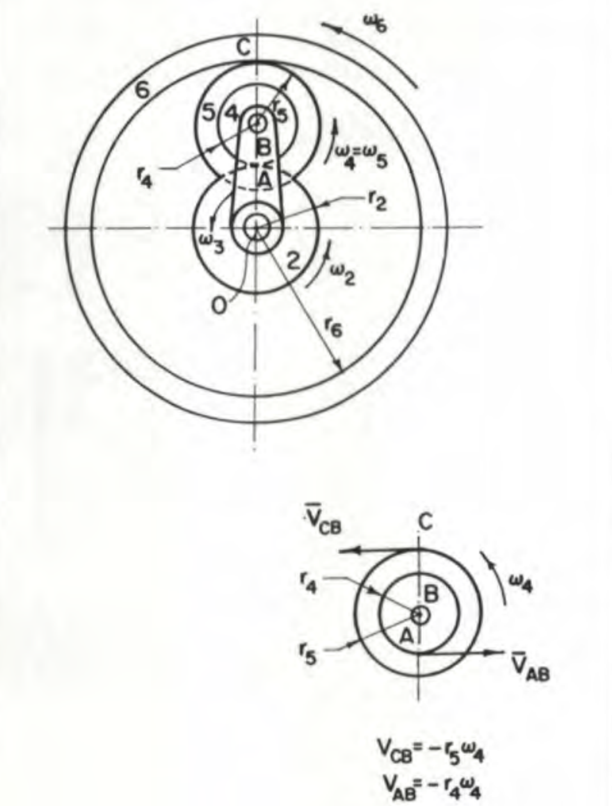

Fig. 4 – Velocities in the compound planetary gear train.

and substituting $r_2+r_5=r_6-r_4$ into Equation 24, we have the equation of motion

$$r_5r_2\,(\omega_2-\omega_a) + r_6r_4(\omega_6-\omega_a)=0 \tag{27}$$

which, in terms of n_i and N_i, becomes

$$e = -\frac{N_2N_5}{N_4N_6} = \frac{n_6-n_a}{n_2-n_a} \tag{28}$$

This is in the form of Equation 17 with $q=1$, $n_F=n_2$, $n_L=n_6$, $N_F=N_2$, $N_L=N_6$.

The speed of the planet gear, n_4, is defined in terms of n_2 or n_6 by Equations 22 or 23 as

$$n_4 = n_a - \frac{N_2}{N_4}\,(n_2-n_a) \tag{29}$$

or

$$n_4 = n_a + \frac{N_6}{N_5}\,(n_6-n_a) \tag{30}$$

which are also in the forms of Equations 18 and 19, respectively.

Caution: For ordinary gear trains in which the planet arm is fixed, $n_a=0$ and

$$e = \frac{n_L}{n_F} = \frac{1}{S_R} \tag{31}$$

indicating that the train value for ordinary gear trains is the inverse of the speed reduction ratio S_R generated by the gear train. Hence, when a planetary gear train is driven by an ordinary gear train, the ordinary gear train must be analyzed first to determine the input speed to the planetary gear train.

Input-Output Torque Relation

The power equilibrium of a gear train defines the torque multiplication factor (or the mechanical advantage) the planetary gear train generates. Thus,

$$T_{in}\,\omega_{in} + T_{out}\,\omega_{out} = 0 \tag{32}$$

or

$$\frac{T_{out}}{T_{in}} = -\frac{n_{in}}{n_{out}} = -S_R \tag{33}$$

where the subscripts "in" and "out" designate "input" and "output", respectively. Considering the mechanical efficiency of the gear train as η_t, input torque required to generate an output torque is

$$T_{in} = -\frac{T_{out}}{\eta_t S_R} \tag{34}$$

Gear train manufacturers[1] suggest that the mechanical efficiency of two well-lubricated, mating precision gears is about 0.98; for two and three stage speed reductions, it is about 0.97 and 0.96, respectively. Therefore, if there are R sets of gears in a gear train, its mechanical efficiency may be approximated by

$$\eta_t = (0.98)^{[1+0.5(R-1)]} \tag{35}$$

Reduction in efficiency due to energy loss in bearings should also be considered.

Example of Analysis of a Compound Planetary Gear Train

Following is the analysis needed to find the input torques required for each output shaft operation. The gear train shown in Fig. 5 is used to lift 10^6lbf load where the lift ropes wrap around 10" diameter drums. Mechanical efficiency of the system is 0.93. The input shaft of two-lead-worm rotates at 1000 rpm.

Gears 2 and 3 form an ordinary gear train generating the input speed n_4 of the planetary gear train. Hence, $n_4 = n_F$ must be determined first. It is $n_F = 100/3$ rpm.

Since the train value equation (No. 17) contains three shaft speeds, it can only be solved for one speed when the other two speeds are specified. Then, in the planetary gear train of Fig. 5, we must find a sub-planetary gear train receiving two input speeds. Since in this arrangement, gear 9 is fixed, and $n_9 = 0$, the sub-planetary gear train we are looking for is of gears 4, 5, 6, 7, 8 and 9 with $n_L = n_9 = 0$. This train generates the speed of the planet arm, which is one of the output speeds. Thus, for this train we have

$$e_{4-9} = (-1)^2 \frac{N_4 N_6 N_8}{N_5 N_7 N_9} = \frac{n_9 - n_a}{n_4 - n_a}$$

With the tooth numbers shown it reduces to

$$0.00766 = \frac{-n_a}{100/3 - N_a}$$

and

$$n_a = -0.260587 \text{ rpm}$$

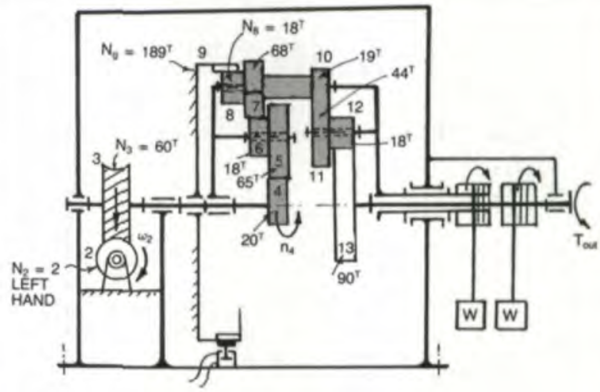

Fig. 5 – Composite planetary gear train analyzed.

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The planet arm is rotating in the direction opposite that of gear 4, generating a speed reduction ratio of

$$S_{R_1} = \frac{n_2}{n_a} = -\ 3837.498$$

The second sub-planetary gear train in the system is of gears 4, 5, 6, 7, 10, 11, 12 and 13. Its equation of motion is

$$e_{4-13} = (-1)^4 \frac{N_4 N_6 N_{10} N_{12}}{N_5 N_7 N_{11} N_{13}} = \frac{N_{13}-n_a}{N_4-n_a}$$

or

$$0.00703 = \frac{n_{13} + 0.260587}{100/3 + 0.260587}$$

$$n_{13} = -0.024422 \text{ rpm}$$

and

$$S_{R_2} = \frac{n_2}{n_{13}} = -\ 40946.73$$

Speeds of gears 5, 7 and 12 are determined using Equations 18 and 19 as

$$n_5 = n_a - \frac{N_4}{N_5}(n_4 - n_a) = -10.5972 \text{ rpm}$$

$$n_7 = n_a + \frac{N_4 N_6}{N_5 N_7}(n_4 - n_a) = 2.4756 \text{ rpm}$$

$$n_{12} = n_a - \frac{N_{13}}{N_{12}}(n_{13} - n_a) = -1.4414 \text{ rpm}$$

Input torque required to lift the load by the planet arm shaft is

$$T_{in_1} = \frac{-\ 5 \times 10^6}{(0.93)\,(S_{R_1})} = 1401 \text{ in}-\text{lbf}$$

To lift the load by the shaft of gear 13 it is

$$T_{in_2} = \frac{-\ 5 \times 10^6}{(0.93)\,(S_{R_2})} = 131.3 \text{ in}-\text{lbf}$$

Bevel Gear Planetary Gear Trains

The value of e is determined as shown above. Its sign, however, must be determined by releasing all the gears, retaining the planet arm fixed, and observing the direction in which the last gear rotates relative to the direction of the input rotation. A right hand rule can be followed. Thus, consider the planetary gear train of a robot joint shown in Fig. 6, where for the planetary gear train of gears 2, 4, 5

$$e_{2-5} = -\frac{N_2 N_4}{N_4 N_5} = \frac{0 - n_a}{n_2 - n_a} = -\frac{1}{4}$$

and

$$n_a = n_2/5$$

generating speed reduction.

For the planetary gear train of gears 2, 4, 6, 7

$$e = -\frac{N_2 N_6}{N_4 N_7} = \frac{n_7 - n_a}{n_2 - n_a} = -2.3935$$

and

$$n_7 = -1.7148 n_2$$

generating a speed increase (over drive ratio).

A bevel gear planetary gear train with only sun and planet gears is shown in Fig. 7. The equation of motion for the train is given by Equation 19 as

$$n_4 = n_a - \frac{N_2}{N_4}(n_2 - n_a)$$

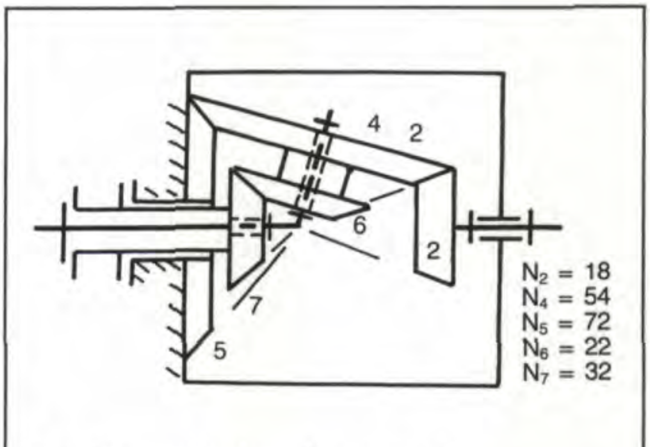

Fig. 6 – Compound bevel gear planetary gear train.

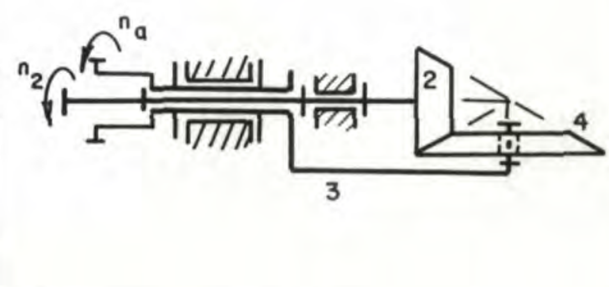

Fig. 7 – Planet gear motion generator.

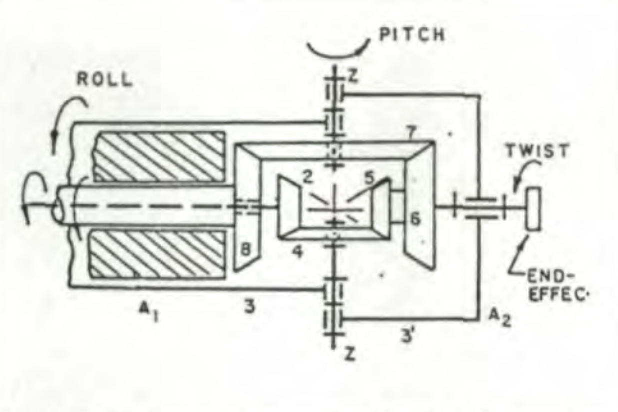

Fig. 8 – Planetary gear train generating roll, pitch and twist motions of a robot wrist joint.

Robot Wrist Joint Using Planetary Gear Trains

Fig. 8 shows another bevel gear planetary gear train used in robot wrist joints. It has three input shafts, shafts of arm A_1, gears 2 and 8[2]. Arm A_1 rotates the joint to reposition the z-z axis about which the arm A_2 rotates when gear 2 or 8 or both rotate. The shaft of gear 5 is the end-effector. With $n_{A_1}=0$,

$$n_4 = -\,n_2\frac{N_2}{N_4} \tag{36}$$

and

$$n_7 = n_8\frac{N_8}{N_7} \tag{37}$$

provide input to the planetary gear train of gears 4, 5, 6, 7 and arm A_2. Its train value is

$$e_{4-7} = -\frac{N_4}{N_5}\frac{N_6}{N_7} = \frac{n_7 - n_{A_2}}{n_4 - n_{A_2}} \tag{38}$$

which defines speed of arm A_2. Substituting n_4 and n_7 from Equations 36 and 37, n_{A_2} is defined in terms of the input speeds as

$$n_{A_2} = \frac{1}{1+\dfrac{N_4 N_6}{N_5 N_7}}\left[\, n_8\frac{N_8}{N_7} - n_2\,\frac{N_2 N_6}{N_4 N_5}\right] \tag{39}$$

The speed of the end-effector is

$$n_5 = n_{A_2} - \frac{N_4}{N_5}(n_4 - n_{A_2}) \tag{40}$$

As observed in Equation 39, in order to retain $n_{A_2} = 0$, n_2 and n_8 must satisfy

$$n_8 = \frac{N_2 N_6 N_7}{N_4 N_5 N_8}\,n_2 \tag{41}$$

Synthesis of Compound Planetary Gear Trains

As observed in the examples for analysis of planetary gear trains, *Equation 17 can be solved to generate only one shaft* speed by a sub-planetary gear train. The objective of synthesis is to find the number of mating gears in the train, their tooth numbers and the number of external contacts q to yield the value of e. The three speed reductions a compound gear train can generate with single input speed are

$$S_{R1} = \frac{n_F}{n_a} = \frac{e-1}{e},\ n_L=0 \tag{42}$$

$$S_{R2} = \frac{n_F}{n_L} = \frac{1}{e},\ n_a=0 \tag{43}$$

$$S_{R3} = \frac{n_L}{n_a} = (1-e),\ n_F = 0 \tag{44}$$

The split input case with two inputs forms the fourth type of speed generation. Inversion generating S_{R2} is a reverted ordinary gear train. S_{R1} and S_{R2} are large speed reductions. S_{R3} is the smallest reduction. Depending on the sign of e, either S_{R1} or S_{R2} is the forward and the largest reduction. For example, if $e<0$, $S_{R1}>0$ and largest, $S_{R2}<0$ and intermediate; if $e>0$, $S_{R2}>0$ and largest, $S_{R1}<0$ and intermediate. S_{R3} is always the smallest forward reduction when $e<1$.

During the synthesis one should keep in mind that the tooth ratio between any two mating spur gears and straight bevel gears should be retained below 8 to assure proper contact, sufficiently large contact ratio and low noise level. Larger tooth ratios may be tolerated for internal and helical gears due to increased contact ratio.

Example: Let us synthesize a compound planetary gear train to generate a speed reduction ratio of +577 from the planet arm shaft with $n_L=0$. By Equation 17

$$e = \frac{n_L - n_a}{n_F - n_a} = \frac{\dfrac{1}{-577}}{1-\dfrac{1}{577}} = -\frac{1}{576}$$

It can be factored into

$$e_1 = -\frac{1}{576} = -\left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{8}\right) = (-1)^3\ \frac{N_2}{N_4}\frac{N_5}{N_6}\frac{N_7}{N_8}\frac{N_9}{N_{10}}$$

or

$$e_2 = -\left(\frac{1}{8}\right)\left(\frac{1}{6}\right)\left(\frac{1}{12}\right) = (-1)^3\ \frac{N_2}{N_4}\frac{N_5}{N_6}\frac{N_7}{N_8}$$

The last gear in e_1, gear 10, must be an internal gear; in e_2, gear 8 must be an external gear to generate $q=3$. These two

gear trains are shown in Figs. 9 and 10, respectively. In Fig. 9 let

$N_2=20,\ N_4=120,\ N_5=20,\ N_6=80,\ N_7=24,\ N_8=72,\ N_9=24,\ N_{10}=192$

In terms of center distance vectors

$$\overline{C}_{25} = (r_2 + r_4)\,\overline{U}_{25}$$

$$\overline{C}_{57} = (r_5 + r_6)\,\overline{U}_{57},\ \overline{C}_{79} = (r_7 + r_8)\,\overline{U}_{79},$$

and $\overline{C}_{29} = (r_{10} - r_9)\,\overline{U}_{29}$, where $\overline{U}_{ij}$ is the unit vector.

$$|\overline{C}_{29}| \leq |\overline{C}_{25} + \overline{C}_{57} + \overline{C}_{79}| \geq \overline{C}_{29} \qquad \text{(a)}$$

must be satisfied when mounting the planet gears on the planet arm. (See Fig. 9b.) Considering the same diametral pitch for all the gears, Equation (a) demands that, depending on the locations of shafts of gears 5 and 7,

$$N_2 + N_4 + N_5 + N_6 + N_7 + N_8 \geq N_{10} - N_9$$
$$N_2 + N_4 + N_5 + N_6 - N_7 - N_8 \leq N_{10} - N_9$$

and

$$N_2 + N_4 - N_5 - N_6 + N_7 + N_8 \leq N_{10} - N_9$$

must be satisfied.

In Fig. 10 let

$N_2=20,\ N_4=160,\ N_5=20,\ N_6=120,\ N_7=18,\ N_8=216$

which must satisfy

$$|\overline{C}_{27}| \leq |\overline{C}_{25} + \overline{C}_{57}| \geq |\overline{C}_{27}| \qquad \text{(b)}$$

which demands that

$$N_2 + N_4 - N_5 - N_6 \leq N_7 + N_8$$

and

$$N_2 + N_4 + N_5 + N_6 \geq N_7 + N_8$$

must be satisfied.

To generate this speed reduction an ordinary planetary gear train will require $N_L/N_F=278$. With $N_F=20$, $P=6$, one notices the unbelievable gear size: $N_L = 5520$, $d_L = 920''$. This defect with ordinary planetary gear trains is minimized by forming series-connected ordinary planetary gear trains.

The planet arm and the planet gears must be balanced, which is commonly achieved by mounting symmetrical planet gears.

Multiple Speed Reduction Generation

Synthesis of a compound planetary gear train is performed in two forms:

Form 1. All the desired speed reductions are generated simultaneously, without requiring shifting for gear arrangements within the gear train.

Form 2. Only one desired speed reduction is generated at a time, requiring shifting of gears, releasing and activating clutches.

The second form of synthesis yields costly systems, since it requires complex mechanisms, connections and clutches to shift the gears and change the shafts. In general it requires a series connection in which the output of one unit is used to drive the next unit.

Example for Form 1 Synthesis

Let us synthesize a compound planetary gear train to generate the speed reduction ratios, 10, −4 and 2.5 simultaneously.

The largest reduction is generated by S_{R1} or S_{R2}. In a compound planetary gear train it is a preferred trade to retain the

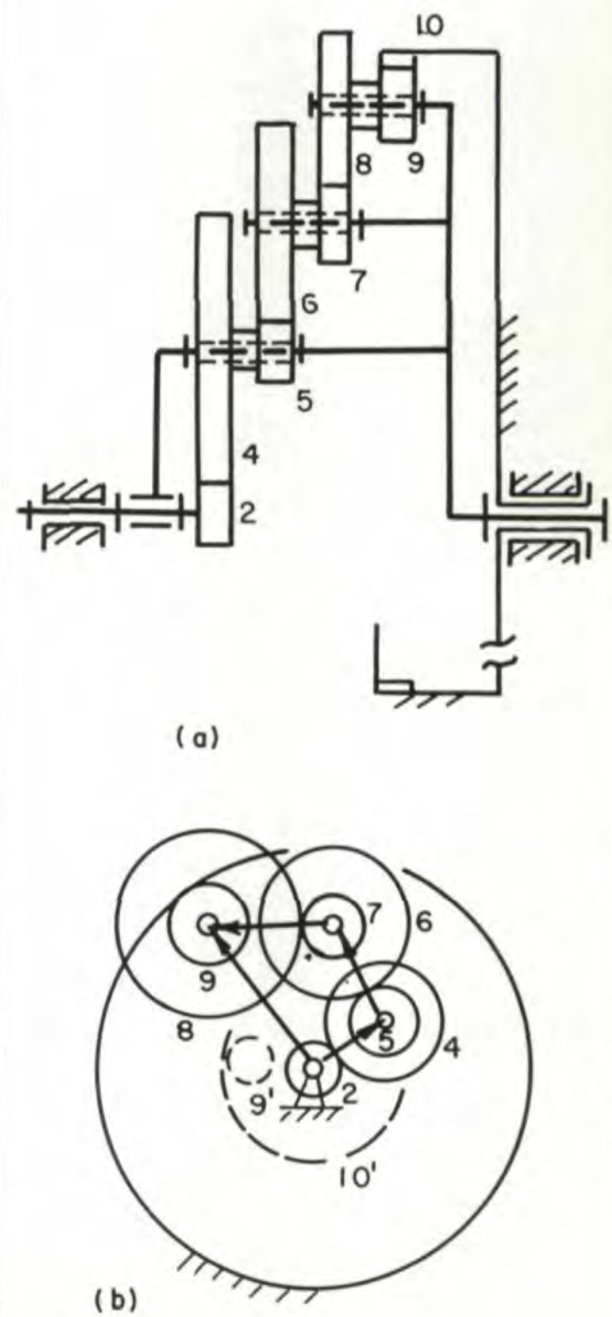

Fig. 9 – Compound planetary gear train generating speed reduction ratio of 577 with eight gears.

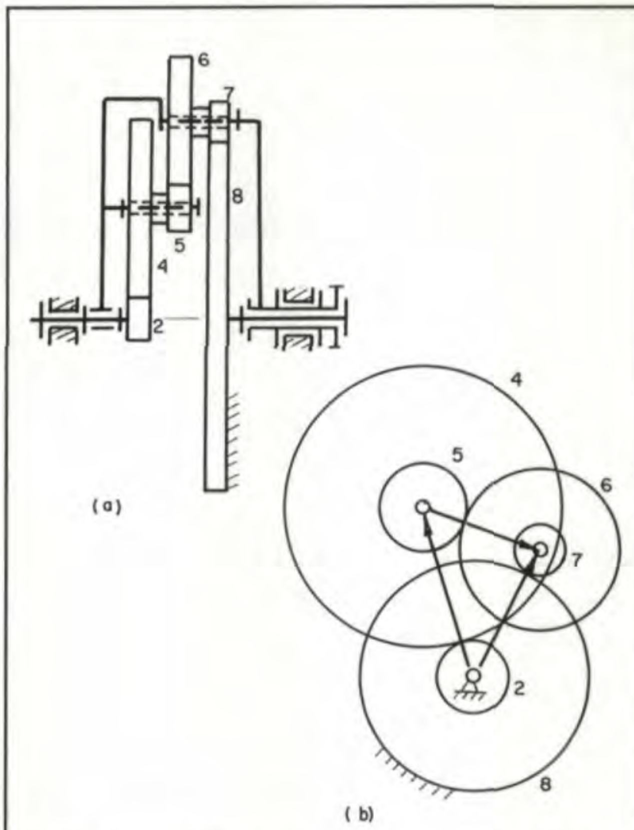

Fig. 10 – Compound planetary gear train generating speed reduction ratio of 577 with six gears.

planet arm active. So let S_{R1} in Equation 42 generate speed reduction 10. Hence,

$$10 = \frac{e_1 - 1}{e_1} \tag{a}$$

and

$$e_1 = -\frac{1}{9}$$

or

$$e_1 = \frac{-\frac{1}{10}}{1 - \frac{1}{10}} = -\frac{1}{9} \tag{b}$$

Using four gears, the first two having external contact, we form

$$e_1 = \left(-\frac{1}{3}\right)\left(\frac{1}{3}\right) = \left(-\frac{N_2}{N_4}\right)\left(\frac{N_5}{N_6}\right) \tag{c}$$

which states that gear 6 is an internal gear. This portion of the gear train is shown is Fig. 11(a). Considering $N'_2=1$, $N'_4=3$, $N'_5=1$, $N'_6=3$, the constraint equation

$$a\,(N'_2 + N'_4) = b\,(N'_6 - N'_5) \tag{d}$$

must be satisfied, where

$$a = \frac{1}{2P_{24}}, \; b = \frac{1}{2P_{56}}.$$

To use the same diametral pitch

$$a\,(1+3) = b(3-1) \tag{e}$$
$$2a = b$$

Letting $a=1$, $b=2$, we have

$$1 + 3 = 6 - 2 \tag{f}$$

defining $N'_2=1$, $N'_4=3$, $N'_5=2$, and $N'_6=6$. Now, both sides of (f) are multiplied by the same number to define the actual tooth numbers. Multiply by 40 (or 20, 24, 30, etc.) to have $N_2=40$, $N_4=120$, $N_5=80$, $N_6=240$.

Using the same planet arm, the other two speed reductions are generated. Thus, by Equation 18, and noting that

$$-\frac{N_2}{N_4} = -\frac{1}{3}$$

must also exist in the sub-gear trains, speed reduction ratio (-4) is generated by

$$e_2 = \left(-\frac{N_2}{N_4}\right) e_B = \frac{-\frac{1}{4} - \frac{1}{10}}{1 - \frac{1}{10}} = -\frac{14}{36} = \left(-\frac{1}{3}\right)\left(\frac{14}{12}\right) \tag{g}$$

where

$$e_B = \frac{14}{12}$$

Its positive sign requires another internal gear, gear 8, or two sets of external gears. Choosing the latter,

$$e_B = \frac{14}{12} = \left(\frac{2}{2}\right)\left(\frac{7}{6}\right) = \left(\frac{N_7}{N_8}\right)\left(\frac{N_9}{N_{10}}\right) \tag{h}$$

Tooth numbers in (h) must satisfy

$$N_2 + N_4 \leq N_7 + N_8 + N_9 + N_{10}$$

and

$$N_2 + N_4 + N_7 + N_8 \geq N_9 + N_{10}$$

Hence,

$$160 \leq C\,(2 + 2 + 7 + 6) \leq 240$$

and

$$160 \geq C(7 + 6 - 2 - 2)$$

and

$$9 \leq C \leq 17$$

letting $C=10$, $N_7 = 20$, $N_8 = 20$, $N_9 = 70$ and $N_{10} = 60$.

This sub-train expands the gear train to the form shown in Fig. 11(b), gears 7 and 8 being the same size.

Speed reduction ratio (+2.5) is generated by

$$\left(-\frac{N_2}{N_4}\right) e_c = \frac{\frac{1}{2.5} - \frac{1}{10}}{1 - \frac{1}{10}} = \frac{3}{9} = \left(-\frac{1}{3}\right)\left(-\frac{1}{1}\right)$$

where

$$e_c = -\frac{1}{1} = -\frac{N_{11}}{N_{12}}$$

gear 12 being an external gear. Since

$$N_2+N_4=160 = N_{11}+N_{12} = d(1+1)$$

$$d=80, \text{ and}$$

$$N_{11} = N_{12} = 80$$

The final form of the gear train is shown in Fig. 11(c). The gear train may drive three units with the three speeds it generates, or only one unit may be driven by simply using an external clutch coupling or shaft splines shifted axially as shown in Fig. 11(c).

It should be noted that the second speed reduction (−4) could also be generated, forming a gear train that uses $n_7=n_2$ and the planet speed (1/10) of the first reduction as the split inputs, where the planet arm drives the last gear.

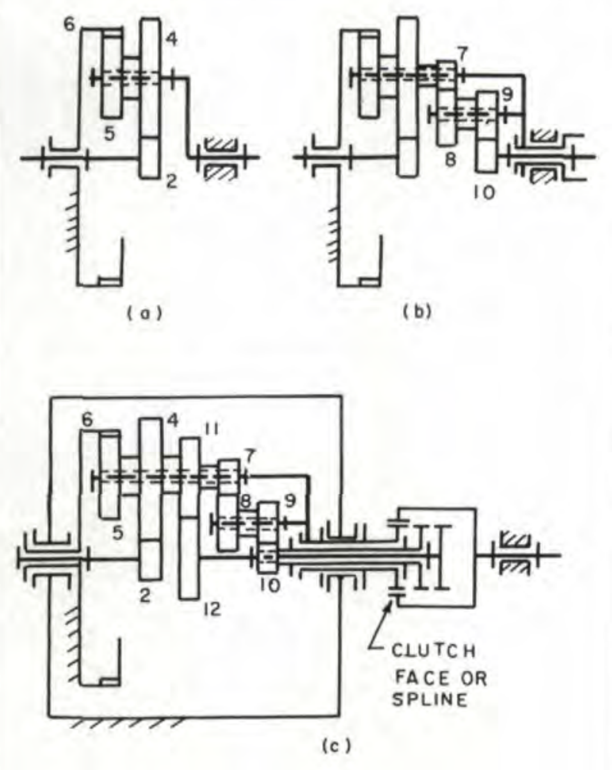

Fig. 11 – Compound planetary gear train synthesized to generate speed reduction ratios 10, −4 and 2.5, simultaneously by form 1 synthesis.

Thus,

$$e_2 = \frac{\frac{1}{10} + \frac{1}{4}}{1 + \frac{1}{4}} = \frac{7}{25} = \left(-\frac{2}{5}\right)\left(-\frac{7}{10}\right) = \left(-\frac{34}{85}\right)\ -\frac{49}{70}$$

$$= \left(-\frac{N_7}{N_8}\right)\left(-\frac{N_9}{N_{10}}\right)$$

yielding a second compound planetary gear train with $N_7 = 34$, $N_8 = 85$, $N_9 = 49$ and $N_{10} = 70$, which satisfy $(r_7 + r_8) = (r_9 + r_{10})$ with $P_{79} = P_{9,\,10}$. In this case the planet arm of the first unit must drive gear 10, requiring a more complex shaft arrangement.

Speed reduction 2.5 may be generated similary using split inputs $n_{11} = n_2$ and $n_L = 1/10$. Thus,

$$e_3 = \frac{\frac{1}{10} - \frac{4}{10}}{1 - \frac{4}{10}} = -\frac{1}{2} = -\frac{40}{80} = -\frac{N_{11}}{N_{13}}$$

yielding a simple planetary gear train as the third unit with $N_{11} = 40$, $N_{12} = 20$, $N_{13} = N_{13} = 80$. The new solution has three planetary gear train units with three planet arms and more complex shaft connections. It may be a much larger and more costly large drive system compared to the solution in Fig. 11c.

As noted, Form 1 synthesis is a *stepwise synthesis* for simultaneous speed reduction generation.

Example for Form 2 Synthesis

In this case three gear trains in series will be formed with different shaft connections for each speed reduction generated. Since there are three speeds to be generated, three products of speed ratios are formed using the forms of speed reductions in Equations 42 – 44. One can form 729 combinations of three products of S_{R1}, S_{R2} and S_{R3}, considering each unit causing speed reduction. Noting that the inverse of a speed reduction is an overdrive condition, with the possibility of two overdrive units out of three units, 243 other combinations are possible.

Let us generate speed ratios 10, −4 and 2. Choose the following products:

$$\left(\frac{e_1-1}{e_1}\right)\left(\frac{e_2-1}{e_2}\right)\left(\frac{1}{e_3}\right) = 10 \qquad \text{(a)}$$

$$\left(\frac{e_1-1}{e_1}\right)\left(\frac{1}{e_2}\right)\left(\frac{1}{e_3}\right) = -4 \qquad \text{(b)}$$

$$\frac{(e_1-1)}{e_1}\,\frac{(1)}{e_2}\,(1-e_3) = 2 \qquad \text{(c)}$$

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where the last gear of unit 1 is fixed, its planet arm drives the first gear of unit 2, whose planet arm drives the first gear of unit 3, whose arm is fixed, and its last gear generates $S_{R1}=10$ in (a). Unit 1 does not change its fixed shaft condition. In (b) and (c) the arm shaft of unit 1 drives the sun gear of unit 2, whose last gear drives the first gear of unit 3 in (b); it drives the last gear of unit 3 in (c), the first gear being fixed. Several clutches and shaft coupling units are needed to form the shaft arrangements in this case. Solving (a), (b) and (c) simultaneously one finds

$$e_2 = -1.5$$

and

$$e_3^2 - e_3 - 0.5 = 0$$

or the two values of e_3 as

$$e_{31} = 1.3661$$

$$e_{32} = -0.3661$$

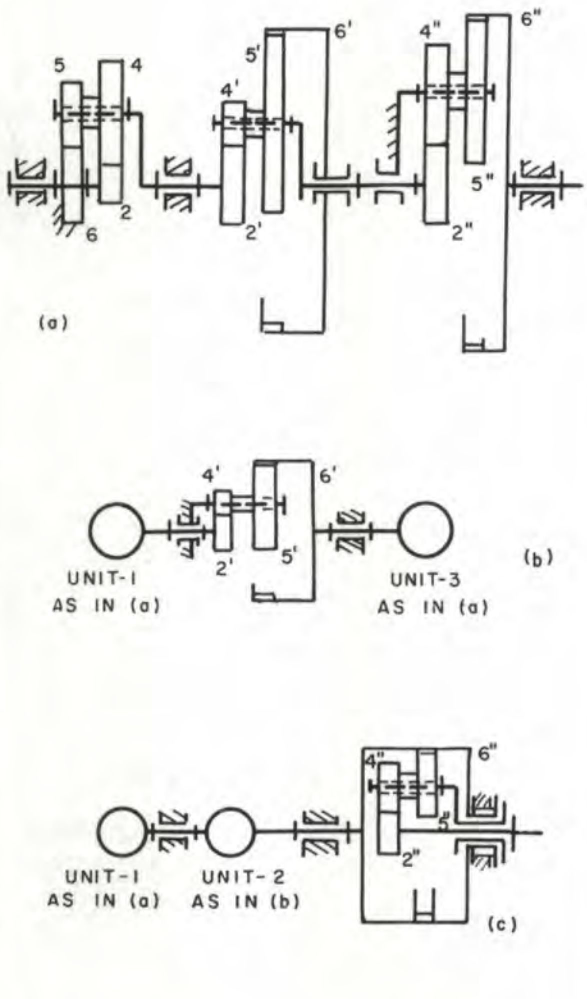

Fig. 12 – Compound planetary gear train synthesized to generate speed reduction ratios 10, −4 and 2 one at a time by form 2 synthesis.

Then,

$$e_{1i} = \frac{1}{1+4\,e_2 e_{3i}}\,,\ i=1,2$$

leading to two solution gear trains with

$$e_{1_1} = -0.1390, \text{ and } e_{1_2} = 0.3129$$

The number of gears, their tooth numbers and the value of q for each solution unit is determined independently of the others. Thus, for unit 2

$$e_2 = -\frac{N'_2\,N'_5}{N'_4\,N'_6} = -1.5 = \left(\frac{-3}{1}\right)\left(\frac{1.5}{3}\right)$$

and N'_6 must be an internal gear. Satisfying

$$N'_2+N'_4 = N'_6-N'_5,\ a\,(3+1)=b(3-1.5),$$

$a=3$, $b=8$; $N'_2=54$, $N'_4=18$, $N'_5=72$, $N'_6=144$ for unit 2. Values e_1 and e_3 are satisfied approximately.

Consider the second solution. For unit 1

$$e_{1_2} = 0.3129 = \frac{1}{3.1959} \simeq \left(-\frac{1}{2}\right)\left(\frac{1}{1.6}\right) = \frac{1}{3.2} =$$

$$\left(\frac{-N_2}{N_4}\right)\left(-\frac{N_5}{N_6}\right)$$

in which gear 6 is an external gear, satisfying

$$a(N_2+N_4) = b(N_5+N_6)$$

$$N_2=26,\ N_4=52,\ N_5=30,\ N_6=48.$$

For unit 3

$$e_{3_2} = -\ 0.3661 \cong -\frac{1}{2.7315} \cong \left(\frac{-1}{1.3}\right)\left(\frac{1}{2.1}\right) =$$

$$\left(-\frac{N''_2}{N''_4}\right)\left(\frac{N''_5}{N''_6}\right)$$

in which gear 6 is an internal gear, and satisfying

$$a(N''_2+N''_4) = b(N''_6-N''_5)$$

$$N''_2=110,\ N''_4=143,\ N''_5=230,\ N''_6=483.$$

The three gear trains with their shaft connections to generate the three speed ratios are shown in Figs 12(a), (b) and (c). With the approximated e_3 and e_1, the system generates:

$$10.01,\ -4.004, \text{ and } 2.039.$$

Observing the sizes of gears in unit 3, and noting the existence of three planet arms and several clutch and coupling units and shifting or actuating mechanisms within the system in form 2 synthesis, synthesis as in form 1 is more economical and advantageous. Other products of e_{32} should be searched. A smaller drive with a larger speed error may be formed. For example, try

$$e_{32} \cong \left(-\frac{1}{1.7}\right)\left(\frac{1}{1.6}\right) = -0.36765 = \left(-\frac{10}{17}\right)\left(\frac{10}{16}\right)$$
$$= \left(-\frac{N''_2}{N''_4}\right)\left(\frac{N''_5}{N''_6}\right)$$

and $N''_2 = 30$, $N''_4 = 51$, $N''_5 = 135$, $N''_6 = 216$, generating the speed reduction ratios as 9.9547, −3.98188, 2.00216.

Readers should form solution gear trains for e_{31} and e_{11}.

Differential Gear Trains

When a planetary gear train is driven with two inputs to generate the third speed (split input drive), it is in general called a "differential gear train". Some differential gear trains were already synthesized in the foregoing compound planetary gear train examples. Here, we will see how a differential with two inputs can be synthesized to generate the sum of two variables as a mechanical computer. Rewrite Equation 17 for the planet arm as

$$n_a = A\,n_L - B\,n_F \tag{45}$$

where

$$A = \frac{1}{1-e}, \quad B = \frac{e}{1-e}$$

A planetary gear train can be synthesized to generate a function of the form

$$z = ax + by \tag{46}$$

by the planet arm, x and y being the input functions $n_L \equiv x$, $n_F \equiv y$ and $n_a = z$. Consider the following example.

Example: Let us generate

$$z = 2x - 4y \tag{47}$$

Comparing Equation 45 and 47 we have

$$A = \frac{1}{1-e} = 2\,,\; B = \frac{e}{1-e} = 4$$

Using A or B, find e being sure that the other coefficient, B or A, is larger than its required value so that an overdrive unit is not formed. In this case, we use B to find e as

$$e = \frac{4}{5}$$

which yields

$$A = 5$$

Therefore, the input to the last gear of the train must be supplied as

$$x/m$$

where

$$m = \frac{A_{formed}}{A_{desired}} = \frac{5}{2} = 2.5 \tag{48}$$

$e = 4/5$ is generated by four external gears to supply y. Thus,

$$e = \frac{4}{5} = \left(-\frac{N_2}{N_4}\right)\left(-\frac{N_5}{N_6}\right) = \left(-\frac{36}{36}\right)\left(-\frac{32}{40}\right)$$

and $N_2 = 36$, $N_4 = 36$, $N_5 = 32$, $N_6 = 40$. The desired differential gear train is shown in Fig. 13(a), where $N_7 = 20$, $N_8 = 30$, $N_9 = 50$.

Equation 17 may also be written to generate z by the shaft of the last gear, supplying x on the arm shaft, y on the first gear shaft, as

$$n_L = Dn_a + En_2 \tag{49}$$

with $D = (1-e)$, $E = e$. In that case, to generate the function

in Equation 47, $e = -4$, $n_a \equiv x/2.5$ and $n_F = y$. A bevel gear train generating z for this case is shown in Fig. 13 (b), where

$$e = -4 = -\frac{N_2 N_5}{N_4 N_6} = -\left(\frac{40}{80}\right)\left(\frac{20}{40}\right)$$

The reader should form another gear train using $e = -1$ supplying $n_a = x$, $n_F = 4y$ as overdrive.

Vehicle Rear End Differentials

The objective in designing a vehicle rear end differential planetary gear train is to generate $e = -1$, so that when the vehicle is making a turn without the planet arm rotating, the outer wheel goes one unit rotation forward as the inner wheel goes one unit rotation backward. Fig. 14 shows a commonly used bevel gear rear end differential, where gear 7 is driven by the universal shaft to supply the input by the planet arm. $N_2 = N_5$; N_4 is of any suitable number and $e = -1$. When gear 2 is held stationary, $n_2 = 0$, and gear 5 is lifted for balancing, one observes

$$-1 = \frac{n_5 - n_a}{-n_a},\ n_5 = 2n_a$$

Vehicle rear-end differentials can also be formed using spur gears, provided $e = -1$ is maintained. Figures 15(a) and (b) show two such rear-end differentials, where planet gears

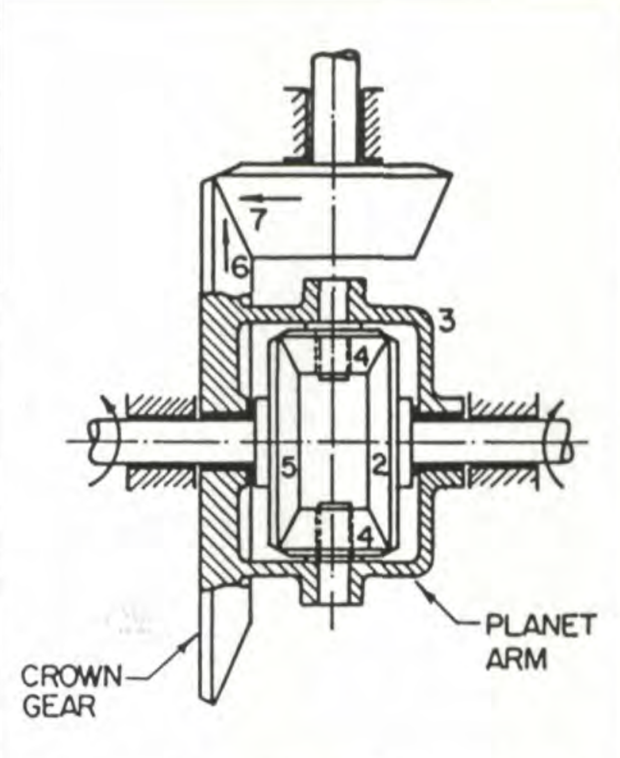

Fig. 14 – Bevel gear planetary gear train differential for vehicle rear-end with $e = -1$.

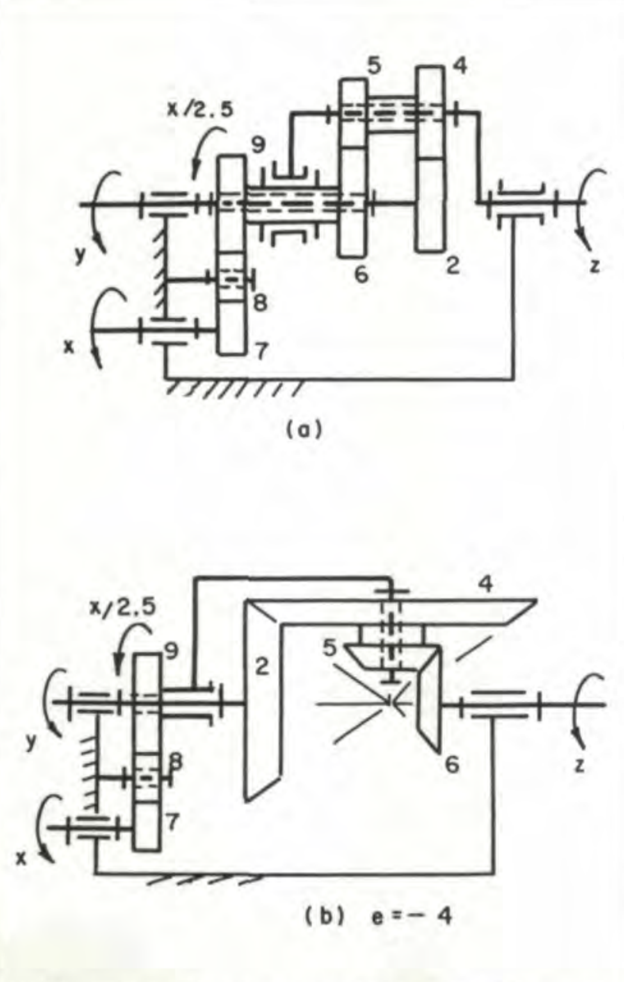

Fig. 13 – $z = ax + by$ function generating planetary gear differentials.

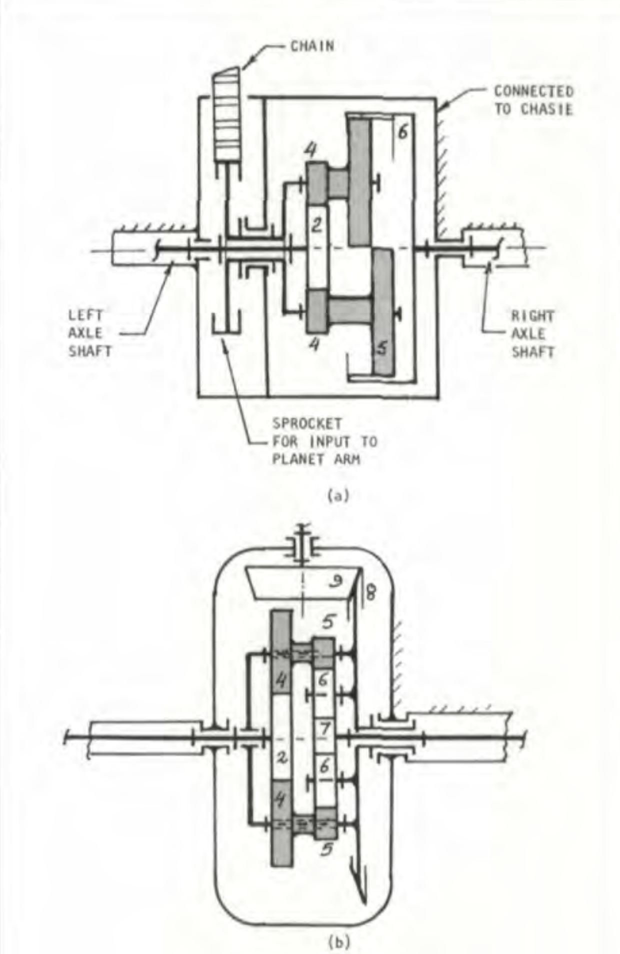

Fig. 15 – Rear-end differentials formed of spur gears, $e = -1$.

are symmetrically mounted for balancing. In (a) chain drives the planet arm; in (b) bevel gears do the same. In (a)

$$-1 = \left(\frac{-N_2}{N_4}\right)\left(\frac{N_5}{N_6}\right) = \left(\frac{-40}{20}\right)\left(\frac{60}{120}\right)$$

In (b)

$$-1 = \left(\frac{-N_2}{N_4}\right)\left(\frac{-N_5}{N_6}\right)\left(\frac{-N_7}{N_8}\right) = \left(\frac{-60}{60}\right)\left(\frac{-20}{40}\right)\left(\frac{-40}{20}\right)$$

Differentials for fractional values of e, such as $e = -0.8$ for a race car to rotate the outer wheel less than the inner wheel, are used in order to generate stabilizing traction torque that tends to retain the vehicle on the straight line. In that case, $e = -0.8$ should be satisfied instead of -1 as done above.

Automatic Transmissions

An automatic vehicle transmission in general has two or more simple planetary gear trains whose shafts can be coupled or held stationary to form different arrangements. Input is supplied through one or two fluid couplings (torque converters). Fig. 16 shows a Hydramatic 4L60 (THM 700-R4) automatic transmission by General Motors that generates two forward and one reverse speed reductions. The skeleton of a five-speed automatic transmission with three simple planetary gear trains driven with two torque converters is shown in Fig. 17. It has three clutches at G, H, and P; four sprag overrunning clutches at J, I, N and O.

Observe its operation:

Neutral position. Small converter B-C is empty, gear 3 is held stationary and clutches G and H are open. The planet

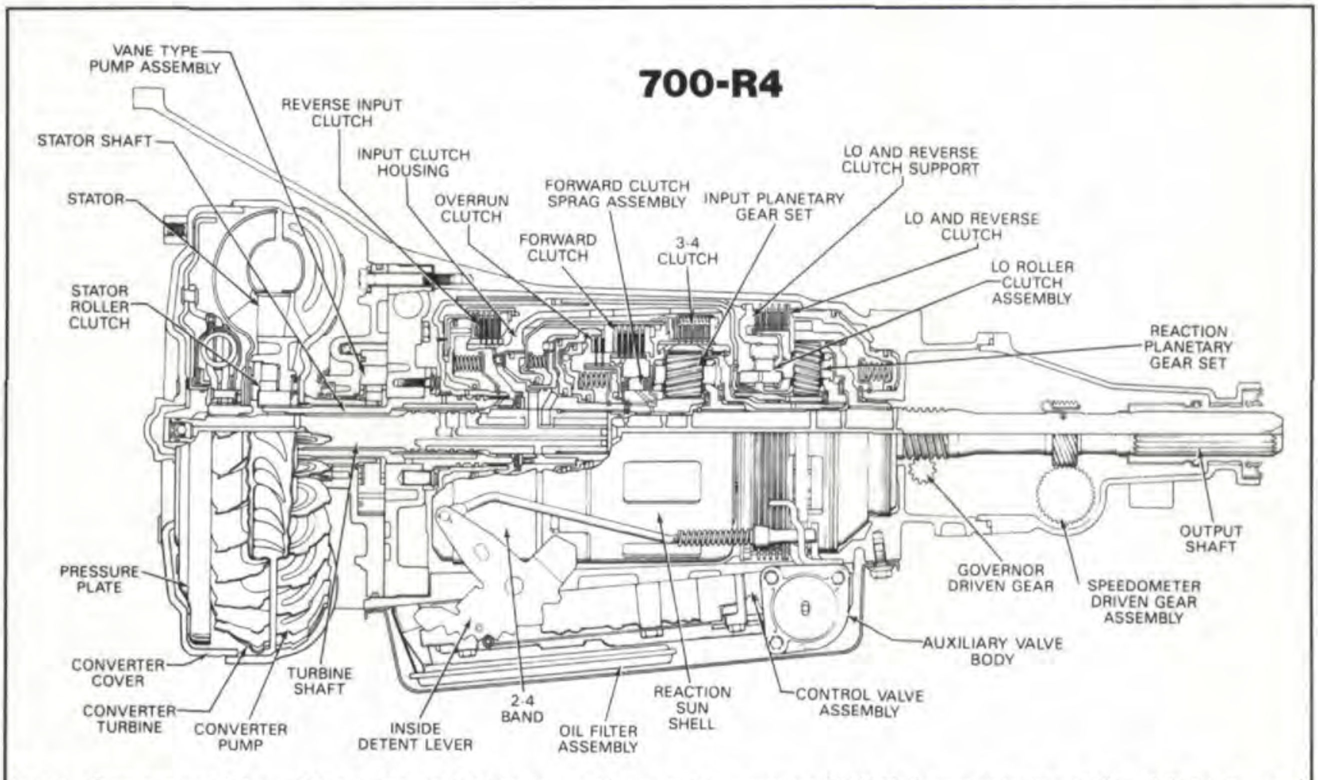

Fig. 16 – Cross section of General Motors 4L60 (THM 700-R4) model automatic transmission.

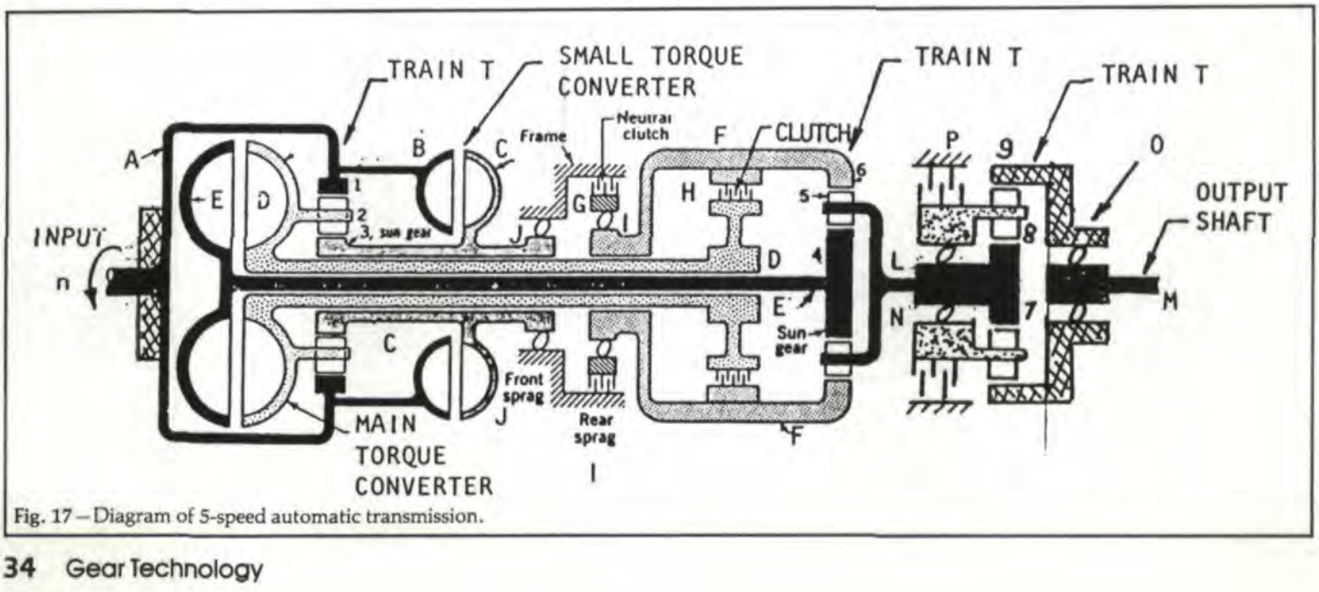

Fig. 17 – Diagram of 5-speed automatic transmission.

arm D rotates freely, $n_4=n_D=n_6$, and shaft L idles, and $n_L=n_6$. P and O are open, N is engaged, $n_7=n_9$ and $n_M=0$.

Low-Low Forward Reduction. B-C is empty, D rotates E and gear 4. Clutches G, J, I retain $n_3=n_6=0$. P is open; N, O are engaged. From train 1

$$e_1 = -\frac{N_1}{N_3} = \frac{0-n_D}{n_1-n_D}, \; n_D = \frac{e_1}{e_1-1}\, n_1 \qquad \text{(a)}$$

for $e_1=-1.3$ and $n_D=0.5833n_1$.
In train 2, $n_4 = n_D$, and

$$e_2 = -\frac{N_4}{N_6} = \frac{0-n_L}{n_4-n_L} \qquad \text{(b)}$$

with $e_2 = -0.7143$ and $n_M=n_L=0.243n_1$.

Second Forward Reduction. H, E are open, G, I are engaged and $n_6=0$. Small converter B-C is full driving gear 3, $n_1=n_3=n_4=n_D$; P is open, N, O are engaged, and from (b) above

$$n_M = n_L = \frac{e_2}{e_2-1}\, n_1 = 0.4167n_1$$

Third Forward Reduction. J is engaged and $n_3=0$. G and H are engaged, I is open, B-C is empty, P is open, N, O are engaged, and $n_6=n_D=n_E$. Then from (a) above

$$n_M = n_L = n_D = 0.5833n_1$$

Fourth Forward Speed. In this case the entire gear train rotates as one body at the engine speed n_1. I, J, P are open; G, H, N, O are engaged, B-C is full and $n_M = n_L = n_1$.

Reverse Reduction. B-C is full and the system is engaged as for the fourth forward speed, except that P and O are engaged and N is open. From the train value of the third gear train with $n_a=0$,

$$e_3 = -\frac{N_7}{N_9} = \frac{n_M}{n_1}$$

with

$$e_3 = -1/3, \; n_M = -n_1/3$$

Conclusions

Offered in the foregoing with illustrative examples are efficient methods of analysis and synthesis of compound planetary gear trains and planetary differentials. The methods use the train values of component planetary gear trains, and lead to very simple design processes. As expected, by introducing worm-gear sets in a compound planetary gear train, very large speed reductions can be generated with fewer number of gears in the train.

References

1. PLANTARY GEAR REDUCERS, Crichton Manufacturing Co., Johnstown, PA.
2. MACHINE DESIGN, Vol. 54, No. 18, August 12, 1982, p.55.

Back To Basics

Manufacturing of Forged and Extruded Gears

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Introduction

Traditional methods of manufacturing precision gears usually employ either hobbing or shaper cutting. Both of these processes rely upon generating the conjugate tooth form by moving the workpiece in a precise relation to the tool. Recently, attention has been given to forming gear teeth in a single step. Advantages to such a process include reduced production time, material savings, and improved performance characteristics. Drawbacks include complicated tool designs, non-uniformity of gears produced throughout the life of the tooling, and lengthy development times.

Through projects funded by the U.S. Army Tank Automotive Command, Battelle's Columbus Division developed a method for designing spiral bevel, spur, and helical gear forming dies. This article discusses this method and summarizes the current state of the art regarding the manufacturing of forged and extruded gears.

Traditional Gear Forming Methodology

Design/test/modify. The traditional method of forging and extrusion die design and manufacture is based on experience and trial and error. A preliminary die is made, and a few parts are formed. Measurements are taken of the finished part, and the die is modified accordingly. A second series of trials is conducted, and so on, until the final die geometry is obtained. Such a development program is required for every new design, which makes the precision forming process economically less attractive, especially when complex and precise geometries are involved, as with gears. Therefore, methods need to be developed to apply advanced computer-aided design and manufacturing (CAD/CAM) technologies (analysis of loads, stresses, and temperatures using finite element method based approaches, etc.) to gear forming die design and manufacture. This approach benefits from the capabilities of the computer in performing complex mathematical analysis and information storage, and allows the die designer to examine the effects of various changes of process variables on the die design, without trying out each new change on the shop floor.

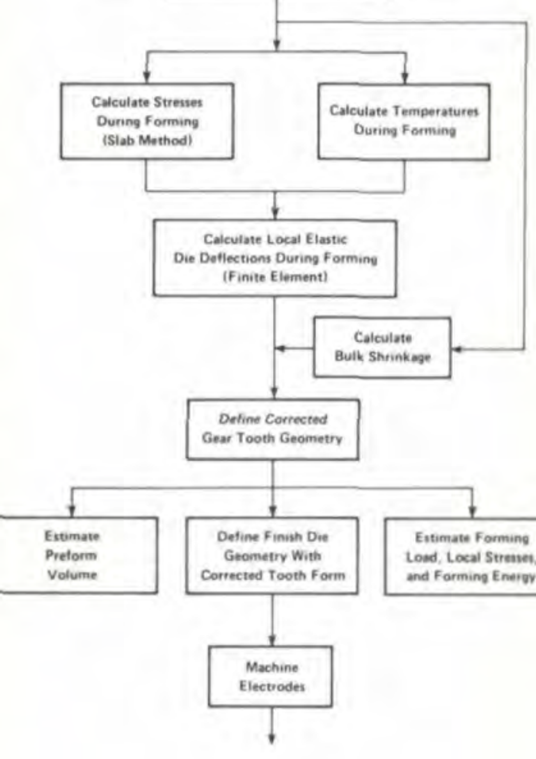

Fig. 1 – Methodology for designing gear forming dies.

CAD/CAM Applied To Forging and Extrusion

In recent years, CAD/CAM techniques have been applied to various forging processes. The experience gained in all of these applications implies a certain overall methodology for CAD/CAM of dies for precision and/or near-net shape forming. This computerized approach is also applicable to precision cold and hot forming of spiral bevel, spur, and helical gears, as seen in Fig. 1. The procedure uses as input: the process variables and the part (gear) geometry. The former consist of:

(1) data on billet material under forming conditions (billet and die properties, such as flow stress as a function of strain, strain-rate, temperatures, and heat transfer coefficients),

(2) the friction coefficient to quantify the friction shear stress at the material and die interface, and

(3) forming conditions, such as temperatures, deformation rates, and suggested number of forming operations.

Using the process variables and the part geometry, a preliminary design of the finish forming die can be made. Next, stresses necessary to finish form the part and temperatures in the material and the dies are calculated. The elastic die deflections due to temperatures and stresses can be estimated and used to predict the small corrections necessary on the finish die geometry. Knowledge of the forming stresses also allows the prediction of forming load and energy. The estimation of die geometry corrections is necessary for obtaining close tolerance formed parts and for machining the finish dies to exact dimensions. The corrected finish die geometry is used to estimate the necessary volume, and the volume distribution in the billet or the preform. Ideally, a simulation of the metal flow should be conducted for each die design. This is a computerized prediction of metal flow at each instant during forming. This simulation allows the determination of the cavity filling without excessive loading of the dies and prediction of defects.

Process Variables

Billet Material Characterization.[1] For a given material composition and deformation/heat treatment history (microstructure), the flow stress, and the workability (or formability) in various directions (anisotropy), are the most important material variables in analysis of a metal forming process. For a given microstructure, the flow stress, σ, is expressed as a function of strain, ϵ, strain rate, $\dot{\epsilon}$, and temperature, T:

$$\sigma = f(\epsilon,\dot{\epsilon},T) \tag{1}$$

To formulate the constitutive equation (Equation 1) it is necessary to conduct torsion, plane-strain compression, and uniform axisymmetric-compression tests. During any one of these tests, plastic work creates a certain increase in temperature of the billet material, which must be considered in evaluation and use of the test results.

Tooling and Equipment. The selection of a machine for a given process is influenced by the time, accuracy, and load/energy characteristics of that machine. Optimum equipment selection requires consideration of the entire forming system, including lot size, conditions at the plant, environmental effects, and maintenance requirements, as well as the requirements of the specific part and process under consideration.

The tooling variables include design and geometry, surface finish, stiffness and mechanical and thermal properties under conditions of use.

Friction and Lubrication at the Tool/Workpiece Interface. The mechanics of interface friction are very complex. One way of expressing friction quantitatively is through a friction coefficient, μ, or a friction shear factor, m. Thus, the frictional shear stress, τ, is

$$\tau = \sigma_n \cdot \mu \tag{2}$$

or

$$\tau = m \cdot \sigma\sqrt{3} = f \cdot \sigma \tag{3}$$

where σ_n is the normal stress at the interface, σ is the flow stress of the deforming material, and f is the friction factor.

Friction estimation. The friction between the workpiece and the tooling is dependant upon the die surface, the temperature, and the type of lubricant. A standard test, called the ring test, is used to determine the friction shear factors of various lubricants. This test involves taking a ring of known dimensions and temperature and upsetting it (thickness reduction) between flat dies. Measurement of the final inside and outside diameters plus thickness enable one to calculate the friction shear factor of the lubricant. Thus, to accurately predict the loads when forming gears, it is important that data be obtained regarding the friction associated with the lubricant to be used.

Estimation of Die Corrections. *Temperature Effects.* Temperature affects all forgings in two ways. First, when the workpiece is heated to be able to form the part, it expands. In addition, the material will always increase in temperature due to deformation heating. After the part is formed, it shrinks during cooling. Thus, the final part size will always be smaller than the die in which it was formed. The second temperature effect comes from the tooling. The tooling will always be slightly heated due to the transfer of heat from the workpiece. In addition, the die may be intentionally heated if the part is to be formed at elevated temperatures. This prevents premature tool failure or defect formation in the final part. As a result, the part formed will always be larger than the size of the die manufactured at "room temperature". Both of

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these temperature effects must be compensated for when designing tooling which one expects to use to form near-net parts.

Shrink fitting. Shrink fitting refers to the technique of placing the forming die, which contains the cavity in which the part is formed, under a residual compressive stress to permit higher forming loads. This compressive stress is generated by using an interference between the die and one or more outer rings. As a result of this shrink fitting, the dimensions of the die cavity will be smaller than when it was machined. This dimensional shrinkage must also be accounted for when designing the forming dies for near-net shape manufacturing.

Elastic Deflection. (Mechanics of Plastic Deformation/Slab Method). During forming, pressure on the workpiece causes the die to expand, and the material moves out to meet this expanded surface. To compensate for this geometry change, the die must be corrected, depending on the estimated pressure. To calculate this pressure, an elementary plasticity theory developed by Sachs(2) and Siebel(3), known as slab theory, can be applied to practical metal forming problems. The following reasonable assumptions are made(1):

- The deforming material is isotropic and incompressible.
- The elastic deformations of the deforming material and of the tool are neglected.
- The inertial forces are small and are neglected.
- The friction shear stress is constant at the die/material interface and is defined as in Equation 3 above.
- The material flow criterion is defined by TRESCA's maximum shear stress criterion:

$$\sigma_1 - \sigma_3 = \sigma \qquad (4)$$

where σ_1 and σ_3 are the maximum and minimum principal stresses respectively.

- The flow stress, σ, and the temperature are constant within the analyzed portion of the deforming material.

The basic approach for using the slab method is as follows:

1. The region of material undergoing plastic deformation is identified. Thus, the boundary between the elastic and plastic regions of the workpiece material is identified.
2. One of the three principal stresses (assuming TRESCA's yield condition will be used) should be determined. In most cases it will be easy to determine certain points where one of the principal stresses must be zero. For example, on a simple forward extrusion process, the axial stress should be zero at the exit of the die. It will also be necessary to determine the nature of this stress in the deformation zone (tensile or compressive).
3. In order to apply the TRESCA yield criterion, the maximum and minimum principal stresses must be ascertained.
4. The equilibrium equations of stresses in the deformation zone are then formulated assuming the deformation zone is completely filled. Knowing the friction and flow stress of the material in the deformation zone, the load required for the operation can be computed.

Case 1: Extrusion. Fig. 2 shows the basic arrangement of the tooling for extruding a spur or helical gear, as well as the friction forces present.

The force required to extrude the gear is given by:

$$F_p = F_{id} + F_{sh} + F_f \qquad (5)$$

where:

F_p = total punch force,
F_{id} = ideal deformation force,
F_{sh} = force due to shearing of the material at the entry and exit of the die, and
F_f = friction force along the die walls and the punch.

The friction force, F_f, is equal to the sum of the friction forces shown in Fig. 2. The remaining forces, F_{id} and F_{sh}, are computed from measurements of the flow stress of the material being formed. Once the total punch force is calculated, the punch pressure, as well as the radial pressure in the die cavity, can be determined. This last parameter is required in order to determine and compensate for die deflections.

Case 2: Forging. A typical tool setup for forging spur or helical gears is given in Fig. 3 In this case, the slab method is applied in similar fashion to extrusion, except that in this case, each tooth can be thought of as being extruded individually, and the total force calculated by multiplying by the number of teeth in the gear. The procedure for computation of loads for forging spiral bevel gears is also similar. (Fig. 4.)

Generating the Gear Tooth Geometry

To define the tooth geometry, certain gear and/or cutting tool parameters must be specified. Some additional data pertaining to the mating gear may also be required in certain instances. All the

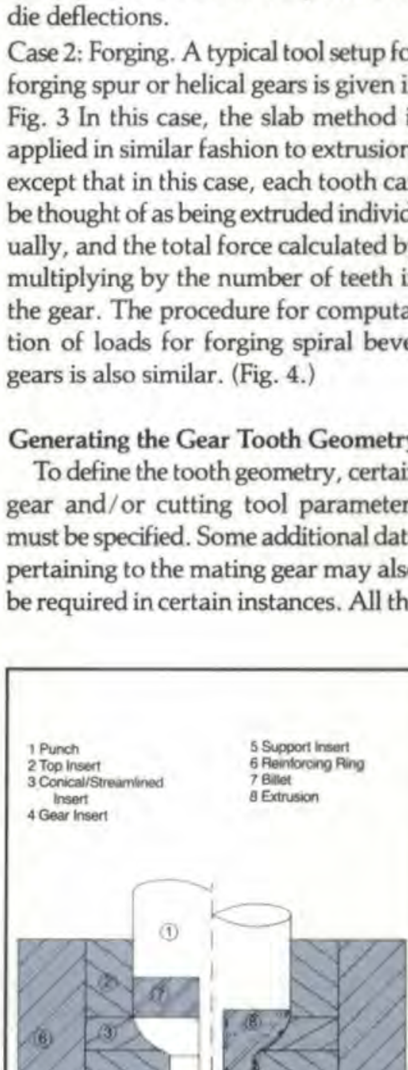

Fig. 2 – Tooling arrangement for extruding parallel axis gears and the associated friction forces.

data required for the computations and the geometry of the cutting tool can be obtained from a "summary sheet" developed by gear designers. With this data, standard gear equations are used to calculate the X and Y coordinates of the points describing the gear tooth profile.

Cutting the Die. A common method of die manufacture is called Electrical Discharge Machining (EDM). The process uses an electrode, usually made of graphite or brass, which is the negative of the die geometry. The electrode is brought close to, but not in contact with, the die material. An electrical current is allowed to arc across the gap which "burns" away the die material. Another form of this method of manufacture is called wire EDM. Current is passed through a straight wire that moves in two dimensions, burning the die geometry as it moves. Spur and straight bevel gear dies are the only ones which can be cut using the wire EDM process. In all cases, a corrected set of gear tooth profile coordinates is needed. Under contract from the United States Army Tank-Automotive Command, Battelle Columbus Division engineers developed two computer programs which calculate corrected gear tooth geometries and assist in the overall die design process. These programs, called SPBEVL and GEARDI, are used for spiral bevel gear forging and spur/helical gear extrusion/forging respectively.

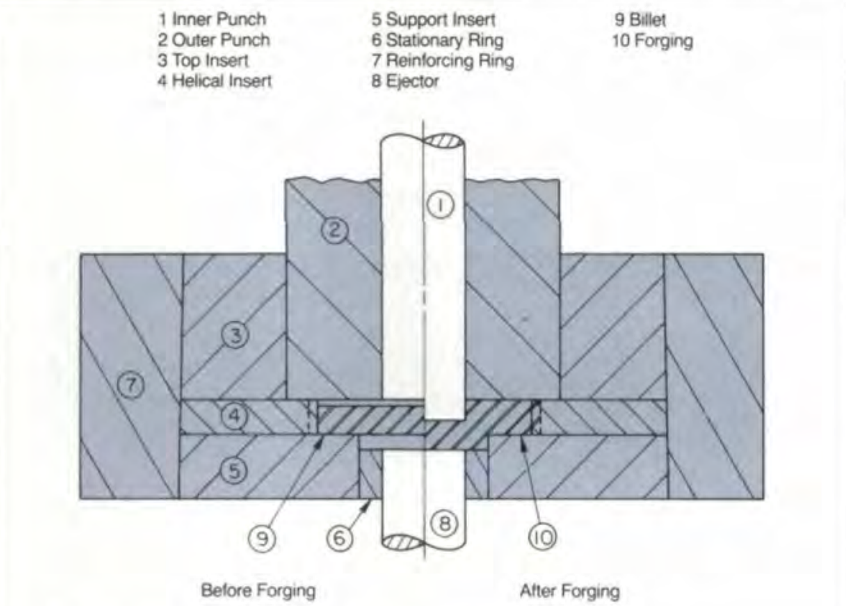

Fig. 3 – Tooling arrangement for forging parallel axis gears.

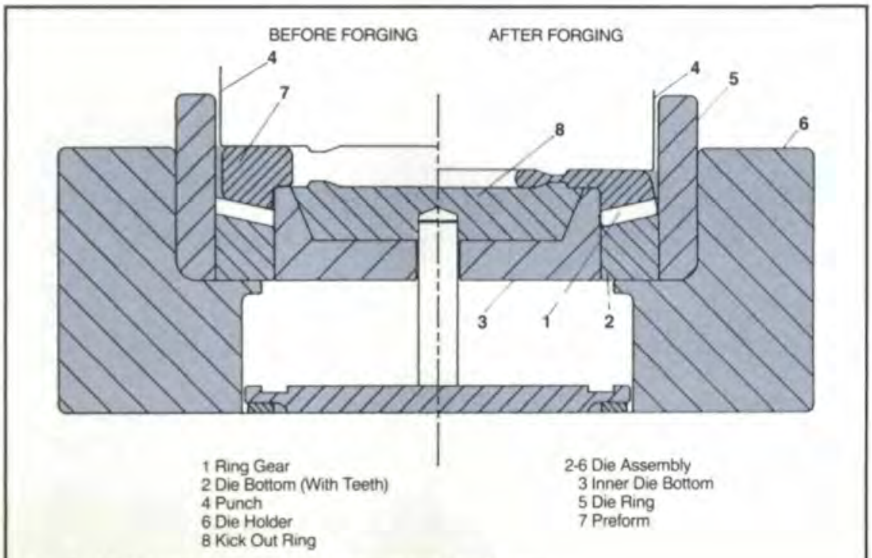

Fig. 4 – Tooling arrangement for forging spiral bevel gears.

Computer Program "GEARDI". The main functions of GEARDI are

- Define the exact tooth form of a spur or helical gear,
- Compute the forming load required to produce the current gear design,
- Compute the coordinates of the corrected gear geometry necessary for machining the EDM electrodes by taking into account the change in the die geometry due to temperature differentials, load stresses, and shrink fitting, and,
- Determine the specifications of a tool which can cut the altered tooth geometry on a conventional hobbing or shaper cutter machine.

This program enables the user to design spur and helical gears, predict tooling loads and pressures, estimate metal flow for forming the gear, and define the geometry required to manufacture the tooling using conventional or wire EDM. Several examples of gears currently being forged in industry were tested in the GEARDI computer program. The predicted forging loads were within acceptable limits of the actual loads measured during production runs.

GEARDI is an interactive, graphics-oriented program which was developed on a Digitial Equipment Corporation

VAX/ VMS 11/750 computer. It is a menu-driven program that allows the user to select various options from a pre-defined list. The GEARDI program has powerful application possibilities, not only in the area of metal forming, but also in the area of gear and gear train design, with its ability to design hobs and shaper cutters and to modify the fillet from a trochoidal shape to a circular shape.

Computer Program "SPBEVL". The main functions of the computer program "SPBEVL" are

- Determine the tooth geometry for spiral bevel gears and pinions normally machined on Gleason generators. The full implementation in the program is for FORMATE™ gears only. This is accomplished by an analysis of the kinematics of the machine, blank and cutter dimensions, and initial settings proposed from the Gleason summary charts.
- Correct the tooth geometry for the forging process parameters, such as temperature of forging, initial die temperatures, etc. The corrections are made for temperature differentials, loading of the dies, die manufacturing techniques (electrode overburn, shrink-fitting, etc.)
- Give new machine settings, blank, and tool geometries determined from the modified tooth geometry. These are used to machine the EDM graphite electrodes on the same Gleason machines that are used to cut the gears.

SPBEVL is an interactive graphics program operational on VAX/VMS systems.

Extrusion Trials

Spur/Helical Gears. Tooling designed using the GEARDI program was used to conduct spur and helical gear extrusion trials at Battelle's Columbus Division using a 700-ton hydraulic press. These trials were intended to validate the capabilities of the GEARDI program and demonstrate the practicality of extruding parallel axis gears. The spur gear was a 15 tooth, 5 DP gear. The gears were extruded using a "push-through" concept. Each gear is first partially formed and left in the die while the punch is retracted. A second billet is placed on top of the partially formed gear and the press is cycled again. During this cycle, the partially formed gear is finished formed and pushed through the die, dropping out the bottom of the die. Fig. 5 shows the sequence of parts in the tooling. Once formed, the teeth on the gear are not machined further. A fixture which holds the gear on the pitch line of the teeth is used to finish-machine the inside and ends of the gear. The spur gear formed in these trials is intended to be an AGMA quality class 8 gear. Measurements taken on the extruded gears indicated a gear of between AGMA quality 7 and 8.

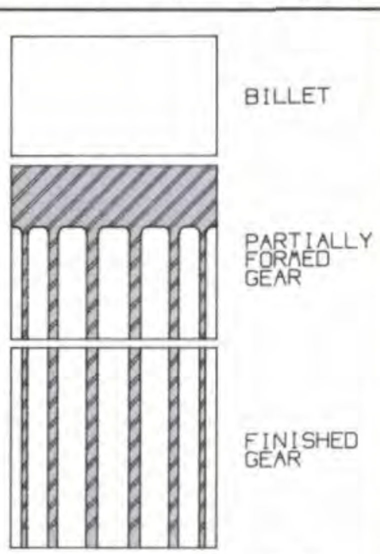

Fig. 5 – Sequence of steps in extruding parallel axis gears.

The helical gear chosen for the extrusion trials was a 32 tooth 10 DP gear with a helix angle of 30°. During the forming trials, several dies were damaged from internal tooth breakage. This was due largely to the relatively high helix angle and the fine pitch of the part. Several die designs were used before parts were finally extruded successfully. Yet even these gears were observed to have poor surface finish and were less accurate than desired.

Spiral Bevel Gear. Refer to Fig. 4 for a schematic of some initial forging tooling. The preform geometry is given in Fig. 6. Hot work steel H-13 was used as the die material. Six electrodes were needed for the spark erosion of the die. The gear forging trials were conducted at Eaton Corporation's Forging Division in Marion, Ohio. A 3,000-ton, mechanical forging press was used for the trials, based upon a forging load estimate of 2,500 tons. The preforms were machined for these trials, though they could be made by forming methods. The preforms were heated in an induction coil that was specifically designed for this purpose. The preform was heated for 200 seconds under a protective atmosphere to the required forging temperature of 2,000°F (1,093°C). The forging loads were monitored using

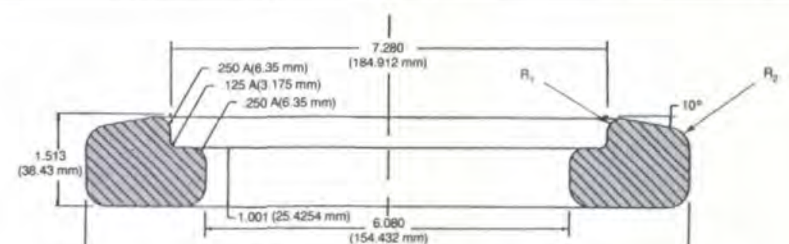

Fig. 6 – Preform geometry for spiral bevel gear forging.

load transducers attached to the frames of the press.

Die lubrication, used during the forging trials, consisted primarily of a water-base graphite material sprayed with pressurized air. After forging, the gears were placed teeth down in a sand-grapite mixture to reduce oxidation of the teeth during cooling. The back surfaces were still left exposed to the air so that the cooling rates would not be excessively slow. Gears were forged with a machining allowance of 0.007" (0.178 mm) on both tooth surfaces. A specially designed nest located the teeth so that the back surfaces are machined to be reference surfaces. This reference surface was used in the finish machining of the teeth in a conventional gear cutting machine.

The forged and machined gear was checked in a Zeiss coordinate measuring machine. Fig. 7 shows the relative deviation of the forged tooth profile as compared to a "master gear tooth" produced by conventional machining. Note that the relative error at the center of the profile is zero; i.e., the variations were measured relative to the center of the coast and drive surface of the master gear. The maximum variation was 0.003" (0.0762 mm). This difference can be easily compensated for in the machining of the pinion.

The above procedure was repeated to forge a 16" ring gear. These trials showed the applicability of the above approach in designing dies for spiral bevel gear forging.

Conclusions & Future Work

Parallel Axis Gearing. The GEARDI program has demonstrated the feasibility of extruding and forging spur and helical gears. Several U.S. gear manufacturers are currently using this program to design their dies on an ongoing basis. Still, there is much to be determined concerning the limits of this process in terms of part geometry and process conditions. Those companies which currently are forming gears using this

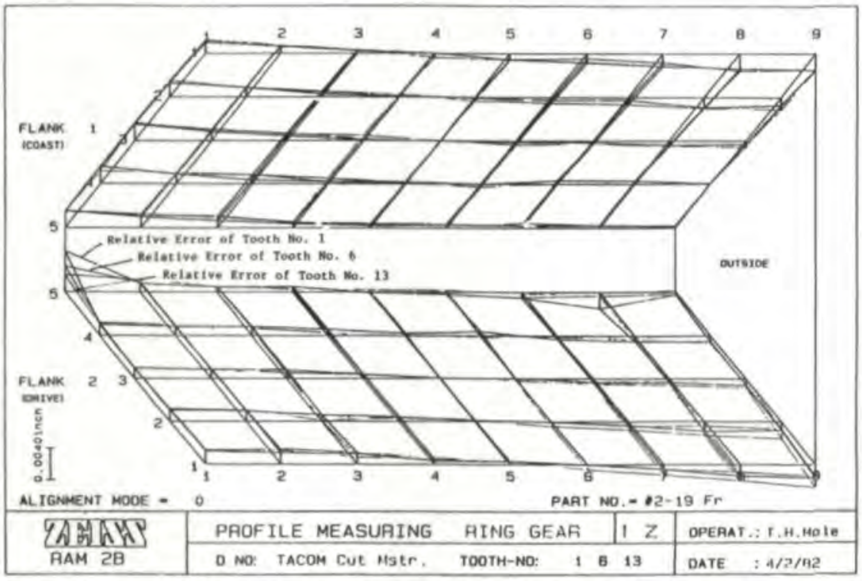

Fig. 7 – Zeiss gear tooth plots comparing the tooth form variation of a forged gear versus the cut master gear.

technology have gained considerable experience to date. There is a definite learning curve which one must travel in order to get into the business of forming gears. Research is still needed in many areas in order to generally quantify the process, including the following:

- gear materials which are formable,
- lubrication,
- tool life,
- variance in part dimensions between the first and last pieces from a given tool set, and
- limitations on part geometry.

The GEARDI program should also be improved to run on other computers, as well as refined in its ability to modify the fillet form and to more accurately estimate forming loads and generate die corrections.

Bevel Gearing. One of the major problems in spiral bevel gear forging is the extension of the above approach to other types of spiral bevel gears (generated tooth, for example). Though the modifications of the tooth surface can perhaps be determined by a rigorous mathematical treatment of the problem, the cost factor for making dies with an electrode to be machined in a five-axis machine is still unknown. Further, the computer-aided methods should consider the distortion of the gear tooth during cooling. Tool life is another unknown factor. The fact that spiral bevel gears are forged around the world (with or without using the above method) is an indication that the method suggested in this article will be a good first step for the initial design of the dies without much trial and error. The future work is currently directed towards extending the above concept to gears that are cut with both modified roll and tilt mechanisms using Gleason machines.

References:

1. ALTAN, T., OH, S., and GEGEL, H. *Metal Forming Fundamentals and Applications*, ASM, Metals Park, Ohio, 1983, pp. 6-8, 132.
2. SACHS, G. "On the Theory of the Drawing Process". (in German), Z. *angew. Math. u. Mech.*, 7, 1927, pp. 235-236.
3. SIEBEL, E. *Metal Forming in Plastic Condition* (in German), Düsseldorf, Verlag Stahleisen, 1932.

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TECHNICAL CALENDAR

SEPTEMBER 5-13, 1990. IMTS-90, McCormick Place, Chicago, IL. Largest trade show in the Western hemisphere with exhibitors from around the world. A new feature this year will be the 120,000 sq. ft. Forming and Fabricating Pavilion. For more information, contact: IMTS-90, 7901 Westpark Drive, McLean, VA 22102-4269. Ph:(703) 893-2900. Fax: (703) 898-1151.

SEPTEMBER 11-13, 1990. Short Course on Gear Noise. The Ohio State University, Columbus, OH. Discussions will cover gear noise cause and reduction, measurement and analysis, transmission error methods, dynamic modeling, gear noise signal analysis, and shaft and housing dynamics. Case history workshops and laboratory demonstrations. Featured speakers include Profs. D. R. Houser and R. Singh of OSU and Dr. Robert Munro of Huddersfield Institute of Technology, England. For more information contact: Mr. Richard D. Frasher, Director, Continuing Education, College of Engineering, 2070 Neil Ave., Columbus, OH 43210. (614) 292-8143.

OCTOBER 29-31, 1990. AGMA Fall Technical Meeting. Hilton International Hotel, Toronto, Canada. Papers and seminars on gear design, analysis, manufacturing, applications, gear drives, and related products. Contact AGMA, 1500 King St., Suite 201, Alexandria, VA, 22314. (703) 684-0211.

TRANSMISSION ERRORS . . .

(continued from page 13)

process of its generation. That may be accomplished by a computer-controlled machine for cutting.

Synthesis of Spiral Bevel Gears with a Predesigned Parabolic Function of Transmission Errors

Generally, spiral bevel gears generated on Gleason machinery are designed and manufactured with non-conjugate tooth surfaces. By varying the machine tool settings, it is possible to obtain a lead function of transmission errors, a parabolic function with pinion lagging, or a parabolic function with gear lagging. Only a parabolic function with gear lagging is good for applications. The problem encountered is that it is very difficult to reduce the level of the parabolic function of transmission errors with gear lagging. Litvin et al.[5] proposed a method for generation of spiral bevel gears with conjugate tooth surfaces. Gears with such surfaces can transform rotation with constant gear ratio. Since the gear misalignment will cause a piece-wise linear function of transmission errors, it is necessary to predesign even for conjugate spiral bevel gears a parabolic function of transmission errors. Such a function will absorb the linear function of transmission errors. This goal can be achieved by modifying of the pinion tooth surface slightly. This is done by varying the machine tool settings for the conjugate gear tooth surfaces.

Rearranging (1), we can obtain

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 + \Delta\phi_2(\phi_1) \qquad (12)$$

Differentiating (9) by ϕ_1, we obtain

$$m_{21}(\phi_1) = \frac{N_1}{N_2} + \Delta\phi_2^1(\phi_1) \qquad (13)$$

where $m_{21}(\phi_1)$ is the instantaneous gear ratio.

Differentiating again (10) by ϕ_1, we receive

$$m_{21}'(\phi_1) = \Delta\phi_2''(\phi_1) \qquad (14)$$

To obtain a parabolic function with gear lagging, the value of m_{21}' must be negative.

The determination of machine tool settings is based on the local synthesis of the gears. This synthesis is based on the following ideas: (i) The gear tooth surfaces are in contact at the chosen mean point. (ii) The instantaneous gear ratio at the mean contact point is equal to the given value. (iii) The derivative of m_{21} is a negative that yields a predesigned parabolic function with gear lagging. (iv) The tangent to the path contact has the prescribed direction. (In the discussed case the above-mentioned tangent is directed along the surface.) (v) The principal curvatures and directions of the gear tooth surfaces must be related to satisfy the requirements to m_{21}, the direction of the tangent, and the dimensions of the contact ellipse. The meshing of the gears and their bearing contact in the region of meshing can be investigated by the TCA (Tooth Contact Analysis) Program.[6]

Example 4. The pinion is right-handed, $N_1 = 10$, $N_2 = 41$, diametral pitch is 5.559" in^{-1}, pinion root angle is 12°1', gear root angle is 72°25', mean spiral angle is 35°.

Table 7 shows the basic machine tool settings for conjugate gears and for gears with the predesigned parabolic function. The transmission errors for the gear convex side (drive side) when the gears are not misaligned are given in Table 8. Table 9 shows the resulting transmission errors for the gears with the crossing offset 0.002". Table 10 shows the resulting transmission errors for the gear convex side with the shift of the pinion axis 0.002". The transmission errors in both cases are represented by a parabolic function.

The bearing contact of the synthesized gears is shown in Fig. 12.

Conclusion

Modified tooth surfaces for spur, helical, and spiral bevel gears with a predesigned parabolic function of transmission errors have been proposed. The predesigned function allows absorption of a linear function of transmission errors that are caused by misalignment and errors of manufacturing. Methods for generation of the modified gears have been also developed.

Table 7. Machine-Tool Settings.

	Gear (LH)	Pinion (RH)	
	Both Sides	Concave Side	
	Both Cases	Conjugate Gears	Nonconjugate Gears
Cutter Radius	3.9571	3.8957	3.8953
Cutter Width	0.08	—	—
Blade Angle	20°	16.7979°	16.5919°
Machine Root Angle	72.4098°	12.0236	12.0236°
Radial	3.3775	3.3153	3.3031
Setting Angle	73.6837°	75.1337°	76.8170°
Machine Offset	0	0.00508	0.01391
Machine Ctr. to Back	0	−0.00041	−0.01130
Sliding Base	0	−0.00009	−0.00235
Ratio of Roll	0.9738	0.2375	0.2371
Tilt	0°	0°	0°
Swivel	0°	0°	0°

Table 8. Predesigned Function of Transmission Errors.

$\phi_1^{(°)}$	-17	-11	-5	1	7	13	19
$\Delta\phi_2^{('')}$	-7.66	-3.06	-0.61	-0.02	-1.10	-3.67	-7.60

Table 9. Function of Transmission Errors.

$\phi_1^{(°)}$	-32	-26	-20	-14	-8	-2	4
$\Delta\phi_2^{('')}$	-2.76	1.75	4.14	4.71	3.65	1.13	-2.69

Table 10. Function of Transmission Errors.

$\phi_1^{(°)}$	-24	-18	-12	-6	0	6	12
$\Delta\phi_2^{('')}$	-6.71	-1.89	0.62	1.16	0	-2.68	-6.71

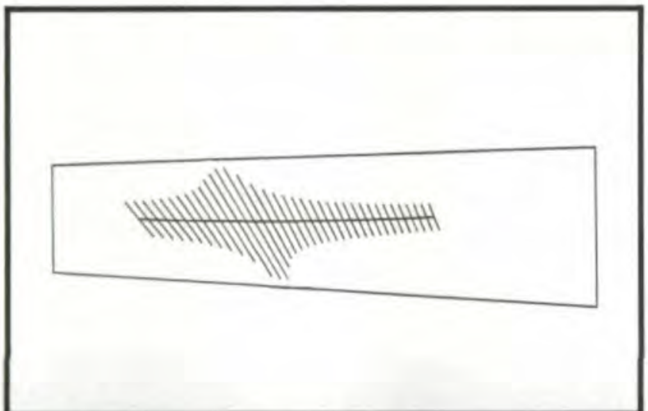

Fig. 12 – Bearing contact on the gear surface.

References:

1. CHIRONIS, N.P. "Design of Novikov Gear," *Gear Design and Application*, N.P. Chironis (ed.), McGraw-Hill, New York.
2. LITVIN, F.L. 1968, Theory of Gearing, 2nd edition, Moscow (in Russian). The English version is in press, NASA publications.
3. LITVIN, F.L., PERVEZ, R., and GOLDRICH, R.N. "Mathematical Models for the Synthesis and Optimization of Spiral Bevel Gear Tooth Surfaces." NASA Contractor Report 3553, 1982.
4. LITVIN, F.L. and TSAY, C. "Helical Gears with Circular Arc Teeth : Simulation of Conditions of Meshing and Bearing Contact," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 107, No. 4, pp. 556-564.
5. LITVIN, F.L., ZHANG, J., et al., "Crowned Spur Gears: Optimal Geometry and Generation," AGMA paper, 87FTM7. 1987.
6. LITVIN, F.L., TSUNG, W.-J. and LEE, H.-T. "Generation of Spiral Bevel Gears with Conjugate Tooth Surfaces and Tooth Contact Analysis," NASA Contractor Report 4088, AVSCOM Technical Report 87-C-22, 1987.
7. LITVIN, F.L. and TSAY, C.B., 1987, "Helical Gears with Circular Arc Teeth: Generation, Geometry, Precision and Adjustment to Errors, Computer Aided Simulation of Conditions of Meshing, and Bearing Contact." NASA Contractor Report 4089. AVSCOM Technical Report 87-C-18, 1987.
8. LITVIN, F.L., TSUNG, W.J., COY, J.J., and HEINE, C. "Method for Generation of Spiral Bevel Gears with Conjugate Gear Tooth Surfaces." *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 109, pp. 163-170. 1987.
9. WILDHABER, E., 1962, Method and Machine for Producing Crowned Teeth, U.S.A. Patent 3,046,844.

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