Reverse Engineering of Pure Involute Cylindrical Gears Using Conventional Measurement Tools

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Introduction

Designing a gear set implies a considerable effort in the determination of the geometry that fulfills the requirements of load capacity, reliability, durability, size, etc. When the objective is to design a new set of gears, there are many alternatives for the design, and the designer has the freedom to choose among them. Reverse engineering implies an even bigger challenge to the designer, because the problem involves already manufactured gears whose geometry is generally unknown. In this case, the designer needs to know the exact geometry of the actual gears in order to have a reference for the design.

Using advanced measurement machines, the profile of the tooth can be checked and compared with a reference surface; therefore, using a trial and error scheme it is possible to approximate the actual geometry of the gears. Unfortunately, these machines are expensive and seldom available to the designer, so the need for a method using conventional measurement tools is justified, especially when the measurement has to be done in the field.

This article presents a methodology based on measurement over wires and span measurement to determine the geometry of a pure involute gear.

Background

For the complete specification of a cylindrical gear, it is necessary to know the following:

Number of teeth
Pressure angle at a reference diameter
Outside diameter
Root diameter
Helix angle at a reference diameter
Circular tooth thickness at a reference diameter
Face width

It is well known that the operating surface of the tooth is uniquely defined by the base radius and the base helix angle. This surface is limited by the outside diameter, form diameter and face width. The tooth thickness is defined by the relative position of two symmetric tooth surfaces.

In order to define the geometry of a gear, it is useful to divide its characteristics into three groups. The first group includes characteristics that can be directly measured with conventional tools. The second group is integrated by properties that require special tools or procedures for their determination. The third one is formed by those properties which require some additional calculations for their determination. This division is shown in Table 1.

Nomenclature	
MOB	Measurement over balls
N	Total number of teeth
N_s	Number of spaces in the span measurement
N_t	Number of teeth in the measurement over balls
OD	Outside diameter
OM	Outside measurement
P	Transverse diametral pitch
P_n	Normal diametral pitch
R	Theoretical pitch radius
R_2	Auxiliary radius during measurement over balls
R_b	Base radius
RD	Root diameter
RM	Root measurement
R_w	Radius of the wire (ball for helical gears)
SM	Span measurement
T	Transverse tooth thickness at reference radius R
ϕ	Theoretical normal pressure angle
ϕ_2	Auxiliary pressure angle during measurement over balls
ϕ_τ	Theoretical transverse pressure angle at reference radius R
ψ	Theoretical helix angle at reference radius R
ψ_β	Base helix angle

The proposed method is based on the measurement of the four properties in Group I and two in Group II (measurement over balls and span measurement).

The procedure to calculate measurement over balls and span measurement is available in the literature (Refs. 1–3). Fig. 1 shows the transverse plane of a gear; from this figure, it may be observed that given the transverse tooth thickness 'T' of the gear at a given radius 'R', the measurement over balls may be calculated as follows:

$$MOB = 2R_w + R_2 \sqrt{2 - 2Cos\left(\frac{2\pi N_t}{N}\right)} \quad (1)$$

Where:

$$R_2 = \frac{R_b}{Cos(\phi_2)} \quad (2)$$

And ϕ_2 is calculated from:

$$Inv(\phi_2) = \frac{T}{2R} + Inv(\phi_t) + \frac{R_w}{R_b} - \frac{\pi}{N} \quad (3)$$

In the case of helical gears, the effect of the helix angle has to be considered, and Equation 3 becomes:

$$Inv(\phi_2) = \frac{T}{2R} + Inv(\phi_t) + \frac{R_w}{R_b Cos(\psi_b)} - \frac{\pi}{N} \quad (4)$$

From Fig. 2, the span measurement for a given gear may be calculated by the equation:

$$SM = R_b Cos(\psi_b)\left[\frac{T}{R} + \frac{2\pi N_s}{N} + 2Inv(\phi_t)\right] \quad (5)$$

Again, in Equation 5, the base helix angle is included to compensate for the fact that in helical gears the measurement is actually performed in the normal plane. The number of teeth considered during the measurement (N_s) is limited by two conditions: contact of the caliper with the root of the teeth and contact of the caliper with the tip of the teeth.

Fig. 3 shows the limiting number of teeth for different numbers of teeth and pressure angles in standard spur gears. As may be observed, there is always a range of teeth numbers to use for the determination of the span measurement.

Tools

During the measurement process the following tools are required:

• A conventional caliper or micrometer of suitable size for the gear to be measured. If a digital caliper is available, it is recommended, although this is not strictly necessary.

• A set of pins for the measurement of gears. It is convenient to remember that helical gears must

Group I (Directly Measured)	Group II (Requires Special Tools/Procedures)	Group III (Calculated)
Number of teeth	Chordal thickness & addendum	Pressure angle
Outside diameter	Measurement over balls	Diametral pitch
Root diameter	Span measurement	Helix angle
Face width		

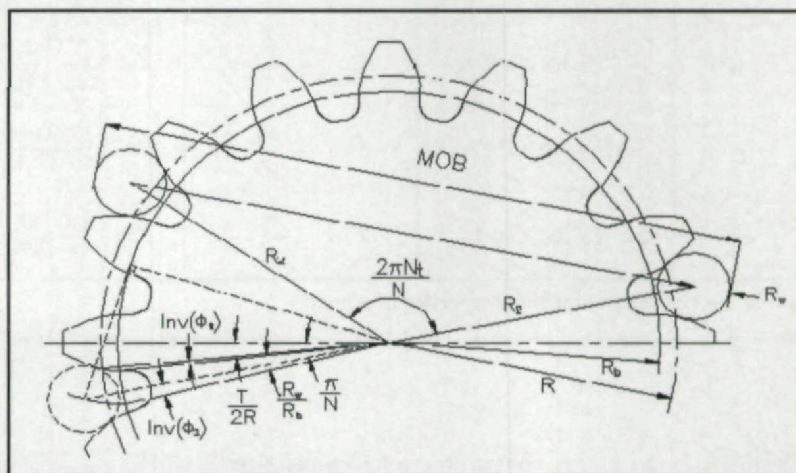Table 1 – Division of the geometrical characteristics in a gear

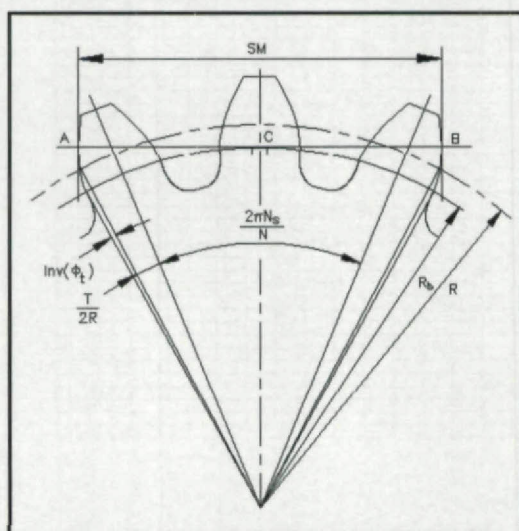Fig. 1—Measurement over balls in a spur gear.

Fig. 2—Span measurement in a spur gear.

be measured with balls instead of pins. The recommended "standard" ball diameter is defined as 1.728/Diametral Pitch or 1.68/Diametral Pitch. If a set of "standard" pins is not available, a set of steel balls of nominal diameter close to the standard may be used instead.

• A set of disc calipers is desirable, although in most of the cases this tool can be replaced with conventional calipers.

• A calculator or a portable computer with the program for measurement. The computer is recommended only because the calculations can be done faster, but the procedure can be easily done with the calculator.

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Calculate / Given	ϕ	ϕ_t	ψ	R	P_n	P
ϕ	ϕ	$Sin^{-1}\left(\dfrac{Sin(\phi)}{Cos(\psi_b)}\right)$	$Cos^{-1}\left(\dfrac{Tan(\phi)}{Tan(\phi_t)}\right)$	$\dfrac{R_b}{Cos(\phi_t)}$	$\dfrac{N}{2RCos(\psi)}$	$P_nCos(\psi)$
ϕ_t	$Sin^{-1}(Sin(\phi_t)Cos(\psi_b))$	ϕ_t	$Cos^{-1}\left(\dfrac{Tan(\phi)}{Tan(\phi_t)}\right)$	$\dfrac{R_b}{Cos(\phi_t)}$	$\dfrac{N}{2RCos(\psi)}$	$P_nCos(\psi)$
ψ	$Sin^{-1}(Sin(\phi_t)Cos(\psi_b))$	$Cos^{-1}\left(\dfrac{R_b}{R}\right)$	ψ	$\dfrac{Tan(\psi)\cdot R_b}{Tan(\psi_b)}$	$\dfrac{N}{2RCos(\psi)}$	$P_nCos(\psi)$
R	$Sin^{-1}(Sin(\phi_t)Cos(\psi_b))$	$Cos^{-1}\left(\dfrac{R_b}{R}\right)$	$Tan^{-1}\left(\dfrac{RTan(\psi_b)}{R_b}\right)$	R	$\dfrac{N}{2RCos(\psi)}$	$P_nCos(\psi)$
P_n	$Sin^{-1}(Sin(\phi_t)Cos(\psi_b))$	$Cos^{-1}\left(\dfrac{R_b}{R}\right)$	$Sin^{-1}\left(\dfrac{NTan(\psi_b)}{2P_nR_b}\right)$	$\dfrac{N}{2P_nCos(\psi)}$	P_n	$P_nCos(\psi)$
P	$Sin^{-1}(Sin(\phi_t)Cos(\psi_b))$	$Cos^{-1}\left(\dfrac{R_b}{R}\right)$	$Tan^{-1}\left(\dfrac{NTan(\psi_b)}{2PR_b}\right)$	$\dfrac{N}{2P}$	$\dfrac{P}{Cos(\psi)}$	P

Table 2 – Calculation of the properties of a gear.
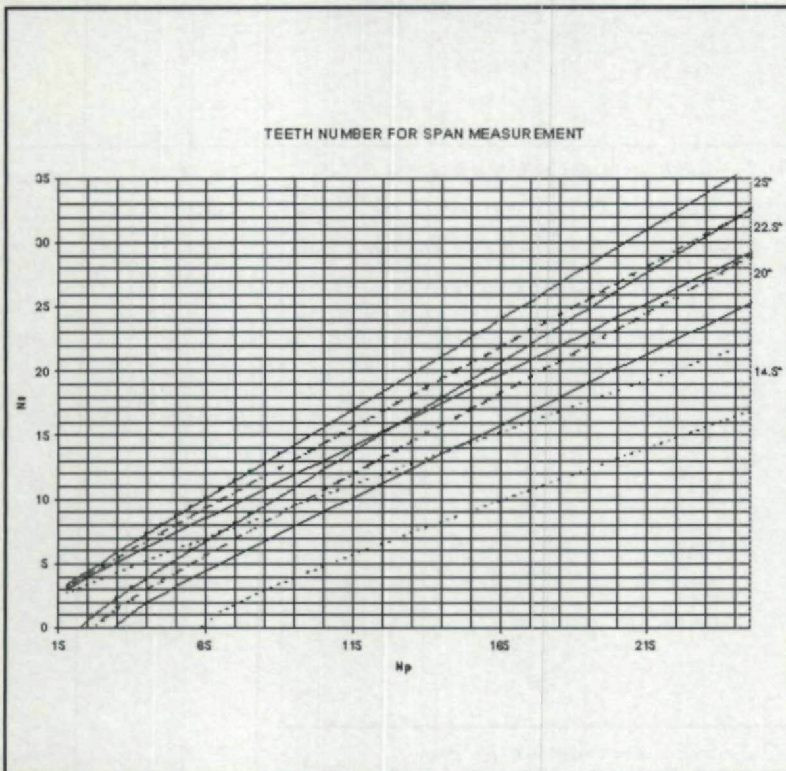

Fig. 3—Limiting number of teeth for span measurement.

Procedure

The proposed procedure assumes it is possible to take at least two span measurements. These measurements will be called SM_1, SM_2, etc. and are obtained considering N_{s1}, N_{s2}, etc. spaces. The geometry of the gear is calculated as follows:

1. Determine the number of teeth in the gear.
2. Determine the outside diameter of the gear.

From Fig. 4, if the number of teeth is even, the outside diameter corresponds to the outside measurement of the gear OD=OM. If the number of teeth in the gear is odd, or there are some missing teeth, the outside diameter may be calculated using Equation 6:

$$OD = OM \sqrt{\frac{2}{1 - Cos\left(\frac{2\pi N_s}{N}\right)}} \qquad (6)$$

Where N_s is the number of spaces considered when performing the measurement (see Fig. 4a).

3. Determine the root diameter of the gear, taking into account the same considerations as in the outside diameter, RD=RM for even number of teeth and for odd number of teeth:

$$RD = RM \sqrt{\frac{2}{1 - Cos\left(\frac{2\pi N_t}{N}\right)}} \qquad (7)$$

Where N_t is the number of teeth considered when performing the measurement (see Fig. 4b).

4. Determine the face width of the gear.
5. Select a suitable ball diameter.
6. Determine the measurement over balls of the gear.
7. Determine at least two span measurements in the gear registering the number of spaces used for every measurement.
8. Perform the following calculations:

Assuming two measurements are available, then from Equation 5 a constant K_1 may defined as:

$$K_1 = R_bCos(\psi_b)$$

$$= \frac{(SM_2 - SM_1)N}{2\pi(N_{s2} - N_{s1})} \qquad (8)$$

Because of the fact that the tooth thickness is the same for all the teeth, it is possible to define a constant K2 as:

$$K_2 = \frac{T}{R} + 2\mathrm{Inv}(\phi_t)$$

$$= \frac{SM_1}{K_1} - \frac{2\pi N_{s1}}{N}$$

$$= \frac{SM_2}{K_1} - \frac{2\pi N_{s2}}{N} \qquad (9)$$

Then from Equation 3:

$$\mathrm{Inv}(\phi_2) = \frac{K_2}{2} + \frac{R_w}{K_1} - \frac{\pi}{N} \qquad (10)$$

And from Equation 1,

$$R_2 = \frac{MOB - 2R_w}{\sqrt{2 - 2\mathrm{Cos}\left(\dfrac{2\pi N_t}{N}\right)}} \qquad (11)$$

Then, from Equation 2:

$$R_b = R_2 \cdot \mathrm{Cos}\,(\phi_2) \qquad (12)$$

If it is possible to measure with balls of different radius, the value of R_b would be the average of the individual values obtained applying Equations 10–12 for each ball diameter.

Using the definition for K_1, it is possible to obtain ψ_b by:

$$\psi_b = \mathrm{Cos}^{-1}\left(\frac{K_1}{R_b}\right) \qquad (13)$$

It is well known that it is possible to manufacture a gear with a given pressure angle at a reference diameter using a hob with different pressure angle by properly pulling or pushing the hob during the manufacturing process. Based on this, it is possible to assign an arbitrary value to any one of the following properties and determine the rest of the parameters in the gear.

ϕ	Normal pressure angle
ϕt	Transverse pressure angle
ψ	Theoretical helix angle
R	Theoretical pitch radius
Pd_n	Normal diametral pitch
Pd_t	Transverse diametral pitch

The equations relating these parameters may be found in References 2 & 3. Six cases are possible, and the equations for each of them are listed in Table 2. Once these parameters have been defined, the tooth thickness may be calculated applying Equation 9 as:

$$T = R \cdot [K_2 - 2\mathrm{Inv}(\phi_t)] \qquad (14)$$

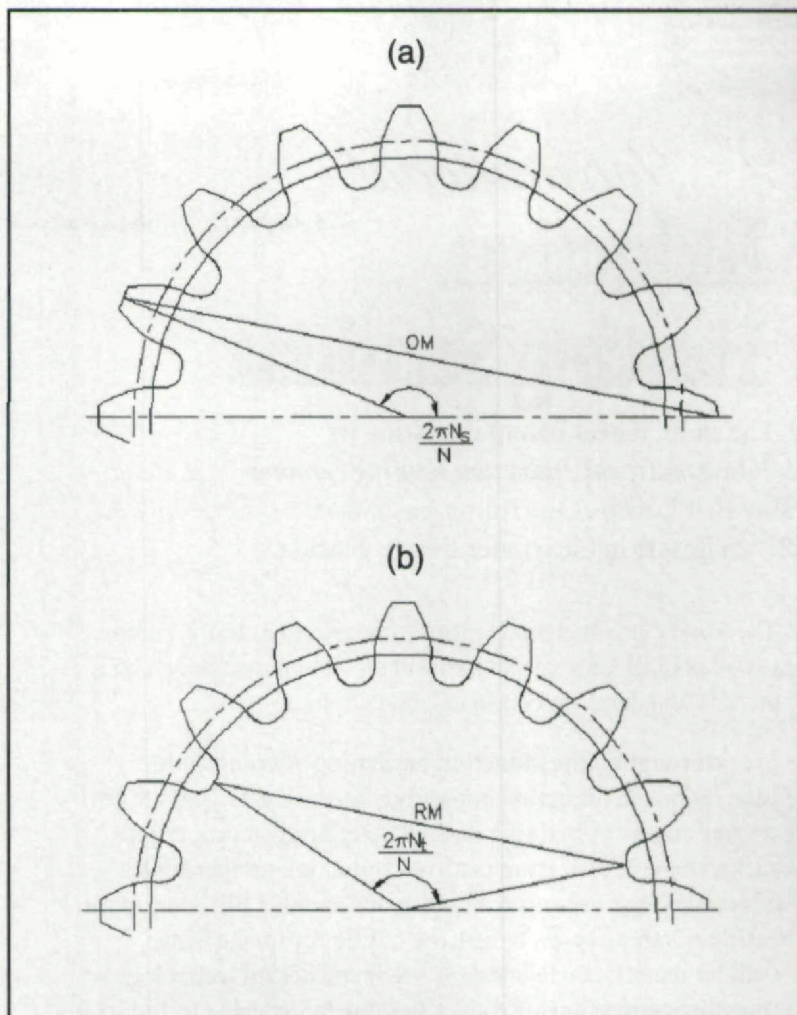*Fig. 4—Outside and root measurement for a gear with odd or missing teeth.*

Bibliography

1 Lynwander, Peter. "Gear Drive Systems, Design and Applications," Marcel Dekker, Inc. 1983.

2 Dudley, Darle W. "Dudley's Gear Handbook," McGraw-Hill, 1991.

3 American Gear Manufacturers Association. "ANSI/AGMA 2002-B88; Tooth Thickness Specification and Measurement," 1988.