Tooth Flank Corrections of Wide Face Width Helical Gears that Account for Shaft Deflections

Shan Chang, Dr. Donald R. Houser, Jonny Harianto

Management Summary

The tooth flank correction of power transmission helical gears with wide face width is studied using a finite element based shaft deflection analysis program in conjunction with a numerical load distribution analysis procedure. The load distributions along the line of action, the elastic deflections and transmission errors of gear pairs are obtained by solving the equations of compatibility of displacement and equilibrium of forces.

This paper discusses the influence of tooth flank corrections (tip relief, root relief, load modification, end relief and their combinations) on gear stresses and transmission errors due to shaft deflections. The technique used in the paper has the capability of modeling all significant geometric and elastic contributions due to tooth contact of the pair being analyzed as well as other gears mounted on the same shafts. The results show that it is possible to optimize at the design stage the gear micro-geometry for minimum stresses and transmission errrors without changing the gear macro-geometry.

Nomenclature

W_{1k}	total elastic deformation of point k on the first body
W_{2k}	total elastic deformation of point k on the second body
e_k	initial separation between the first and second body at point k
$R\theta$	rigid body approach
F_k	force acting on point k
r_b	distance of point k from the center line or base radius
T	applied torque
[S]	N x N matrix of influence coefficients or compliances
{F}	N x 1 vector of forces
{i}	N x 1 vector of ones
[I]	N x N identity matrix
{Y}	N x 1 vector of slack variables
{e}	N x 1 vector of initial separations

Introduction

Since 1938, when Walker (Ref. 1) first pointed out the importance of tooth flank corrections for spur gears, much work has been done to improve the load-carrying capacity and reduce the noise of power transmission gears. Terauchi (Ref. 2) obtained the tooth modifications (tip relief) based on the deflection results found using the combined approach of 3-D elastic theory and a mapping function. The tooth proportions that mesh on two teeth pairs can be corrected according to loaded deformation. Lee and Lin (Refs. 3 & 4) studied the influence of tooth flank modification and loading conditions on the dynamic tooth load and stress for high contact ratio spur gears. Maruyama (Ref. 5) achieved reduced transmission errors for automobile gears by optimum tooth crowning. Sigg (Ref. 6) provided a set of rules for profile and lead modifications required to provide smooth load distribution to correct for shaft bending and torsional deflections. Conry and Seireg (Ref. 7) provided an optimum design procedure for tooth flank corrections of helical gears. They solved non-linear contact equations of loaded gear pairs by a mathematical programming technique and set load distribution along the contact lines as the objective function. Other researchers (Refs. 8–9) also investigated optimum tooth modifications for spur and helical gears in considering the combination of various kinds of tooth modification types and parameters.

The key problem for gear tooth flank modification is how to get the precise deflections, including the loaded tooth elastic deformations and shaft deflections, and how to get load distribution along the contact lines. This paper's authors have citations to about 200 additional papers on the load distribution of gears, with some of them (Refs. 10–18) giving approaches to evaluate tooth load distribution of wide face width spur and helical gears.

Although it is recognized that shaft deflections and the loading of another gear on the same shaft have significant influences on tooth load distribution, few papers have been published on this topic.

Gopinath (Ref. 19) has developed the finite element-based shaft deflection analysis procedure used in this paper. His method was extended by Merugu (Ref. 20) to predict natural frequencies and mode shapes as well as dynamic response due to transmission error or external excitations of complex geared systems having several shafts. This shaft analysis procedure is used as a preprocessor to obtain influence coefficients and deflections due to outside loadings. These coefficients and deflections are then used in a load distribution solving routine similar to that developed by Conry and Seireg (Ref. 7). This routine predicts load distribution, stresses and transmission error of a gear pair.

This paper demonstrates these procedures for optimizing tooth flank corrections to reduce transmission error, root stresses and contact stresses of wide face width helical gears. The importance of considering shaft deflections and the deflections due to secondary gear loading will be demonstrated. The results in this paper show that very significant improvements can be achieved by careful tooth flank corrections to minimize transmission error and stresses under load.

Calculation Procedure

The load distribution is assumed to be a function of elasticity of the gear system and errors or tooth flank corrections on the gear pair. Below is a list of factors that the load distribution procedure considers in its calculation.

Elastic deformation:

- Bending and shear deflections of contacting teeth
- Base rotation and base translation of contacting teeth
- Local contact deflection
- Bending deflection of gear bodies and supporting shafts
- Flexibility of bearings and housings
- Torsional deflections of gear bodies
- Buttressing effects at tooth ends

Errors or modifications (initial separations):

- Shaft misalignment and shaft runout
- Involute errors or modifications
- Lead errors or modifications

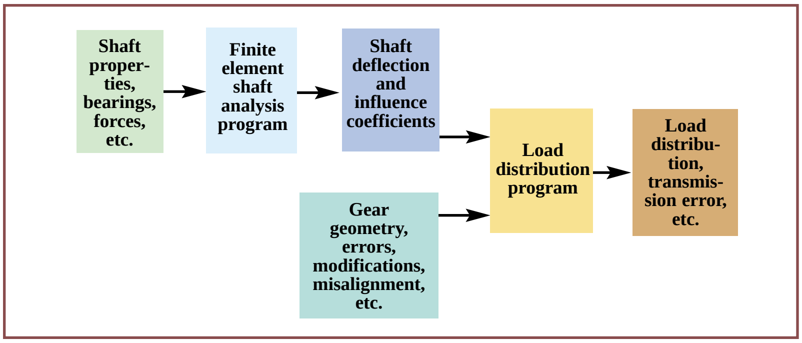

Figure 1—The relationship of LDP and SHAFT module.

- Tooth spacing errors

The analysis procedure is incorporated into a program called the Load Distribution Program (LDP), which can compute the bending and torsion deflections of the gear bodies and supporting shafts, assuming the shafts are simple cylinders supported by two bearings. For more complex shafts, the shaft analysis module (SHAFT) uses a finite element procedure for the calculation of influence coefficients and misalignment. In the SHAFT module, shafts are modeled as beam elements with six degrees of freedom at each node and gears are modeled in a manner similar to shafts with diameter equal to pitch circle diameter. Bearings are modeled using a 6x6 bearing stiffness matrix. Matrix values may be obtained from bearing manufacturers or may be computed using a procedure developed by Lim and Singh (Ref. 21).

The effect of a pinion-gear pair is modeled as a set of forces and displacements for computing the displacements across the face width of another gear mounted on the same shaft. Only one shaft is analyzed at a time. The schematic of the SHAFT module and its interaction with LDP is shown in Figure 1. Although not shown in this paper, the SHAFT program also may perform a forced vibration analysis of multi-shafted transmissions such that natural frequencies, mode shapes and dynamic motions and forces are predicted for the entire system.

Solution of Compatibility and Equilibrium

The relatively complex problem of determining the load distribution between mating gear teeth and the elastic deflection of gear pairs can be solved by setting up compatible equations for displacement and equilibrium of forces for a sufficient number of discrete points representing the contact region along the contact lines. Load distribution is obtained with the method that is based on the work of Conry and Seireg (Ref. 7) for elastic bodies in contact. A simplex type of algorithm is

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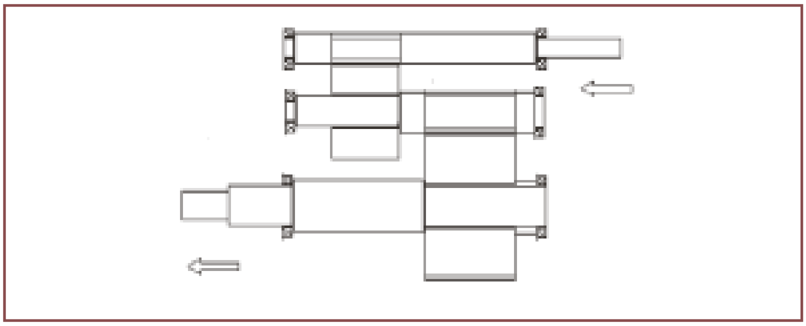

Figure 2—Shaft arrangement of typical two-stage gearbox.

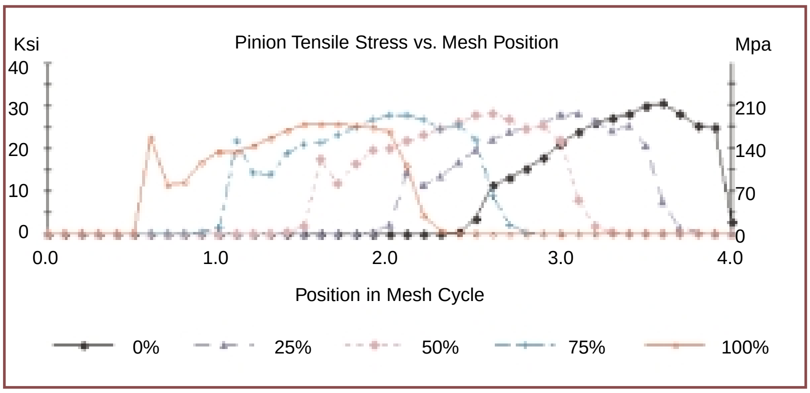

Figure 3—Pinion root stress without considering shaft deflections.

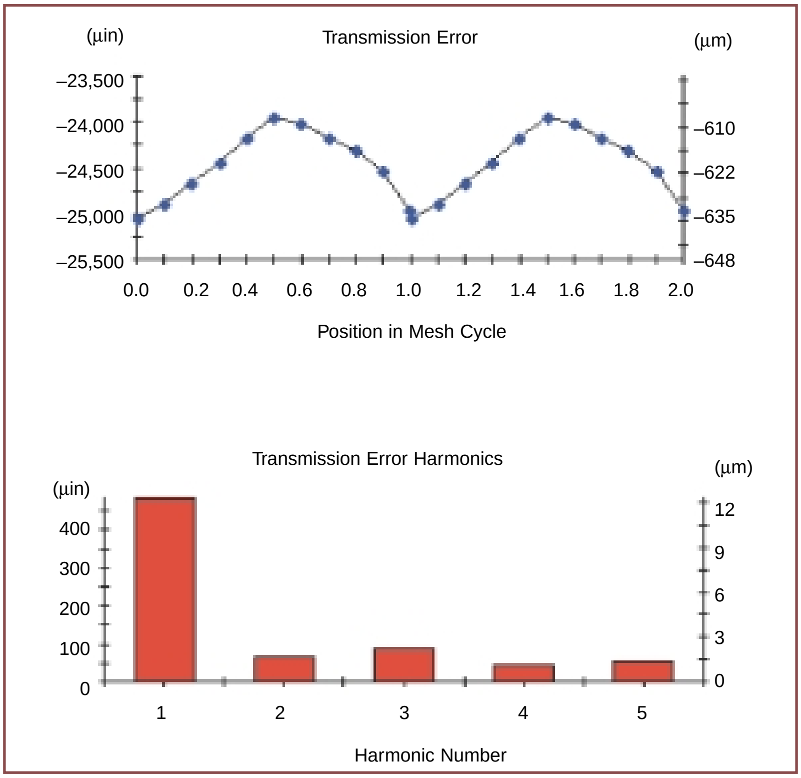

Figure 4—Transmission error without tooth flank modification.

used to compute the load distribution and the rigid body rotation based on the compliance of each point in the contact zone, the applied load, and the initial separations under no load, including errors or tooth flank modifications. The criteria used for the formulation of the solution are:

1.) Compatibility—This specifies the condition under which two points may come into contact.

2.) Equilibrium—This states that the sum of the torques acting on the system is zero.

According to the first criterion, the contributions due to the initial separations and the elastic deformation must exceed the rigid body motion along the line of action, for any discrete point k in the contact zone. Mathematically, this means:

$$W_{1k} + W_{2k} + e_k \geq R\theta \qquad (1)$$

The second criterion may be expressed as:

$$\sum_k (F_k \bullet r_b) + T = 0 \qquad (2)$$

In Equation 1, the inequality may be converted into an equation using a slack variable Y(k) and rewritten as:

$$W_{1k} + W_{2k} + \mathrm{e}_k - R\theta - \mathrm{Y}_k = 0 \qquad (3)$$

For contact between the bodies, Y(k) = 0 and consequently F(k) ≥ 0. If, however, Y(k) > 0, then the two bodies are not in contact at the discrete point k and F(k) = 0. Introducing the compliance coefficient matrix, the problem involves the estimation of the values of F(k), Y(k) and θ using Equations 4 and 5.

$$-[\mathrm{S}][\mathrm{F}] + R\theta[\mathrm{I}] + [\mathrm{I}][\mathrm{Y}] - \{e\} = 0 \qquad (4)$$

$$\{\mathrm{I}\}^T\{\mathrm{F}\}r_b + T = 0 \qquad (5)$$

Equation 4 expresses the conditions for compatibility of displacement at N discrete points and represents N equations in N+1 unknowns. The N+1th equation is provided by the equilibrium condition expressed by Equation 5. Equations 4 and 5 are solved using a modified simplex algorithm (Ref. 7). To obtain the tooth load distribution along the contact lines (Refs. 22–23), there are two places where shaft information is used. First, the LDP procedure needs the shaft deflections (due to the effects of another gear mounted on the same shaft, if any) as one of the components of the initial separation. Secondly, the shaft compliance is added to the compliances for tooth

Table 1—Gear Details.	First Stage		Second Stage	
	Pinion	Gear	Pinion	Gear
Number of teeth	15	63	17	69
Normal module, mm	4.5		6	
Normal pressure angle, deg.	23		23	
Helix angle, deg.	11		9	
Center distance, mm	182.88		266.7	
Outside diameter, mm	82.3	301.5	120.9	436.4
Root diameter, mm	60.45	279.6	92.176	407.4
Face width, mm	152.4	152.4	241.3	241.3
Torque Nm	3,114.4		13,080.5	
Bearing radial stiffness, x10^9 N/m	1.28, 0.86	1.7		1.75, 2.27
Bearing thrust stiffness, x10^9 N/m	0.16, 0.16	0.14		0.28, 0.28
Bearing span, mm	530	485		500
Shaft diameter, mm	82	105		150
Distance from face width center to bearing span center, mm	127	127	90	90

bending and shear, base motion, and Hertzian contact. In this case, the shaft influence coefficients supplied by the SHAFT program include the effects of bearing and housing deflections, as well as shaft torsion and bending. The analytical contact analysis can then be carried out for a specified micro-geometry at a number of positions of mesh to determine load distribution, transmission error, contact stresses and root stresses at given torques and misalignments. The specified micro-geometry can be tip relief, root relief, profile crowning, end relief, face crowning, lead modification, bias modification or topographic correction of the flanks.

Analysis Results

As an example, the load distribution procedure using the SHAFT module is used to analyze a typical two-stage gear reduction gearbox. The pinion of the second stage is mounted on the same shaft with the first stage gear, as shown in Figure 2. The gear details are given in Table 1.

Figure 3 shows the predicted root stresses (done with a hybrid finite element approach) at five positions along the face width of the pinion. Stress traces look quite similar, but the peak stress seems to be higher for the right side of the tooth pair.

Influence of Shaft Deflections on Tooth Contact

To emphasize the importance of making accurate assessments of the misalignment caused by the gear mounted on the same shaft in the wide face width helical gear pair, elastic mesh analysis

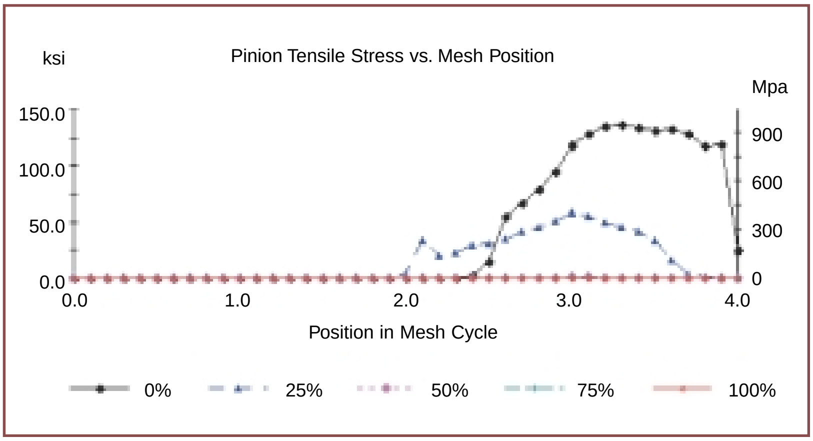

Figure 5—Effect of second-stage pinion deflections on the pinion root stresses along the face width (without tooth modification).

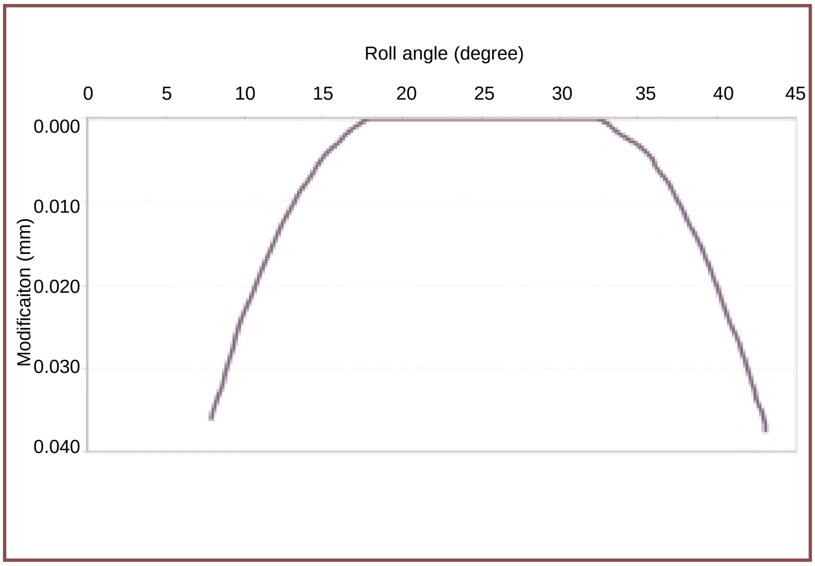

Figure 6—Profile modification for the first stage pinion.

Tooth Flank Corrections of Wide Face Width Helical Gears that Account for Shaft Deflections

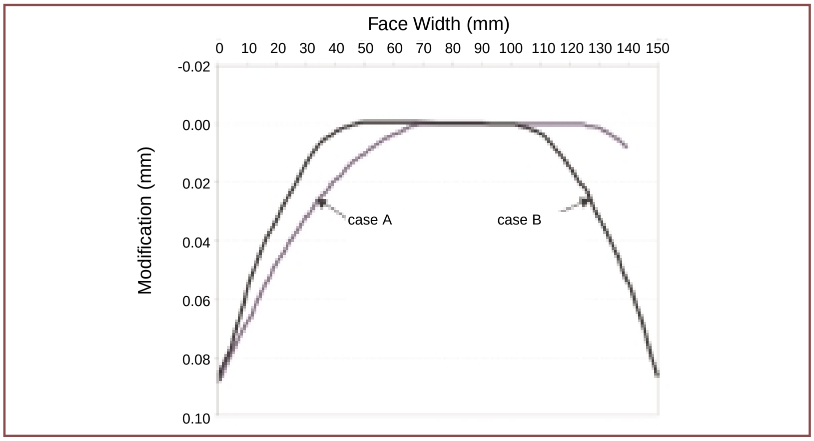

Figure 7—Lead modification for cases A and B.

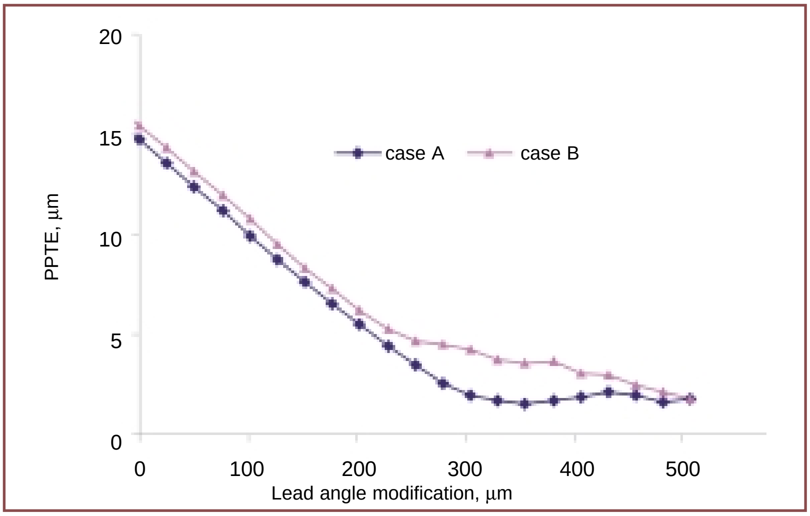

Figure 8—Effect of lead angle modification on peak-to-peak transmission error for two cases of lead crowning.

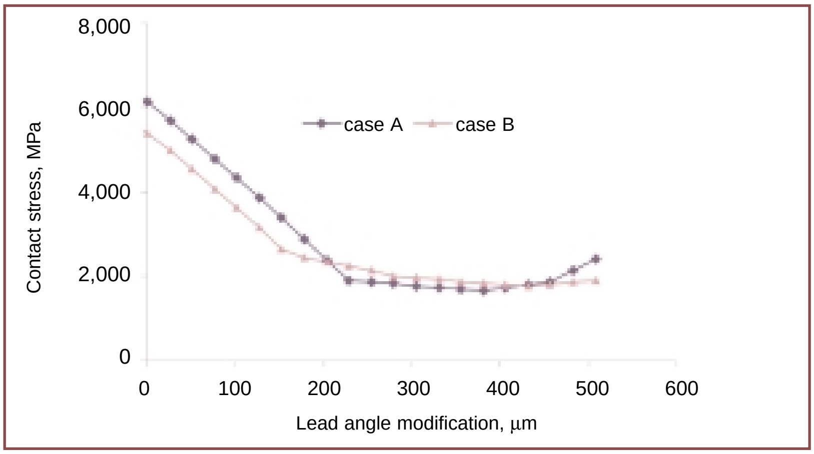

Figure 9—Effect of lead angle modification on peak contact stress for two cases of lead crowning.

has been carried out for the input gear pair. The misalignment along this gear pair's face width can result in an uneven load distribution across the tooth face width.

Using the data in Table 1 to calculate transmission error, load distribution, contact stresses and root stresses of the input gear pair without any tooth flank correction, it is noted that:

- Without tooth flank correction, contact stress is shifted to one side of the tooth and corner contact peak exists.
- Only about 48% of gear face width carries load.
- The maximum contact stress increases 3.6 times and the maximum pinion root stress increases 4.5 times compared with that of the stresses prior to considering shaft deflection due to loading of the second stage pinion (Fig. 5).
- Although not shown, the peak-to-peak transmission error increased significantly due to the misalignment.

Tooth Flank Corrections

The tooth flank corrections are carried out only on the first stage pinion. Tip relief and tooth relief are used to reduce the high contact stresses that occur at the tooth corners (the entering and exiting regions). Tip relief is 38 µm and the starting modification point is at a roll angle 31.6°. Root relief is 38 µm and its starting modification point is at a roll angle of 19°. The profile modification curve is parabolic, as shown in Figure 6.

To discuss the validity of end relief (lead crown) and lead angle modification on wide face width gears, calculated examples are given in this paper. End relief and lead angle modifications are considered according to the various conditions shown in Table 2. The values used in cases A and C–E are based on a procedure recommended by Sigg (Ref. 6). The two types of end relief are shown in Figure 7. The asymmetry of the Sigg form is meant to compensate for the large torsional effect of the small diameter pinion of the first gear pair. All of the calculations used in the simulations in cases A–E include tooth tip relief and root relief.

The peak-to-peak transmission error (PPTE), contact stresses, pinion root stresses and load distribution factor are shown in Figures 8–11 respectively, for cases in which base-line lead crowning was included and the lead slope was varied in order to compensate for shaft deflections. In these cases, all shaft deflections are considered. It is interesting to compare the performance of gear pairs that have different end reliefs. Figures 8–11 clearly show that both cases A and B achieve improved results. The asymmetric end relief can obtain slightly lower values of PPTE and pinion root stress, while contact stress and load distribution factor are about the same for the two types of end relief.

Based on the results in Figures 8–11, it is observed that lead angle modifications between 220 µm and 435 µm provide the best values of PPTE, stresses and load distribution factor. Three lead angle modifications are chosen as listed in Table 2 to optimize the end relief. The results are shown in Figures 12–14.

Figure 12 shows that both lead angle modification and end relief are important for obtaining the lowest PPTE for wide face helical gears. Comparing Figure 13 with Figure 15, the variation of contact stress with tooth flank correction parameters is very similar to that of load distribution factor. Figure 14 shows that when lead modification is much less, it is difficult to lower the root stress by using end relief. From the point of view of improving the load carrying capacity of wide-faced helical gears, lead angle modification is dominant. However, without the combination of tip relief, root relief and end relief, it is very difficult to obtain the favorable load distribution and lower contact stress as shown in Figures 12–14.

When end relief at the torque input end is 51 µm, lead angle modification is 432 µm, and PPTE gets the lowest value of 0.73 µm. On the other hand, the lowest load distribution factor of 1.15 is achieved with different tooth flank correction parameters, that is, with a 63.5 µm end relief at the torque input end and a 355 µm lead angle modification. With the tooth flank corrections that give the lowest PPTE, the TE and pinion root stress distributions of the wide face width gear pair are calculated and shown in Figures 15–16, respectively.

Comparing Figures 3–4 with Figures 15–16, it is observed that:

- The peak-to-peak transmission error is reduced significantly from 28.5 µm to 0.73 µm. The PPTE of this gear pair is very close to that of a modified set in which shaft deflections are not considered.
- Load carrying area increases up to 99% of face width.
- Without tooth flank correction, the contact stress at the corner region is more than 8,300 MPa, also the maximum contact stress posi-

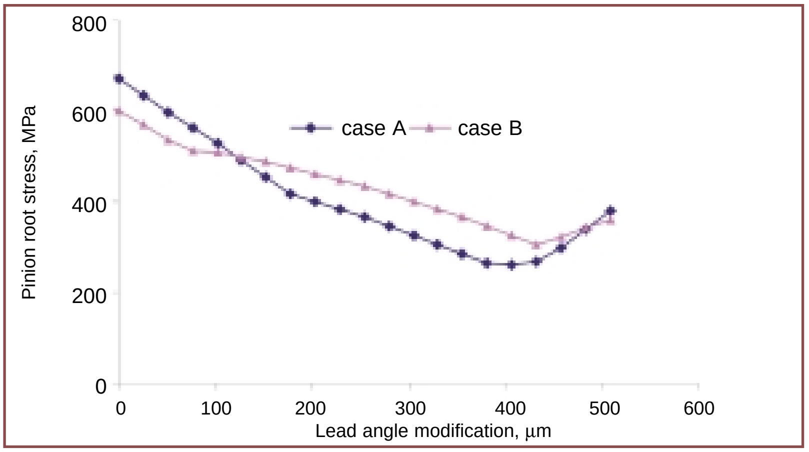

Figure 10—Effect of lead angle modification on pinion root stress for the two cases of lead crowning.

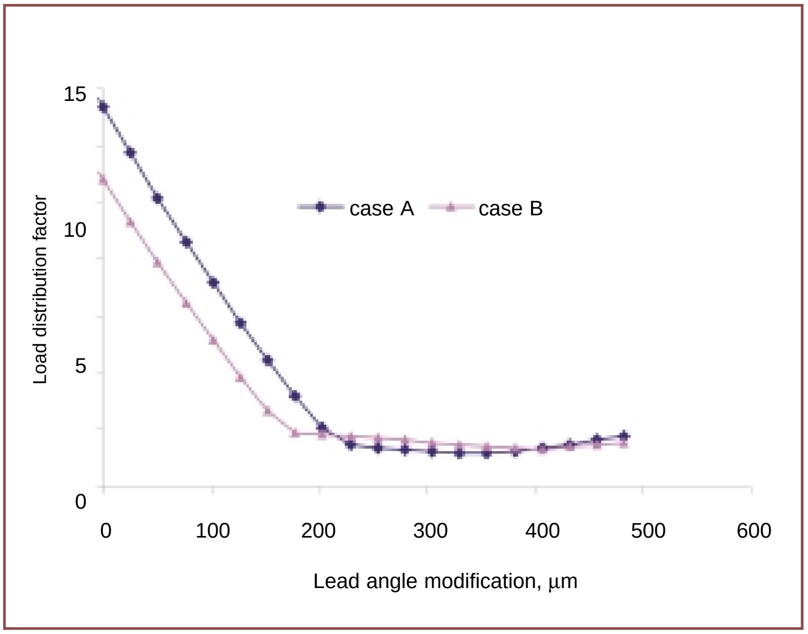

Figure 11—Effect of lead angle modification on load distribution factor for two cases of lead crowning.

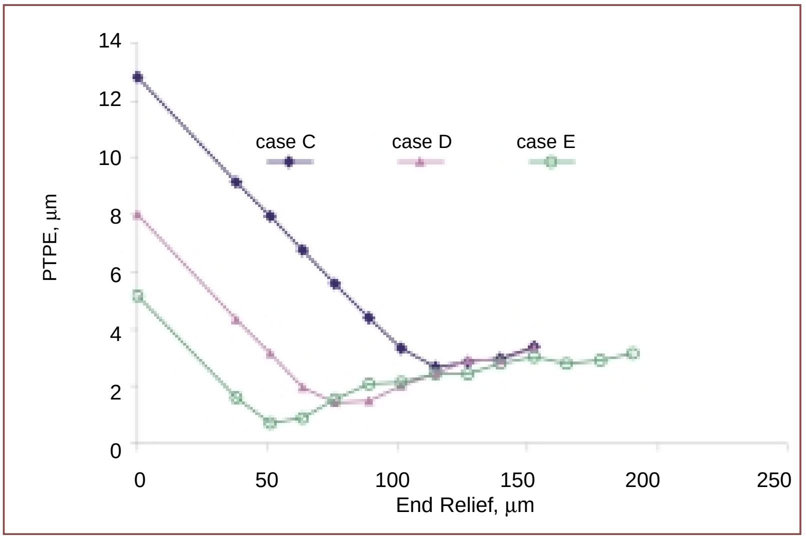

Figure 12—Effect of amplitude of end relief (Sigg's Method) on peak-to-peak transmission error for three different lead angle corrections.

Tooth Flank Corrections of Wide Face Width Helical Gears that Account for Shaft Deflections

Table 2—End Relief and Lead Angle Modification.

	End Relief	Lead Angle Modification
Case A	Relief: 90 µm at torque input end, 25 µm at free end Length: 76.2 mm at torque input end, 25.4 mm at free end Curve: Parabolic curve	Variable
Case B	Relief: 90 µm at torque input end, 90 µm at free end, Length: 51 mm at torque input end, 51 mm at free end Curve: Parabolic curve	Variable
Case C	Relief: Variable The relief at torque input end is three times that of free end. Length: 76.2 mm at torque input end, 25.4 mm at free end Curve: Parabolic curve	228 µm
Case D	Relief: Variable The relief at torque input end is three times that of free end. Length: 76.2 mm at torque input end, 25.4 mm at free end Curve: Parabolic curve	355 µm
Case E	Relief: Variable The relief at torque input end is three times that of free end Length: 76.2 mm at torque input end, 25.4 mm at free end Curve: Parabolic curve	432µm

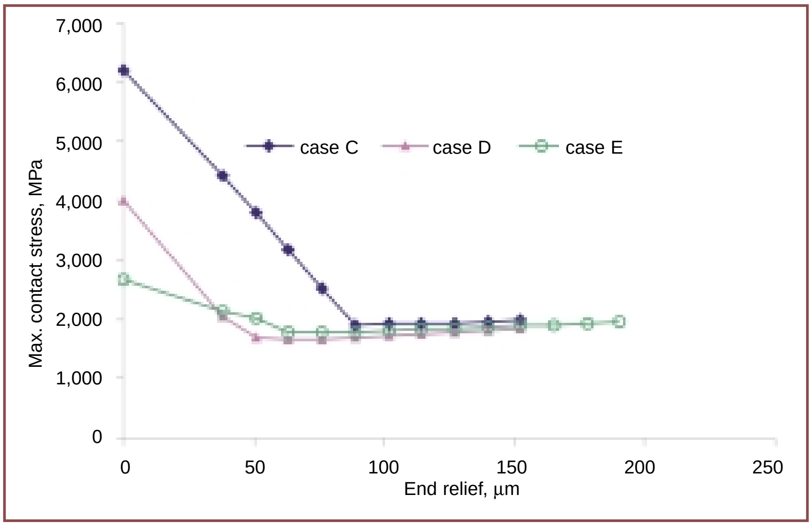

Figure 13—Effect of amplitude of end relief (Sigg's Method) on peak contact stress for three different lead angle corrections.

tion. After tooth flank modification, the contact stress at this position reduces to 490 MPa, and the contact point that has the maximum contact stress (2,018 MPa) shifts to the center region in the tooth face. A bit larger lead correction could further reduce this maximum stress and relocate it closer to the center of the tooth.

- Maximum root stress reduces greatly from 961 MPa to 313 MPa, and the root stress distribution along the face width improves. Again, a further shifting of the lead correction could further reduce the peak stress value.

Conclusions

The use of an elastic mesh model that is coupled with a detailed finite element model of the supporting shafting has been shown to be a useful tool for improving the geometric design of gearing. From the results of the above examples, the following conclusions are made:

1.) Shaft deflections and misalignments caused by another gear mounted on the same shaft have a significant effect on load carrying capacity and performance for wide face gear pairs.

2.) The optimum tooth flank correction for helical gears with wide face widths can achieve a good balance between contact stress, root stress, peak-to-peak transmission error and load distribution factor.

The technique described can be applied to spur and helical gears with wide face widths and to gear systems with complex shaft arrangements. The method is capable of computing the influence of another gear mounted on the same shaft. Properly applied, it can achieve significant reduction in transmission error and root bending stress and contact stress by optimizing the gear micro-geometry at the design stage without changing the gear macro-geometry.

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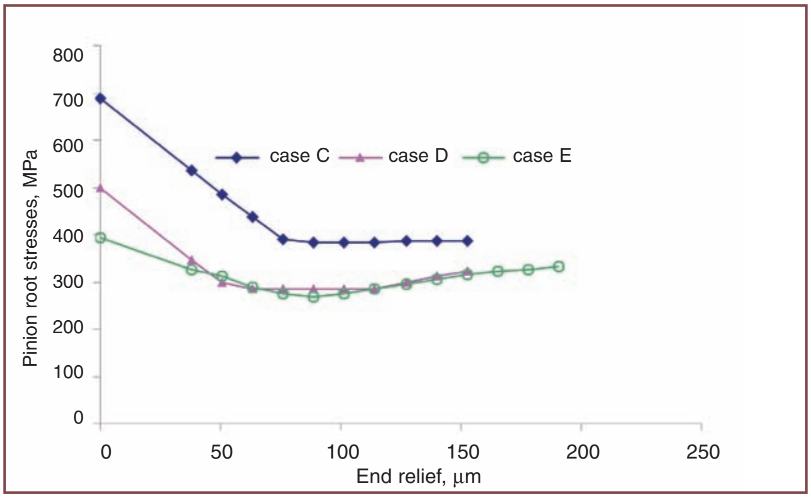

Figure 14—Effect of amplitude of end relief (Sigg's Method) on peak pinion root stress for three different lead angle corrections.

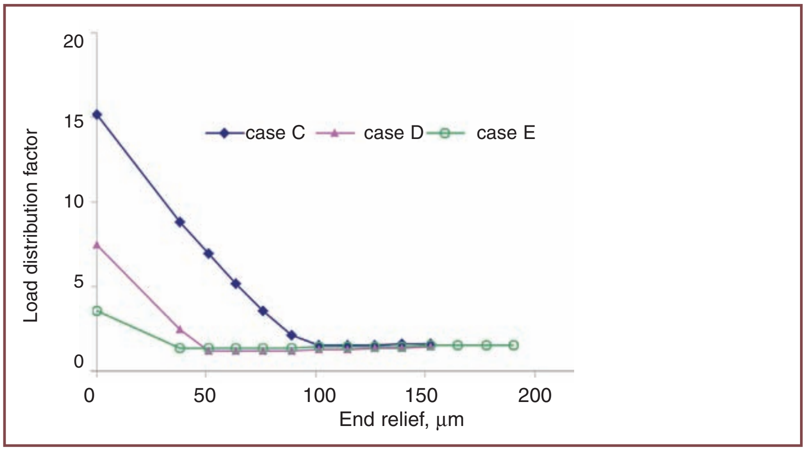

Figure 15—Effect of amplitude of end relief (Sigg's Method) on load distribution factor for three different lead angle corrections.

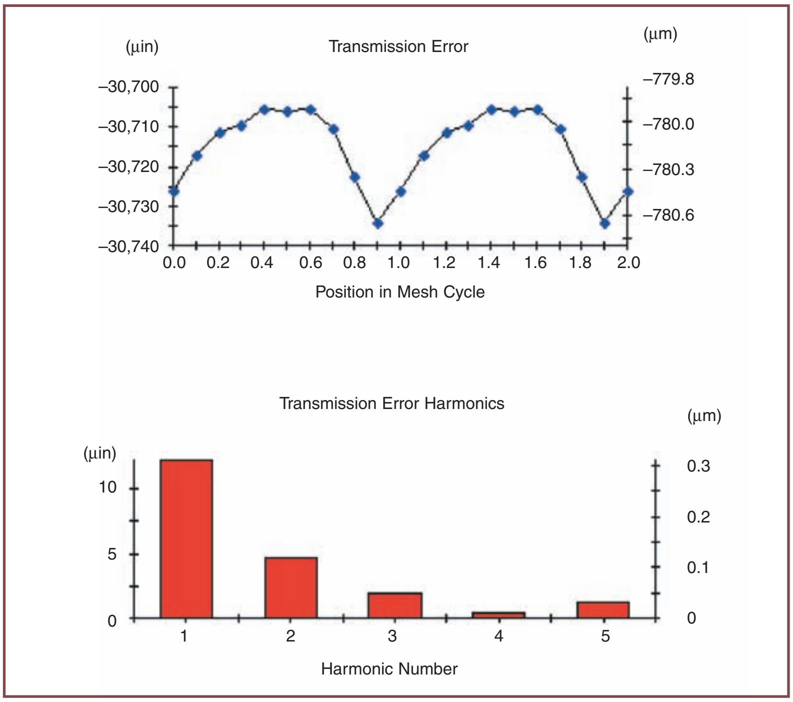

Figure 16—Transmission error after tooth flank correction.