Measurement of Directly Designed Gears with Symmetric and Asymmetric Teeth

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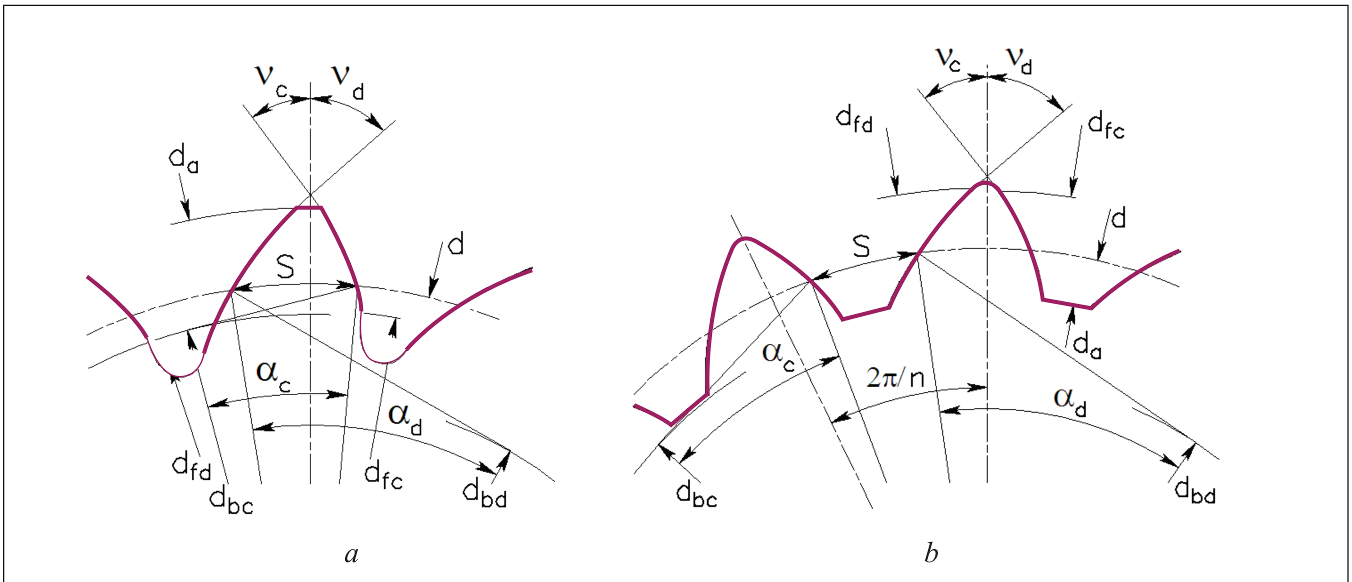

Figure 1—Gear tooth profile: *a* = external gear; *b* = internal gear; d_a = tooth tip circle diameter; d_b = base circle diameter; d_f = form circle diameter; *d* = reference circle diameter; *S*= circular tooth thickness at the reference diameter; α = involute profile (or pressure) angle at the reference diameter; *v* = involute intersection profile angle; *n* = number of teeth; subscripts "d" and "c" are for the drive and coast flanks of the asymmetric tooth.

Management Summary

In comparison with the traditional gear design approach based on preselected, typically standard generating rack parameters, the Direct Gear Design method provides certain advantages for custom high-performance gear drives that include: increased load capacity, efficiency and lifetime; reduced size, weight, noise, vibrations, cost, etc. However, manufacturing such directly designed gears requires not only custom tooling, but also customization of the gear measurement methodology.

This paper presents definitions of main inspection dimensions and parameters for directly designed spur and helical, external and internal gears with symmetric and asymmetric teeth.

Measurement Over (Between) Balls or Pins

Spur gears. The Direct Gear Design method (Refs. 1–2) presents the gear tooth by two involutes of two base circles with the angular distance between them and tooth tip circle arc (Fig.1). The equally spaced n teeth form the gear. The fillet between teeth is designed independently, thus providing minimum bending-stress concentration and sufficient clearance with the mating tooth-tip in mesh. If the two base circles are identical, the gear teeth are symmetric; if they are different, the teeth are asymmetric.

Measurement over (between) balls or pins for spur gears is defined based on the given:

- Number of teeth n
- Reference circle diameter d
- Involute profile angles at the reference diameter α_d and α_c; for symmetric gears involute profile angle at the reference diameter $\alpha = \alpha_d = \alpha_c$
- Circular tooth thickness at the reference diameter S
- Gear tooth-tip diameter d_a

Initially selected ball or pin diameter D can be adjusted based on the calculation results. The relation between angles v_d and v_c, and α_d and α_c is:

$$\frac{\cos\nu_d}{\cos\nu_c} = \frac{\cos\alpha_d}{\cos\alpha_c} = \frac{d_{bd}}{d_{bc}}, \qquad (1)$$

where: $d_{bd} = d \text{ x } \cos\alpha_d$ and $d_{bc} = d \text{ x } \cos\alpha_c$.

Angles v_d and v_c are defined from equations:
For external gear:

$$inv(\nu_d) + inv(\nu_c) = inv(\alpha_d) + inv(\alpha_c) + \frac{2 \text{ x } S}{d}, \qquad (2)$$

For internal gear:

$$inv(\nu_d) + inv(\nu_c) = inv(\alpha_d) + inv(\alpha_c) + 2 \text{ x} \frac{\pi}{n} - \frac{S}{d}, \qquad (3)$$

where: $inv(x) = \tan(x) - x$ is involute function and x is involute profile angle in radians. The centers of the ball or the pin are located on the diameter d_p (Fig. 2), which is:

$$d_p = \frac{d_{bd}}{\cos\alpha_{pd}} = \frac{d_{bc}}{\cos\alpha_{pc}}, \qquad (4)$$

where the angles α_{pd} and α_{pc} are defined by equations (Ref. 3):

For external gear:

$$inv(\alpha_{pd}) + inv(\alpha_{pc}) = inv(\nu_d) + inv(\nu_c) + \frac{D}{d_{bd}} + \frac{D}{d_{bc}} - \frac{2\pi}{n}, \qquad (5)$$

For internal gear:

$$inv(\alpha_{pd}) + inv(\alpha_{pc}) = inv(\nu_d) + inv(\nu_c) - \frac{D}{d_{bd}} - \frac{D}{d_{bc}}. \qquad (6)$$

The ball or pin touches the gear tooth in the points T_d and T_c. They should be always located on the involute flanks. This condition is described by the following equation:

For external gears:

$$\arccos\frac{d_{bd}}{d_{fd}} < \alpha_{td} < \arccos\frac{d_{bd}}{d_a}, \qquad (7)$$

and:

$$\arccos\frac{d_{bc}}{d_{fc}} < \alpha_{tc} < \arccos\frac{d_{bc}}{d_a}; \qquad (8)$$

For internal gears:

$$\arccos\frac{d_{bd}}{d_a} < \alpha_{td} < \arccos\frac{d_{bd}}{d_{fd}} \qquad (9)$$

and:

$$\arccos\frac{d_{bc}}{d_a} < \alpha_{tc} < \arccos\frac{d_{bc}}{d_{fc}} \qquad (10)$$

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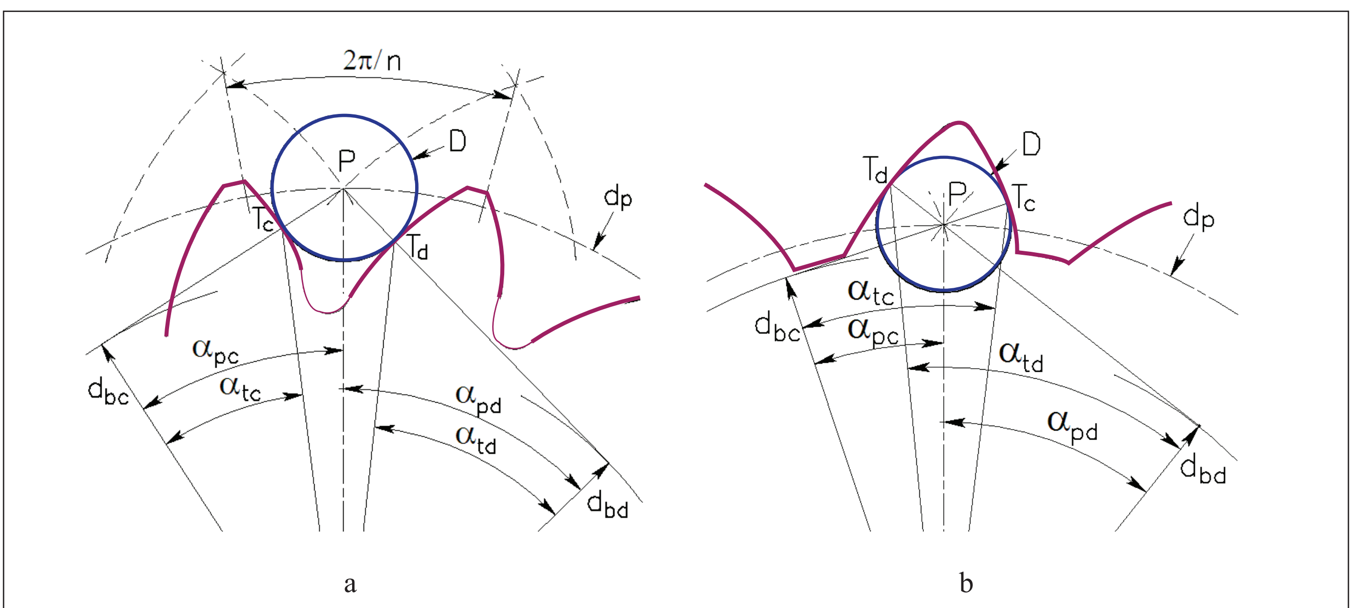

Figure 2—Ball or pin position: *a* = external gear; *b* = internal gear; *D* = ball or pin diameter; *P* = center of the ball or pin; α_{pd} and α_{pc} = involute profile angles at the center of the ball or pin; d_p = ball or pin center location diameter; T_d and T_c = contact points of the ball or pin with the tooth drive and coast tooth flanks; α_{td} and α_{tc} = involute profile angles at the contact points.

The measurement over two balls or pins for the external gear is for even number of teeth (Fig. 3a):

$$M = d_p + D; \tag{11}$$

For odd number of teeth (Fig. 3b):

$$M = d_p \,.\, \cos\frac{\pi}{2n} + D. \tag{12}$$

The measurement between two balls or pins for the internal gear is for even number of teeth (Fig. 4a):

$$M = d_p - D; \tag{13}$$

For odd number of teeth (Fig. 4b):

$$M = d_p \,.\, \cos\frac{\pi}{2n} - D. \tag{14}$$

For inspection convenience the measurement over balls or pins for external gears should be $M > d_a$ and the measurement between balls or pins for internal gears should be $M < d_a$. These and conditions (Eqs. 7–10) define the ball or pin diameter.

Helical gears. Measurement over (between) balls or over pins for helical gears is defined based on the given:

- Number of teeth n
- Reference circle diameter d
- Normal involute profile angles at the reference diameter α_{nd} and α_{nc}; for symmetric gears $\alpha_n = \alpha_{nd} = \alpha_{nc}$
- Normal circular tooth thickness at the reference diameter S_n
- Helix angle at the reference diameter β
- Gear tooth-tip diameter d_a

Cylindrical pins cannot be used to measure the internal helical gears, because the pin surface cannot be tangent to the internal helical gear flanks. The transverse tooth thickness at the reference diameter S is:

$$S = S_n / \cos\beta. \tag{15}$$

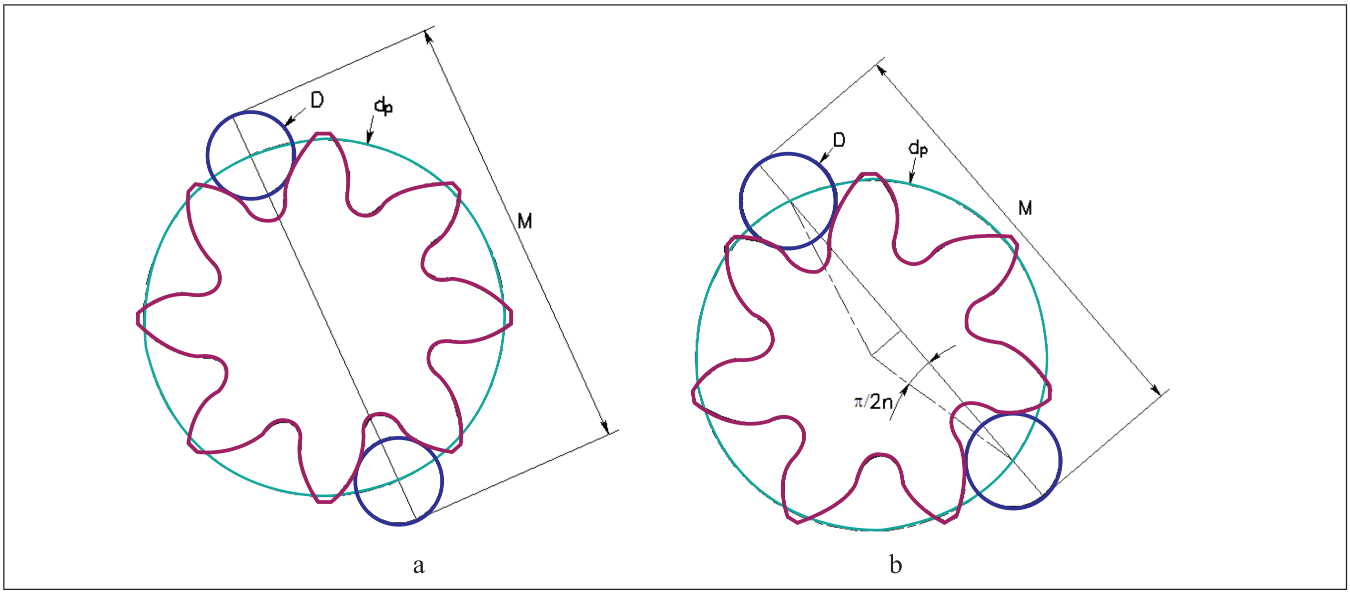

Figure 3—Measurement over balls or pins for external gears: *a* = even number of teeth; *b* = odd number of teeth.

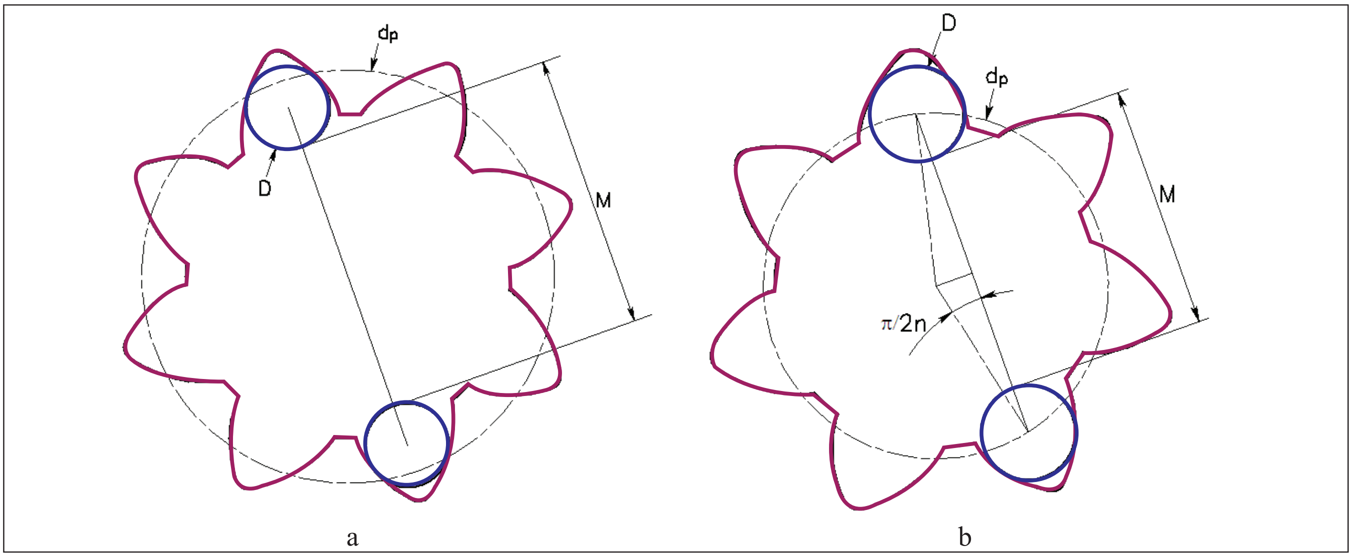

Figure 4—Measurement between balls or pins for internal gears: *a* = even number of teeth; *b* = odd number of teeth.

The transverse involute profile angles at the reference diameter α_d and α_c are:

$$\alpha_d = \arctan\frac{\tan\alpha_{nd}}{\cos\beta}, \quad (16)$$

$$\alpha_c = \arctan\frac{\tan\alpha_{nc}}{\cos\beta}. \quad (17)$$

The helix angles at the drive and coast base diameters β_{bd} and β_{bc} are:

$$\beta_{bd} = \arctan(\tan\beta \text{ x } \cos\alpha_d), \quad (18)$$

$$\beta_{bc} = \arctan(\tan\beta \text{ x } \cos\alpha_c). \quad (19)$$

The centers of the ball or the pin (for external gear with even number of teeth) are located on the diameter d_p that, defined by the equation (4), where the angles α_{pd} and α_{pc} are defined by:

For external helical gear:

$$inv(\alpha_{pd}) + inv(\alpha_{pc}) = inv(\nu_d) + inv(\nu_c) + \frac{D}{d_{bd} \text{ x } \cos\beta_{bd}} + \frac{D}{d_{bc} \text{ x } \cos\beta_{bc}} - \frac{2\pi}{n}, \quad (20)$$

For internal helical gear (for measurement over balls):

$$inv(\alpha_{pd}) + inv(\alpha_{pc}) = inv(\nu_d) + inv(\nu_c) - \frac{D}{d_{bd} \text{ x } \cos\beta_{bd}} - \frac{D}{d_{bc} \text{ x } \cos\beta_{bc}}. \quad (21)$$

The ball or pin diameters should also satisfy Equations 7–10. Measurements over two balls for external helical gears (Fig. 5) and between two balls for internal helical gears (Fig. 6) are defined by Equations 11–13 and 14, accordingly.

Measurement over two pins for external helical gears with even number of teeth is also defined by Equation 11.

For external helical gears with odd number of teeth, the shortest distance L between the pin centers does not lay in the transverse section of the circle diameter d_p. This distance and measurement over two pins for external helical gears with odd number of teeth definition is described in Reference 4. The transverse distance L_t between the ball centers, in case of the odd number of teeth, is always greater than the distance L that is (Fig.7):

$$L = \frac{d_p}{2 \text{ x } \tan\beta_p}\sqrt{\lambda^2 + 4 \text{ x } (\tan\beta_p \text{ x } \cos(\frac{\pi}{2n} + \frac{\lambda}{2}))^2}, \quad (22)$$

where the helix angle at the pin center diameter βp is:

$$\beta_p = \arctan(\frac{d_p}{d} \text{ x } \tan\beta), \quad (23)$$

and the angle λ is a solution of the equation:

$$\frac{\lambda}{\tan\beta_p} - \sin(\frac{\pi}{n} + \lambda) = 0. \quad (24)$$

Then the measurement over two pins for external helical gears with odd number of teeth (Fig. 8) is:

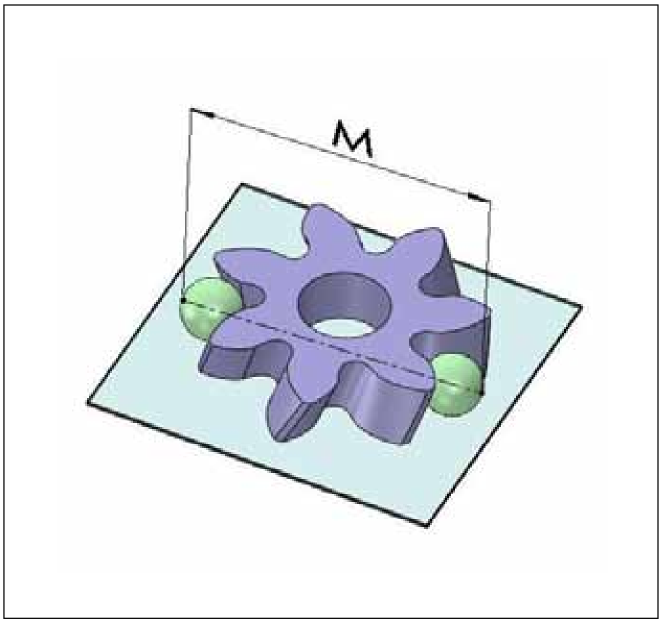

Figure 5—Measurement over balls of the external helical gear.

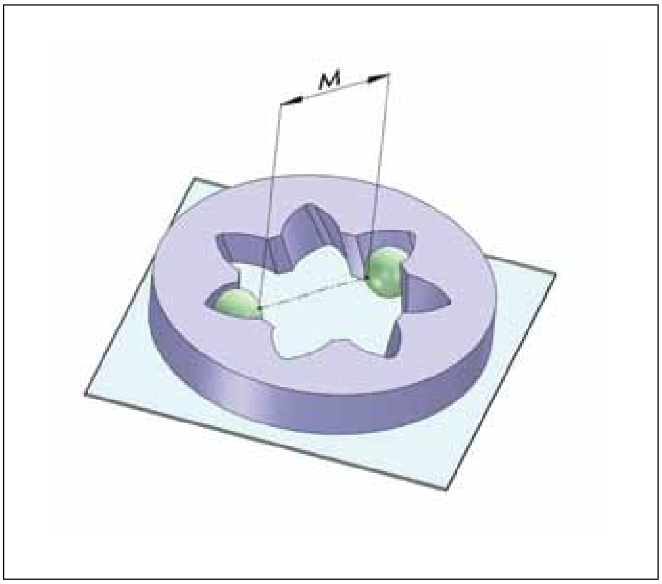

Figure 6—Measurement between balls of the internal helical gear.

$$M = L + D. \quad (25)$$

Span Measurement

Span measurement is the measurement of the distance across several teeth, along a line tangent to the base cylinder (Ref. 5). This kind of inspection is used for gears with external teeth. It is also applied only for gears with symmetric teeth, because it is impossible to have a common tangent line to two concentric base cylinders of asymmetric tooth flanks.

Span measurement over n_w teeth (Fig. 9) is:

$$W = (S_b + (n_w - 1) \text{ x } p_b) \text{ x } \cos\beta_b, \quad (26)$$

where S_b is the tooth thickness at the base diameter:

$$S_b = S \text{ x } \cos\alpha + d_b \text{ x } inv(\alpha), \quad (27)$$

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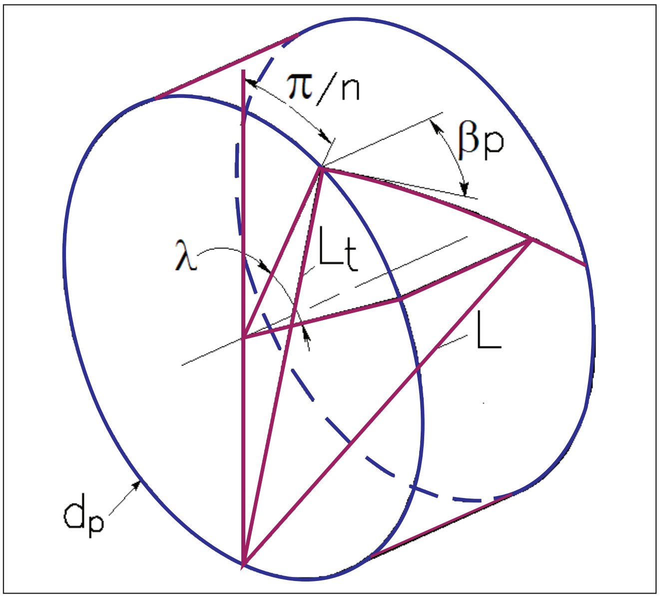

Figure 7—Definition of the distance between the pin centers for the helical gears with odd number of teeth.

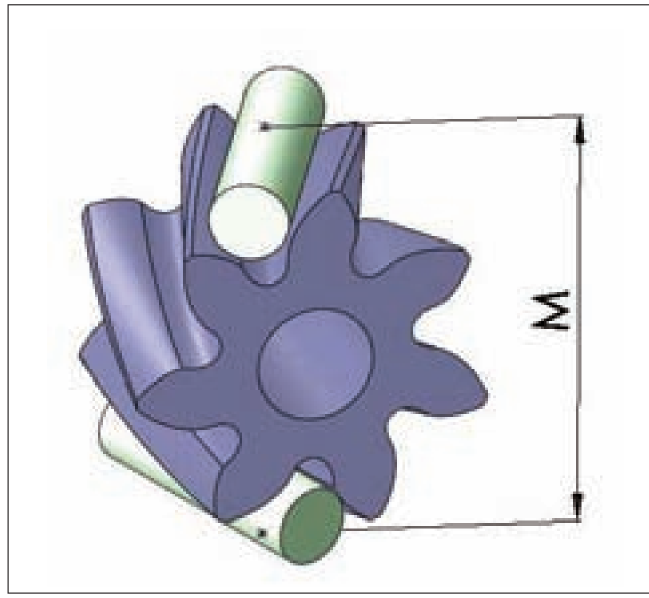

Figure 8—Measurement over pins of the external helical gear with odd number of teeth.

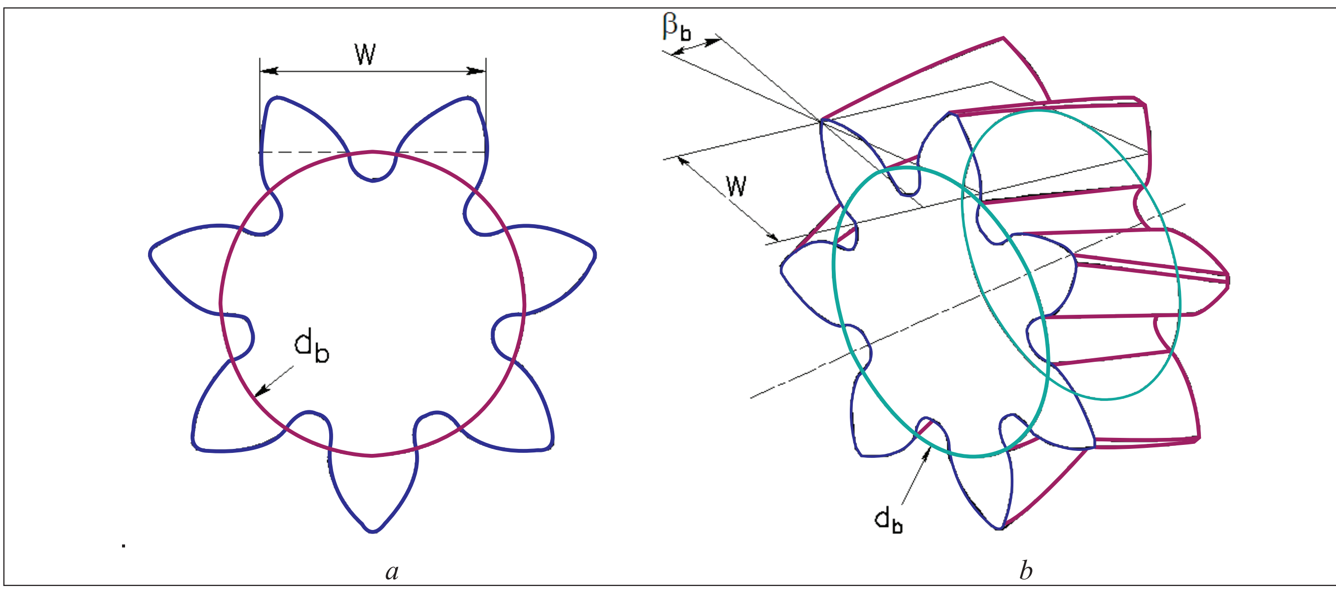

Figure 9—Span measurement; *a* = spur gear; *b* = helical gear.

Figure 10—CMM measurement of asymmetric gear.

p_b is the circular pitch at the base diameter

$$p_b = \frac{\pi \text{ x } d_b}{n}, \tag{28}$$

n_w is number of teeth for span measurement

$$2 \leq n_w \leq n_{wmax}, \tag{29}$$

n_{wmax} is maximum number of teeth

$$n_{wmax} = \frac{\sqrt{d_a^2 - d_b^2} - S_b}{p_b}. \tag{30}$$

Calipers, micrometers or special gages are used for span measurement.

CMM Gear Inspection

CMM gear inspection (Fig. 10) allows mapping the whole surface of all teeth including the fillet profiles. However, it is typically used to control the involute accuracy. Although the gear tooth fillet is an area of maximum bending stress concentration, its profile and accuracy are marginally defined on the gear drawing by typically very generous root diameter tolerance and, in some cases, by the minimum fillet radius. The Direct Gear Design method optimizes the gear tooth fillet profile for minimum bending stress concentration (Ref. 6). For such critical- application gears the tooth fillet profile must be clearly specified, toleranced and inspected.

The whole tooth (including the fillet) CAD profile at the average material condition presented as the B-spline or the tangent arcs accompany the gear drawing for the CMM inspection. The data set also includes the involute flank and fillet profile tolerances that are established by the designer depending on the gear accuracy and also the manufacturing technology. The CMM is programmed to indicate if the inspected tooth profile points lay within the corridor defined by the CAD tooth profile ± profile tolerance. A similar inspection technique is used to inspect curved surfaces, for example, of the airfoil air compressor or gas turbine blades.

Summary and Conclusion

This paper has covered the measurement specifics of the symmetric and asymmetric gears that are designed using the Direct Gear Design method. They are:

- A defined measurement over (or between) balls and pins for external and internal gears
- A defined span measurement for external gears with symmetric teeth
- Descriptions of some CMM inspection issues for directly designed gears

Presented materials should be helpful for manufacturing custom gears with symmetric and asymmetric teeth.

References:

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Dr. Alexander L. Kapelevich *possesses more than 30 years custom gear research and design experience, as well as over 100 successfully accomplished projects for a variety of gear applications and clients. His company, AKGears, provides consulting services—from complete geartrain design (for customers without sufficient gear expertise) to retouching (typically tooth and fillet profile optimization) of existing customers' designs—in the following areas: traditional or direct gear design; current design refinement; R&D; and failure-and-testing analysis. The company provides gear drive design optimization for increased load capacity; size and weight reduction; noise and vibration reduction; higher gear efficiency; backlash minimization; increased lifetime; higher reliability; cost reduction; and gear ratio modification and adjustment. Kapelevich is the author of numerous technical publications and patents, and is a member of the AGMA Aerospace and Plastic Gearing Committees, SME, ASME and SAE International. He holds a Ph.D. in mechanical engineering from Moscow State Technical University and a Masters Degree in mechanical engineering from the Moscow Aviation Institute.*