Tribology Aspects in Angular Transmission Systems

Part IV: Spiral Bevel Gears

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(This article is part four of an eight-part series on the tribology aspects of angular gear drives. Each article will be presented first and exclusively by Gear Technology, but the entire series will be included in Dr. Stadtfeld's upcoming book on the subject, which is scheduled for release in 2011.)

Dr. Hermann Stadtfeld *received a bachelor's degree in 1978 and in 1982 a master's degree in mechanical engineering at the Technical University in Aachen, Germany. He then worked as a scientist at the Machine Tool Laboratory of the Technical University of Aachen. In 1987, he received his Ph.D. and accepted the position as head of engineering and R&D of the Bevel Gear Machine Tool Division of Oerlikon Buehrle AG in Zurich, Switzerland. In 1992, Dr. Stadtfeld accepted a position as visiting professor at the Rochester Institute of Technology. From 1994 until 2002, he worked for The Gleason Works in Rochester, New York—first as director of R&D and then as vice president of R&D. After an absence from Gleason between 2002 to 2005, when Dr. Stadtfeld established a gear research company in Germany and taught gear technology as a professor at the University of Ilmenau, he returned to the Gleason Corporation, where he holds today the position of vice president-bevel gear technology and R&D. Dr. Stadtfeld has published more than 200 technical papers and eight books on bevel gear technology. He holds more than 40 international patents on gear design and gear process, as well as tools and machines.*

Design

If two axes are positioned in space and the task is to transmit motion and torque between them using some kind of gears, the following cases are commonly known:

- Axes are parallel → cylindrical gears (line contact)
- Axes intersect under an angle → bevel gears (line contact)
- Axes cross under an angle → crossed helical gears (point contact)
- Axes cross under an angle (mostly 90°) → worm gear drives (line contact)
- Axes cross under any angle → hypoid gears (line contact)

The axes of spiral bevel gears in most cases intersect under an angle of 90°. This so-called shaft angle can be larger or smaller than 90°; however, the axes always intersect, which means they have, at their crossing point, no offset between them (*Author's Note: see upcoming chapter on "hypoid gears"*). The pitch surfaces are cones calculated with the following formula:

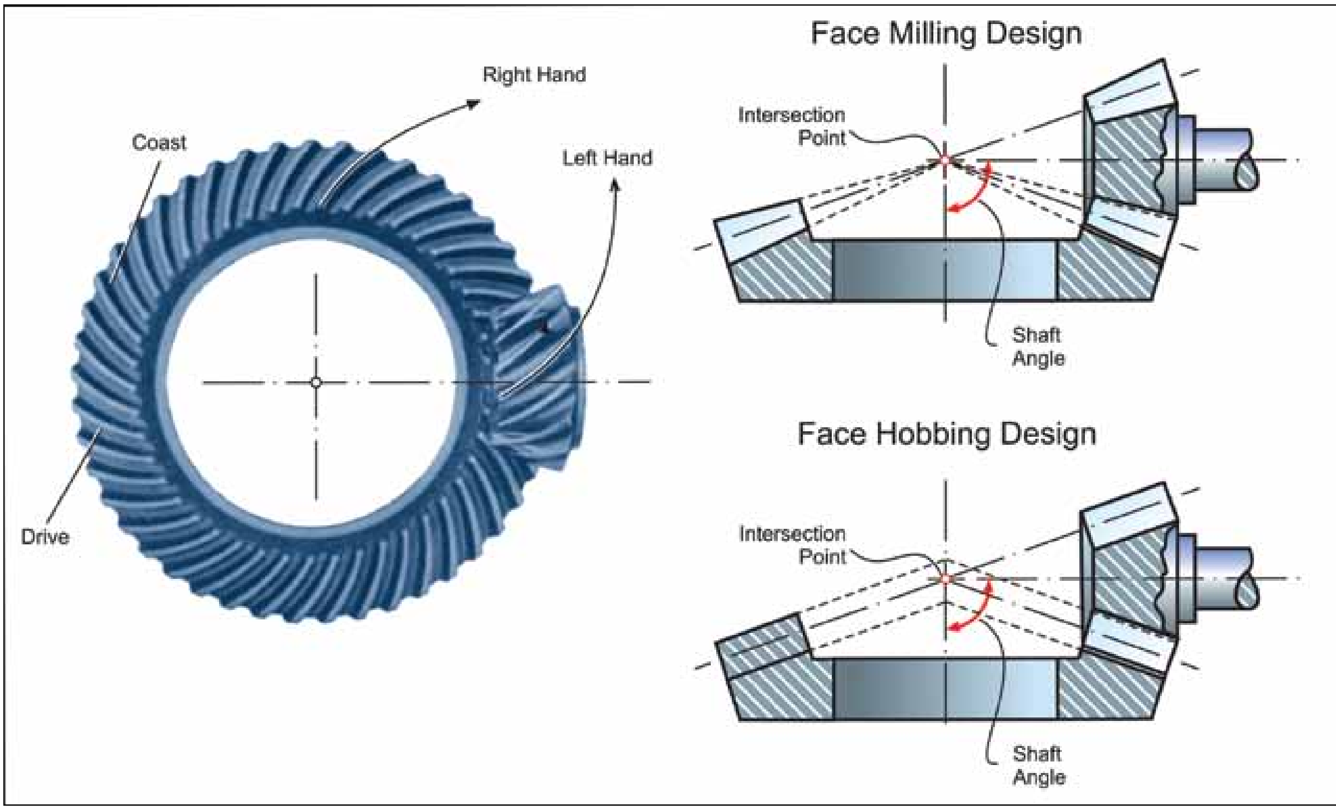

Figure 1—Spiral bevel gear geometry.

$$z_1/z_2 = \sin\gamma_1/\sin\gamma_2$$

$$\Sigma = \gamma_1 + \gamma_2$$

in case of $\Sigma = 90°$:

$$\gamma_1 = \arctan(z_1/z_2)$$
$$\gamma_2 = 90° - \gamma_1$$

where:

z_1 Number of pinion teeth
z_2 Number of gear teeth
γ_1 Pinion pitch angle
Σ Shaft angle
γ_2 Gear pitch angle

Spiral bevel gears have a parallel-depth profile along the face width if they are manufactured in the continuous face hobbing process or a tapered-depth profile along the face width if the manufacturing is done using the single-indexing face milling process.

Spiral bevel gear teeth follow in the face width direction a curve on the conical gear and pinion body that lies under an angle to a cone element (spiral angle). The tooth lead function in the face width direction—if unrolled into a plane—is an epicycloid or a circle, depending on the manufacturing method. The tooth profile is a spherical involute in a parallel-depth tooth system; it is an octoid in a tapered-depth tooth system. The tooth form with a spherical involute will result in line contact between two mating flanks in each angular position if no crowning has been applied. With an octoid, there will be an initial, "natural-profile" crowning and, depending on the machining set-up, some flank twist. Both effects are utilized with certain corrective machine settings in order to generate the desired crowning (*see also "General Explanation of Theoretical Bevel Gear Analysis"*).

The photo of a spiral bevel gear set in Figure 1 explains the definition of right-hand and left-hand spiral direction, and indicates the coast- and drive-side gear flanks. The cross-sectional drawings to the right in Figure 1 illustrate the blank design for face milling on top (tapered-depth teeth) and face hobbing design at the bottom (parallel-depth teeth).

Analysis

Since the mentioned distortions in tapered-depth tooth systems are detected by comparison with conjugate mating flanks, it is possible to define potential contact lines that would apply if the distortions are removed or if load-affected deflections allow for a contact spread. In order to allow for deflections of tooth surfaces, shafts, bearings and gearbox housing without unwanted edge contact, a crowning in face width and profile direction is applied. A

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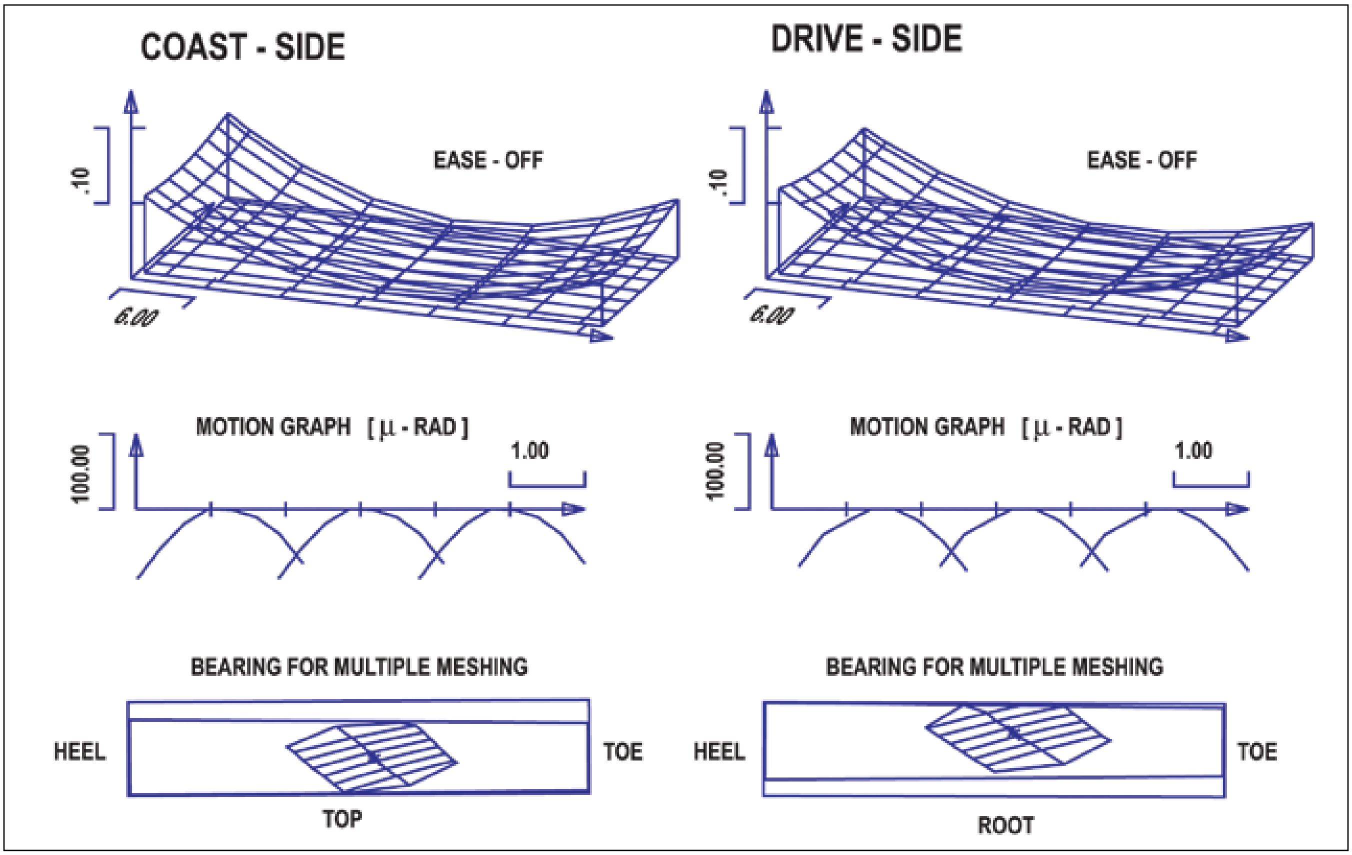

Figure 2—Tooth contact analysis of a spiral bevel gear set.

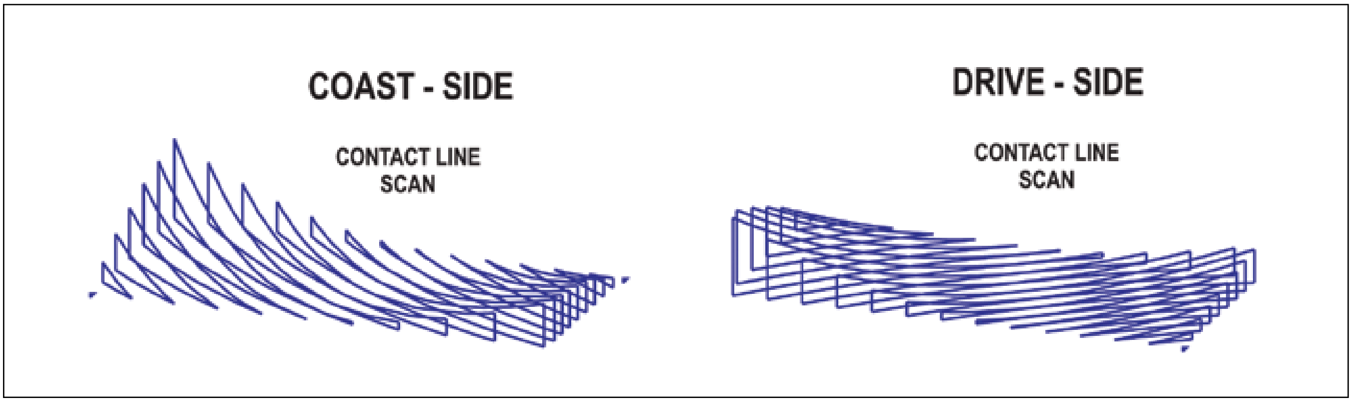

Figure 3—Contact-line scan of a spiral bevel gear set.

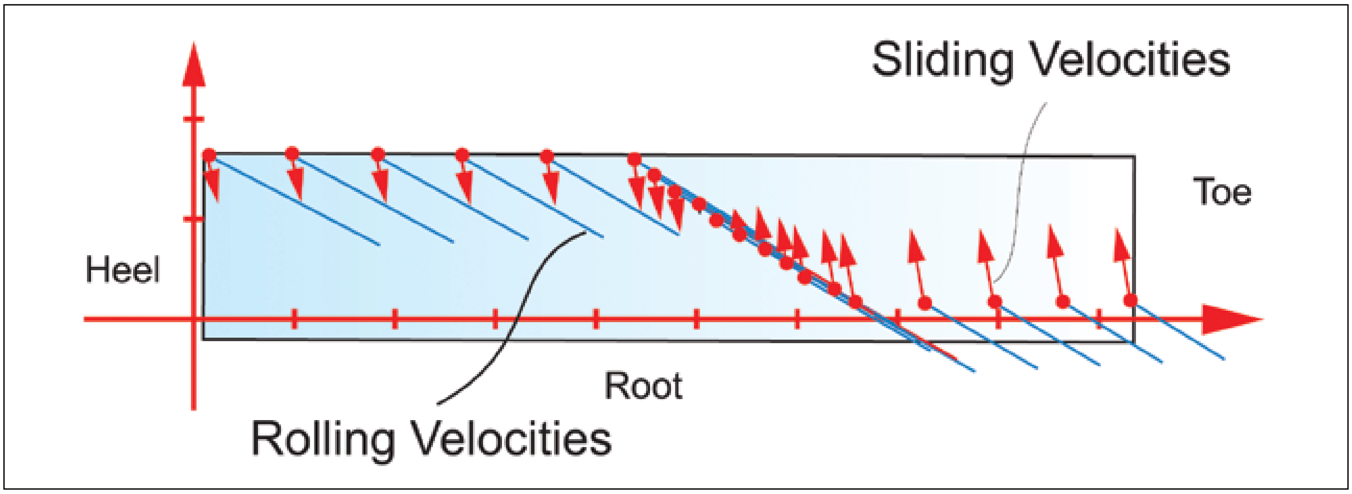

Figure 4—Rolling- and sliding-velocities of a spiral bevel gear set in path-of-contact points.

theoretical tooth contact analysis (TCA) previous to gear manufacturing can be performed in order to observe the effect of the crowning in connection with the basic characteristic of the particular gear set. This also allows the possibility to return to the basic dimensions in order to optimize them if the analysis results show any deficiencies. Figure 2 shows the result of a TCA of a typical spiral bevel gear set.

The two columns in Figure 2 represent the analysis results of the two mating flank combinations (*see also "General Explanation of Theoretical Bevel Gear Analysis*"). The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs in Figure 2 have a combination of length- and profile-crowning to the extent that a clearance along the boundary of the teeth is established.

Below each ease-off, the motion transmission graphs of the particular mating flank pair are shown. The motion transmission graphs show the angular variation of the driven gear in the case of a pinion that rotates with a constant, angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning in order to prevent edge contact and to allow for load-affected deflections, the crowning in turn causes proportional amounts of angular motion variation of about 60 micro radians in this example.

At the bottom of Figure 2, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero-load and a virtual-marking-compound film of 6 μm thickness. This basically duplicates the tooth contact by rolling the real version of the analyzed gear set under light load on a roll tester, while the gear member is coated with a marking-compound layer of about 6 μm thickness. The contact lines lie under an angle to the face width direction, depending basically on the spiral angle. The path-of-contact connects the beginning and end of meshing, and its orientation is nearly perpendicular to the contact lines.

The crowning reflected in the ease-off results in a located contact zone within the boundaries of the gear tooth. A smaller tooth contact area generally results from large magnitudes in the ease-off and motion graph, and vice versa.

Figure 3 shows 20 discrete, potential contact lines with their individual crowning amounts along their length (contact-line scan). The gap geometry in contact-line direction can be influenced by a change in ease-off topography and optimized regarding the gap kinematic cases (*see also "General Explanation of Theoretical Bevel Gear Analysis," Fig. 8*). The gap geometry perpendicular to the contact-line direction (not exactly the same as the path-of-contact direction) does not significantly depend on the ease-off topography; rather, it is mainly dominated by the geometry of the mating tooth profiles.

Figure 4 shows the sliding- and rolling-velocity vectors of a typical spiral bevel gear set for each path-of-contact point for the 20 discussed roll positions; each vector is projected to the tangential plane at the point-of-origin of the vector. The velocity vectors are drawn inside the gear tooth projection plane. The points-of-origin of both—rolling- and sliding-velocity vectors—are grouped along the path-of-contact, which is found as the connection of the minima of the individual lines in the contact-line scan graphic (Fig. 4). The velocity vectors can be separated in a component in contact-line direction and a component perpendicular to that in order to investigate the hydrodynamic lubrication properties. This is accomplished by utilizing the information from the contact-line scan (curvature and curvature change) and the tooth surface curvatures perpendicular to the contact-line direction (*see also "General Explanation of Theoretical Bevel Gear Analysis," Figure 8, cases 1–6*).

Regarding the discussed spiral bevel gear set, the sliding-velocity vectors are basically profile-oriented. In the top area (left, Fig. 4), the sliding vectors point down. Moving along the path-of-contact from top to bottom (left-to-right, Fig. 4), the sliding velocity reduces its magnitude and reaches a magnitude of zero at the pitch line. Below the pitch line, the sliding velocity develops, growing positive magnitudes (towards the bottom of the gear tooth). The maximal magnitude of the sliding velocities (top-versus-root) is a result of the distance from the pitch line. As such, the distance between the lowest active flank line to the pitch line is larger than the distance from the pitch line to the top. The rolling-velocity vectors point to the bottom and to the right, and all have basically the same orientation. The orientation is a result of the spiral angle (zero spiral angle delivers profile-oriented sliding). The larger magnitude of the rolling velocity at the heel-top (left) is caused by the larger circumferential speed at the outer diameter.

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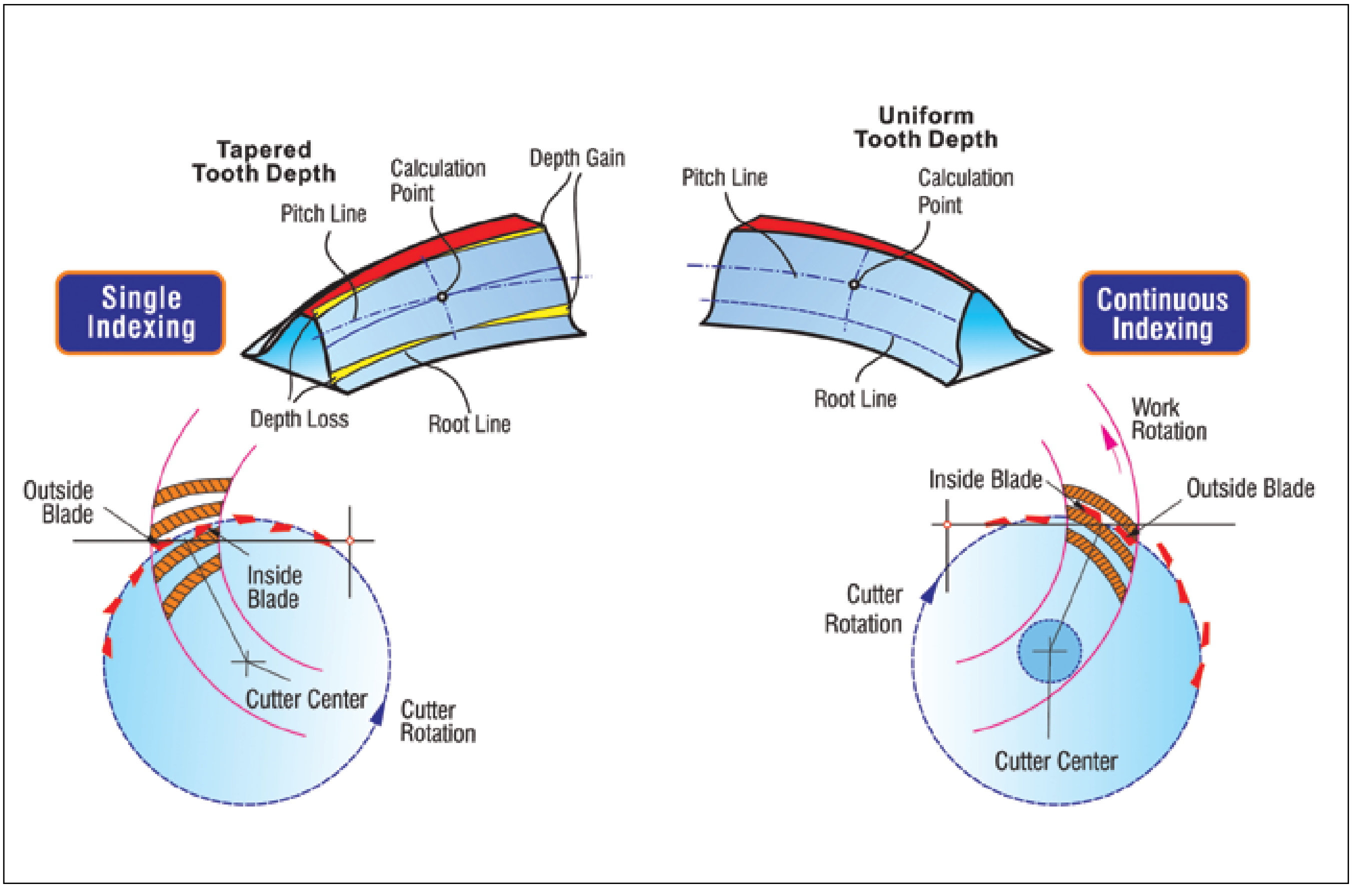

Figure 5—Left: face milling; Right: face hobbing.

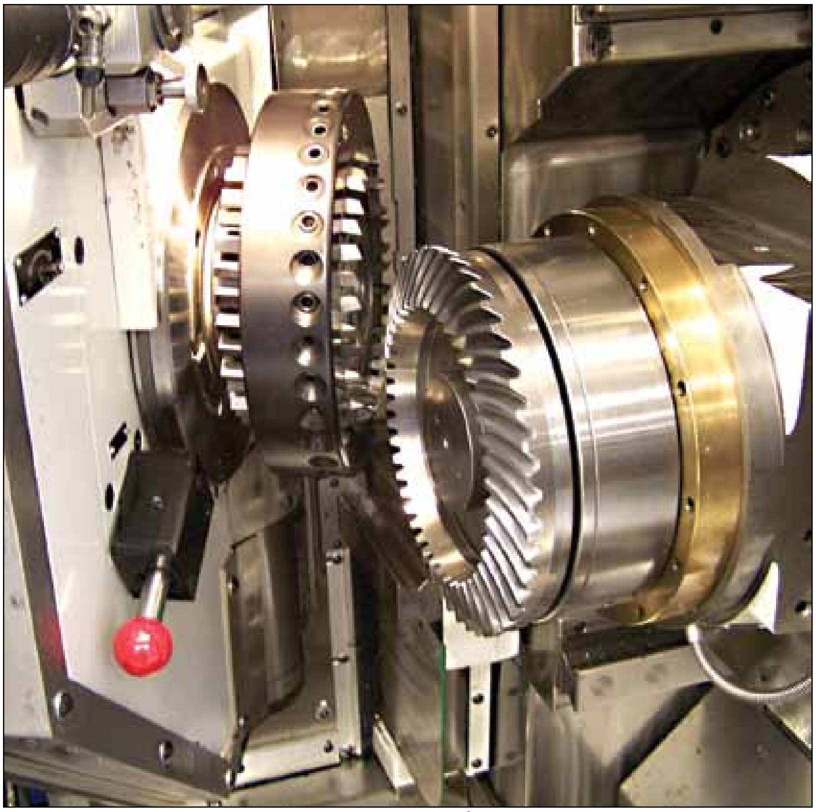

Figure 6—Spiral bevel gear cutting with face hobbing cutter (continuous process).

It thus becomes evident that a complex gap-and-velocity evaluation of a variety of discrete points considering the two principal curvature directions is important in spiral bevel gears to attain reliable results about the lubrication mechanics.

Manufacturing

Spiral bevel gears are manufactured in a continuous-indexing face hobbing process or in a single-indexing face milling process. In the face milling process, the blades are oriented around a circle and pass through one slot (while they plunge or generate the flanks of that particular slot), as illustrated in Figure 5, left. The work is not performing any indexing rotation. At the blade tip and in equidistant planes—normal to the cutter head axis—the slot width produced has a constant width between toe and heel. In order to achieve a proportionally changing slot width (and tooth thickness), the root line of face milled bevel gears is inclined versus the pitch line (Fig. 5, left). This modification has to be implemented in both members, which is why the face angle requires the same modification as the root angle of the mating member.

In face hobbing (Fig. 5, above), a group of mostly one inside and one outside blade passes through one slot, while the work rotates with:

$\omega_{Work} = \omega_{Cutter}$ • (Number of Cutter Blade Groups) / (Number of Work Teeth)

The blades in one group are positioned along a spiral where the sum of the blade groups is oriented around a circle with equal distance to the cutter head center. Due to the described kinematic, the flank lines of the outer- and inner-flank are epicycloids that divide slot width and tooth thickness in equal fractions of the circumference—at any point along the face width. The result is a "natural" slot width taper proportional to the distance from the pitch apex. A root angle modification is not required—or useful—due to the already-existing perfect fit of mating teeth and slots.

Figure 6 is a photo of the view into the work chamber of a free-form bevel and hypoid gear cutting machine during high-speed dry-cutting of a spiral bevel gear. The face cutter head has coated carbide stick blades which are arranged in blade groups for a continuous face hobbing process.

Hard-finishing after heat treatment of face milled spiral bevel gears is generally done by grinding. The grinding wheel resembles the cutter head geometry, while the grinding machine uses the same set-up geometry and kinematics as the cutting machine for the previous soft machining. Hard-finishing of face hobbed spiral bevel gears is generally done by lapping. Pinion and gear are rolled under light torque while a lapping compound of a silicone-carbide oil mixture is present between the flanks. Lapping causes abrasive grain to become imbedded in the flank surfaces, which might lead to several problems such as wear, temperature and lowered efficiency.

Application

Most spiral bevel gears to be used in power transmissions are manufactured from carburized steel and undergo case-hardening to a surface hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. Because of the higher pinion revolutions, it is advisable to give the pinion a higher hardness than the ring gear (e.g., pinion 62 HRC, gear 59 HRC).

Regarding surface durability, spiral bevel gears are very similar to helical gears. At the pitch line, the sliding velocity is zero and the rolling velocity under certain loads cannot maintain a surface-separating lubrication film. The result is pitting along the pitch line that can destroy the tooth surfaces and even lead to tooth flank fracture. However, it is possible that the pitting can be stabilized if the damage-causing condition is not prevalent in the duty cycle. Figure 7 is a photograph of typical pitch-line-pitting on a spiral bevel ring gear flank surface. This effect is not as common as in straight bevel gears due to the more favorable direction of the rolling velocities.

Spiral bevel gears have axial forces that can be calculated by applying a normal-force vector at the position of the mean point at each member (*see also "General Explanation of Theoretical Bevel Gear Analysis"*). The force-vector normal to the transmitting flank is separated in its X, Y and Z components (Fig. 8).

The relationship in Figure 8 leads to the following formulas, which can be used to calculate bearing force components in a Cartesian coordinate system and assign them to the bearing load calculation in a CAD system:

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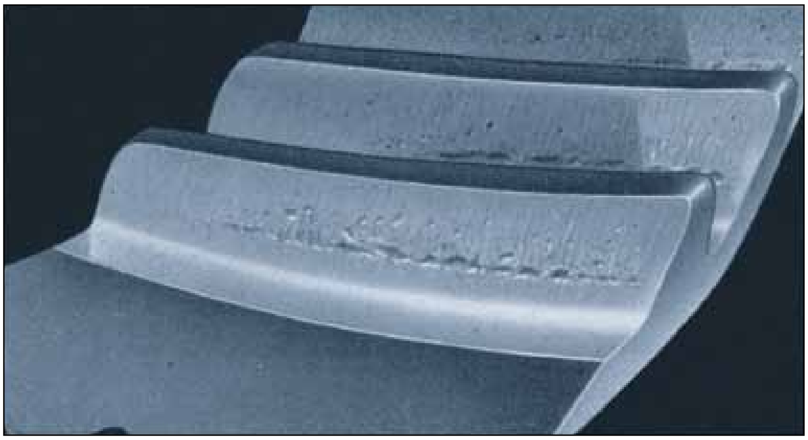

Figure 7—Pitch-line pitting on a spiral bevel gear surface.

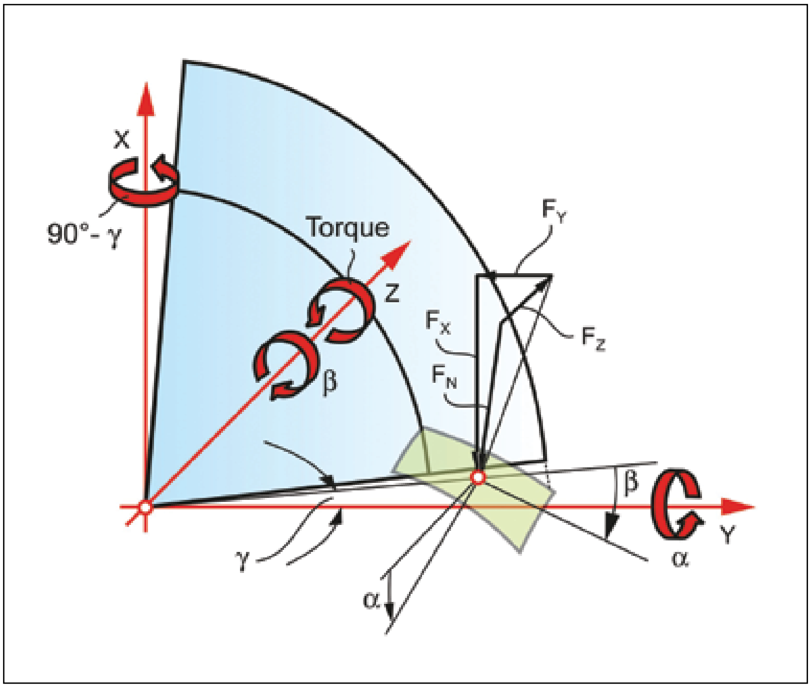

Figure 8—Force diagram for calculation of bearing loads.

$$F_x = -T / (A_m \cdot \sin\gamma)$$

$$F_y = -T \cdot (\sin\gamma \cdot \sin\beta \cdot \cos\alpha + \cos\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

$$F_z = -T \cdot (\cos\gamma \cdot \sin\beta \cdot \cos\alpha - \sin\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\beta \cdot \cos\alpha)$$

where:

T	Torque of observed member
A_m	Mean cone distance
γ	Pitch angle
β	Spiral angle
α	Pressure angle
F_x, F_y, F_z	Bearing load force components

The bearing force calculation formulas are based on the assumption that one pair of teeth transmits the torque with one normal force vector in the mean point of the flank pair. The results are good approximations that reflect the real bearing loads for multiple teeth meshing within an acceptable tolerance. A precise calculation is, for example, possible with the Gleason bevel and hypoid gear software.

The introduction of spiral angles lead to a face-contact-ratio that in turn reduces the tooth root thickness. The tooth thickness counts are squared in a simplified root bending stress calculation using a deflection beam analogy. The thickness reduces by: cos (spiral angle). The face contact ratio increases are simplified by: tan (spiral angle). Those formulas applied to a numerical example will always show an advantage of the spiral angle in root bending strength. However, the crowning of real bevel gears will always cause one pair of teeth to transmit an over-proportionate high share of the load, while the one or two additionally involved tooth pairs will only share a small percentage of the load. Finite element calculations can be useful in finding the optimal spiral angle for maximal root strength. In general, bevel gears not ground or lapped after heat treatment show the highest root strength and the lowest spiral angles. This explains why in those cases the straight bevel gear remains the bevel gear of choice.

Spiral bevel gears can operate with regular transmission oil or, in case of low RPMs, with a grease filling. In cases of circumferential speeds above 10 m/min, a sump lubrication with regular transmission oil is recommended. The oil level has to cover the face width of those teeth lowest in the sump. More oil causes foaming, cavitations and unnecessary energy loss; there is no requirement for any lubrication additives. The preferred operating direction of spiral bevel gears is the drive-side, where the convex gear flank and the concave pinion flank mesh. In the drive-direction (Fig. 8), the forces between the two mating members bend the pinion sideways and axially, away from the gear, generating the most backlash. Coast-side operation reduces the backlash—in extreme cases—to zero, which interrupts any lubricant flank separation and leads to immediate surface damage, often followed by tooth fracture.

References

Stadtfeld, H.J. "Handbook of Bevel and Hypoid Gears," Rochester Institute of Engineering, Rochester, New York, March 1993.

Hotchkiss, R.G. "The Theory of Modern Bevel Gear Manufacturing," AGMA Gear Manufacturing Symposium, Cincinnati, Ohio, April 1989.

(*Ed.'s Note: Next issue—"Face Gears"*)