Practical Considerations for the Use of Double-Flank Testing for the Manufacturing Control of Gearing - Part I

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Part I of this paper describes the theory behind double-flank composite inspection, detailing the apparatus used, the various measurements that can be achieved using it, the calculations involved and their interpretation. Part II, which will appear in the next issue, includes a discussion of the practical application of double-flank composite inspection, especially for large-volume operations. Part II covers statistical techniques that can be used in conjunction with double-flank composite inspection, as well as an in-depth analysis of gage R&R for this technique.

Double-flank composite inspection (DFCI) is a valuable technique that can functionally provide quality control results of test gears quickly and easily during manufacturing. However, the successful use of DFCI requires careful planning from product design, through master gear design and gage control methods in order to achieve the desired result in an application. This document explains the practical considerations in the use of double-flank testing for the manufacturing control of spur, helical and crossed-axis helical gearing.

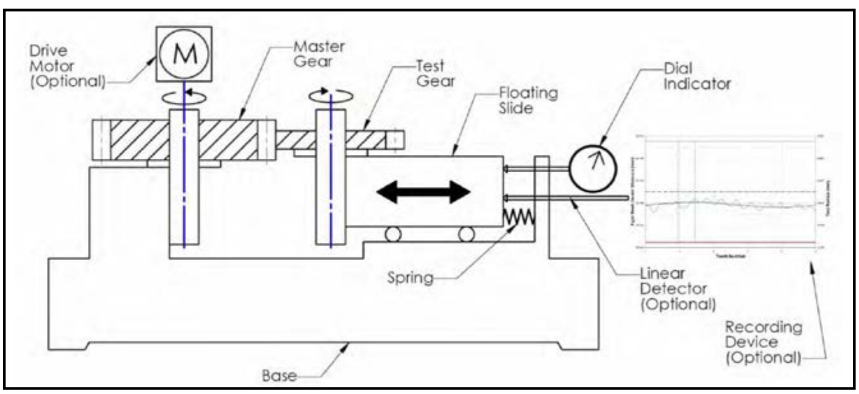

Figure 1 General arrangement of a double-flank composite tester.

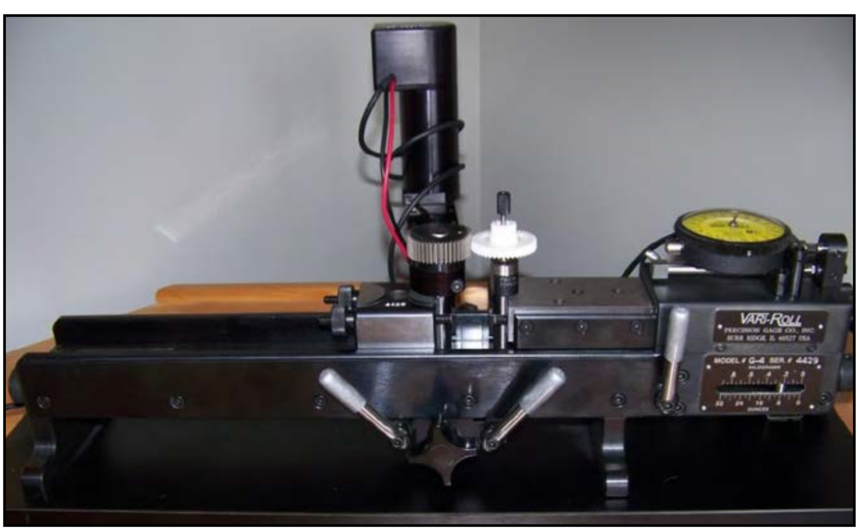

Figure 2 An actual double-flank composite tester in tight mesh (courtesy of Web Gear Services Ltd.).

Description of Double-Flank Composite Inspection

Double-flank testing is a technique that has been used in the gear industry to identify potential manufacturing defects in the design intent of the gear. It is a practical, fast and effective screening tool that can identify when the gear manufacturing process has deviated from an ideal condition that could result in a loss of conjugate action, a change in backlash, or an unwanted noise in a gear mesh.

The test itself involves an apparatus of general layout as shown in Figure 1 and of actual configuration as shown in Figure 2. A master gear of known precision is mounted on a fixed base with only rotational freedom. The test gear is mounted on a floating slide mechanism that allows rotation of the test gear and movement along an axis between the line of centers of the master and test gear. A spring (with a preset force) pushes the floating slide, resulting in zero-backlash, double-flank contact (i.e., on both left and right flanks) on both the test gear and the master gear.

As the master gear is rotated (by hand or by motor), the test gear follows. Involute theory dictates that perfectly formed teeth will prevent any movement of the floating slide between the line-of-centers. However, since no gear can be manufactured in absolutely perfect condition, there will always be some movement of the floating slide as the gears rotate. The magnitude of this movement is measured with either a mechanical indicator or electronic detector that contacts the slide mechanism. If the measuring instrument is calibrated to an actual distance reading between the centers of the gears, then an actual tight mesh center distance result can be obtained.

In order to maintain accuracy in the measurement, intimate double-flank contact must be maintained at all times. Therefore the selection of the pre-set spring force and the speed-of-rotation of the gears should be given careful consideration to limit measurement errors.

The pre-set force may need to be selected specifically for the test gear's design, taking into account the material's ability to resist deformation under load (i.e., plastic gears), where a large pre-set force may distort the gear into conformity. In addition, if there is excessive resistance coming from the mounting of either the master gear on its mandrel, or, more commonly, of the test gear on its mandrel, then a low, pre-set spring force will result in separation of the two gears out of double-flank contact, creating an error in the measured values. The correct pre-set spring force is the minimum force needed to maintain continuous, double-flank contact without distorting the test gear.

The speed of rotation of the gears should be selected by taking into account the natural response of the mechanical and electrical (if so equipped) elements of the tester. It is generally recommended that at least 20 data points per tooth are available in the data set collected to ensure sufficient sensitivity of the results.

The types of measurements that can be made on a double-flank tester are shown in Figure 3 and will be explained in the following sections.

Total composite variation. Measurement of the total composite variation (error) is the difference between the maximum and minimum indicator (or linear detector) readings during one rotation cycle of the test gear (Fig. 3). The total composite variation result includes effects of runout in the gear, plus anomalies in the tooth pitches, profiles and helix. It also reports the total effect in terms of this linear change as variation in tight mesh center distance. It is not possible to accurately establish the magnitude of each individual effect on the total composite variation using the double-flank test alone. Hence, the double-flank test is very good at screening production quality and flagging potential errors, but the results may not identify the specific nature of the problem. Other tests, (such as analytical inspections) would need to be performed in order to more closely identify the exact nature of the defect.

Tooth-to-tooth composite variation. The tooth-to-tooth composite variation (error) is defined as the greatest deviation indicator reading within a single, circular tooth pitch. This result is based on the worst tooth on the entire gear. The gear tested in Figure 3 shows this to be in the zone around tooth 3.

As the number of teeth in a gear becomes smaller, the ratio of the tooth-to-tooth error to total composite error generally increases. In the extreme condition, a single-start worm (i.e., one tooth) will have a tooth-to-tooth composite error equivalent to its total composite error. As the number of teeth increases, the tooth-to-tooth results are considered to be a better indicator of anomalies in the tooth pitch, profile and helix.

Errors in gear pressure angle will result in a repeated pattern of arches similar to that shown between teeth 5 and 7 in Figure 3. Use of tooth-to-tooth test limits also helps to control burrs and nicks in gears that are not always detected by analytical measurement techniques.

Tight mesh center distance. One of the most powerful uses of the double-flank test is to measure and control gears not only for total composite variation, but also for tight mesh center distance. The ability to measure this variation gives the necessary insight to control both tooth size and composite parameters simultaneously. Since AGMA and ISO accuracy standards do not include the effect of tooth size, tight mesh center distance is not discussed in those standards. However, the effectiveness of this tooth size measurement should not be overlooked when evaluating backlash in a gear mesh.

Tight mesh center distance, as shown in Figure 3, can be measured if an additional calibration step is performed during the

Double Flank Inspection of Sample Part

Part #	Sample #	Result	Specification			Result			
3020568	14		Mid Value	Max Value	Min Value	Mid Value	Max Value	Min Value	
Total Composite (mm)		Accept		0.100			0.022		Accept
Runout (mm)		Ref.		0.080			0.008		
Tooth to Tooth Eccentricity Included (mm)		Accept		0.046			0.016		
Tight Mesh Center Distance (mm)		Accept	23.125	23.202	23.048	23.104	23.115	23.093	
Test Radius (mm)		Accept	3.850	3.928	3.773	3.829	3.840	3.818	

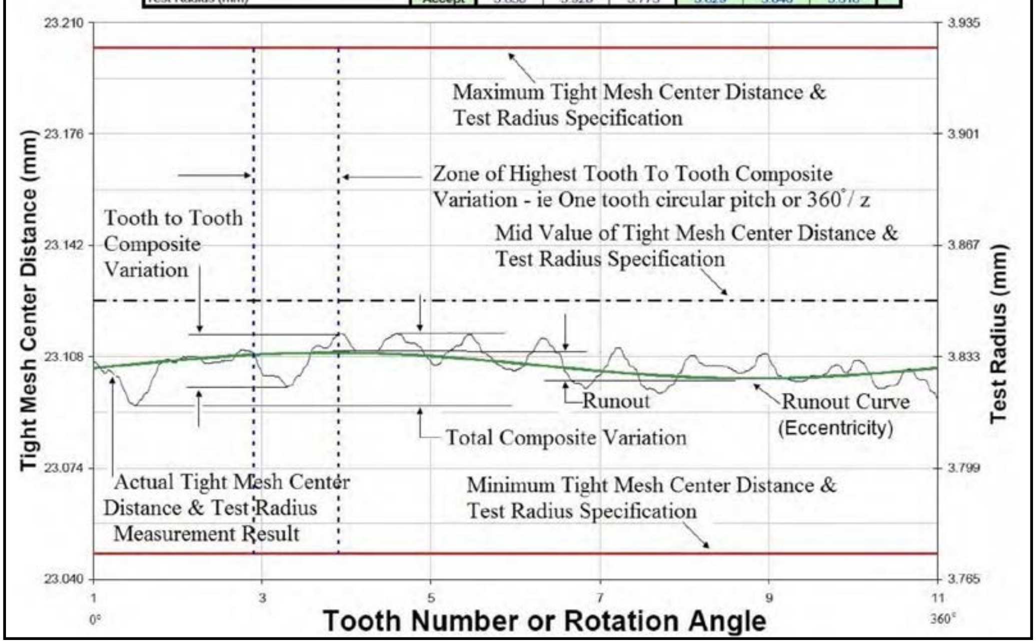

Figure 3 Double-flank inspection report (courtesy of Web Gear Services Ltd.).

Figure 4 Calibration for tight mesh center distance (courtesy of Web Gear Services Ltd.).

gage set-up. If the dial indicator or detector is calibrated to a known center distance reading between the spindles prior to measurement (Fig. 4), then the actual tight mesh center distance will be the difference between the calibrated value and the rolling variation. Maximum and minimum test limits must be established for tight mesh center distance. These limits are shown by the horizontal red lines on the chart in Figure 3. Every portion of the actual tight mesh center distance measurement must be within the minimum and maximum boundaries for a test gear to be acceptable. When properly calculated through the gear design process, adherence to these boundaries will ensure maximum and minimum operational backlash levels in the gear mesh.

The Calculation of Tight Mesh Center Distance Limits for Spur, Helical, Crossed-Axis Helical and Worm Gears

In order to establish the maximum and minimum tight mesh center distance limits for external or internal spur, helical, crossed-axis helical and worm gears, the following gear design data must be available:

m_n Is the normal module of the system, mm
z_w Is the number of teeth on the test gear
Note: For external gears, use a positive value for z_w, and for internal gears, use a negative value.
z_3 Is the number of teeth on the master gear
β_w Is the helix angle of the test gear, degrees or radians
β_3 Is the helix angle of the master gear, degrees or radians
Note: For spur gears, $\beta_w = \beta_3 = 0$, degrees or radians
Note: For right-hand helical gears, worms and worm gears, use a positive value for the helix angle. For left-hand helical gears, worms and worm gears, use a negative value for the helix angle.
α_n Is the normal pressure angle for the mesh, degrees or radians
$s_{nw\,max}$ Is the maximum normal circular tooth thickness of test gear, mm
$s_{nw\,min}$ Is the minimum normal circular tooth thickness of test gear, mm
s_{n3} Is the normal circular tooth thickness of master gear, mm
F_{idTw} Is the total composite tolerance for the test gear, mm

The calculation procedure that follows is sufficiently general to account for gears with non-standard tooth thicknesses and heavily modified profiles.

Step 1. ***Calculation of the standard center distance, a.*** The standard center distance, a, of an external or internal test gear when meshed with an external master gear on a double-flank tester is:

$$a = \frac{z_w}{|z_w|} \frac{m_n}{2} \left[\frac{z_w}{\cos\beta_w} + \frac{z_3}{\cos\beta_3} \right] \tag{1}$$

where
a is the standard center distance between the test gear and the master gear, mm.
Note: These equations are sufficiently general to account for external or internal spur, helical, crossed-axis helical and worm gears.

Parallel-axis double-flank tight mesh center distance limits. The following additional steps are needed for the calculation of tight mesh center distance test limits for external and internal parallel-axis spur and helical gear meshes.

Step 2. ***Calculation of the transverse pressure angle,*** α_t. The transverse pressure angle, α_t, for the mesh on the double-flank tester is:

$$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta_w}\right) \tag{2}$$

where
α_t is the transverse pressure angle for the mesh in degrees or radians
Note: For spur meshes $\alpha_t = \alpha_n$

Step 3. ***Calculation of the maximum tight mesh center distance limit, $a_{d\,max}$.*** The maximum tight mesh center distance, $a_{d\,max}$, of the test gear with the master gear for a spur and parallel-axis helical double-flank mesh is:

$$a_{d\,max} = \frac{a\cos\alpha_t}{\cos\left\{\mathrm{inv}^{-1}\left[\mathrm{inv}\,\alpha_t - \frac{z_w}{|z_w|}\left(\frac{\pi\,m_n - s_{n3} - s_{nw\,max}}{2\,a\cos\beta_w}\right)\right]\right\}} + \frac{z_w}{|z_w|}\,\frac{F_{idTw}}{2} \quad (3)$$

where

$a_{d\,max}$ Is the maximum tight mesh center distance of the test gear with the master gear, mm

inv φ Is the involute function and inv $\varphi = \tan\varphi - \varphi$ with φ expressed, radians

$\mathrm{inv}^{-1}\,x$ Is the inverse involute function where $x = \mathrm{inv}\,\varphi = \tan\varphi - \varphi$.

Therefore, the result of the function $\mathrm{inv}^{-1}\,x = \varphi$, where φ is an angle.

For more information on the calculation of this function, see AGMA 930-A05, Annex E (Ref. 1).

Step 4. ***Calculation of the minimum tight mesh center distance limit, $a_{d\,min}$.*** The minimum tight mesh center distance, $a_{d\,min}$, of the test gear with the master gear for a spur and parallel-axis helical double-flank mesh is:

$$a_{d\,min} = \frac{a\cos\alpha_t}{\cos\left\{\mathrm{inv}^{-1}\left[\mathrm{inv}\,\alpha_t - \frac{z_w}{|z_w|}\left(\frac{\pi\,m_n - s_{n3} - s_{nw\,max}}{2\,a\cos\beta_w}\right)\right]\right\}} + \frac{z_w}{|z_w|}\,\frac{F_{idTw}}{2} \quad (4)$$

where

$a_{d\,min}$ is the minimum tight mesh center distance of the test gear with the master gear, mm.

Note: For internal gears, Equation 3 will actually give a minimum value result and Equation 4 will give the maximum value result.

When specifying tight mesh center distance limits, it is important to also include a definition of the master gear's number of teeth and normal circular tooth thickness upon which the tight mesh center distance limits are based.

Crossed-axis helical and worm gear double-flank tight mesh center distance limits. The calculation for crossed-axis and worm gear double-flank meshes differs from other cylindrical gear meshes because the gears "see" each other in a way that is analogous to two racks in mesh, as opposed to two involute gears in mesh. Crossed-axis helical gears include the case where the driving member is a master worm (Fig. 5) used to measure a helical gear at right angles. The calculations presented here are also sufficiently general to include the scenario where two helical gears mesh at shaft angles other than 90°, as well as a situation (Fig. 6) where a plastic test worm is meshed against a master spur gear. In the case of worm gears, the master gear would actually be a cylindrical worm mounted at a right angle to the worm gear.

Note: The formulas presented here allow for meshing on the double-flank tester at any shaft angle.

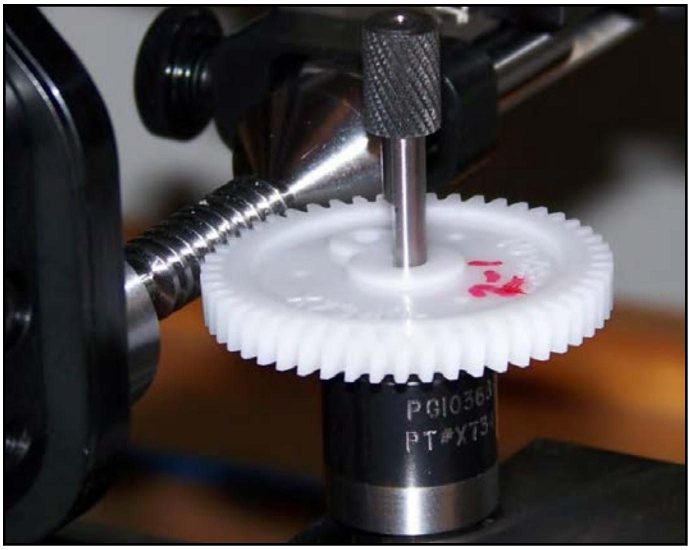

Figure 5 Master worm in double-flank mesh with a plastic helical gear (courtesy of Web Gear Services Ltd.).

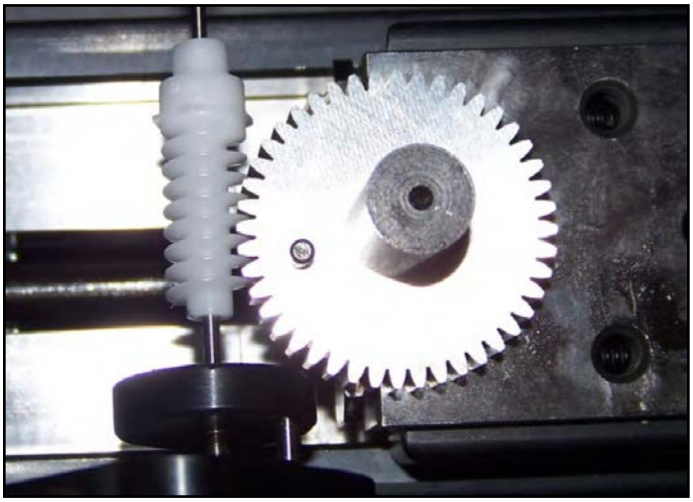

Figure 6 Plastic test worm in double-flank mesh with a master spur gear at an offset shaft angle (courtesy of Web Gear Services Ltd.).

Step 2. ***Calculation of the meshing shaft angle on the double-flank tester, ψ.*** The shaft angle ψ, on the double-flank tester for a given crossed-axis helical gear or worm gear mesh, is calculated as follows:

$$\psi = \beta_w + \beta_3 \quad (5)$$

where

ψ is the meshing shaft angle on the double-flank tester, degrees or radians.

Note: *Careful adherence to the sign of each of the helix angles (i.e., right- and left-hand) is crucial in this calculation.*

Step 3. ***Calculation of the maximum tight mesh center distance limit, $a_{d\,max}$.*** The maximum tight mesh center distance, $a_{d\,max}$, of the test gear with the master gear for a crossed-axis helical or worm gear double-flank mesh is:

$$a_{d\,max} = \frac{(s_{n3} - s_{nw\,max} - \pi\,m_n)}{2\tan\alpha_n} + a + \frac{F_{idTw}}{2} \quad (6)$$

Step 4. ***Calculation of the minimum tight mesh center distance limit, $a_{d\,min}$.*** The minimum tight mesh center distance, a_d

Table 1 Numerical example of the effect of master gear normal circular tooth thickness and number of teeth on the test radius

	Master gear A	Test gear	Master gear B
Module, mm	1.0	1.0	1.0
Number of teeth	38	20	50
Pressure angle, degrees	20	20	20
Total composite tolerance µm	-	96	-
Normal circular tooth thickness, mm	50% of circular pitch 1.5708 ±0.000 mm	40% of circular pitch 1.2566 ±0.020 mm	60% of circular pitch 1.8850 ± 0.000 mm
Test radius limits, mm	9.539 ±0.080		
		9.568 ± 0.075	

$_{min}$, of the test gear with the master gear for a spur and parallel axis helical double-flank mesh is:

$$a_{d\,min} = \frac{(s_{n3} - s_{nw\,max} - \pi\, m_n)}{2\tan\alpha_n} + a + \frac{F_{idTw}}{2} \quad (7)$$

Some software programs incorrectly calculate tight mesh center distance for crossed-axis helical gears and worm gears using the parallel axis approach in the previous section, instead of this method. If the sum of the normal circular tooth thicknesses between the master gear and test gear are close to the normal pitch, the calculation procedure detailed in the previous section may present results that are close to the actual values. However, as the sum of these tooth thicknesses deviates from the normal pitch, calculation error becomes increasingly significant. As such, the method shown in this section is always preferred for crossed-axis helical and worm gears.

Test radius. Test radius can be measured on a double-flank tester, as is demonstrated in Figure 3. Test radius is similar to tight mesh center distance in terms of set-up and calibration. However, it differs in that it is calculated as the tight mesh center distance of the mesh, minus the test radius of the master gear, as shown in the following equation:

$$R_{rw} = a_d - \frac{z_w}{|z_w|} R_{r3} \quad (8)$$

where

- R_{rw} Is the instantaneous test radius of the external or internal test gear (i.e., working gear), mm
- a_d Is the instantaneous tight mesh center distance of the mesh on the double-flank tester, mm
- R_{r3} Is the test radius of the master gear, as seen by a rack (see next section for further explanation), mm

Thus the scale between the left-side vertical axis in Figure 3 and right-side vertical axis is shifted by the magnitude of the master gear test radius.

One of the reasons why test radius is specified instead of tight mesh center distance is due to the common misconception that the master gear's number of teeth and its normal circular tooth thickness have no influence on the limits of a test gear's test radius. The assumption is that, regardless of the master used, the test radius limits of a test gear are constant. If this statement were true, it would then obviously be an advantage in circumstances where a manufacturer may have a different master gear than the purchaser. However, in reality, careful analysis of Equations 1–8 shows that there is some difference in the test radius results, depending on the master gear's number of teeth and normal circular tooth thickness. An illustrative example is shown in Table 1, where master gears A and B have different numbers of teeth and normal circular tooth thicknesses, resulting in significantly different test radius limits on the same test gear.

There is therefore no practical advantage in specifying test radius instead of tight mesh center distance. In both cases the master gear's number of teeth and normal circular tooth thicknesses must be defined to make the specification valid. It is common to report either tight mesh center distance or test radius, but not necessarily both. Tight mesh center distance has greater international usage as compared to test radius. Most North American electronic versions of double-flank testers available will report tight mesh center distance and test radius, while European or Asian equipment often does not include test radius results with their equipment.

Test radius of the master gear, R_{r3}. In order to calculate the result in Equation 8, the test radius of the master gear, R_{r3}, must be determined. Unfortunately, there are several methods by which the test radius of a master gear is defined — all having

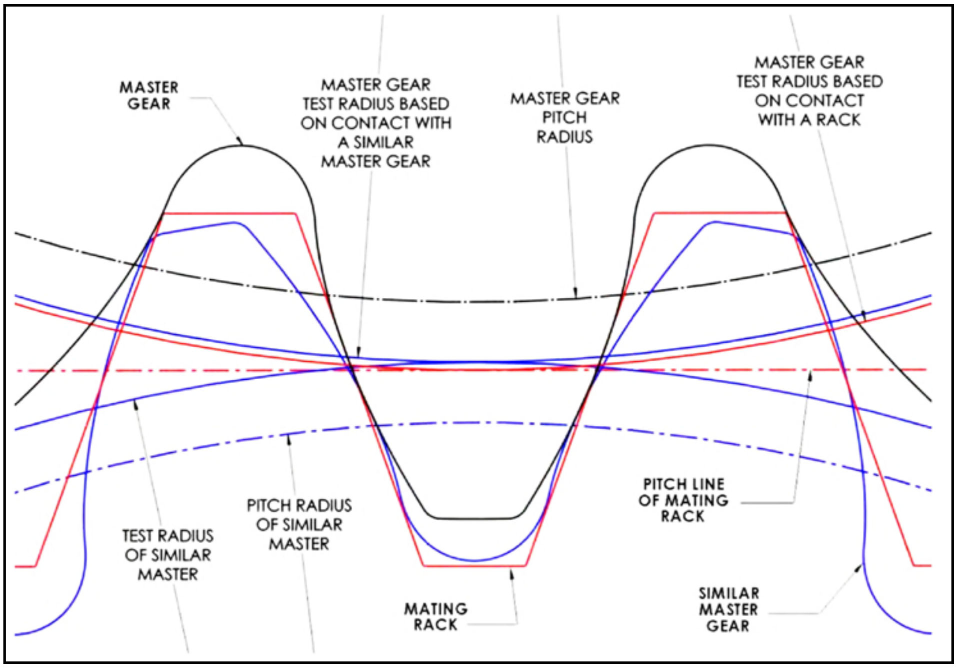

Figure 7 Test radius of a master gear (in black) against a rack (in red) or a similar master (in blue).

potentially different results — thus creating even more confusion in the industry.

The practical issue in the definition is that the test radius may be different, depending on whether the master gear is defined by its action against a rack or itself (Fig. 7), or against another cylindrical gear. Furthermore, if using a definition based on its action against another cylindrical gear, a single master gear may have many different test radius values, depending on the cylindrical gear it mates with. Hence, based on Equation 8, the test radius of a test gear will change, depending on the test radius of the master. Therefore if a master gear has an ambiguous test radius definition, the part gear test radius will also be inherently ambiguous. Specifying tight mesh center distance as opposed to test radius will remove this ambiguity.

However, if test radius must be used, the most common practice to avoid ambiguity is to define the test radius of a master gear by the tight mesh radial distance between the master gear center and the pitch line of a mating rack, whose pitch line is defined as the location where the tooth thickness is equal to its space width. In making such a standardized definition, a master gear will have a single test radius, regardless of the gear it mates with.

The equation for the test radius of a master gear, R_{r3}, as seen by a rack, is as follows:

$$R_{r3} = \frac{m_n z_3}{2\cos\beta_3} + \frac{(s_{n3} - 0.5\,\pi\, m_n)}{2\tan\alpha_n} \quad (9)$$

where

R_{r3} is the test radius of the master gear as seen by a rack, mm.

Test radius limits of a test gear. The test radius of a test gear is related to tight mesh center distance by the following equations:

$$R_{rw\,max} = a_{d\,max} - \frac{z_w}{|z_w|} R_{r3} \quad (10)$$

and

$$R_{rw\,min} = a_{d\,min} - \frac{z_w}{|z_w|} R_{r3} \quad (11)$$

where

$R_{rw\,max}$ Is the maximum test radius limit of the test gear, mm

$R_{rw\,min}$ Is the minimum test radius limit of the test gear, mm

Eccentricity (double-flank runout). In electronic (and computer-driven) gages, it is possible to use a Fourier transform calculation to extract the first-order, sinusoidal wave component from the measured double-flank data. The first-order component is shown as the green sinusoidal wave in Figure 3. By using this technique, the magnitude and orientation of the test gear's eccentricity can be established. In the Figure 3 example, the "runout" result (i.e., the peak-to-peak amplitude) is reported as 0.008 mm. This data may be useful in identifying how to improve net-shape gears (such as plastic, powder metal, or die cast gears) where the location of the mounting datum (i.e., bore or journal) to the gear geometry can sometimes be adjusted through tooling changes. The Figure 3 example would therefore report that the gear's datum as mounted on the double-flank tester is eccentric from the gear's teeth (as a single set) by one-half of the runout or, in this case, 0.004 mm.

The term runout is a misnomer when it is derived by this double-flank method. To be more precise, this is a double-flank runout and should not be confused with the runout result one would obtain by actually inserting a pin or ball between the flanks of the teeth and comparing the maximum and minimum result of the individual readings. The two methods may yield slightly different results. When using double-flank runout methods, the test reports should indicate the identifier "double-flank runout" instead of just "runout."

Master Gear Design Considerations

Master gears used in double-flank composite measurements must meet the following criteria in order to mesh properly with a test gear.

- The tip of the master gear must not contact the test gear below the form diameter of the test gear. This applies to initial contact and to any type of secondary contact in the fillet zone due to inadequate clearance.
- The tip of the test gear must not contact the master gear below the form diameter of the master gear. This applies to initial contact and to any type of secondary contact in the fillet zone due to inadequate clearance.
- The minimum contact ratio of the double-flank test must not be less than 1.0 when accounting for maximum tooth thickness, minimum outside diameter, maximum root diameter and maximum tip radius of the test gear. Should the contact ratio drop below 1.0, the meshing action of the gears on test will generate an immediate jump in the double-flank result for every tooth meshing cycle. This happens when the spring of the slide on the composite tester compensates for the loss of mesh force by abruptly pushing the gears together.
- The master gear and the test gear must have the same normal base pitch. In most cases, this is the case when the normal module and normal pressure angle match between the master and the test gear. However, mathematically it is possible to mesh a master gear with a different normal module and normal pressure angle than the test gear if the following equation is satisfied:

$$\pi\, m_{nw} \cos\alpha_{nw} = \pi\, m_{n3} \cos\alpha_{n3} \quad (12)$$

where

m_{nw} Is the normal module of the test gear, mm

m_{n3} Is the normal module of the master gear, mm

α_{nw} Is the normal pressure angle of the test gear, degrees or radians

α_{n3} Is the normal pressure angle of the master gear, degrees or radians

This may be useful in some special circumstances, depending on product design.

- For parallel-axis helical gear double-flank arrangements, the master gear must have an equal helix angle to the test gear but of opposite hand.
- For crossed-axis helical gear double-flank arrangements, the shaft angle setting on the double-flank tester must fulfill Equation 5.

In addition, the following recommendations for good practice may also be of use:

- The maximum contact ratio of the double-flank test should be less than 2.0 when taking into account minimum tooth thickness, maximum outside diameter, minimum root diameter, and minimum tip radius of the test gear. High contact ratios on the double-flank tester promote more overlapping of the mesh and may hide errors in the test gear that may otherwise exist. Helical gears, due to their face widths, may have an overall contact ratio greater than 2.0 when run against a master gear covering its full face width. In such cases a decision should be made to either accept the possible smoothing out of errors that would result with this high contact ratio, or to possibly reduce the face width of the master gear and measure the helical gear in different contact zones along the test gear's axis while maintaining an overall contact ratio of less than 2.0.

- In the case of crossed-axis helical double-flank meshes where the driver is a worm, a worm can also be considered for the master gear. This may provide an advantage for the test gear in that only the functional zone is measured and other tooth errors that will not even be seen in the actual product mesh will be ignored. In some cases, a narrow-face-width helical gear master may provide a similar result in a parallel-axis arrangement. In applications where a worm master is used, it may be necessary to add lubrication to the double-flank mesh to assist a sliding action in the mesh without causing reading errors.
- The extent of the master gear's reach (i.e., the master gear's outside diameter) into the test gear should be carefully chosen. Although, as stated above, the mesh under test must have a minimum contact ratio of 1.0 and a maximum contact ratio of less than 2.0, there must also be no contact of the master beyond the form diameter of the test gear. This may afford a wide range of choices in between those requirements when establishing the outside diameter of the master gear. The decision on what master design to use may be based on the cost and availability of existing or commercially available master gears, or it may be based on measuring a test gear to at least its start of active profile location in the actual application.
- Every combination of master gear and test gear should be checked at all tolerance levels to make sure the mesh meets the criteria described here. Just because an off-the-shelf master gear is commercially available does not mean it will mesh properly with a specific test gear.
- In order to machine and produce high-quality master gears by grinding, the bore on the master would need to be sufficiently large enough for a stable mandrel to hold the master gear during machining. Ground master gears with bores less than 6 mm should be carefully considered for the effect on master gear precision from a small-diameter machining mandrel.

Product Design Considerations

Tight mesh center distance and test radius have been described as a means of using double-flank composite inspection to control functional tooth thickness. The functional tooth thickness is the tooth thickness as perceived by a mating gear and therefore includes effects of all tooth deviations as previously described. The nominal tooth thickness (sometimes referred to as "design tooth thickness") does not include any tooth deviations other than allowance for thickness variation at the standard pitch diameter without runout.

As a result, the inspection of tight mesh center distance or test radius will provide information on operational backlash expected in a gear mesh if both gears are double-flank tested individually and the actual mounting center distance is known.

When designing gears, one of the goals is to control backlash. Too little backlash may result in power loss, heat build-up, wear and noise. Too much backlash may result in excessive lost motion and potentially abnormal noise upon direction reversal. One of the common errors in gear design is to ignore the effect that total composite variation has on backlash. As an example: in an external gear mesh, when the tight mesh center distance of two gear positions peak simultaneously, the result will be a minimum backlash condition. On the other hand, the same gears with minimum simultaneous tight mesh center distance positions will result in a maximum backlash condition. Housing center distance variation will further contribute to the backlash result.

When designing gears, the total composite tolerances need to be established simultaneously with the tooth thickness selection criteria in order to establish the true design backlash. The selection of total composite tolerances as an afterthought at the end of the design process may result in an inappropriate level of backlash or a potential for gear binding in the assembly.

Calculation of Backlash on External Spur and Helical Gears, Including the Effects of Total Composite Tolerances

When designing external spur and helical gears, the following calculation procedure may be useful in establishing backlash goals when taking total composite tolerances into consideration.

Calculation of the standard pitch diameters. The standard pitch diameter of a pinion or gear is:

$$d_k = \frac{z_k m_n}{\cos\beta} \tag{13}$$

where

- k Is a general subscript with value k = 1 for the pinion, and k = 2 for the gear
- d_k Is the standard pitch diameter of the pinion or gear, mm
- z_k Is the number of teeth on the pinion or gear
- m_n Is the normal module of the pinion and gear, mm
- β Is the helix angle of the pinion and gear, degrees or radians

Calculation of the transverse pressure angles. The transverse pressure angle, α_t, of the pinion and gear is:

$$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \tag{14}$$

where

- α_t Is the transverse pressure angle of the pinion or gear, mm
- α_n Is the normal pressure angle, degrees or radians

Calculation of the base circle diameters. The base circle diameter, d_{bk}, of a pinion or gear is:

$$d_{bk} = d_k \cos\alpha_t \tag{15}$$

where

- d_{bk} Is the base circle diameter of the pinion or gear, mm

Calculation of functional operating pitch diameters. The functional operating pitch diameters of the pinion and gear differ from the operating pitch diameters typically calculated in other documents in that the effect of the total composite tolerances are included.

The maximum and minimum functional operating pitch diameters are:

$$d_{wk\,max\,functional} = \frac{z_k(2\,a_{max} + F_{idT1} + F_{idT2})}{(z_1 + z_2)} \tag{16}$$

and

$$d_{wk\,min\,functional} = \frac{z_k(2\,a_{min} - F_{idT1} - F_{idT2})}{(z_1 + z_2)} \tag{17}$$

where

$d_{wk\ max\ functional}$ Is the maximum functional operating pitch diameter of the pinion or gear, mm

$d_{wk\ min\ functional}$ Is the minimum functional operating pitch diameter of the pinion or gear, mm

a_{max} Is the maximum mesh center distance, mm

a_{min} Is the minimum mesh center distance, mm

F_{idT1} Is the total composite tolerance of the pinion, mm

F_{idT2} Is the total composite tolerance of the gear, mm

Calculation of functional operating transverse pressure angles. The functional operating transverse pressure angle is calculated at the functional operating pitch diameter positions as follows:

$$\alpha_{wtk\ max\ functional} = \cos^{-1}\left(\frac{d_{bk}}{d_{wk\ max\ functional}}\right) \tag{18}$$

and

$$\alpha_{wtk\ min\ functional} = \cos^{-1}\left(\frac{d_{bk}}{d_{wk\ min\ functional}}\right) \tag{19}$$

where

$\alpha_{wtk\ max\ functional}$ Is the maximum functional operating transverse pressure angle, degrees or radians

$\alpha_{wtk\ min\ functional}$ Is the minimum functional operating transverse pressure angle in degrees or radians

Calculation of maximum and minimum transverse circular tooth thicknesses at the functional operating pitch diameter. The maximum and minimum transverse circular tooth thicknesses at the functional operating pitch diameter can be calculated based on the following equations:

$$s_{wtk\ max\ functional} = d_{wk\ max\ functional}\left(\frac{s_{nk\ min}}{d_k \cos\beta} + \text{inv}\,\alpha_t - \text{inv}\,\alpha_{wtk\ max\ functional}\right) \tag{20}$$

and

$$s_{wtk\ min\ functional} = d_{wk\ min\ functional}\left(\frac{s_{nk\ max}}{d_k \cos\beta} + \text{inv}\,\alpha_t - \text{inv}\,\alpha_{wtk\ min\ functional}\right) \tag{21}$$

where

$s_{wtk\ max\ functional}$ Is the maximum transverse circular tooth thickness at the maximum functional operating pitch diameter for the pinion or gear, mm

$s_{wtk\ min\ functional}$ Is the minimum transverse circular tooth thickness at the minimum functional operating pitch diameter for the pinion or gear, mm

Calculation of mesh backlash. The maximum and minimum transverse circular backlash at the functional operating pitch diameter is:

$$j_{wt\ max} = \frac{\pi\, d_{w2\ max\ functional}}{z_2} - s_{wt1\ max\ functional} - s_{wt2\ min\ functional} \tag{22}$$

and

$$j_{wt\ min} = \frac{\pi\, d_{w2\ min\ functional}}{z_2} - s_{wt1\ min\ functional} - s_{wt2\ min\ functional} \tag{23}$$

where

$j_{wt\ max}$ Is the maximum transverse circular backlash at the function operating pitch diameter

$j_{wt\ min}$ Is the minimum transverse circular backlash at the function operating pitch diameter

Conclusion, Part I

(*Ed.'s Note: Part II of this paper will appear in the March/April issue of* Gear Technology.)

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