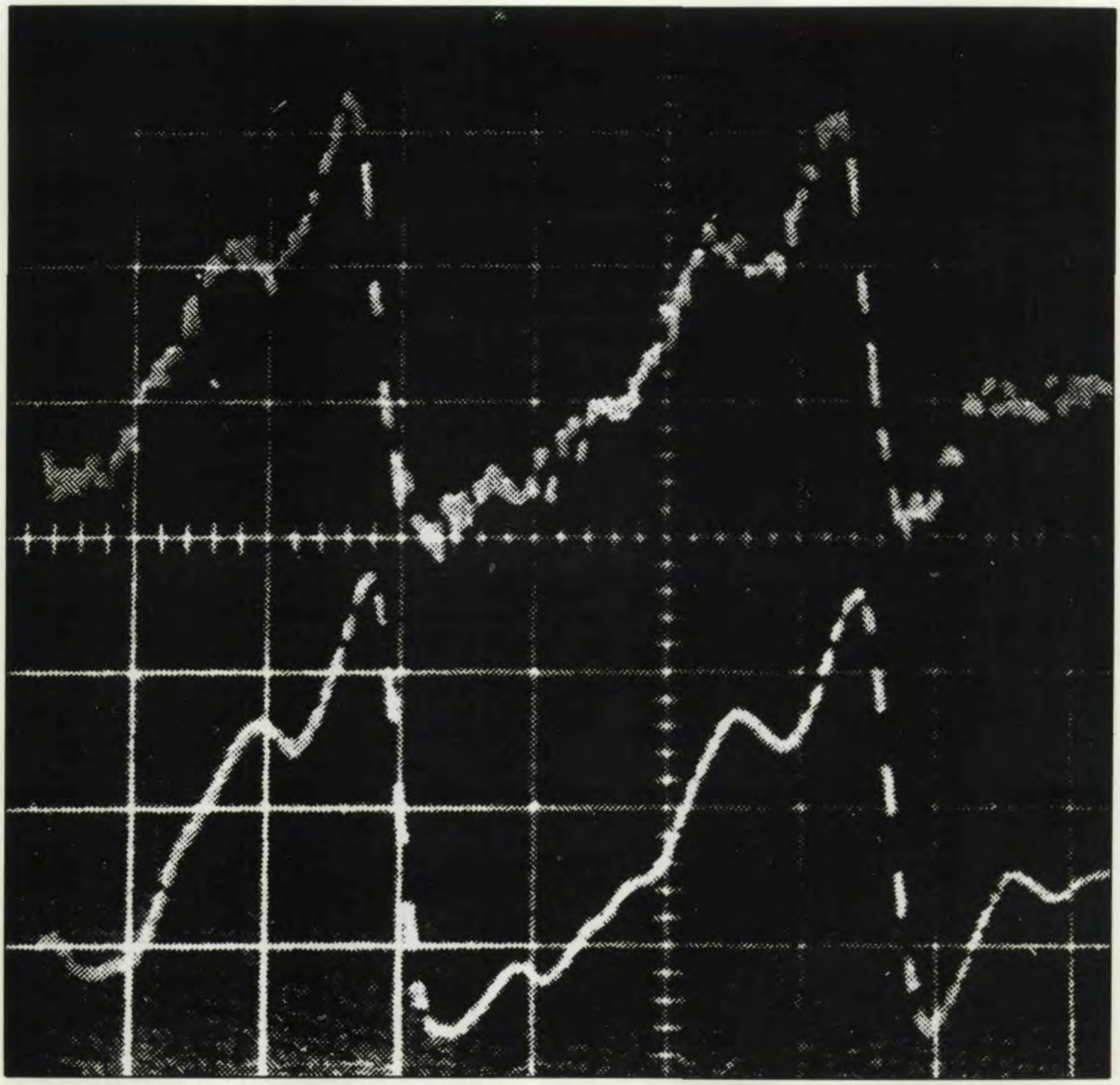
Lubricant Jet Flow Phenomena in Spur and Helical Gears

L. S. Akin
California State University
Long Beach, CA

D. P. Townsend
National Aeronautics and Space Administration,
Lewis Research Center, Cleveland, Ohio



Abstract:

The work reported is an extension from a previous study which was limited to standard centers and tooth proportions only. This article includes long and short addendums and modified center distances. The analysis develops the equations for the limit values of variables necessary to remove prior severe limitations or constraints necessary to facilitate computer analysis. A new computer program, IMPOUT2, has been developed using these newly established "Limit Formulas" to prevent negative impingement on the pinion. The industrial standard nozzle orientation usually found where the offset $S = 0$ and inclination angle $\beta = 0$ will often cause the pinion to be deprived of primary impingement, which can be an important cause of incipient scoring failure in high-speed drives.

Introduction

In the gearing industry, gears are lubricated and cooled by various methods. At low to moderate speeds and loads, gears may be partly submerged in the lubricant which provides lubrication and cooling by splash lubrication.[1] With splash lubrication, power loss increases considerably with speed.[1] This is partially because of churning losses. It is shown that gear scoring and surface pitting can occur when the gear teeth are not adequately lubricated and cooled.[2] The results of spur gear oil jet lubrication[3] show that as the gear pitch line velocity increases at a constant into-mesh lubrication condition, the limiting tooth load that will cause gear scoring is drastically reduced. This is primarily because the method of lubrication does not provide adequate cooling at the increased pitch line velocity. In a study of high-speed, heavy-duty gears,[4] the authors showed that the oil jet location and amount of oil flow to the gears varies nearly linearly with gear pitch line velocity and also with tooth load. The authors[5] showed that with radial jet lubrication, the gear tooth temperature at various speeds and loads can be considerably reduced by increasing the lubricant jet pressure and flow rate to obtain better cooling. It was shown[5,6] that the oil jet lubrication is the most effective method when the jet is directed in the radial direction with adequate pressure and flow. Many gears are lubricated by directing the oil jet at the engaging side of the mesh (into-mesh lubrication) or at the disengaging sides of the gear mesh (out-of-mesh lubrication). The authors analyzed oil jet lubrication when the oil jet is directed at the engaging side of the gear mesh, and it was shown that there is an optimum oil jet velocity and oil jet location to obtain the best lubrication and cooling for into-mesh lubrication.[7,8]

The oil jet lubrication for out-of-mesh lubrication was analyzed,[9] and the oil jet impingement depth was determined for standard gearing dimensions. Also in the same reference, the oil jet location and direction were limited to a no-offset condition (directed at the pitch point only) and in a direction normal to the line of centers. This method will give good results for standard gear dimensions with gear ratios close to unity. However, when nonstandard dimensions, spread center distance, etc. and large gear ratios are used, the oil jet direction and location should be changed to provide the optimum oil jet impingement depth and maximum cooling conditon.

The objective of the work reported herein is to analyze the out-of-mesh jet lubrication with most of the simplifying constraints removed.[9] Since most high-performance gears require addendum modifications and sometimes spread centers in addition, the analysis presented herein set out to include these and other related conditions. One practical constraint is added: A nozzle orientation that allows a pinion or gear to be missed provides no primary cooling to the missed member. Such solutions are not permitted in this analysis since they are not of practical value and can mislead an inexperienced gear design engineer.

Brief Description of the Impingement Cycle

The beginning of the pinion impingement cycle is about to start as the leading edge of the top land of the gear is passed as shown in Fig. 1. Gear tooth rotation continues toward the jet stream until the jet reaches the trailing edge of the gear tooth as shown in Fig. 2. At this position, the time t is set at zero ($t=0$) and the geometrical position of the gear θ_{g1} is calculated. Also the initial position of the pinion θ_{p1} is calculated at ($t=0$). Then, the geometry of the lowest impingement point on the pinion is established by setting the time of flight of the jet stream head equal to the time of rotation on the pinion from ($t=0$) until impingement on the pinion takes place at θ_{p2}, see Fig. 3, (@$t_f=t_w$). Fig. 4 shows the initial position of the jet head (@$t=0$) when the impingement flow toward the lowest impingement point on the gear is initiated. Here the initial position of the pinion is θ_{p4} and θ_{g2} for the gear. Again the times of flight and rotation are equated ($t_f=t_w$). Impingement at the lowest point on the gear (L_g) is shown in Fig. 5. The gear position is θ_{g3} at this lowest point of impingement.

Thus the depth of primary impingement on the gear is d_g as shown in the figure. When the jet velocity V_g is reduced below $V_{f(min)}$, the pinion is missed, and the impingement depth on the gear is not increased as expected, but reduced.

AUTHORS:

MR. D.P. TOWNSEND *is a resident NASA gear consultant for gear systems for NASA and the military along with numerous industrial companies. Townsend earned a BSME from the University of West Virginia. During his career at NASA he has authored over fifty papers in the gear and bearing research area. For the past several years, he has served in active committee roles for ASME. Presently he is a member of the ASME Design Engineering Executive Committee.*

DR. LEE S. AKIN *has been working in mechanical engineering since 1947, specializing principally in technologies related to rotating machinery. About half of this time has been spent in the gear industry and the other half in the aerospace industry, concentrating on mechanisms involving gears and bearings as well as friction, wear, and lubrication technologies.*

Since 1965, when he received his Ph.D. degree in mechanical engineering, he has been extensively involved in gear research especially related to the scoring phenomena of gear tooth failure. In 1971 he joined forces with Mr. Dennis Townsend of NASA Lewis Research Center, and together they have produced numerous papers outlining aspects of their research on technologies related to gear scoring.

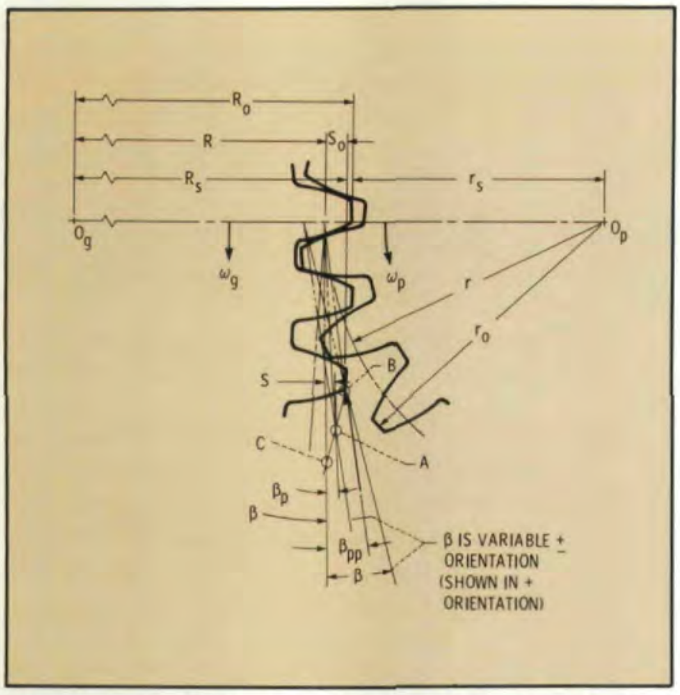


Fig. 1—Jet coordinate origins for impingement on pinion: A = General case where $0<S<S_0$; B = special case where $S = S_0$; C = classic case where $S = 0$ deg.

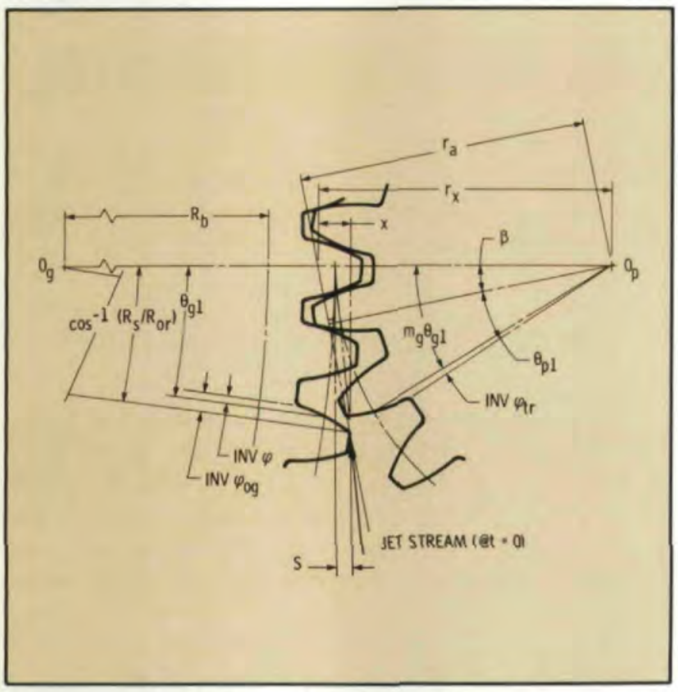


Fig. 2—Impingement on pinion ($@t = 0$)

Model Controls and Restraints

Development of the mathematical model used in reference 1 was restricted to spur gears, and the nozzle position was restricted to the arbitrary offset distances $S=0$ and arbitrary inclination angle $\beta=0$ orientation. The geometric definition of S and β are described below and shown in Fig. 1. The foregoing restrictions have been removed in this mathematical development. Also the origin for the jet stream trajectory is defined as the position where the jet crosses the gear outside diameter (O.D.) at A, B, and C. (Fig. 1) Position A shows

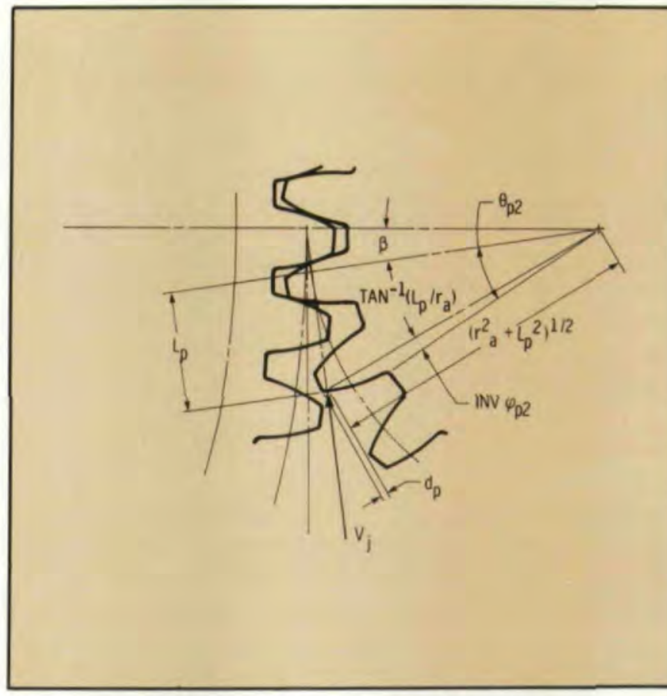


Fig. 3—Impingement on pinion ($@t_f = t_w$)

the general case where $0<S<S_0$; B is a special case where $S=S_0$; and C is the classic case where $S=0$ pointing at the pitch point and perpendicular to the line of centers.

In this article, the value of the arbitrary offset S where the jet line crosses the gear I.D. is restricted to $S_{(min)} \leqslant S \leqslant S_0$ where

$$S_0 = \frac{(a_{gr} m_g - a_{pr})}{m_g + 1} + \frac{a_{gr}^2 - a_{pr}^2}{2 r_r (m_g + 1)} \quad (1)$$

as shown in Fig. 1. (See Nomenclature for variable definitions not described in text). Thus the operating (or running) offset S_0 to the crotch or common intersection of the outside diameters is the maximum value allowed for the offset S to remain within the geometric definitions described in this article. Further when the addendum modification Δa_{pr} and Δa_{gr} are unequal and extreme enough to cause S_0 to be negative, then $S_0 \leqslant S \leqslant 0$. Also, when $\Delta a_p = \Delta_g = \Delta a = 0$ with standard centers, S_0 reduces to

$$S_0\ (\text{std}) = (1/(P_n \cos \psi))((N_g/N_p) - 1) / ((N_g/N_p) + 1) = a(m_g - 1)/(m_g + 1)$$

Also the inclination angle β is confined to a point at the line of centers between the confines of the outside diameters of the pinion and gear, respectively, as shown in Fig. 1.

The inclination angle β is considered positive when slanted from right to left through the point of origin A, B, or C as shown in Fig. 1. At $\beta=0$ the jet is pointed perpendicular to the line of centers, and when β is negative, it is slanted from left to right through the point of origin on the O.D. of the gear, not shown in Fig. 1. It is not usually considered wise to use negative β angles with near-standard proportion gears, lest we starve the gear of adequate coolant.

The mathematical definition of the arbitrary inclination

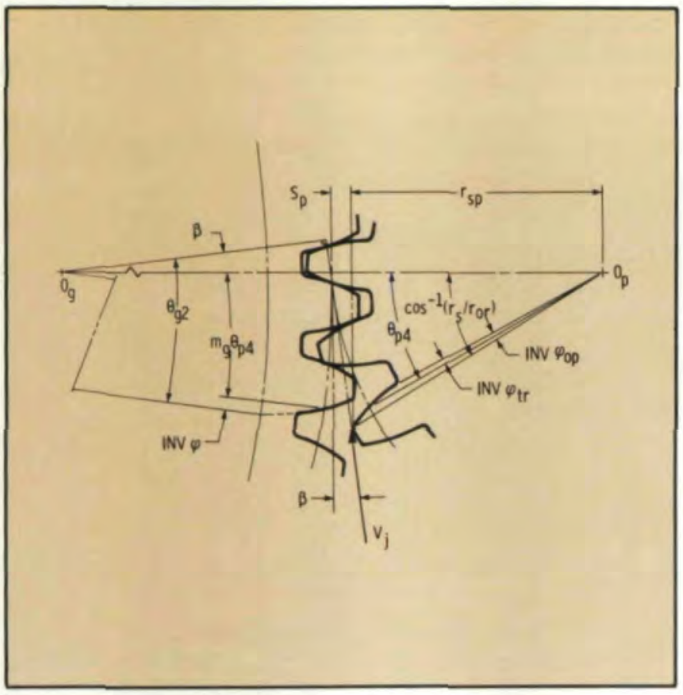


Fig. 4—Impingement on pinion ($@t = 0$)

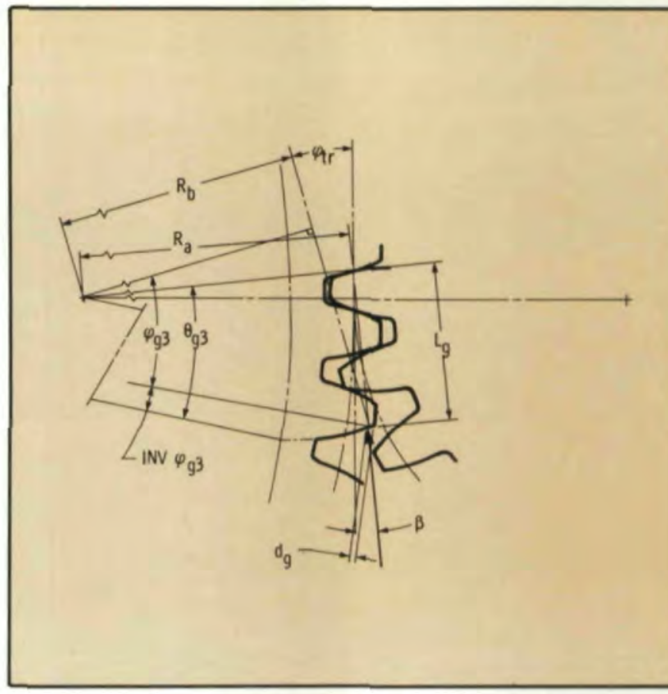


Fig. 5—Impingement on gear ($@t_f = t_w$)

angle β is:

$$\beta = \tan^{-1}(x/R_i) \tag{2}$$

where:

$R_i = (R^2_{or} - R^2_s)^{1/2}$ and

x = an arbitrary offset distance from S position to where the jet stream crosses the common line of centers.

$R_{or} = R_o + \Delta a_g$ = operating outside radius

$R_0 = N_g/(2P_n \cos\psi) + 1/P_n$ = std. O.D., gear

$R_s = R_r + S$ = radius to offset from gear center

$R_r = C_r N_g/(N_g + N_p)$ = operating pitch radius, gear

$\Delta a_g = \Delta N_p/(2P_n \cos\psi)$ = gear addendum modification

When β is given then,

$x = R_i \tan\beta$, where β is arbitrary

Thus to remain within the confines established for β, β_{max} and β_{min} are defined as:

$$\beta_{max} = \tan^{-1}[(S + a_{pr})/R_i] \text{ and}$$
$$\beta_{min} = \tan^{-1}[(S + a_{gr})/R_i]$$

Also, for any given offset S, the angle β_p to the pitch point (used as a normalizer) is

$$\beta_p = \tan^{-1}(S/R_i) \tag{3}$$

and, further, if $S = S_0$, then the angle to the pitch point is

$$\beta_{pp} = \tan^{-1}(S_0/R_i)$$

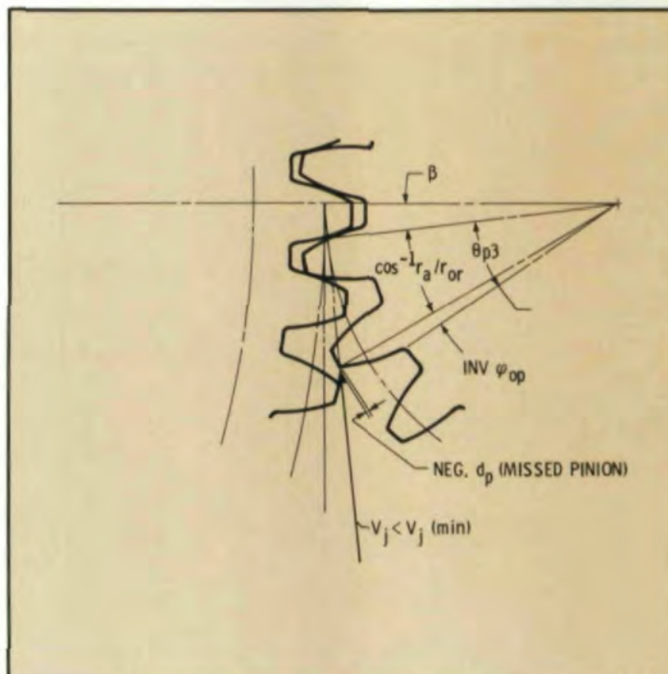


Fig. 6—Missing the pinion when $V_j < V_{j(min)}$

Since β can be arbitrarily selected, it is necessary to provide the user with a normalized (dimensionless) input value for β that will not be out of bounds of the solvable geometry. This is done here using the β_i input parameter where $-1 \leq \beta_i \leq 1$ and is defined in the following way. When β_i is positive (+):

$$\beta_i = \frac{\beta - \beta_p}{\beta_{max} - \beta_p}, \; + \beta = \beta_i(\beta_{max} - \beta_p) + \beta_p \tag{4}$$

and when β_i is negative (−):

$$\beta_i = \frac{\beta - \beta_p}{\beta_p - \beta_{min}}, + \beta = \beta_p - \beta_i(\beta_{min} - \beta_p) \qquad (5)$$

so that when

$\beta_i = 0 : \beta = \beta_p$ (pointing at pitch point)

$\beta_i = 1 : \beta = \beta_{max}$ (pointing at O.D. pin.)

$\beta_i = -1 : \beta = \beta_{min}$ (pointing at O.D. gr.)

Further if the user desires to know the value of β_i that will make $\beta = 0$, he calculates β_{io} from:

$\beta_{io} = \dfrac{\beta_p}{\beta_{min} - \beta_p}$ if β_{io} is negative as is usual.

Similarly, when S is normalized we get

$$S_i = S/S_o \qquad (6)$$

where the value of S_i is confined to $S_{i(min)} \leqslant S_i \leqslant 1$.

Thus the use of the computer program, IMPOUT2, can be entered using S_i and β_i without knowing S or β with some relative feel for where the jet nozzle is pointing.

A further constraint of paramount importance is $S_{i(min)}$ so defined, given β_i or β and Δa_p and Δa_g, so that $d_{p(max)} = 0$, thus making the pinion O.D. barely reachable when the jet velocity V_j approaches infinity. This further confines S_i so that $S_{i(min)} < S_i \leqslant 1$ to maintain impingement at least on the top land of the pinion, where $S_{(min)}$ is found by iterating:

$$\text{abs}[r_{or}^2 - r_a^2 + L_{p(min)}{}^2] \leqq 10^{-3} \qquad (7)$$

as a function of $S_{(min)}$ with given β and $\Delta a_{p,\,g}$ until the inequality is satisfied when the abovementioned restraints on S_i, β_i, Δa_p, and Δa_g are given, the user of the program IMPOUT2 is assured of impingement on the pinion. The definition of $d_{p(max)}$ is given later in this article.

Once $S_{(min)}$ is found from equation (7), $S_{i(min)}$ can be calculated from

$$S_{i(min)} = S_{(min)}/S_o \qquad (8)$$

Development of the Geometric Model for the Pinion

The problem to be solved here depends on what is given. If the jet velocity is given, then we solve implicitly for the impingement depth d_p so that subsequently this depth can be used to determine the cooling effect on the pinion and gear, respectively. On the other hand, if the desired depth of impingement is given, and $d_p < d_{p(max)}$, then the desired jet velocity V_j can be calculated explicitly from the equation:

$$V_{jp} = (((R_{or}{}^2 - R_s{}^2)^{1/2}\sec\beta - p_x \sin\beta - L_p)\omega_p)/(\theta_{p2} - \theta_{p1}) \qquad (9)$$

where

ω_p = angular velocity (rad/s)

ψ = helix angle

N_p = number teeth on pinion

N_g = number of teeth on gear wheel

$\Delta a_p = \Delta N_p/(2P_n\cos\psi)$ = pinion addendum modification

$r_x = r_r - S + x$ = radius to jet stream intersection on line of centers from pinion center (see Fig. 2)

$r_r = C_r N_p/(N_g + N_p)$ = pinion operating pitch radius (in.)

$C_r = C + \Delta a_p + \Delta a_g$ = operating center distance (in.)

$L_p = ((r_{or} - d_p)^2 - r_a^2)^{12}$ = impingement distance (in.) (see Fig. 13)

$r_{or} r_o + \Delta a_p$ = operating outside radius of pinion (in.)

$r_o = N_p/(2P_n\cos\psi) + 1/P_n$ = outside radius of pinion (see Fig. 1)

$r_a = r_x\cos\beta$ = perpendicular distance from pinion center (see Fig. 2)

d_p = impingement depth from pinion O.D. (in.) (see Fig. 3)

$\theta_{p1} = m_g\theta_{g1} + \text{inv}\phi - \beta$ = jet head position $t=0$ (see Fig. 2)

$m_g = N_g/N_{p1}$ = gear ratio (see Fig. 2)

$\phi_{tr} = \cos^{-1}(N_i\cos\phi_t/(N_i + (2p_n\Delta a_i)))$ = operating pressure angle (see Fig. 5)

$N_i = N_p + N_g$

$\Delta a_i = \Delta a_p + \Delta a_g$

$\phi_t = \tan^{-1}(\tan\phi_n/\cos\psi) = \cos^{-1}(R_b/R)$ = transverse pressure angle

ϕ_n = normal pressure angle at pitch radius

$\text{inv}\phi_{tr} = \tan\phi_{tr} - \phi_{tr}$ = radians (see Fig. 4)

$\theta_{g1} = \cos^{-1}(R_s/R_{or}) - \text{inv}\phi_{og} + \text{inv}\phi_{tr}$ (see Fig. 1)

$\phi_{og} = \cos^{-1}(R_b/R_{or})$ = pressure angle at gear O.D.

$\text{inv}\phi_{og} = \tan\phi_{og} - \phi_{og}$ = radians (see Fig. 2)

$\theta_{p2} = \tan^{-1}(L_p/r_a) + \text{inv}\phi_{p2}$ = radians

$\phi_{p2} = \cos^{-1}(r_b/(r_a^2 + L_p^2)^{1/2})$ = pressure angle at impingement point

$\text{inv}\phi_{p2} = \tan\phi_{p2} - \phi_{p2}$ = radians (see Fig. 3)

As will be seen by reviewing the input parameters, a substantial amount of calculation is required before equation (7) for V_{jp} can be solved. Thus a computer program is required to do the job reliably, such as the program IMPOUT2 mentioned above.

Generally solving for the required velocity to obtain a desired depth d_p on out-of-mesh cooling would be unusual, mainly because of the limit imposed by $d_{p(max)}$ which is the limiting depth of impingement when $V_j \rightarrow \infty$ so that $0<d_p<d_{p(max)}$. We calculate $d_{p(max)}$ from

$$d_{p(max)} = r_{or} - (r_a^2 + L_{p(min)}^2)^{1/2} \tag{10}$$

where

$$L_{p(min)} = r_a \tan(\theta_{p1} - \text{inv}\phi_{p2})$$

Note that $\phi_{p2} = f_n(L_p)$ and $L_p = f_n(S+\beta)$ as $V_j \rightarrow \infty$

In real geared systems, the jet velocity V_g is already specified by the system lubricant pressure so that we solve for d_p within the range $0<d_p<d_{p(max)}$ which is usually quite narrow. When V_g is given, it is necessary to solve for d_p implicitly using an iterative technique. This can be accomplished by solving for the impingement distance L_p (See Fig. 3) implicitly, using the equation

$$L_p = \frac{(R_{or}^2 - R_s^2)^{1/2}}{\cos\beta} - \frac{V_j(\theta_{p2} - \theta_{p1})}{\omega_p} - r_x \sin\beta \tag{11}$$

with V_j given and noting that $\theta_{p2} = f_n(L_p)$.

Then the depth of impingement on the pinion is

$$d_p = r_{or} - (r_a^2 + L_p^2)^{1/2} \tag{12}$$

where $0<d_p<d_{p(max)}$ and a negative d_p means that the jet missed the pinion.

Missing the pinion (or neg. d_p) can be avoided by proper placement of the nozzle offset S where $S_{(min)}<S<S_o$ as established above and/or $V_{i(min)}<V_i<\infty$ where (see Fig. 6).

$$V_{j(min)_p} = \frac{((R_{or}^2 - r_s^2)^{1/2} - (R_{or}^2 - r_a^2)^{1/2} - r_x \sin\beta)\omega_p}{(\theta_{p3} - \theta_{p1})} \tag{13}$$

where

$$\theta_{p3} = \cos^{-1}(r_a/r_{or}) + \text{inv}\phi_{op}(\text{rad})$$
$$\theta_{op} = \cos^{-1}(r_b/r_{or}) \text{ and}$$
$$\text{inv}\phi_{op} = \tan\phi_{op} - \phi_{op}(\text{rad})$$

Equation (13) provides the minimum velocity to reach the top land of the pinion for the selected offset S when $S_{(min)}<S$ and $d_p=0$, assuming that S_0 is positive.

Development of the Geometric Model for the Gear

In the case of the gear meshing with the pinion, the jet velocity of the gear is common with that for the pinion; so that even if the velocity V_{jp} is found explicitly to provide a desired or specified depth of impingement on the pinion d_p, the jet velocity for the gear is always provided by $V_{jp} = V_{gp} = V_j$ so that the impingement depth for the gear is always found for the implicit solution for L_p from the equation:

$$L_g = \frac{(r_{or}^2 - r_{sp}^2)^{1/2}}{\cos\beta} + r_x \sin\beta - \frac{V_j(\theta_{g3} - \theta_{g2})}{\omega_g} \tag{14}$$

where we note that

$$\theta_{g3} = f_n(L_p) \text{ (see Fig. 5)}$$
$$r_{sp} = r_r - S_p = \text{offset radius from pinion center (see Fig. 4)}$$
$$S_p = ((r_{or}^2 - r_x^2\cos^2\beta)^{1/2} + r_x\cos\beta)\sin\beta - (x - S) \text{ (see Fig. 4)}$$
$$R_x = R_r + S - x$$
$$\theta_{g2} = \theta_{p4}/m_g + \text{inv}\phi_{tr} + \beta \text{ (see Fig. 4)}$$
$$\theta_{g3} = \tan^{-1}(L_g/R_a) + \text{inv}\phi_{g3} \text{ (see Fig. 5)}$$
$$\theta_{p4} = \cos^{-1}(r_{sp}/r_{or}) - \text{inv}\phi_{or} + \text{inv}\phi_{tr}) \text{ (see Fig. 4)}$$
$$R = R_x \cos\beta \text{ (see Fig. 5)}$$
$$\phi_{g3} = \cos^{-1}(R_b/R_a^2 + L_g^2)^{1/2} \text{ (see Fig. 5)}$$
$$\text{inv}\phi_{g3} = \tan\phi_{g3} - \phi_{g3}(\text{rad}) \text{ (see Fig. 5)}$$
$$\phi_{or} = \cos^{-1}(r_b/r_{or})$$
$$\text{inv}\phi_{or} = \tan\phi_{or} - \phi_{or}(\text{rad})$$
$$R_b = R\cos\phi_t \text{ (see Figs. 3 and 5)}$$

Then the impingement depth on the gear is calculated from

$$d_g = R_{or} - (R_a^2 + L_g^2)^{1/2}$$

Computerized Parametric Study

A rather intricate computer program has been developed which should be useful to the design engineer as well as the researcher performing parametric studies. This program, IMPOUT2, was used in the study for this section of the article.

The out-of-mesh nozzle orientation imposes severe impingement depth problems especially when the gear ratio is larger than one-to-one. This is demonstrated in Fig. 7 where it can be seen that when the gear ratio m_g is equal to 1.0 the depth of jet oil impingement is equal on pinion and gear for a perpendicular jet pointed at the pitch point. However, when m_g is larger than unity, the impingement depth on the pinion is very dependent on the offset S. This is shown in Fig. 7 using the dimensionless offset S_i. As can be seen when $S_i=1.0$, the position at the intersection of the pinion and gear O.D.'s the jet is pointed at the pitch point $\beta_i=0$. Both pinion and gear have near equal impingement depth, but as *the offset S_i is lowered or the gear ratio is increased, the im*pingement on the pinion rapidly disappears. In the figure at $S_i=0.96$, the depth disappears at $m_g=6.0$, and at $S_i=0.863$ it disappears at 2.5. At $S_i=0$ it disappears at 1.2. Obviously, when $S_i=0$, the pinion receives jet impingement only when $m_g \leq 1.2$. This illustrates the idea that for a given pinion/gear tooth combination we need to know $S_{i(min)}$ where no impingement is possible, even when the jet velocity V_j approaches infinity ($V_j \rightarrow \infty$). This then allows solutions for d_p and V_{jp} when $S_{i(min)}<Si<1.0$. The gear depths d_g are also shown for teeth with a working depth of 0.25 in. and, therefore, the primary impingement can only reach about 1/10th the working depth as shown. Fig. 8 shows the effect of the inclination angle β_i on pinion impingement depth d_p. As expected, when the jet pressure is increased, so is the impingement depth, and as the β_i ratio is decreased, the depth is increased up to the maximum depth $d_{p(max)}$ as a function of S_i *and* β_i per Equation 10.

Fig. 9 shows the effect of offset S_i on impingement depth d_p and d_g on a mesh with long and short addendums and a spread center distance. The mesh has been somewhat overcompensated so that S_0 is negative, which is rare, and reverses the situation such that the gear now is the member that can be easily starved if the jet nozzle is not placed properly. Here, as can be seen, the depth of d_g improves as S_i is increased above $S_i=0.6667$, and when optimized by one of the three methods available in IMPOUT2 will still provide an equal impingement depth for both the pinion and gear at dimensionless depth of about $\delta \cong 0.1$, where $\delta = d$/whole depth. In addition, the program option 3 usually has a larger value than option 2, which is also reversed relative to standard mesh conditions.

Fig. 10 shows the results of setting $S_i=0$ and $\beta_i=0$ and adjusting the pinion and gear addendums to realize the balance of depths of impingement desired. Here as is seen in Fig. 10, as Δa_p is reduced, ΔC is reduced, and at $\Delta a_p=0.0662$ an equal (or optimum) impingement depth is reached on the pinion and gear. The connected circles at A depict the depths when $\Delta a_p=0.10$ and $\Delta a_g=-0.0375$. Obviously we must optimize using S_i and β_i as discussed in Fig. 9. The connected circles at C depict the depths when $\Delta a_p=0.08375$ for the pinion and -0.0375 for the gear are closer together than in Case A. In addition, when Δa_p is further reduced to $\Delta a_p=0.06875$, as in Case B, the depths get much closer together, and at $\Delta a_p=0.0662=-\Delta a_g$ when optimized on Δa, the depths are equal.

Fig. 11 shows the effect of β_i on $S_{i(\min)}$. The 21/35 tooth combination was used as the example here. This figure shows that if β is slanted backward in the negative direction, S_i can be made smaller before the pinion is starved. Also if the offset S_i is set at 1.0, it is nearly impossible to starve the pinion at any reasonable inclination angle β or β_i ratio, even when the pressure is modest.

Fig. 12 shows the effect of the dimensionless offset S_i on the minimum velocity or its cause, jet nozzle pressure at the nozzle exit ($V_{j(\min)}$=velocity needed to reach pinion O.D.). As is seen in the figure, $S_{i(\min)}$ establishes the asymptote where $V_{j(\min)}$ approaches infinity. This makes it clear that we cannot set S_i below $S_{i(\min)}$ and expect to obtain primary impingement on the pinion top land or profile at any jet pressure.

Discussion

Figs. 11 and 12 show the most important results of this study, in that they point out the importance of careful placement of the nozzle in both position and pointing direction, especially at higher gear ratios considered in Fig. 7. Also for out-of-mesh nozzle orientation, increasing the oil jet pressure

Fig. 7—Effect of gear ratio on impingement depth, $\beta_i = 0$, $\Delta P = 138$ psi, $n = 5000$ rpm, $N_p = 28$

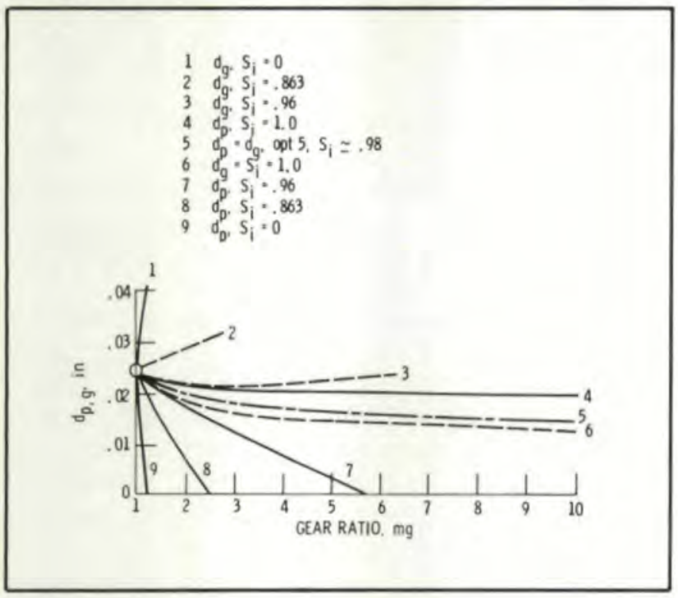


Fig. 8—Effect of β_i on impingement depth, $S_i = 1.0$, 21/35 combination

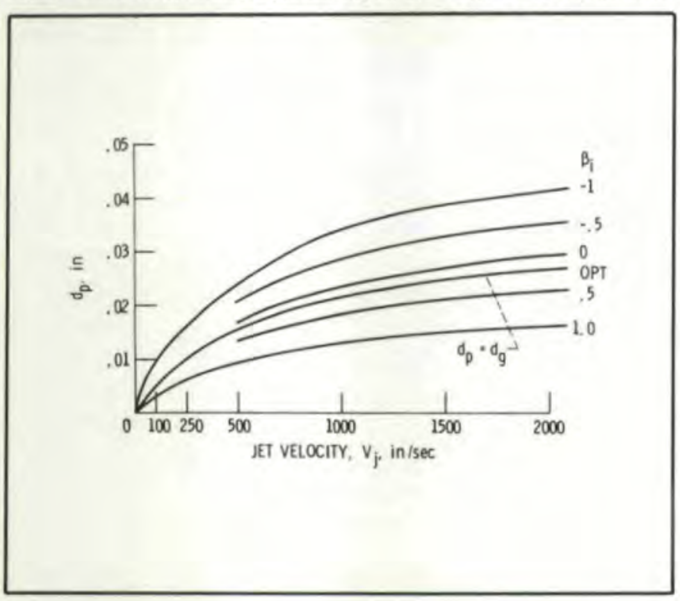
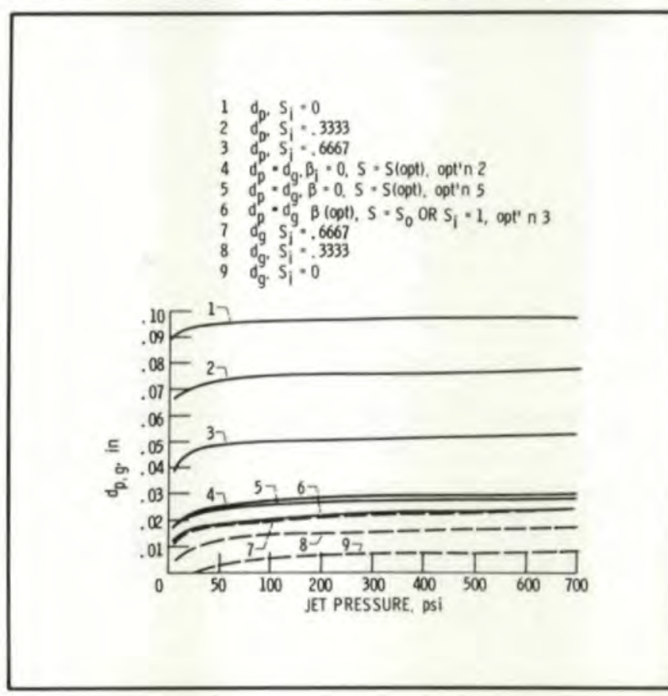


Fig. 9—Effect of offset S_i on impingement depth: 12/43 comb., $\beta_i = 0$, $\Delta a_g = -0.0375$, $\Delta c = 0.0625$, $\Delta a_p = 0.1$, $DP = 8$, $\varphi = 20$ deg



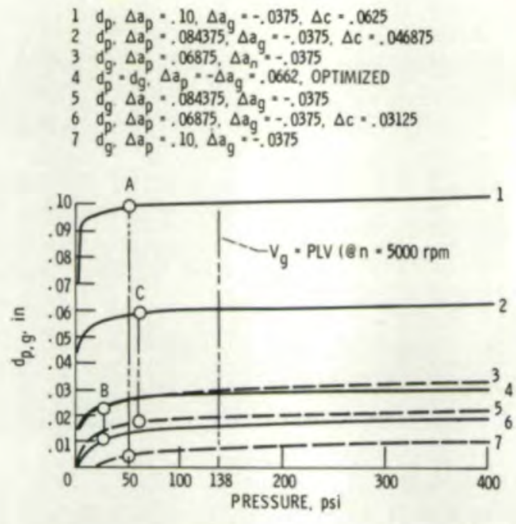


Fig. 10 – Effect of addendum modification and spread centers: $S_i = 0$, $\beta_i = 0$, for 12/43 tooth combination

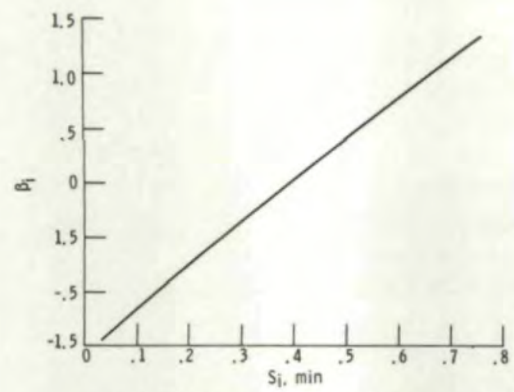


Figure 11. - Effect of β_i on S_i (min), 21/35 combination, φ = 20, P_d = 8.

Fig. 11 – Effect of β_i on $S_{i(min)}$, 21/35 combination, $\varphi = 20$, $P_d = 8$

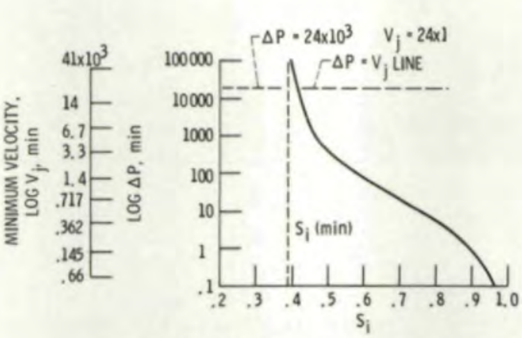


Figure 12. - Minimum differential oil pressure "ΔP (min)" versus dimensionless offset "S_i", 21/35 combination, 8 DP, 20° PA, β_i = 0, $V_j = 156 \sqrt{\Delta P}$, 5000 rpm.

Fig. 12 – Minimum differential oil pressure $\Delta P_{(min)}$ versus dimensionless offset S_i, 21/35 combination, 8 *DP*, 20 deg PA, $\beta_I = 0$, $V_j = 156\sqrt{\Delta P}$, 5000 rpm.

NOMENCLATURE

$C_r = C + \Delta a_p + a_g$ = operating center distance (in.)
L_p = impingement distance for pinion (see Fig. 3) (in.)
L_g = impingement distance for gear (see Fig. 5) (in.)
N_p, N_g = number of teeth in pinion and gear, respectively
R = standard pitch radius of gear
$R_a = R_x \cos\beta$ = perpendicular distance from gear center (in.)
R_b = base radius of gear (in.)
$R_i = (R_{or}^2 - R_s^2)^{1/2}$ offset normal (in.)
$R_{or} = R_0 + \Delta a_g$ operating outside radius of gear (in.)
$R_r = C_r N_g/(N_g + N_p)$ = running or operating pitch radius of gear (in.)
$R_s = R_r + S$ radius to gear offset from gear center (in.)
$R_x = R_r + S - x$ = gear radius to coincidence of O.D.'s (in.)
S = offset of jet stream on gear O.D. (see Figs. 1 and 2) (in.)
$S_i = S/S_0$ = dimensionless offset
$S_i(\min) = S(\min)/S_0$ = minimum theoretical dimensionless offset when $d_p(\max) = 0$ and $V_j = \infty$ simultaneously (see Figs. 11 and 12)
S_0 = offset of jet to crotch (or coincidence) of O.D.'s (see Fig. 1) (in.)
S_p = offset of jet stream on pinion O.D. (see Fig. 4) (in.)
t_w = total time of rotation(s)
x = arbitrary offset distance from S (see Fig. 2) (in.)
β = arbitrary inclination angle of jet stream (rad)
$+\beta_i = (\beta - \beta_p)/(\beta_{max} - \beta_p)$ = normalized positive inclination angle
$-\beta_i = (\beta - \beta_p)/(\beta_p - \beta_{min})$ = normalized negative inclination angle
β_{max} = maximum inclination angle permitted
β_{min} = minimum inclination angle permitted
β_p = inclination angle when jet passes through pitch point (rad)
β_{pp} = inclination angle when jet passes through crotch of O.D.'s and pitch point ($S = S_0$) (rad)
$\delta = dP_n/2$ = dimensionless impingement depth, pinion or gear
$\Delta a_g = \Delta N_g/(2P_n \cos\psi)$ = gear addendum modification (in.)
$\Delta a_p = \Delta N_p/(2P_n \cos\psi)$ = pinion addendum modification (in.)
ΔN_g = tooth addendum modification for Δa_g
ΔN_p = tooth addendum modification for Δa_p
ΔP = differential jet pressure at nozzle exit (psi)
ω_p = angular velocity of pinion (rad/s)
Φ_n = normal pressure angle (rad)
V_j = oil jet velocity from nozzle exit (in./s)

and velocity to high levels may not always improve the primary impingement depth appreciably as shown in Figs. 9, 10, and 12, and in practical fact, may sometimes cause flooding in the gear case housing. Further, if the inclination angle β is adjusted to an extreme, per Fig. 8, to improve the impingement depth on the pinion, then the gear depth is dimished usually unacceptably.

Even when the position is fixed in the historically standard orientation position which will not allow primary impingement on the pinion for even modest gear ratios (at any jet pressure), the pinion and gear addendums can be adjusted per Fig. 10 to provide adequate impingement on both mesh members. This has been done in the past to control incipient scuffing or scoring, often without the designer realizing he was also controlling the impingement and cooling phenomena favorably.

Summary of Results

An analysis was developed for the lubrication jet flow in the out-of-mesh condition. The analysis provides for the in-

V_{jmin} = jet velocity when $d_p=0$ for given ω_p (in./s)
$a_{gr}=R_{or}-R_r$ = operating gear addendum (in.)
$a_{pr}=r_{or}-r_r$ = operating pinion addendum (in.)
d_g = depth of impingement on gear (in.)
d_p = depth of impingement on pinion (in.)
$d_{p(max)}$ = maximum theoretical depth on pinion $V_g=\infty$
$\text{inv}\phi_{og}=\tan\phi_{og}-\phi_{og}$ = involute function of ϕ_{og} (typical) (rad.)
m_g = gear reduction ratio
p_n = normal diametral pitch
r = standard pitch radius of pinion
$r_a=r_x \cos\beta$ = perpendicular distance for pinion center (in.)
$r_b=r \cos\phi_t$ = base radius of pinion (in.)
$r_{or}=r_0+\Delta a_p$ = operating outside radius of pinion (in.)
$r_r=C_rN_p/(N_p+N_g)$ = running or operating pitch radius of pinion (in.)
$r_{sp}=r_r-S_p$ = offset radius from pinion center (in.)
$r_x=r_r-S+x$ = pinion radius to coincidence of O.D.'s (in.)
t = generalized time of jet flight and gear rotation(s)
t_f = total time of flight to jet impingement(s)
ϕ_{og} = pressure angle at O.D. of gear (rad)
ϕ_{p2} = pressure angle at lowest impingement point on pinion (rad)
ϕ_{op} = pressure angle at O.D. of pinion (rad)
ϕ_t = tangential standard pressure angle (rad)
ϕ_{tr} = tangential operating pressure angle (rad)
ψ = helix angle (deg)
θ_{gl} = initial position of gear tooth at beginning of pinion impingement cycle (t=0) (rad)
θ_{g2} = initial position of gear tooth at beginning of gear impingement cycle (rad)
θ_{g3} = final position of gear at max. impingement depth ($t_f=t_w$) (rad)
θ_{p1} = initial angular position of pinion tooth at beginning of pinion impingement cycle ($t=0$) (rad)
θ_{p2} = final position of pinion tooth at maximum impingement depth ($t_s=t_w$) (rad)
θ_{p3} = final position of pinion tooth when just missed by jet stream ($t=0$) (rad)
θ_{p4} = initial position of pinion tooth at beginning of gear impingement cycle ($t=0$) (rad)

clusion of modified center distances and modified addendums. The equations are developed for the limit values of variables necessary to remove the severe limitations or constraints necessary to facilitate computer analysis. A computer program was developed using these limit formulas to prevent negative impingement (missing) on the pinion. The following results were obtained:

1. The industrial standard nozzle orientation usually found where the offset $S=0$ and inclination angle $\beta=0$ will often cause the pinion to be deprived of primary impingement, which can be important cause of incipient gearing failure in high-speed drives.

2. For ratios larger than 1:1, the oil jet will only impinge on the gear teeth unless a minimum calculated jet velocity is provided to lubricate the pinion teeth.

3. When a minimum oil jet velocity is provided, the oil jet offset must be equal to or greater than a minimum calculated offset to assure impingement on the pinion.

4. As the oil jet velocity is increased above the calculated minimum value, the impingement depth will increase, but

at a decreasing rate. The maximum impingement depth will generally not exceed 10 percent of the tooth profile depth.

References

1. UMEZAWA, K., INOH, T., and KATAH, H., "Power Loss of Automobile Transmissions with Exact Measurement of Total Heat Rejection," *The International Symposium on Gearing and Power Transmissions*, JSME, Tokyo, Aug. 30-Sept. 3, 1981, pp. 287-292.
2. DUDLEY, D. W., "Characteristics of Regimes of Gear Lubrication," *The International Symposium on Gearing and Power Transmission*, JSME, Tokyo, Aug. 30-Sept. 3, 1981, pp. 319-324.
3. NARUSE, C., HAIZUKA, S., and NEMOTO, R., "Studies on the Limitation Load for Scoring of Gears," *The International Symposium on Gearing and Power Transmissions*, JSME, Tokyo, Aug. 30-Sept 3, 1981, pp. 371-376.
4. DOI, Y., et al., "On Bulk Temperature in High Speed and Heavy Duty Gears," *The International Symposium on Gearing and Power Transmissions*, JSME, Tokyo, Aug. 30-Sept. 3, 1983, pp. 247-252.
5. AKIN, L., MROSS, J. J., and TOWNSEND, D. P., "Study of Lubricant Jet Flow Phenomena in Spur Gears," ASME *Journal of Lubrication Technology*, Vol. 97, No. 2, Apr. 1975, pp. 283-288.
6. TOWNSEND, D., and AKIN, L., "Analytical and Experimental Spur Gear Tooth Temperature as Affected by Operating Variables," ASME *Journal of Mechanical Design*, Vol. 103, No. 4, Jan. 1981, pp. 219-226.
7. AKIN, L. S., and TOWNSEND, D. P., "Into Mesh Lubrication of Spur Gears with Arbitrary Offset. Part 1: For Oil Jet Velocity Less Than or Equal to Gear Velocity," ASME *Journal of Mechanisms, Transmissions and Automation in Design*, Vol. 105, No. 4, Dec. 1983, pp. 713-718.
8. AKIN, L. S., and TOWNSEND, D. P., "Into Mesh Lubrication of Spur Gears with Arbitrary Offset. Part 2: For Oil Jet Velocity Equal To or Greater Than Gear Velocity," ASME *Journal of Mechanisms, Transmissions and Automation in Design*, Vol. 105, No. 4, Dec. 1983, pp. 719-724.
9. TOWNSEND, D., and AKIN, L., "Study of Lubricant Jet Flow Phenomena in Spur Gears—Out-of-Mesh Condition," ASME *Journal of Mechanical Design*, Vol. 100, No. 1, Jan. 1978, pp. 61-68.

This article was previously presented by the ASME-DED Power Transmission and Gearing Committee at the October, 1984, Design Engineering Technical Conference. Paper No. 84-DET-96.

CALL FOR PAPERS . . .

from the American Gear Manufacturers' Association for its Fall Technical Meeting to be held on October 4-7, 1987.

Papers will be accepted from all sectors of the gear industry—machine tool builders, gear manufacturers, consultants and educators. The subject matter of papers should relate to the latest advances in the selection, design, manufacture and application of mechanical power transmission products.

One page abstracts of articles should be submitted ASAP. None will be accepted after Jan. 31, 1987.

For further information contact:
Harry Kidd
American Gear Manufacturers' Ass'n
1500 King Street, Suite 201
Alexandria, VA 22314
(703) 684-0211