Describing Nonstandard Gears — An Alternative to the Rack Shift Coefficient

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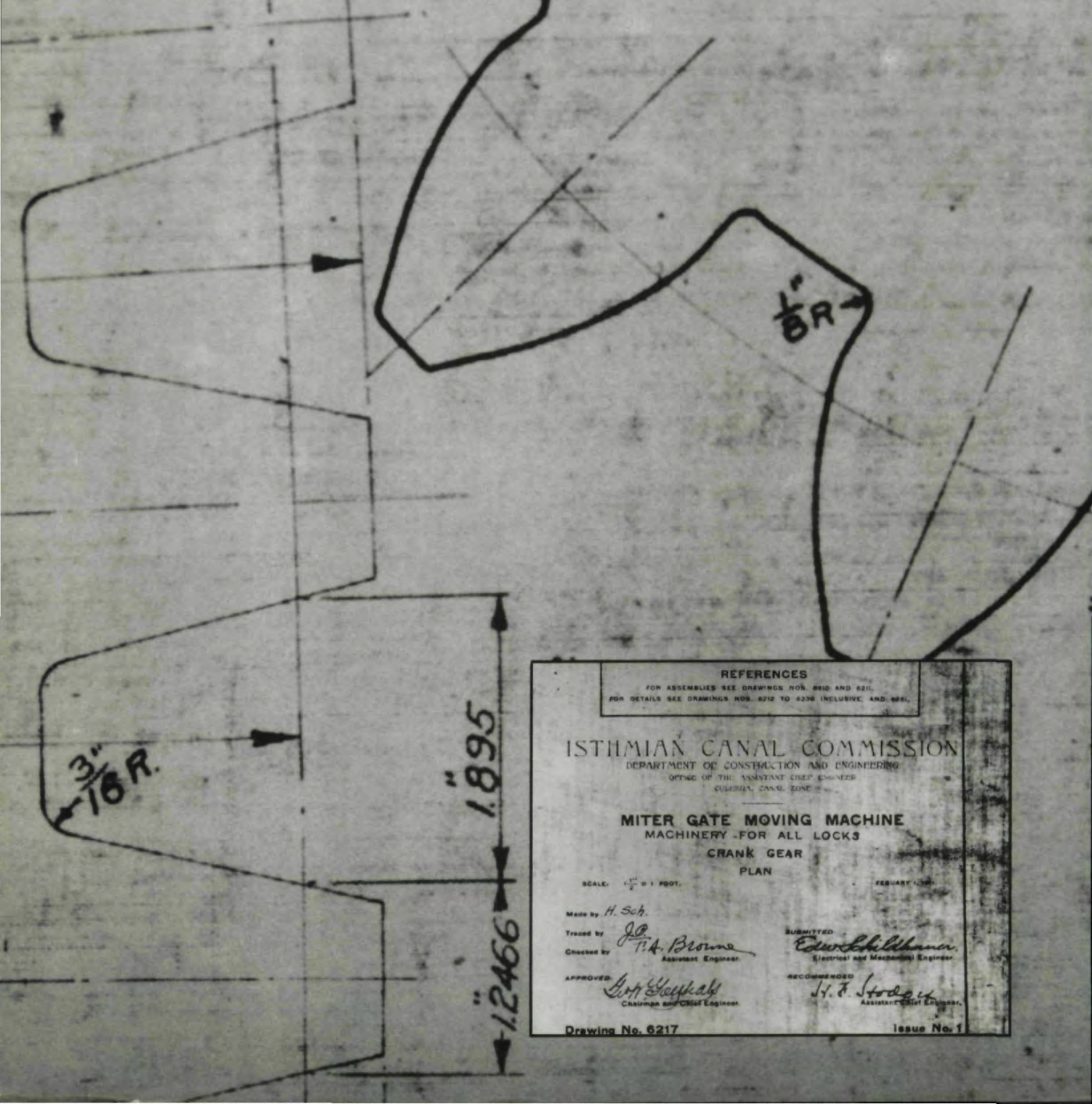

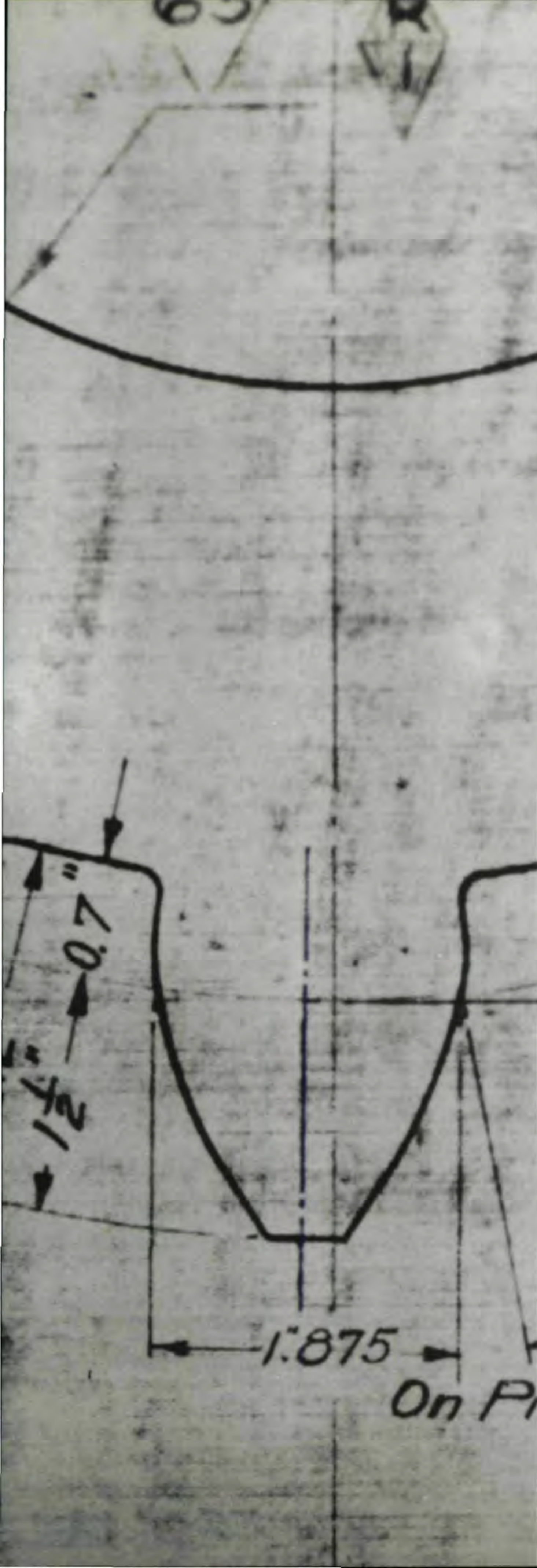

Abstract

The rack shift method of describing gears made with nonstandard addendum diameters (tip diameters or nonstandard tooth thickness) has serious limitations, since the nominal pitch of the cutting tool, which is required for the calculation of rack shift coefficient, may not be available to the designer, may vary as the tool is sharpened or may be different for the gear and its mate.

The details of the calculation of rack shift coefficient are not standardized, so there is little agreement between analysts about the value of "x" for a given nonstandard gear or gear pair. Two common methods to calculate rack shift coefficient are described in this article.

This article proposes a different nomenclature for nonstandard gears, using the involute function of the transverse pressure angle at the diameter where the space width is equal to the circular tooth thickness. This is calculated from the fundamental relationships between number of teeth, normal base pitch, axial pitch and normal base tooth thickness.

This dimensionless value, called "T", is used with the addendum diameter and face width to describe a single gear. A pair of gears is described by the same system, by adding center distance and backlash to the parameters.

Introduction

The use of dimensionless factors to describe gear tooth geometry seems to have a strong appeal to gear engineers. The stress factors I and J, for instance, are well established in AGMA literature. The use of the rack shift coefficient "x" to describe nonstandard gear proportions is common in Europe, but is not as commonly used in the United States. When it is encountered in the European literature or in the operating manuals for imported machine tools, it can be a source of confusion to the American engineer.

Even those who use the rack shift method do not agree on how to evaluate the rack shift coefficient of a specific gear set. As a test, the author sent a set of data for a simple spur gear set to seven gear engineers in the U.S. and Europe with the request that they evaluate the "x" factor. Of the six replies, no two results were the same! The value of the rack shift coefficient "x" varied by more than 20%. The gear data and the results are tabulated in Appendix C.

This article proposes that these nonstandard gears be described by a different parameter, the involute function of

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the transverse pressure angle where the tooth thickness equals the space width.

Before describing the proposal, the rack shift method of describing nonstandard involute gears and some of its limitations are discussed.

Rack Shift Coefficient – Two Definitions

Outside Diameter Method. One definition of rack shift coefficient is given by Maag:[1]

"The amount of the radial displacement of the datum line from the position touching the reference circle is termed *addendum modification*. This amount is given a positive sign in the calculations when the displacement is away from the center of the gear, and a negative sign when towards the center of the gear. . . The *addendum modification coefficient* $\underline{x}$ is the amount of addendum modification measured in terms of the diametral pitch or module.

$$\text{Addendum Modification} = \frac{x}{p} \text{ (or x.m)} \qquad (1)$$

The dimensions of the gears can be determined as the so-called zero backlash gearing by means of a system of equations. (In practice the backlash is obtained by the tolerances on the theoretical, nominal dimension.)"

This definition, which will be called the outside diameter (OD) method, is useful because, if root clearances are kept standard, the center distance can easily be calculated without confusing the calculation with considerations of operating backlash. (See Figs. 1 & 2.) It is clear that, even without using a dimensionless coefficient

$$C = \frac{D_{o1} + D_{r2}}{2} + c \qquad (2)$$

where

D_{o1} = Outside diameter of pinion
D_{r2} = Root diameter of gear
C = Center distance
c = Root clearance.

The geometry is calculated as if the gear were cut by feeding a hob in from the outside diameter to standard depth, then side cutting to achieve the desired backlash.

The gear data block includes "backlash allowance in this gear", which specifies the amount of this side cutting. For gears which are to run on widely spread centers, this backlash

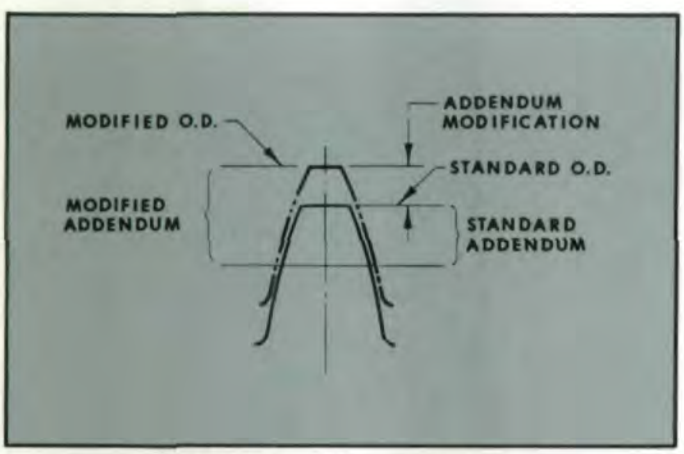

Fig. 1 – The outside diameter method.

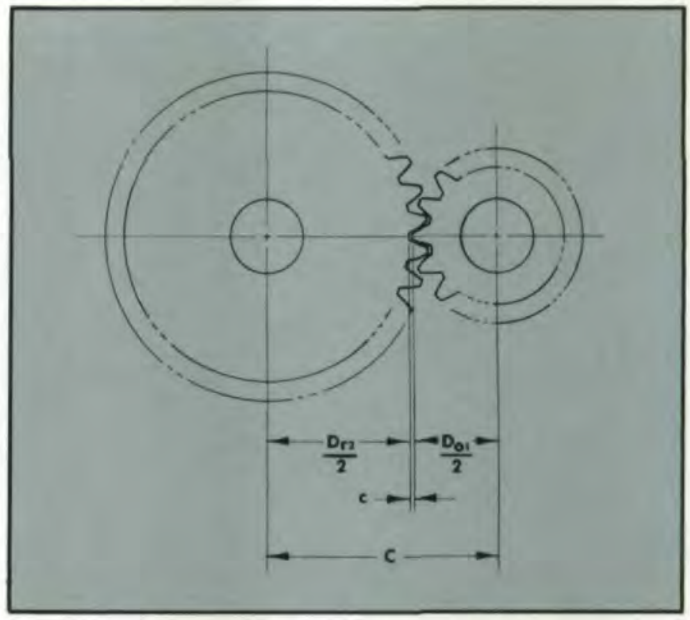

Fig. 2 – Center distance using constant radial clearance.

allowance must be negative to provide reasonable operating backlash. Negative backlash allowance is a difficult concept which can lead to false expectations of root clearance.

This is a simple mathematical convention, but it is not the way gears are made. In real life, the cutter is fed in until the desired tooth thickness is achieved. The resulting root diameters are not the same as those assumed in the original convention, so the root clearances on which the convention is based are not standard.

Nonstandard root clearances are not much of a problem if they are greater than expected, but they can lead to interference problems in those gears which have negative "backlash allowance" in order to operate properly on spread centers. (See Fig. 3.)

The interference can be alleviated by reducing the outside diameter to restore the standard clearance, but this throws the engineer into a loop, since the x factor is based on the OD. Lorenz[2] introduces a K factor (tip modification factor) to clean this up without recalculating the x factor.

This definition has a more subtle problem, since helical gears and spur gears with the same normal generating tooth thicknesses do not have the same x factor, even though it seems they should. This question will be covered later in this article.

Tooth Thickness Method. Another definition of rack shift is related to the actual position of the cutting tool when cutting the thickest allowable teeth for the design under consideration.

$$x = \frac{(P_{nd}t_{sn} - \pi/2)}{2 \tan \phi_c} \qquad (3)$$

where

t_{sn} = Tooth thickness at standard (generating) pitch diameter.
P_{nd} = Normal diametral pitch
ϕ_c = Cutter profile angle

This is dimensionless, since the tooth thickness is multiplied by the diametral pitch or divided by the module. We shall call this the tooth thickness (TT) method. (See Fig. 4)

The TT method eliminates the problems with negative backlash allowance and misleading root clearance assumptions inherent in the OD method. It offers little help in calculating center distance if tooth thicknesses are known, or in calculating tooth thicknesses to operate at a specified center distance. In this method, the outside diameters of the gears are calculated from the center distance, the root diameters and the required root clearances.

$$D_{o1} = 2(C - c) - D_{r2} \qquad (4)$$

If two gears are operated on a center distance equal to the sum of their standard pitch radii plus the sum of their rack shifts calculated by the TT method, the operating backlash will vary in a complex manner with the sum of the rack shifts. Backlash is zero when the sum of rack shifts is zero, but it must be calculated from operating pitch diameters and operating tooth thicknesses when the sum of rack shifts is not zero. The time required to perform this lengthy iterative process usually precludes the development of an optimum design.

Most of those who responded to the sample problem assumed a "backlash allowance" and adjusted the tooth thicknesses arbitrarily to define a hypothetical "zero backlash set". The differences in their assumptions account for some of the differences in their calculated "x" factors.

Cutter Standards and Standard Racks. In order to make any of the definitions of rack shift dimensionless, the designer must assume a cutter diametral pitch or module. This is usually done in terms of a hypothetical standard rack by assuming that the rack geometry is "standard" where the tooth thickness and the space width of the rack are equal to one half of the rack's circular pitch.

This is dangerous ground. Tool designers and gear machines operate with base pitch, regardless of what is marked on the end of the tool. Short pitch hobs, single tooth rack shaper cutters, single tooth grinders and two wheel grinders are

Fig. 3 – An example of gears interfering in the root with backlash. The hob was fed in to standard depth.

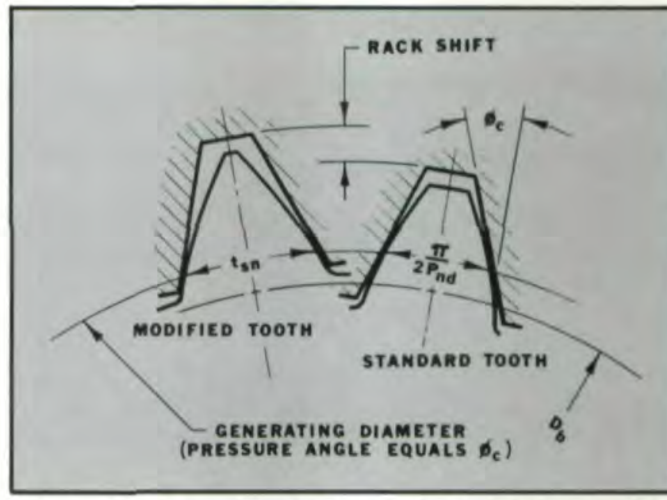

Fig. 4 – Addendum modification factor by the tooth thickness method.

beyond standardization by the "half circular pitch" method.

What is the designer to do? Usually an assumption is made by the designer and ignored by the manufacturing department. The result can be a surprise, usually discovered on the assembly floor.

Earlier Publications

Finding an easy way to calculate the dimensions of nonstandard gears has been the subject of countless papers and articles. As early as 1911, the Isthmian (Panama) Canal Commission[3] used 1 DP oversize spur pinions in the gate mechanisms for the Gatun and Miraflores locks. These gears were made on special Gleason gear planers from a layout, another method which is hard to describe in terms of a standard rack form.

As the advantages of nonstandard gears became better recognized, many authors tried to develop systems for "standardized" nonstandard systems. Maag offers pages of charts, nomographs and advice on this subject. An interesting paper by Grosser[4] provides eight pages of charts to help with these calculations. Farrel[5] also provides guidance in selecting the proportions of gears intended to operate on spread centers. Most of this work dates from the days of mechanical calculators and tables of logarithms, when the difficulty of finding the value of an angle from its involute function was avoided whenever possible to save calculation time.

Computers and the Inverse Involute

Today we can find the arc involute as easily as we find the arc cosine. Even pocket calculators are easily programmed to give an accurate result. There are many algorithms. Cooper[6], Errichello[7] and Laskin[8] all work and are readily available. Easy access to the arc involute frees us up to optimize nonstandard gears if we can find a more direct way to calculate center distance, backlash and limit diameters from tooth thicknesses, or vice versa.

Summary of This Article

This article will demonstrate that the involute function of the transverse pressure angle, where the tooth thickness is equal to the space width, called the "T" factor, is a useful way to describe nonstandard gears. With this parameter, a series of simple equations can be developed, which simplify nonstandard gear geometry calculations and reduce iteration. The "T" factor is

- Dimensionless
- Easily calculated
- Independent of helix angle
- Independent of the cutter geometry
- Independent of the mating gear
- Unique for the geometry of the gear described.

Nomenclature and Definitions

Definitions. Symbols are defined where they are first used in this article. For convenience, a list of symbols is provided in Table 1.

The subscript "1" refers to the pinion, and the subscript "2" refers to the gear.

Fundamental Parameters of the Involute Tooth Form

The involute portions of a meshing gear pair can be completely described in terms of a short list of fundamental parameters.

Numbers of Teeth. The number of teeth on each gear must be known before the geometry can be defined.

Base Pitch. The base pitch must be determined from the cutting tool or arbitrarily if the gear is to be made on a machine where base pitch is adjustable. Base pitch can be measured in the normal plane or the plane of rotation.

Axial Pitch. Axial pitch, which determines the helix angles of the gear set, is required to define the geometry.

Base Tooth Thickness. Base tooth thickness, which determines all of the tooth thicknesses of a gear, must be established for each gear. Base tooth thicknesses can be measured in the normal or transverse plane.

Center Distance and Backlash. If both base tooth thicknesses of a gear pair are known, either center distance or backlash is required. The other can be calculated. If three of the four; t_{b1}, t_{b2}, C and B, are known, the fourth can be calculated.

Blank Dimensions. The outside diameters of the gear blanks are usually determined by the involute geometry chosen, the cutter addendum and the root clearances desired. They are required, along with the face widths, to completely define the gear geometry.

Root Fillet Geometry. The root trochoidal form is determined by the cutting conditions. It is beyond the scope of this article, since it does not affect the involute portion of the teeth, but it is fundamental to the strength of the teeth. For good control, the root fillet coordinates or the cutting method and cutter form must be specified.

Table 1 — Nomenclature

Symbol	Description	Where First Used
B	Backlash in the plane of rotation	Eq. 5
B_N	Backlash normal to involute profile	Eq. 7
C	Center distance	Eq. 2
c	Root clearance	Eq. 2
D_b	Base circle diameter	Eq. 5
D_{o1}	Outside diameter of pinion	Eq. 2
D_{r2}	Root diameter of gear	Eq. 2
m	*Metric module*	Eq. 1
N_1	Number of teeth in pinion	Eq. 6
N_2	Number of teeth in gear	Eq. 6
P	Diametral pitch	Eq. 1
P_{nd}	Normal diametral pitch	Eq. 3
p	Transverse circular pitch	Eq. 14
p_b	Transverse base pitch	Eq. 5
p_N	Normal base pitch	Eq. 7
p_x	Axial pitch	Eq. 9
T	*Tooth thickness factor (inv ϕ_1)*	Eq. 15
t	Transverse tooth thickness	Eq. 14
t_b	Transverse base tooth thickness	Eq. 5
t_{bn}	Normal base tooth thickness	Eq. 7
t_{sn}	Tooth thickness at standard (generating) pitch diameter	Eq.. 3
x	Rack shift coefficient	Eq. 1
ϕ	Transverse pressure angle	Eq. 14
ϕ'	Transverse operating pressure angle	Eq. 5
ϕ_1	*Transverse pressure angle where space width equals tooth thickness*	Eq. 13
ϕ_c	Cutter profile angle	Eq. 3
ψ_b	Base helix angle	Eq. 7

A Pair of Gears

Gear designers are usually intersested in a pair of gears in mesh, since, like Adam without Eve, one gear is more ornamental than useful. It can be demonstrated that for two gears in mesh:

$$\text{inv}\ \phi' = \frac{t_{b1} + t_{b2} + B - p_b}{D_{b1} + D_{b2}} \qquad (5)$$

where

ϕ' = Transverse operating pressure angle
t_{b1} = Transverse base tooth thickness of pinion
t_{b2} = Transverse base tooth thickness of gear
p_b = Transverse base pitch = $\pi D_b/N$
B = Backlash measured in the transverse plane and in the plane of action
D_{b1} = Base circle diameter of pinion
D_{b2} = Base circle diameter of gear

(Derivations for Equations 5, 12, 13, 14 and 21 are included in Appendix A.)

For either gear or pinion:

$$D_b = \frac{N\ p_b}{\pi} \qquad (6)$$

where

N = Number of teeth

If Equation 6 is combined with Equation 5 and the result is multiplied by $\cos\psi_b/\cos\psi_b$, we have the surprising result:

$$\text{inv}\,\phi' = \frac{\pi\,(t_{bn1} + t_{bn2} + B_N - p_N)}{p_N(N_1 + N_2)} \qquad (7)$$

where

ψ_b = Base helix angle
t_{bn} = Normal base tooth thickness of pinion or gear
p_N = Normal base pitch
B_N = Normal backlash measured normal to involute profile. This is the backlash which would be measured by a feeler gauge inserted between the teeth.

Equation 7 demonstrates that ϕ' is independent of ψ_b. This implies that as a pair of spur gears are made "more helical" by increasing the helix angle, holding the normal base tooth thickness constant and holding the axes parallel, the center distance increases in inverse proportion to the cosine of the base helix angle and the operating transverse pressure angle remains constant. A similar equation appears in Section 1.35 of Maag for checking the calculation of base tangent length.

The tight mesh condition, as with a cutting tool or a master gear, is a special case where the backlash is zero.

If the center distance is known:

$$\cos\phi' = \frac{D_{b1} + D_{b2}}{2C} = \frac{p_N(N_1 + N_2)}{2\pi\, C\cos\psi_b} \qquad (8)$$

$$\psi_b = \sin^{-1}\left(\frac{p_N}{p_x}\right) \qquad (9)$$

where

p_x = axial pitch.

Note: For spur gears, the base helix angle is zero. The value of $\cos\psi_b$ is 1.0. Most computers will accept a very large value of p_x, such as 1.0E+9, without significant error, and eliminate the need for a separate routine for spur gears.

If Equation 7 and Equation 8 are solved for ϕ' and equated:

$$\phi' = \cos^{-1}\left(\frac{p_N(N_1+N_2)}{2\pi\, C\cos\psi_b}\right) = \text{inv}^{-1}\left(\frac{\pi(t_{bn1}+t_{bn2}+B_N-p_N)}{p_N(N_1+N_2)}\right) \qquad (10)$$

If the cutter and numbers of teeth are fixed, this simplifies to:

$$\phi' = \cos^{-1}\left(\frac{K_1}{2C\cos\psi_b}\right) = \text{inv}^{-1}\left(\frac{t_{bn1}+t_{bn2}+K_2}{K_1}\right) \qquad (11)$$

where

K_1 = a constant, $\frac{p_N(N_1+N_2)}{\pi}$

K_2 = a constant, $B_N - p_N$

This equation is particularly helpful when a gear set must be designed to fill a given center distance, since the combined effects of increasing tooth thickness and increasing base helix angle (or decreasing axial pitch) are shown in one equation.

Once the sum of the base tooth thicknesses is known, it must be divided between the gear and pinion in accordance with the designer's priorities for balanced strength, sliding velocities and whole depth.

Gear Pairs With Non-Parallel Axes. Equation 7 is derived from the assumption that the axes of rotation of the mating gears are parallel. (The base helix angles are equal.) If this is not the case, as in hobbing, shaving and in spiral gears, a more general relationship must be sought, in which the normal operating circular pitches of the mating parts are equal at the operating pitch diameters, and the sum of the normal operating tooth thicknesses plus the normal backlash is equal to the operating normal circular pitch. This leads to Equation 12, which must be solved by iteration.

$$N_1\text{inv}\phi'_1 + N_2\text{inv}\phi'_2 = N_1\left(\frac{t_{b1}}{D_{b1}}\right) + N_2\left(\frac{t_{b2}}{D_{b2}}\right) + \pi\left(\frac{B_N}{p_N}\right) - \pi \qquad (12)$$

It can be seen that if the axes of the gears are parallel, so that the transverse operating pressure angles, ($\phi'_1 = \phi'_2 = \phi'$), and the base helix angles are equal, Equation 12 reduces to Equation 7.

One Gear. It has been shown by Grosser that the pressure angle where the tooth thickness is equal to the space width can be found by

$$\text{inv}\phi_1 = \frac{t_b - .5p_b}{D_b} \qquad (13)$$

where

ϕ_1 = pressure angle where space width equals tooth thickness.

In Grosser's general case:

$$\text{inv}\phi = \frac{t_b - (t/p)p_b}{D_b} \tag{14}$$

where

t = tooth thickness at the diameter where ϕ is measured
p = circular pitch at the same diameter.

The similarity between Equations 5 and 13 has led to a search for a way to express the fundamental proportions of a gear pair in the same terms, as dimensionless ratios of t_b, p_b and N, independent of the cutter geometry or the units of measurement.

"T" Factor

Definition. The author proposes that the involute of the pressure angle in the transverse plane at the diameter where the tooth space is equal to the tooth thickness, be used as the dimensionless comparison factor. This factor, called T, for transverse tooth thickness factor, is dimensionless, easily calculated and extremely useful. It would also be possible to use the pressure angle at this diameter, which is in more familiar units, at the expense of an additional mathematical conversion. This angle will be shown for easy reference in parentheses as (ϕ_T) wherever numerical values for T are shown, for easy reference.

Calculation of T. Equation 13 can be restated by multiplying it by $\cos\psi_b/\cos\psi_b$, in terms of normal tooth thicknesses and normal base pitch, as:

$$T = \text{inv}\phi_1 = \frac{\pi(t_{bn} - .5p_N)}{N^* p_N} \tag{15}$$

This is the most convenient general form, since it is valid for both spur and helical gears.

When two gears are considered, T can be substituted into Equation 7 giving:

$$\text{inv}\phi' = \frac{T_1N_1 + T_2N_2 + \pi(B_N/p_N)}{N_1 + N_2} \tag{16}$$

The term "B_N/p_N" is identical to "B/p_b", so that backlash can be taken in either sense, as long as the value of base pitch is consistent.

Some examples will illustrate the uses of this concept.

Fixed Center Distance Gears. Engineers are often asked to specify a gear set with a specific ratio to mesh at a given center distance and a specified backlash. Usually, the ratio will not fit the center distance with spur gears made to standard proportions and with standard (or available) tooling. The choice, then, is to use oversize spur gears, helical gears or oversize helical gears. The possibilities are endless, and the calculations are repetitive, so the tendency is to use the first reasonable resolution without really considering the alternatives.

A simple calculation procedure using the T factor will be illustrated by a numerical example. In these examples, calculations will be made for theoretical gears, representing the tightest center distance expected and the maximum material condition of the gears.

Consider a 35/23 ratio (1.52174/1) and a 6.500" center distance. Assume that 5 DP 20° (p_N = .5904) tooling is available, and the desired normal backlash is .010".

The author has found that 25° is a reasonable upper limit for the transverse operating pressure angle of sets like this, so calculations will begin there. From Equation 8 ψ_b = 22.311° and from Equation 9 p_x = 1.5552".

Rearranging Equation 16, we get:

$$T_1N_1 + T_2N_2 = \text{inv}\phi'(N_1 + N_2) - \pi\left(\frac{B_N}{p_N}\right) \tag{17}$$

With the value of $T_1N_1 + T_2N_2$ known, the distribution of base thickness between the two gears is completely up to the designer's judgment. Some of the options are:

Balanced Sliding. If a balance between approach and recess action is desired, T_1 should be approximately equal to T_2, and

$$T = \frac{\text{inv}\phi'(N_1 + N_2) - \pi(B_N/p_N)}{N_1 + N_2}$$

$$= \text{inv}\phi' - \frac{\pi B_N}{p_N(N_1 + N_2)} \tag{18}$$

Substituting and solving for T:

$$T = .02905791, \ (\phi_T = 24.7555°)$$

Rearranging Equation 15:

$$t_{bn} = \frac{T\,N\,p_N}{\pi} + .5p_N \tag{19}$$

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Substituting, t_{bn} = .4208" for the 23 tooth gear and .4863" for the 35 tooth gear.

The tooth thickness values are converted to the transverse plane, and the tooth thicknesses at the generating diameters are calculated. For standard tooling and standard root clearances, the gear OD's will be 5.558" and 8.247", and the tip lands will be .141" and .145", respectively.

For the pinion, the arc of recess action is 20.5°, and the arc of approach is 20.2°. The sliding velocity at the gear tip is 90 inches per second at 1200 pinion rpm, and at the pinion tip it is 87 inches per second. The complete geometry of this gear set is shown as Example 1 in Appendix B.

Balanced Strength. If balanced strength is the first consideration, a solution must be found by iteration.

For external gears with moderate ratios, the iteration can be shortened by setting the T values in inverse proportion to the numbers of teeth, so that the product T x N is a constant. This is equivalent to setting the base tooth thicknesses equal to each other. (See Equation 15.)

From Equation 17, values for T are .036638 (ϕ_T = 26.6303°) for 23 teeth and .024077 (ϕ_T = 23.3217°) for 35 teeth.

Substituting these values into Equation 19, the base tooth thicknesses are .45356".

The outside diameters are 5.654" and 8.151". The tip lands are .1225" and .1574".

The resulting J factors for bending strength are .511 and .458. The complete geometry of this gear set is shown as Example 2 in Appendix B.

This is only one of an infinite number of possible solutions to the problem. Although it provides a reasonable balance, since the J factors are within 6% of the mean, better balance between the bending strengths can be obtained by increasing one base tooth thickness slightly at the expense of the other. The sum of the base tooth thicknesses must be held constant to maintain the operating backlash.

For moderate gear ratios, base tooth thickness is nearly proportional to root tooth thickness, and bending strength is proportional to the cube of root tooth thickness. In our example, a 2% change (.009") in base tooth thickness will make the difference, since $(1.02)^3 = 1.06$. The resulting J factor is .48 for pinion and gear. The designer must judge if such a small change is really significant to the performance of the finished parts. The complete geometry of this gear set is shown as Example 2A in Appendix B.

A higher strength rating might be obtained by increasing the operating pressure angle at the risk of higher noise and a smaller contact ratio. The T factor system allows these possiblities to be explored with a minimum of repetition, and a clear idea of the effects of each change.

Tight Mesh Center Distance. Once the geometry of the work gear is established, the engineer next usually needs the inspection data. If the work gear is to be inspected by the master gear-test radius method, the operating center distance with the master gear will be required. In this example a master gear with a standard tooth thickness—the space width equal to the tooth thickness at the generating pressure angle—will be used.

T for the master gear is then equal to the involute of the transverse generating pressure angle.

$$T = \text{inv}\ (\sin^{-1}(\frac{\sin\phi_c}{\cos\psi_b})) \qquad (20)$$

$$T = .019203,\ (\phi_T = 21.6969°)$$

T for the 35 tooth work gear is .024077, from Example 2.

Substituting the appropriate values in Equation 16, inv ϕ' = .022302 and ϕ' = .3972 radians. From Equation 8, the tight mesh center distance is 6.0583". The complete geometry of this gear set is shown as Example 3 in Appendix B.

Shaper Cutting Conditions. If the gear is to be cut with a pinion shaped cutter, the calculation is done as in the previous example, except that allowance must be made for the sharpening condition of the cutter. The operating center distance with the cutter is necessary for the calculation of interference and form diameters with the shaper cutter. It is usually calculated for two conditions, new cutter and thinnest usable worn cutter.

The base tooth thickness of the cutter can be taken from the cutter drawing or from span measurement of the actual cutter. The second alternative is usually best if a worn cutter is to be used to cut the part.

For example, consider a 71 tooth internal spur gear to be cut with a 20 tooth 3 P_{nd} cutter, to a base tooth thickness of .1460". For the gear, T = .015559, ϕ_T = 20.2787°. T, an involute function, must be positive, so the absolute value of N must be used. In the rest of the calculation, the numbers of internal teeth and internal diameters are considered negative.

The results are shown in Table 2.

Table 2

Cutter Data	New Cutter	Worn Cutter
t_{nb}	.6120"	.5247"
T factor	.019156	.005220
ϕ_T	21.6798°	14.2160°
ϕ'	19.668°	21.8441°
C	8.482"	8.605"
Example No.	4	5

Mesh With A Rack. The T factor system can be used to analyze rack and pinion meshes by assuming that the rack is a gear with a very large number of teeth, such as 9999. This eliminates the computational problems caused by infinite values for t_b, C and N_2. The T factor for the rack is equal to its transverse pressure angle. As an example, we will examine a 33 tooth spur pinion, cut with a 5 P_{nd} 14.5° hob to a base tooth thickness of .4131", in mesh with a special rack. The pinion was designed for a special job requiring low noise and high beam strength. The rack pressure angle of 8° was chosen to minimize separating forces and provide a maximum contact ratio for quiet operation.

The rack circular pitch required to match the pinion base pitch is .6143". T is then the involute function of 8°, or .000914.

This unusual gear set was built for a linear motion device, similar to a planer. (See Figs. 5-7.) It illustrates the ability of the T factor to describe the gear geometry and to facilitate the necessary mesh calculations, even if the two parts are made with widely different cutting tools, as is the case here. The complete geometry of this gear set is shown as Example 6 in Appendix B.

Mesh With A Hob. The T factor as derived is valid for meshes where the axes of rotation of the two members are parallel, but not valid in hobbing or shaping with a rack shaped cutter set in the normal plane. In these cases, it is simple to convert the tool geometry to its equivalent rack in the transverse plane by calculating its transverse pressure angle using Equation 17. For example, the 35 tooth helical gear of Example 2 might be cut with a standard hob with a 20° normal profile angle. The transverse pressure angle of the hob, $\phi_T = 21.6971°$ and $T = .019204$. Example 7 in Appendix B shows the complete geometry of this gear set.

Establishing Root and Outside Diameters. The examples given here establish the root diameters of the gear and pinion from the cutting conditions, using the generating diameter and tooth thickness to establish the cutting position of the cutter, and calculating the tip position of the cutter from the actual cutter geometry. This allows the use of different tools for the gear and pinion if desired. The shaper cutter and hob examples show how this works. The outside diameters are chosen to give standard root clearances from the root diameters of the mating parts, except in the cutting tool examples, where the root clearance with the cutter is zero.

The resulting whole depths vary from standard in order to maintain the designed clearances. This system is not required, but it has the advantage of automatically maintaining equal cutting depths for gear and pinion.

Limit Diameter for Minimum Tip Land. Pinions designed to operate on widely spread centers run the risk of having tip lands which are too small for good operation or for proper

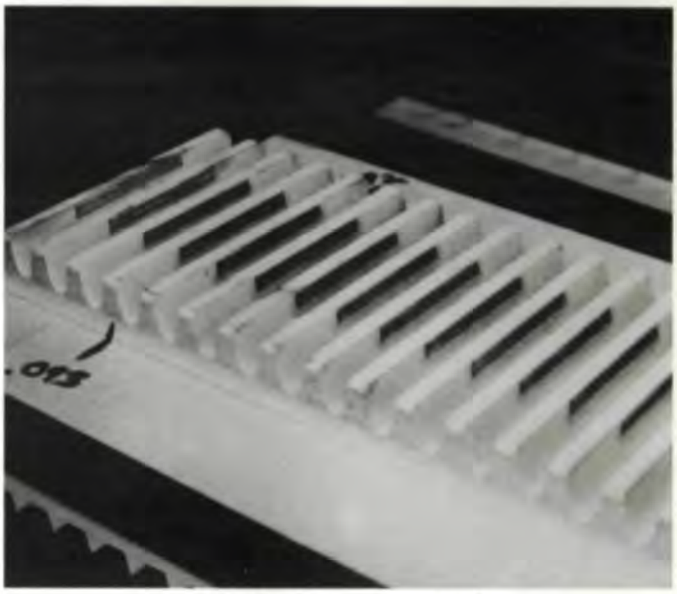


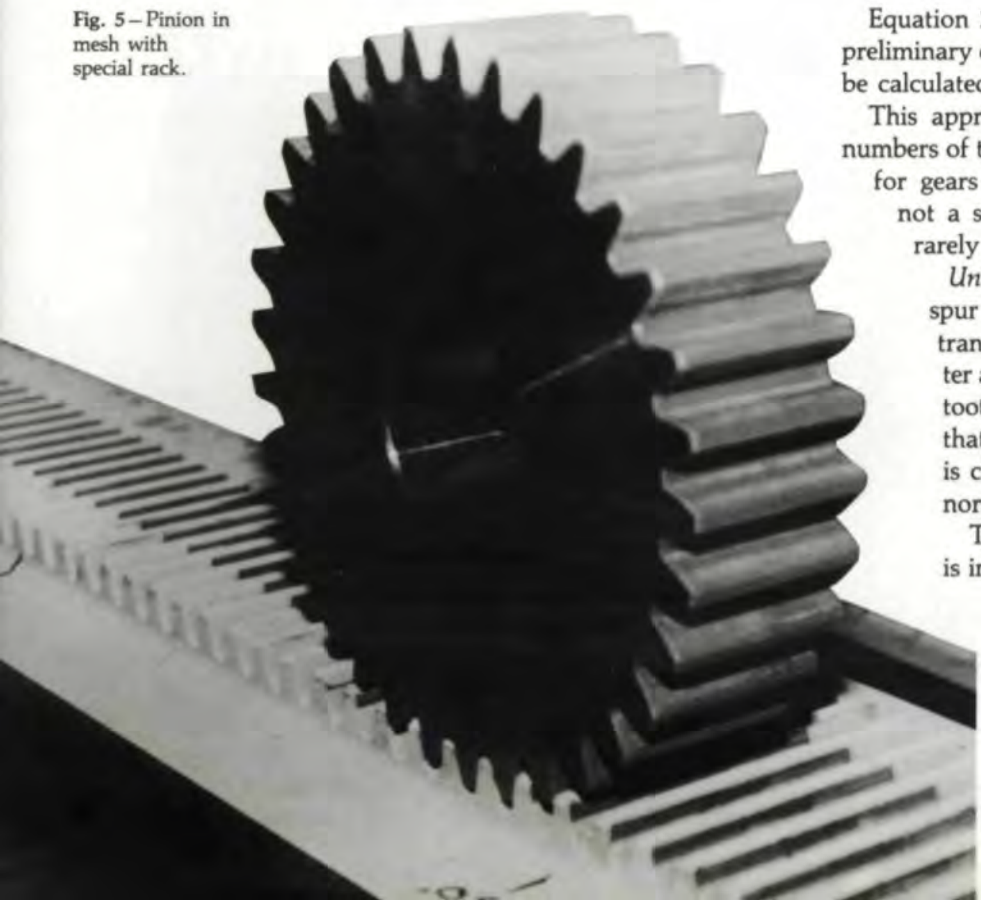
Fig. 6 – Special 8° generated rack.

heat treatment. An adaptation of the T factor allows a quick approximation of the maximum usable diameter limited by minimum tip land. This approximation is based on the premise that a minimum value for tip land should be $.300''/P_{nd}$.

From Equation 14 the involute function at any diameter where the ratio of tooth thickness to circular pitch is known can be calculated. If we assume that the pressure angle at the limiting outside diameter will be approximately 40°, the ratio of base pitch to circular pitch at the outside diameter will be .776, the cosine of 40°. These assumptions let us derive the following:

$$\text{inv}\phi_{Dmax} = \frac{\pi(t_{bn} - .08p_N)}{N\ p_N} \tag{21}$$

Fig. 5 – Pinion in mesh with special rack.



Equation 21 is a good guide to maximum diameter for preliminary design of pinions, but the actual tip land should be calculated before the design is finalized.

This approximation is not useful for gears with large numbers of teeth, since the 40° approximation is unrealistic for gears with more than 40 teeth. This limitation is not a serious one, since the problem is encountered rarely in gears with large numbers of teeth.

Universal Pin Size. Grosser demonstrated that for spur gears, a pin with a diameter equal to half the transverse base pitch will always rest with its center at the diameter where the space width equals the tooth thickness. (See Equation 13). It is easy to show that this is also true for helical gears if the pin size is chosen so that the pin size is equal to half the normal base pitch.

The involute function at that diameter is T, which is independent of the helix angle. The diameter is, of course, a function of D_b, which is inversely proportional to ψ_b. This relationship provides an easy universal way to determine a pin size without tables or long calculation if T is known.

This line of reasoning returns us to the

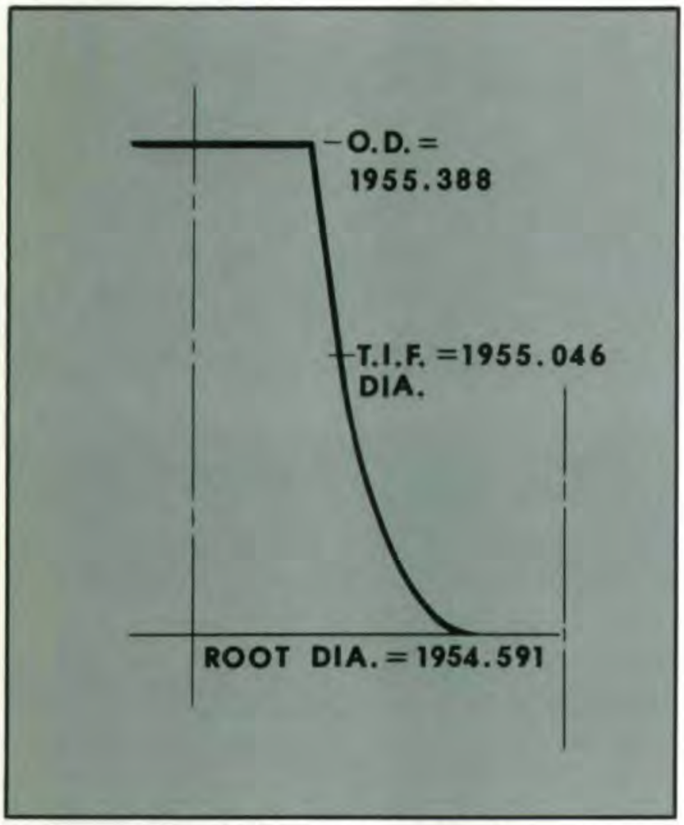


Fig. 7 – Plotted profile of special rack.

concept that a logical relationship should exist between the descriptions of nonstandard helical gears and similar nonstandard spur gears. The T factor does this by giving gears with the same fundamental geometry the same name.

Shaving Cutters. Combining Equation 15 with Equation 12 and eliminating the backlash term, since shaving cutters are in tight mesh,

$$N_1 \text{inv}\phi'_1 + N_2 \text{inv}\phi'_2 = T_1 N_1 + T_2 N_2 \tag{22}$$

Also, since the normal operating pressure angle is identical on both parts,

$$\sin\phi'_1 \cos\psi_{b1} = \sin\phi'_2 \cos\psi_{b1} \tag{23}$$

This pair of equations, which are a rearrangment of those presented by National Broach[9] must be solved by iteration. The solution is beyond the scope of this article, but the equations are presented here to illustrate the value of the T factor in describing a nonstandard gear and simplifying the form of most gear geometry equations.

Conclusions

The equations derived and demonstrated above are not dependent on the T factor. Similar relationships can be derived from the fundamental parameters given at the beginning of this article. Is it really worthwhile to introduce another pet theory into the gear geometry literature? Why not work with t_{bn} and p_N?

At times we need to compare two designs of different pitches to get a sense of how a new design compares to a known set in the field. If we can make the comparison on a nondimensional parametric basis, we can generalize the information and help with future designs too.

The appeal of the x factor system amply demonstrates the need for a comparison factor. The T factor is proposed as a more clearly defined and more useful parameter. By eliminating the assumed cutter definition and the assumed backlash from the parameter, we can compare designs made with nonstandard cutters as easily as we can compare more conventional gears.

Appendix A – Derivations

Symbols

Symbols used are the same as those in the body of the article. The prime symbol (′) denotes values at the operating pitch diameters.

Parallel Axis Gears

Conditions:

- Axes are parallel.
- Transverse operating circular pitches are equal.
- Transverse operating pressure angles are equal.
- Sum of transverse operating tooth thicknesses and transverse operating backlash equals transverse operating circular pitch.

(continued on page 22)

DESCRIBING NONSTANDARD GEARS . . .
(continued from page 20)

Derivation of Equation 5:

$$p' = t'_1 + t'_2 + B'$$

$$p' = D'_1 \left(\frac{t_{b1}}{D_{b1}} - \text{inv}\phi'\right) + D'_2 \left(\frac{t_{b2}}{D_{b2}} - \text{inv}\phi'\right) + B'$$

$$\frac{p'}{D'_1} = \frac{t_{b1}}{D_{b1}} - \text{inv}\phi' + \frac{D'_2}{D'_1}\frac{t_{b2}}{D_{b1}} - \frac{D'_2}{D'_1}\text{inv}\phi' + \frac{B'}{D'_1}$$

$$\frac{p'}{D'_1} = \frac{p_b}{D_{b1}}$$

$$\frac{D'_2}{D'_1} = \frac{D_{b2}}{D_{b1}}$$

$$\frac{B'}{D'_1} = \frac{B}{D_{b1}}$$

Note: B' is transverse backlash at D'.
B is transverse backlash in plane of action.

$$p_b = t_{b1} - D_{b1}\,\text{inv}\phi' + t_{b2} - D_{b2}\,\text{inv}\phi' + B$$

$$\text{inv}\phi' (D_{b1} + D_{b2}) = t_{b1} + t_{b2} + B - p_b$$

$$\text{inv}\phi' = \frac{t_{b1} + t_{b2} + B - p_b}{D_{b1} + D_{b2}}$$

Crossed Axis Gears

Conditions:

- Axis are in parallel planes.
- Normal operating circular pitches are equal.
- Normal operating pressure angles are equal.
- Sum of normal operating tooth thicknesses plus normal circular backlash equals operating normal circular pitch.

Derivation of Equation 12:

Given: t_{bn1}, t_{bn2}, B_n, N_1, N_2, p_N, p_{x1} and p_{x2}

Note that axial pitches are different for the two gears.

$$p'_{n1} = p'_{n2} = t'_{n1} + t'_{n2} + B_n$$

For each gear:

$$p'_t = \frac{\pi D'}{N} \qquad t'_n = t'_t \cos\phi'$$

$$p'_n = p'_t \cos\psi' \qquad D'\cos\psi' = \frac{N\,p'_n}{\pi}$$

$$\frac{t_{tb}}{D_b} = \frac{\pi\, t_{nb}}{N p_N}$$

$$p'_n = p'_{n1} = p'_{t1}\cos\psi'_1 = \frac{\pi\, D'_1\cos\psi'_1}{N_1}$$

$$p'_n = D'_1\cos\psi'_1\left(\frac{\pi\, t_{nb1}}{N_1 p_n} - \text{inv}\phi'_{t1}\right) + D_2\cos\psi'_2\left(\frac{\pi\, t_{nb2}}{N_2 p_n} - \text{inv}\phi'_{t2}\right) + B_n$$

$$p'_n = p'_n\left(\frac{t_{nb1}}{p_N} + \frac{t_{nb2}}{p_N}\right) - \frac{p'_n N_1}{\pi}\text{inv}\phi'_{t1} - \frac{p'_n N_2}{\pi}\text{inv}\phi'_{t2} + B_n$$

$$1 = \frac{t_{nb1}}{p_N} + \frac{t_{nb2}}{p_N} - \frac{N_1}{\pi}\text{inv}\phi'_{t1} - \frac{N_2}{\pi}\text{inv}\phi'_{t2} + \frac{B_n}{p'_n}$$

$$\left(\frac{B_n}{p'_n} = \frac{B_N}{p_n}\right)$$

$$N_1\text{inv}\phi'_{t1} + N_2\text{inv}\phi'_{t2} = \frac{\pi}{p_N}(t_{bn1} + t_{bn2} + B_N) - \pi =$$

$$T_1N_1 + T_2N_2 + \frac{\pi B_N}{p_N} \qquad (22)$$

$$\text{or} \ldots = N_1\left(\frac{t_{b1}}{D_{b1}}\right) + N_2\left(\frac{t_{b2}}{D_{b2}} + \pi\left(\frac{B_N}{p_N}\right) - \pi \qquad (12)$$

Tooth Thickness (See Fig. 8.)

Derivation of Equation 13:

R_1 is the radius where:
"t" = tooth thickness is equal to "s" = tooth space.
ϕ_1 is the pressure angle at that radius.
R_b is the base circle radius.

$$\frac{p_b}{4R_b} + inv\phi_1 = \frac{t_b}{2R_b}$$

$$inv\phi_1 = \frac{t_b - \frac{p_b}{2}}{D_b} \qquad (13)$$

For any value of t_1:

$$inv\phi_1 = \frac{t_b}{2R_b} - \frac{p_b}{2R_b}\,\frac{t_1}{p_1}$$

$$inv\phi_1 = \frac{t_b - p_b t_1/p_1}{D_b} \qquad (14)$$

where p_1, ϕ_1 and t_1 are at D_1

Derivation of Equation 21.

If .300″/p_{nd} is a reasonable limit for tip land, t_o:

$$p_{nd} = \frac{\pi}{p} = \frac{\pi \cos\phi_c}{p_N} \text{ and } t_o = \frac{.300\ p_N}{\pi\cos\phi_c}$$

for 14.5°, $t_o = .099p_N$
for 20°, $t_o = .101p_N$
for 25°, $t_o = .105p_N$

Use $.1p_N$ as an approximation.

$$inv\phi_o = \frac{t_b - p_b(.1p_N)/p_o}{D_b} \qquad (14)$$

If we assume that 40° is a good approximation for ϕ_o, $p_o = p_N/\cos 40°$.

$$inv\phi_{Dmax} = \frac{t_b - p_b(.1p_N .776/p_N)}{p_b}$$

$$inv\phi_{Dmax} = \frac{t_b - .08p_b}{D_b} \qquad (21)$$

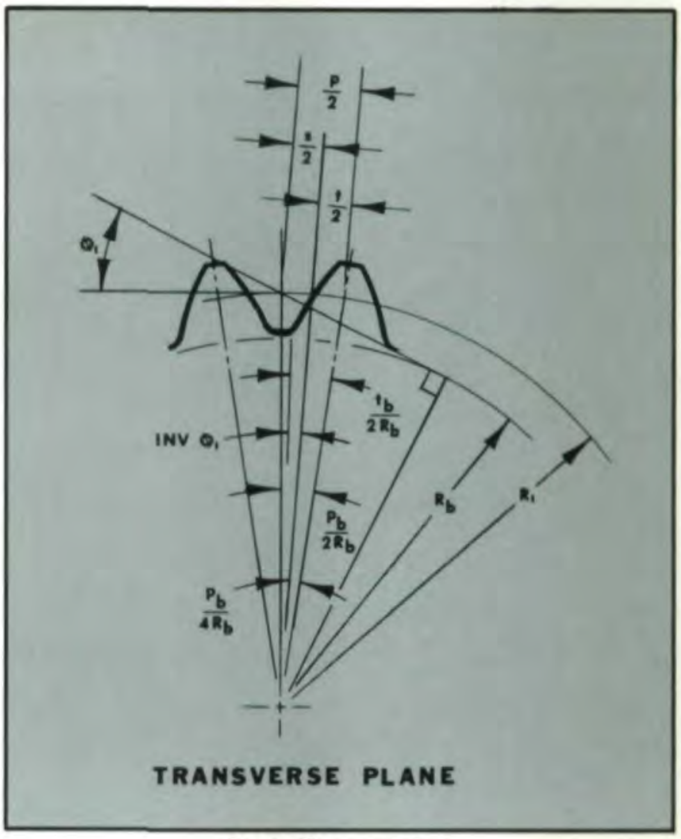


Fig. 8 – Transverse plane.

(continued on page 26)

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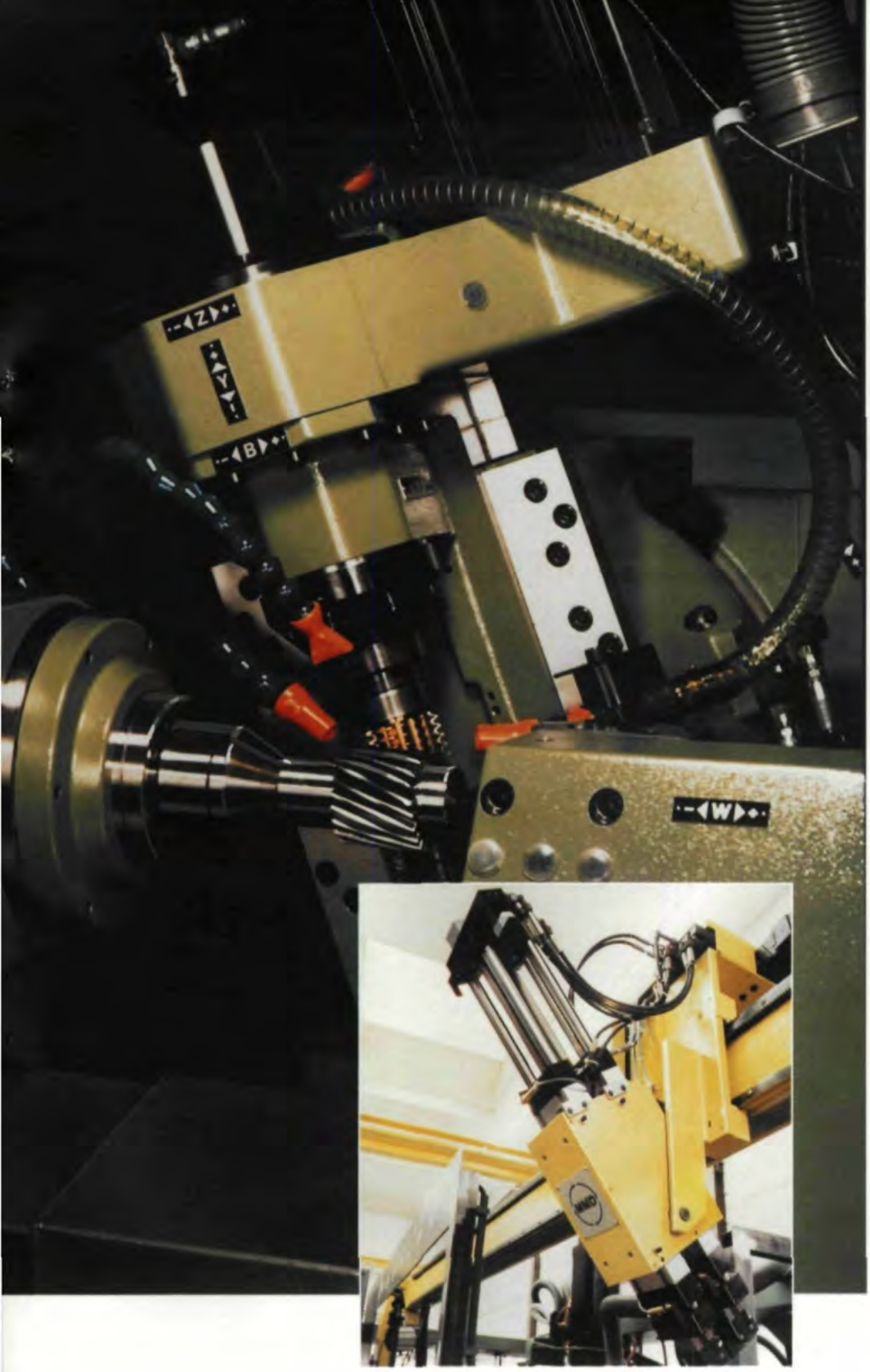

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CIRCLE A-5 ON READER REPLY CARD

DESCRIBING NONSTANDARD GEARS . . .
(continued from page 23)

Appendix B — Numerical Examples

Description Example:	1	2	2A	3	4	5	6	7
No. Pinion Teeth	23	23	23	20	20	20	33	9999
No. Gear Teeth	35	35	35	35	−71	−71	9999	35
Normal Base Pitch	0.5904	0.5904	0.5904	0.5904	0.9840	0.9840	0.6083	0.5904
Norm. Base Tooth Thk., P	*0.4208*	*0.4536*	*0.4450*	*0.3674*	*0.6120*	*0.5247*	*0.4131*	*36.3831*
Norm. Base Tooth Thk., G	0.4863	0.4536	0.4620	0.4536	0.1460	0.1460	2.0747	0.4536
Axial Pitch	1.5552	1.5552	1.5552	1.5552	—	—	—	1.5552
Addendum Dia., Pinion	5.5580	5.6540	5.6290	4.7730	7.5800	7.3300	7.2500	2186.67
Addendum Dia., Gear	8.2470	8.1510	8.1760	8.1510	−23.2000	−23.2000	1955.4570	8.1510
"T" Pinion	0.029054	0.036632	0.034652	0.019203	0.019156	0.005220	0.017047	0.019204
"T" Gear	0.029050	0.024073	0.025356	0.024073	0.015559	0.015559	0.000914	0.024073
"Phi T" Pinion	0.4320	0.4648	0.4567	0.3787	0.3784	0.2481	0.3645	0.3787
Degrees	24.7544	26.6291	26.1686	21.6969	21.6798	14.2160	20.8835	21.6971
"Phi T" Gear	0.4320	0.4070	0.4138	0.4070	0.3539	0.3539	0.1396	0.4070
Degrees	24.7534	23.3205	23.7088	23.3205	20.2787	20.2787	8.0000	23.3205
Base Helix Angle	0.3894	0.3894	0.3894	0.3894	.0000	.0000	.0000	0.3894
Degrees	22.3122	22.3122	22.3122	22.3122	.0000	.0000	.0000	22.3122
Center Dist. Fixed?	1	1	1	0	0	0	0	0
Center Distance	6.5000	6.5000	6.5000	6.0583	−8.4818	−8.6049	981.1751	1096.96
Normal Backlash	*0.0100*	*0.0100*	*0.0100*	*0.0000*	*0.0000*	*0.0000*	*0.0050*	*0.0000*
Inv. Oper. Press. Angle	0.029951	0.029951	0.029951	0.022302	0.014148	0.019613	0.000970	0.019221
Cos. Oper. Press. Angle	0.906355	0.906355	0.906355	0.922136	0.941665	0.928200	0.989880	0.929112
Operating Press Angle	0.4362	0.4362	0.4362	0.3972	0.3433	0.3813	0.1424	0.3788
Degrees	24.9936	24.9936	24.9936	22.7597	19.6668	21.8441	8.1582	21.7033
Gen. Helix Angle, Pinion	0.4159	0.4159	0.4159	0.4159	.0000	.0000	.0000	0.4159
Degrees	23.8297	23.8297	23.8297	23.8297	.0000	.0000	.0000	23.8297
Gen. Helix Angle, Gear	0.4159	0.4159	0.4159	0.4159	.0000	.0000	.0000	0.4159
Degrees	23.8297	23.8297	23.8297	23.8297	.0000	.0000	.0000	23.8297
Tip Land, Pinion	0.1414	0.1225	0.1277	0.1609	0.1096	0.1624	0.1498	0.1442
Tip Land, Gear	0.1449	0.1574	0.1543	0.1574	0.3453	0.3453	0.2608	0.1574
Whole Depth, Pinion	0.452	0.453	0.453	0.450	0.835	0.837	0.228	0.500
Whole Depth, Gear	0.453	0.453	0.453	0.453	0.671	0.671	0.232	0.453
X FACTORS								
X Pinion (Thickness)	0.3112	0.5507	0.4881	.0000	0.1163	−0.2664	0.7339	−0.0005
X Gear (Thickness)	0.4734	0.2341	0.2958	0.2341	−0.0622	−0.0622	−0.0466	0.2341
X Pinion (Diameter)	*0.3233*	*0.5633*	*0.5008*	*0.0006*	*0.3700*	*−0.0050*	*0.6250*	*0.25*
Backlash Allow. Pinion	0.0019	0.0020	0.0020	0.0001	0.0616	0.0635	−0.0113	0.04
X Gear (Diameter)	0.4866	0.2466	0.3091	0.2466	−0.3000	−0.3000	−0.1180	0.2466
Backlash Allow. Gear	0.0021	0.0020	0.0021	0.0020	−0.0577	−0.0577	−0.0072	0.0020
PINION CUTTER DATA								
Normal DP	5.0000	5.0000	5.0000	5.0000	3.0000	3.0000	5.0000	5.0000
Normal Module (mm)								
Profile Angle	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	14.500	20.0000
Profile Angle (RAD)	0.3491	0.3491	0.3491	0.3491	0.3491	0.3491	0.2531	0.3491
Circular Pitch	0.6283	0.6283	0.6283	0.6283	1.0472	1.0472	0.6283	0.6283
Cutter Thickness	0.3142	0.3142	0.3142	0.3142	0.5236	0.5236	0.3142	0.3142
Cutter Addendum	0.2500	0.2500	0.2500	0.2500	0.4167	0.4167	0.2500	0.2500

Appendix B — Numerical Examples (continued)

Description Example:	1	2	2A	3	4	5	6	7
GEAR CUTTER DATA								
Normal DP	5.0000	5.0000	5.0000	5.0000	3.0000	3.0000	5.1141	5.0000
Normal Module (mm)								
Profile Angle	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	8.0000	20.0000
Profile Angle (RAD)	0.3491	0.3491	0.3491	0.3491	0.3491	0.3491	0.1396	0.3491
Circular Pitch	0.6283	0.6283	0.6283	0.6283	1.0472	1.0472	0.6143	0.6283
Cutter Thickness	0.3142	0.3142	0.3142	0.3142	0.5236	0.5236	0.1912	0.3142
Cutter Addendum	0.2500	0.2500	0.2500	0.2500	0.4167	0.4167	0.0500	0.2500
DIAMETERS								
Base Diameter, Pinion	4.6724	4.6724	4.6724	4.0630	6.2643	6.2643	6.3898	2031.28
Base Diameter, Gear	7.1102	7.1102	7.1102	7.1102	−22.2384	−22.2384	1936.1013	7.1102
Generating Dia. Pinion	5.0287	5.0287	5.0287	4.3728	6.6667	6.6667	6.6000	2186.17
Generating Dia. Gear	7.6524	7.6524	7.6524	7.6524	−23.6667	−23.6667	1955.1121	7.6524
Operating Dia. Pinion	5.1552	5.1552	5.1552	4.4060	6.6524	6.7489	6.4551	2186.26
Operating Dia. Gear	7.8448	7.8448	7.8448	7.7106	−23.6160	−23.9586	1955.8950	7.6527
Max. Add. Dia. Pinion	5.6575	5.7196	5.7035	4.9076	7.5661	7.4067	7.3958	2186.86
Max. Add. Dia. Gear	8.3652	8.2973	8.3150	8.2973	ERR	ERR	1956.8395	8.2973
Addendum Dia. Pinion	5.5580	5.6540	5.6290	4.7730	7.5800	7.3300	7.2500	2186.67
Addendum Dia. Gear	8.2470	8.1510	8.1760	8.1510	−23.2000	−23.2000	1955.4570	8.1510
Std. Add. Dia. Pinion	5.4287	5.4287	5.4287	4.7728	7.3333	7.3333	7.0000	2186.57
Std. Add. Dia. Gear	8.0524	8.0524	8.0524	8.0524	−23.0000	−23.0000	1955.5031	8.0524
Root Dia. Pinion	4.6532	4.7490	4.7239	3.8728	5.9109	5.6557	6.7935	2185.670
Root Dia. Gear	7.3417	7.2460	7.2707	7.2460	−24.5415	−24.5415	1954.9939	7.2460
RADIAL CLEARANCE								
Pinion Root Clearance	0.0499	0.0500	0.0500	0.0464	0.1627	0.1673	0.0498	0.0475
Gear Root Clearance	0.0501	0.0500	0.0502	0.0488	−0.0011	0.0009	0.0531	−0.0002
Standard Clearance	0.0500	0.0500	0.0500	0.0500	0.0833	0.0833	0.0500	0.0500
DIMENS. IN TRANSV. PLANE								
Base Pitch	0.6382	0.6382	0.6382	0.6382	0.9840	0.9840	0.6083	0.6382
Operating Circ. Pitch	0.7042	0.7042	0.7042	0.6921	1.0450	1.0601	0.6145	0.6869
Base Tooth Thick., Pin.	0.4549	0.4903	0.4810	0.3971	0.6120	0.5247	0.4131	39.3276
Base Tooth Thick., Gear	0.5257	0.4903	0.4994	0.4903	0.1460	0.1460	2.0747	0.4903
Tooth Thick., Gen. Pin.	0.3930	0.4311	0.4211	0.3434	0.5518	0.4589	0.3901	0.3434
Tooth Thick., Gen. Gear	0.4188	0.3807	0.3905	0.3807	0.5085	0.5085	0.3094	0.3807
Tooth Thick., Std. Pin.	0.3434	0.3434	0.3434	0.3434	0.5236	0.5236	0.3142	0.3434
Tooth Thick., Std. Gear	0.3434	0.3434	0.3434	0.3434	0.5236	0.5236	0.3142	0.3434
Press. Angle, Gen. Pin.	0.3787	0.3787	0.3787	0.3787	0.3492	0.3492	0.2531	0.3787
Degrees	21.6971	21.6971	21.6971	21.6971	20.0070	20.0070	14.5000	21.6971
Press. Angle, Gen. Gear	0.3787	0.3787	0.3787	0.3787	0.3492	0.3492	0.1396	0.3787
Degrees	21.6971	21.6971	21.6971	21.6971	20.0070	20.0070	7.9966	21.6971

(continued on page 48)

DESCRIBING NONSTANDARD GEARS . . .

(continued from page 27)

Appendix C — Survey Results

Five gear designers in the US and two in Europe were asked to determine the x factor for the following gear set, which is used in a high speed, high shock application:

Center Distance	6.000/6.005"	
Spur Gears		
Normal Diametral Pitch	5	
Hob Profile Angle, Normal	20°	
Base Pitch	.5904263"	
Hob Addendum	.280"	
Hob Tooth Thickness	.314"	
	PINION	GEAR
Number of Teeth	23	35
Base Tangent Length	1.590/1.588"	2.257/2.254"
Tooth Thickness at Std. Dia.	.369/.367"	.413/.410"
Outside Diameter	5.130/5.125"	7.655/7.650"

The following data was calculated, but not provided:

Minimum Backlash	.0108"	
X (OD Method)	.3250	.6375
Backlash Allowance	−.0054"	−.0056"
X (TT Method)	.3619	.6759

SURVEY RESPONSES

	X_1	X_2	SUM
A	.3250	.6375	.9625
B	.3619	.6759	1.0375
C	.3688	.6676	1.0364
D	.3804	.6829	1.0633
E	.4000	.7169	1.1169
F	.4994	.8016	1.3010
Average	.3892	.6971	1.0863
Range	+28/−16.5%	+14.9/−8.6%	+19.7/−11.4%

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