Tribology Aspects in Angular Transmission Systems

Part V: Face Gears

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(This article is part five of an eight-part series on the tribology aspects of angular gear drives. Each article will be presented first and exclusively by Gear Technology*, but the entire series will be included in Dr. Stadtfeld's upcoming book on the subject, which is scheduled for release in 2011.)*

Design

If two axes are positioned in space and the task is to transmit motion and torque between them using some kind of gears, then the following cases are commonly known:

- Axes are parallel → Cylindrical Gears (line contact)
- Axes intersect under an angle → Bevel Gears (line contact)
- Axes intersect under an angle → Face Gears (line contact)
- Axes cross under an angle → Crossed Helical Gears (point contact)
- Axes cross under an angle (mostly 90°) → Worm Gear Drives (line contact)
- Axes cross under any angle → Hypoid Gears (line contact)

The axes of face gears in the best sense of the word intersect under an angle of 90°. The face gear geometry is based on a cylindrical pinion (spur or helical) that meshes with a flat-face or crown gear. This naturally requires a shaft angle of 90°. The face gear is derived from the cylindrical pinion, using the principle of conjugate surface generation. Also, shaft angles smaller or larger than 90° can be realized using the same principle, and the face gear becomes a "beveled" gear. Short

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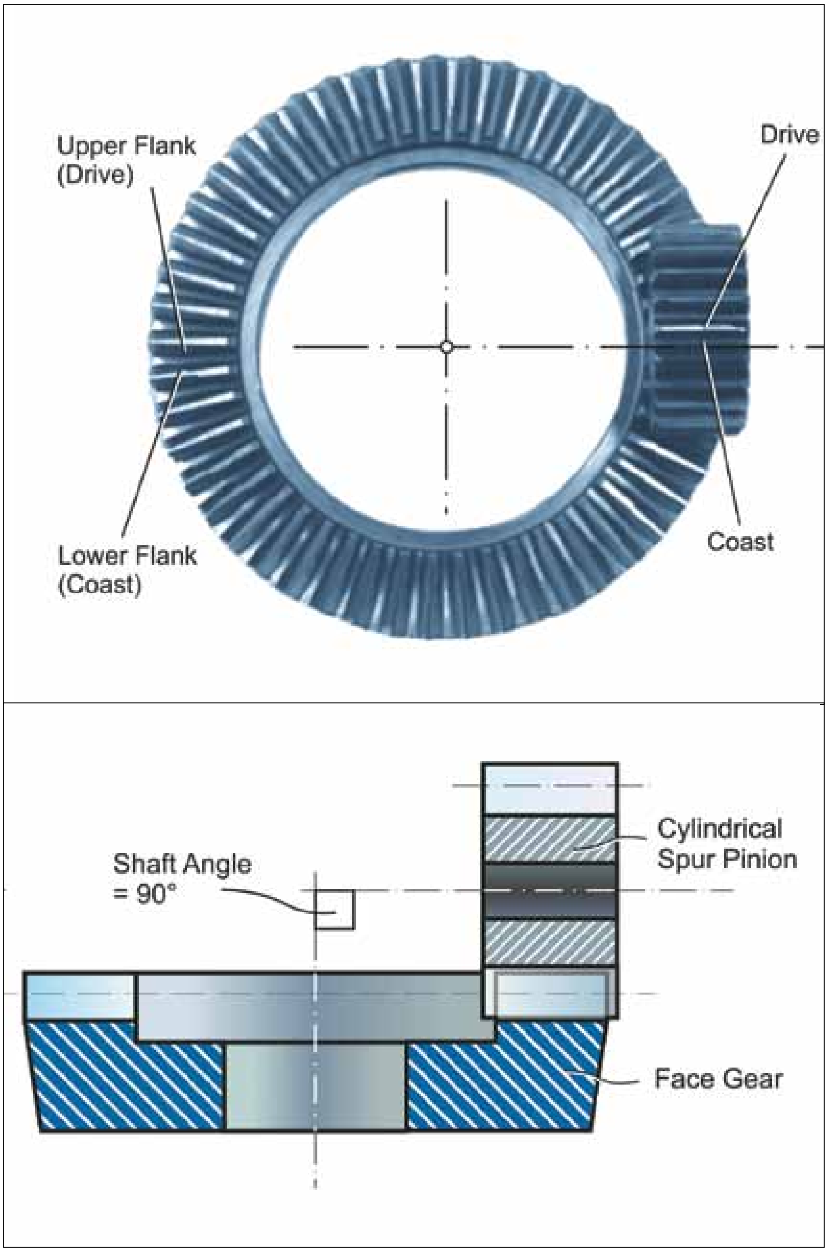

Figure 1—Face gear geometry.

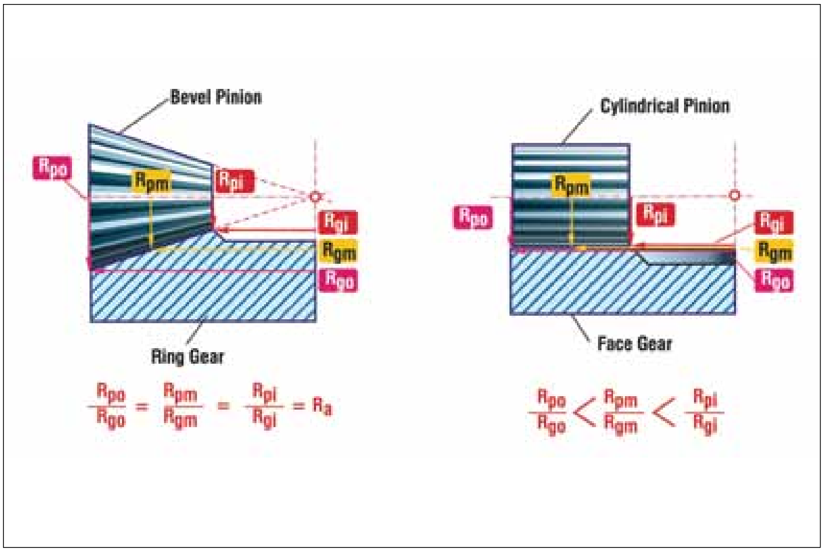

Figure 2—Radii discrepancy in face gears vs. bevel gears.

of a separate name, modified gears in mesh with cylindrical pinions in a non-parallel axis arrangement are called face gears. In beveloid gears it is also possible to use a cylindrical pinion as one member and a beveloid as the second member (manufactured with cylindrical gear cutting and grinding technology). The beveloid in this case is the special case of a face gear with similar flank geometry features.

The pitch surfaces are a cylinder and a cone which are calculated with the following formula:

$$\gamma_1 = 0°$$
$$\Sigma = \gamma_1 + \gamma_2$$
$$\gamma_2 = \Sigma - \gamma_1$$

in case of: $\Sigma = 90°$, $\gamma_2 = 90°$

where:
γ_1 Pinion pitch angle
Σ Shaft angle
γ_2 Gear pitch angle

Face gears are designed and manufactured with parallel tooth depth. The problem is the changing circumference of the face gear member between outer and inner diameter. If the gearing law is applied in order to establish conjugate face gear flank surfaces, it results in heel-pressure angles of up to 45° and toe-pressure angles of down to 0° (in case of 20° pinion-pressure angle). In some area towards the toe—where the pressure angle falls below the limit pressure angle (at about 7°)—the gearing law no longer applies and undercut occurs that extends to the top of the face gear flanks, which is true of a face width commonly used in spiral bevel gears. The solution is to use smaller face width. The profile function of the face gear is not a function found in the family of involutes (octoids).

Figure 1 shows an illustration of a face gear set and a cross-sectional drawing. Face gears with 0° spiral angle have no preferred driving direction. Because of the orientation of the flanks during manufacture, the designations "upper" and "lower" flank are used. Per definition, the calculation programs treat the spur pinion like a left-hand member and the mating face gear like a right-hand member. Consequently there is a drive-side and a coast-side designation for proper definition rather than for implying better suitability of torque and motion transmission.

Analysis

The attempt to apply the gearing law in order to generate a precise mating flank to a given cylindrical pinion flank is possible—but not without obstacles. The straight bevel gear set in the left-side graphic in Figure 2 has identical ratios between the pinion and gear radii along the operating pitch line. This is different for the face gear set to the right in Figure 2 that shows the changing ratio between the pinion and gear radii along the pinion pitch line.

The laws of physics establish the operating pitch line—which is the locus with no sliding and pure rolling—while the correct ratio is transmitted. The operating pitch line in face gear drives is shown in Figure 3. It is identical to the pitch line in an analogue bevel gear set. The inclination between the pinion pitch line and the operating pitch line in Figure 3 leads to a pressure angle distortion. The nominal pressure angle matches only at the crossing point of the operating pitch with the pinion pitch line (in one single point). It increases towards the heel, where it causes pointed topland and reduces towards toe, where it reaches the limit-pressure angle and results in undercut. The face width of face gears is therefore limited in both directions.

The precise conjugate face gear flank surface will result in line contact between the two mating flanks (rolling without any load). In the case of a torque transmission, the contact lines become contact zones (stripes) with a surface stress distribution that shows peak values at the two ends of each observed contact line and where the contact line is limited by the outer end of the tooth (heel) and the undercut zone (toe). In order to prevent this edge contact, a crowning along the face width of the teeth (length crowning) and in profile direction (profile crowning) is introduced into the gear flanks. A theoretical tooth contact analysis (TCA) previous to gear manufacturing can be performed in order to observe the effect of the crowning in connection with the basic characteristics of the particular gear set. This also allows the possibility to return to the basic dimensions in order to optimize them if the analysis reveals any deficiencies. Figure 4 shows the result of a TCA of a typical face gear set.

The two columns in Figure 4 represent the analysis results of the two mating flank combinations (*see also "General Explanation of Theoretical Bevel Gear Analysis"*); however, for face gears the designation "drive" and "coast" are strictly a definition rather than a recommendation. The top graphics show the ease-off topographies. The surface above the presentation grid shows the consolidation of the pinion and gear crowning. The ease-offs in Figure 4 have a combination of length and profile crowning, thus establishing a clearance along the boundary of the teeth.

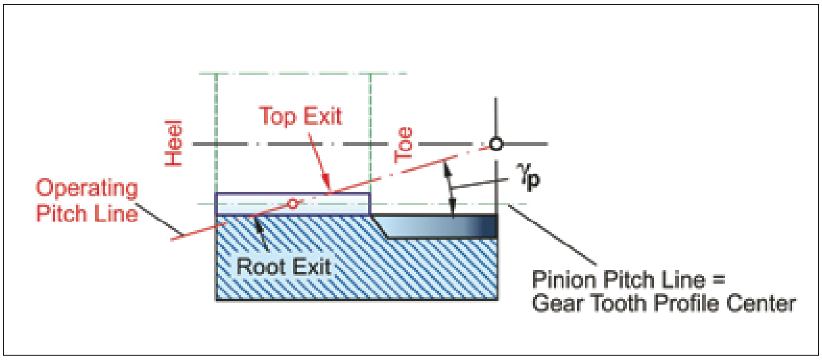

Figure 3—Pinion pitch line and operating pitch line.

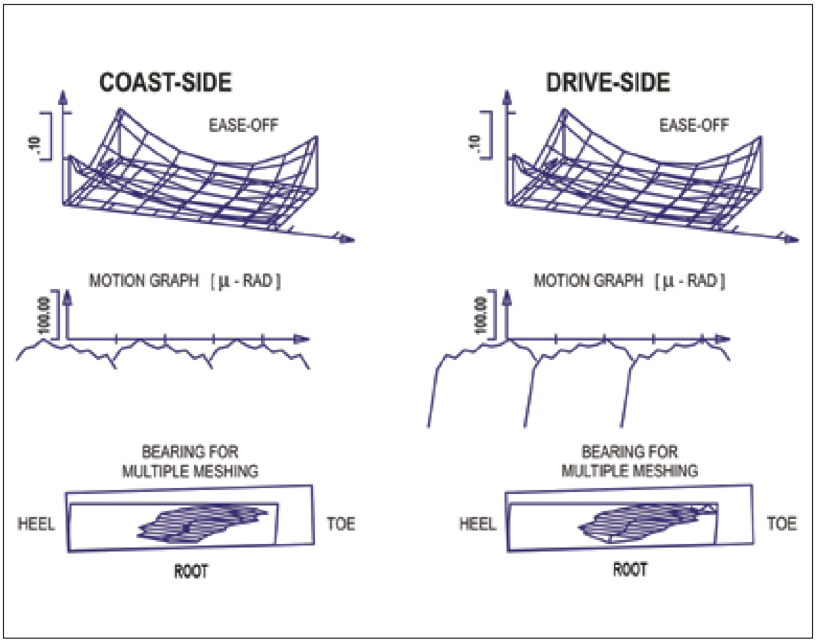

Figure 4—Tooth contact analysis of a face gear set.

Below each ease-off the motion transmission graphs of the particular mating flank pair are shown. The graphs show the angular variation of the driven gear in the case of a pinion that rotates with a constant angular velocity. The graphs are drawn for the rotation and mesh of three consecutive pairs of teeth. While the ease-off requires a sufficient amount of crowning in order to prevent edge contact and allow for load-affected deflections, the crowning in turn causes proportional amounts of angular motion variation of about 50 micro radians.

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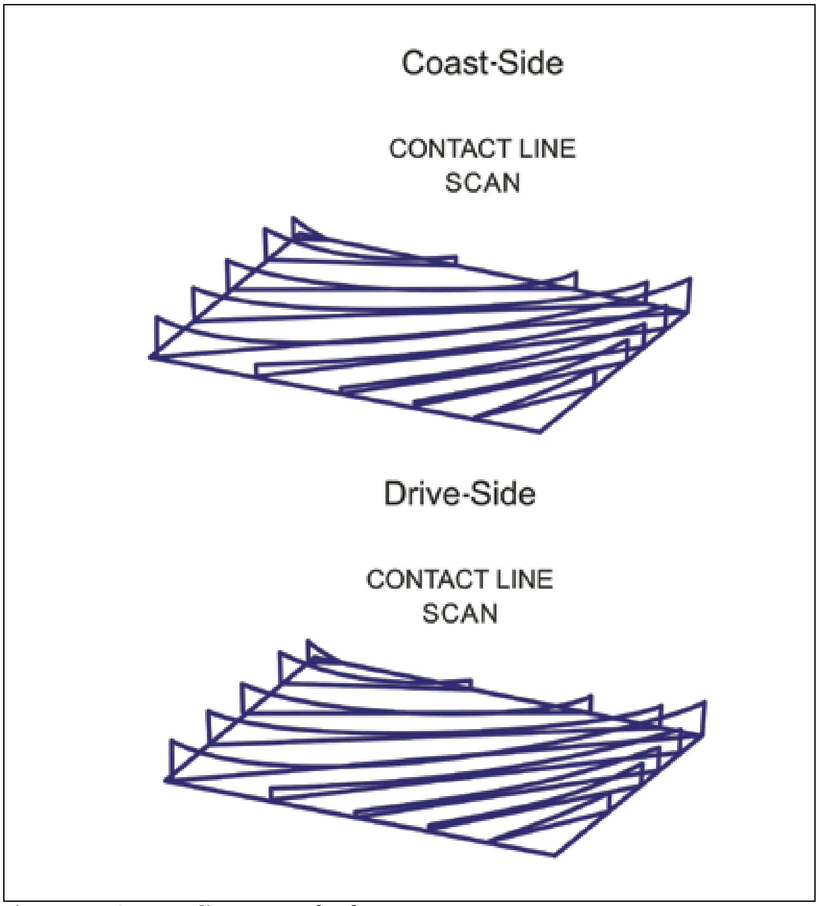

Figure 5—Contact line scan of a face gear set.

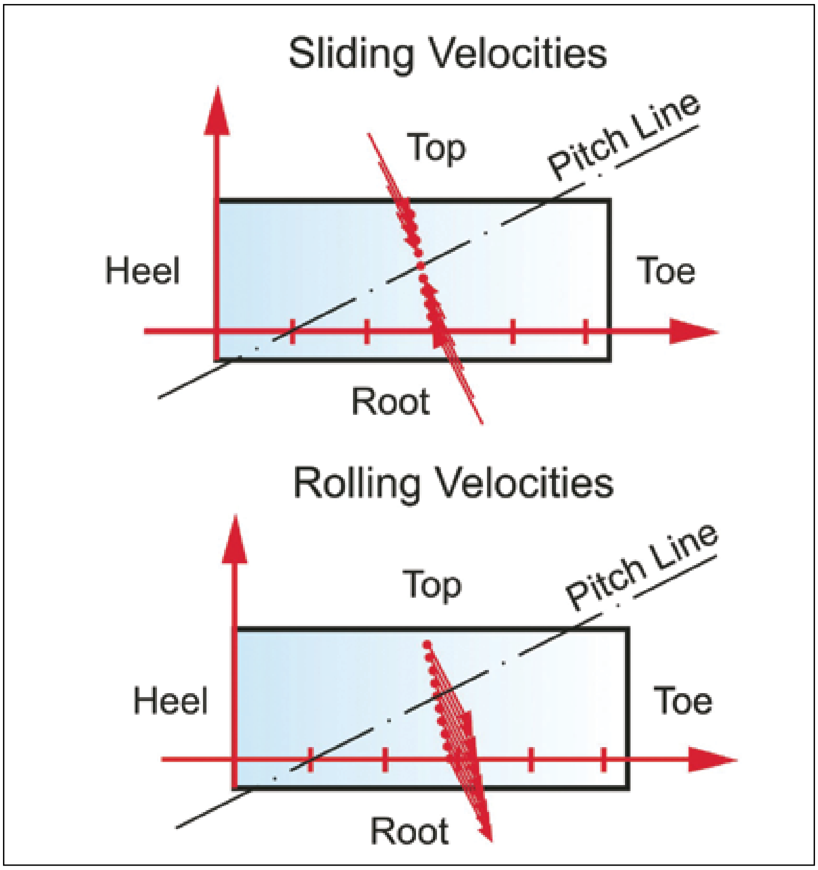

Figure 6—Rolling and sliding velocities of a face gear set along the path of contact.

At the bottom of Figure 4, the tooth contact pattern is plotted inside of the gear tooth projection. These contact patterns are calculated for zero-load and a virtual-marking-compound film of 6 μm thickness. This basically duplicates the tooth contact, one could observe, rolling the real version of the analyzed gear set under light load on a roll tester while the gear member is coated with a thin layer of marking compound. The contact lines show a changing inclination between toe and heel—each of them points to the shaft intersection point. The path-of- contact has a dominating profile orientation; it crossed the central contact line under about 90°.

The crowning reflected in the ease-off results in a located-contact zone inside of the boundaries of the gear tooth. A smaller tooth contact area generally results from large magnitudes of ease-off and motion-graph magnitudes, and vice versa.

Figure 5 shows nine discrete, potential contact lines with their crowning amount along their length (contact-line scan). The orientation of the contact lines is identical to the contact-line orientation in Figure 4. If the gear set operates in drive direction, then the contact zone (instant-contact line) moves from the top heel area to the root toe area.

The graph in Figure 6 illustrates the rolling and sliding velocity vectors. Each vector is projected to the tangential plane at the point of origin of the vector. The velocity vectors are drawn inside the gear tooth boundaries (axial projection of one ring gear tooth). Figure 6 shows at the top the sliding velocity vectors with their tips grouped along the path of contact points. The rolling velocity vectors are drawn in the lower graphic in Figure 6 with their origin grouped along the path of contact. Contrary to spiral bevel and hypoid gears, the directions of both, sliding and rolling velocities, are oriented perpendicular to the pitch line. The rolling velocities in all points are directed to the root while the sliding velocities point to the root, above the pitch line, and to the top, below the pitch line. At the pitch line the rolling velocity is zero, just like in the case of straight bevel gears.

Face gears have properties very similar to straight bevel gears, but for the exception that the rectangular tooth area is under an angle to the pitch line (Fig. 3). Sliding and rolling velocity vectors are pointing perpendicular to the pitch line direction (Fig. 6), which will shift the contact lines in Figure 5 strictly perpendicular to the pitch line. This means that the crowning of the contact lines has no significant influence on the lubrication case (*see "General Explanation of Theoretical Bevel Gear Analysis"*), but only the relative profile curvature perpendicular to the pitch line will define the lubrication case and the hydrodynamic condition. This will lead to lubrication case 3 above the pitch line, and case 2 below the pitch line. In contrast to straight bevel gears, the sliding is high in the toe and heel region caused by the inclined pitch line and the increased distance between the pitch line and the observed flank region. As a result, the efficiency is lower.

Manufacturing

The soft manufacturing processes of face gears are hobbing, shaping or generating with a disk cutter. The hard-finishing processes are continuous-threaded-wheel grinding, single-index generating grinding and skive-shaping.

One example of a cutting and grinding process of face gears is shown in Figure 7. The blades of the disk-shaped cutter (top, Fig. 7) represent with their cutting edges a plane perpendicular to the axis of rotation. A generating motion with a non-constant ratio of roll around the virtual axis of the mating pinion forms the face gear flank profile. The cutter represents a pinion flank meshing with the face gear like the mating pinion. This is a single-indexing process that requires an upper and lower cutting arrangement in order to cut both flanks (*see also "Straight Bevel Gears"*). Despite the single-indexing and single-side cutting, the process is rather fast because of the possibility of high-speed dry-cutting and the fact that the cutter disk only plunges and rolls in one position (without traversing along the face width). The described process leaves an arc in the root along the face width.

The following grinding process for hard-finishing—if required for a particular application (Fig. 7, bottom)—uses a CBN-coated grinding disk that essentially duplicates the cutter-blade silhouette. An advantage of such a related process combination is the fact that soft and hard geometry in flank and root area match perfectly.

Application

Most face gears used in power transmissions are manufactured from carburized steel and undergo a case hardening to a surface

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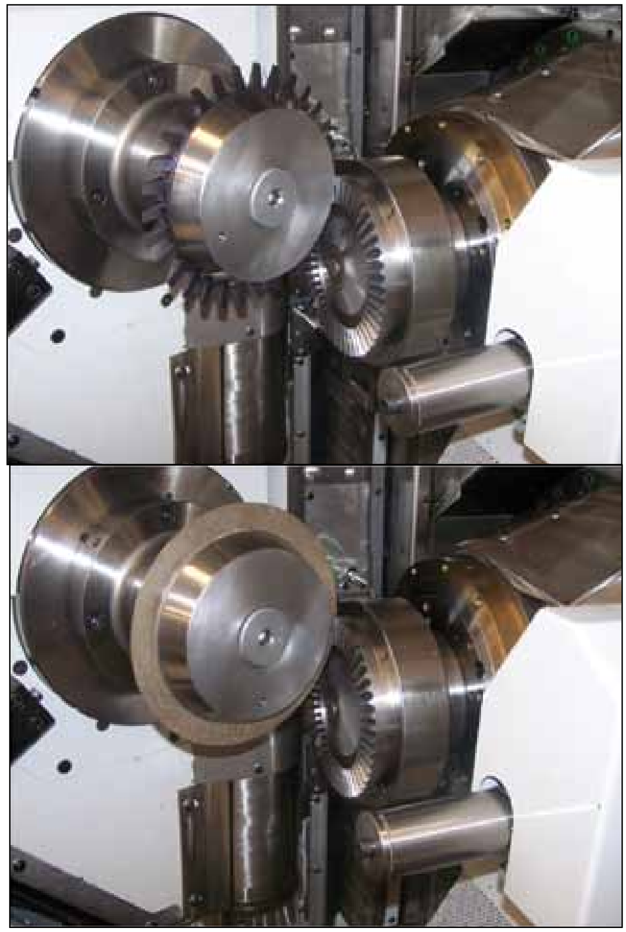

Figure 7—Face gear cutting (top) and grinding (bottom) in free form machine.

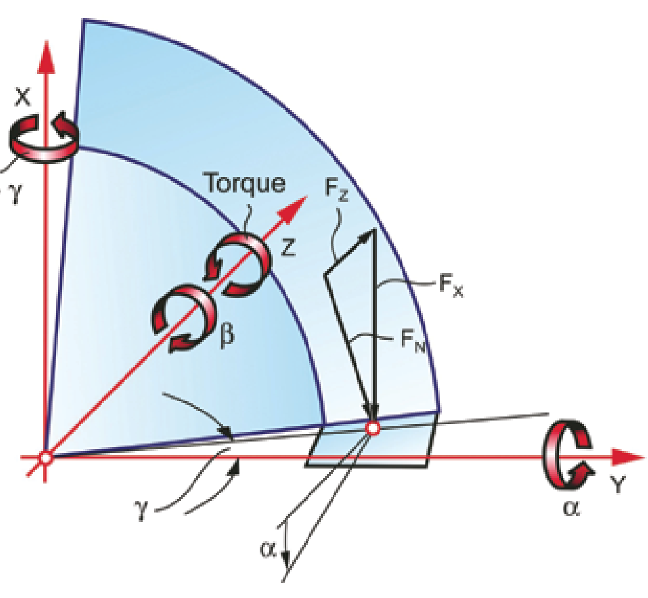

Figure 8—Force diagram for calculation of bearing loads.

hardness of 60 Rockwell C (HRC) and a core hardness of 36 HRC. Because of the higher pinion revolutions, it is advisable to give the pinion a higher hardness than the ring gear (e.g., pinion 62 HRC, gear 59 HRC).

Regarding surface durability, face gears are very similar to straight bevel gears and spur gears. At the pitch line, the sliding velocity is zero and the rolling velocity under certain loads cannot maintain a surface-separating lubrication film. The result is pitting along the pitch line that can destroy the tooth surfaces and even lead to tooth flank fracture. However, it is possible that the pitting can stabilize if the damage-causing condition is not constant in the duty cycle.

In a face gear set, the pinion is a standard spur or helical pinion. Its axial position will not change the backlash between pinion and gear and it is not critical to the gear set's performance. This reduces the requirements for bearings and assembly accuracy, which is welcome in many applications.

The spur pinion of a face gear set has no axial forces. All axial forces a face gear is subjected to are caused by the pressure angle, which is similar to straight bevel ring gears with a simmilar pitch angle γ. The axial force components—due to the spiral and pitch angles—are, for both members—zero. The zero spiral angle minimizes the face contact ratio of face gears to zero, but results in the maximal tooth root thickness.

The bearing forces of face gears can be calculated by applying a normal force vector at the position of the mean point (see "General Explanation of Theoretical Bevel Gear Analysis"). The force vector normal to the transmitting flank is separated in its X and Z components (Fig. 8).

The relationship in Figure 8 leads to the following formulas, which can be used to calculate bearing force components in a Cartesian coordinate system and assign them to the bearing load calculation in a CAD system:

General

$$F_x = -T / (A_m \cdot \sin\gamma)$$

$$F_y = -T \cdot (\cos\gamma \cdot \sin\alpha / (A_m \cdot \sin\gamma \cdot \cos\alpha)$$

$$F_z = T \cdot (\sin\gamma \cdot \sin\alpha) / (A_m \cdot \sin\gamma \cdot \cos\alpha)$$

Cylindrical pinion

$$F_x = T / A_m$$

$$F_y = T \cdot \tan\alpha / A_m$$

$$F_z = 0$$

Face gear in case of Σ = 90°

$$F_x = -T / A_m$$

$$F_y = 0$$

$$F_z = -T \cdot \tan\alpha / A_m$$

where:

where:	T	Torque of observed member
	A_m	Mean cone distance
	Σ	Shaft angle
	γ	Pitch angle
	α	Pressure angle
	F_x, F_y, F_z	Bearing load force components

The bearing force calculation formulas are based on the assumption that one pair of teeth transmits the torque, with one normal-force vector in the mean point of the flank pair. The results are good approximations that reflect the real bearing loads for multiple tooth meshing within an acceptable tolerance. A precise calculation is, for example, possible with the Gleason face gear design and analysis software.

Face gears can operate with regular transmission oil or, in the case of low RPM's, with a grease filling. In case of circumferential speeds above 10m/min, a sump lubrication with regular transmission oil is recommended. The oil level has to cover the facewidth of those teeth lowest in the sump. More oil causes foaming, cavitations and unnecessary energy loss. There is no requirement for any lubrication additives at normal speeds. However, in the case of circumferential speeds above 500 m/min, the higher sliding velocities—compared to straight bevel and spiral bevel gears—might require a high-pressure oil with additives. Because the two flank types in a face gear—upper and lower—are mirror images of each other, there is no preferred operating direction. This is advantageous for many industrial applications.

References

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(Next issue: "Beveloid and Hypoid Gears")