Synthesis of Spiral Bevel Gears

Faydor L. Litvin
University of Illinois
Chicago

There are different types of spiral bevel gears, based on the methods of generation of gear-tooth surfaces. A few notable ones are the Gleason's gearing, the Klingelnberg's Palloid System, and the Klingelnberg's and Oerlikon's Cyclo Palliod System. The design of each type of spiral bevel gear depends on the method of generation used. It is based on specified and detailed directions which have been worked out by the mentioned companies. However, there are some general aspects, such as the concepts of pitch cones, generating gear, and conditions of force transmissions that are common for all types of spiral bevel gears.

Pitch Cones

Consider that rotation is transformed between two intersected axis, Oa_1 and Oa_2, which make an angle γ (Fig. 1). The angular velocities in rotation about these axes are $\omega^{(1)}$ and $\omega^{(2)}$. The instantaneous axis of rotation (OI) is the line of action of the relative angular velocity

$$\boldsymbol{\omega}^{(12)} = \boldsymbol{\omega}^{(1)} - \boldsymbol{\omega}^{(2)} \qquad (1)$$

or

$$\boldsymbol{\omega}^{(21)} = \boldsymbol{\omega}^{(2)} - \boldsymbol{\omega}^{(1)} \qquad (2)$$

The instantaneous axis of rotation is the line of tangency of the pitch cones that roll over each other without slipping. The apex angles of the pitch cones γ_1 and γ_2 are represented by the following equations:

$$\cot \gamma_1 = \frac{m_{12} + \cos \gamma}{\sin \gamma} \qquad (3)$$

$$\cot \gamma_2 = \frac{m_{21} + \cos \gamma}{\sin \gamma} \qquad (4)$$

Here

$$m_{12} = \frac{\omega^{(1)}}{\omega^{(2)}} = \frac{N_2}{N_1} \text{ and } m_{21} = \frac{\omega^{(2)}}{\omega^{(1)}} = \frac{N_1}{N_2}$$

are the gear ratio; N_1 and N_2 are the number of gear teeth.

For the most common case when $\gamma = 90°$, we obtain

$$\cot \gamma_1 = m_{12} \qquad \cot \gamma_2 = m_{21} \qquad (5)$$

Plane II is a tangent plane to the pitch cones (Fig. 1). We may imagine that plane II rotates about axis Oa_g with angular velocity $\omega^{(g)}$, while the pitch cones rotate with angular velocities $\omega^{(1)}$ and $\omega^{(2)}$ about axes Oa_1 and Oa_2, respectively. Plane II, limited with the circle of radius OI, may be considered as a particular case of a pitch cone surface having an apex γ_i, which approaches 90° and has an outer cone distance equal to OI.

Generating Gear: Types of Spiral Bevel Gearing

Consider that a generating surface Σ_g is rigidly connected to the pitch plane II. Surface Σ_g rotates with the pitch plane II about Oa_g (Fig. 1) while gear blanks rotate about Oa_1 and Oa_2, respectively. Surface Σ_g generates tooth surfaces Σ_1 and Σ_2 on gears 1 and 2. Such a generating process provides conjugate gear-tooth surfaces Σ_1 and Σ_2 which contact each other along a line at every instant. The instantaneous line of contact moves over surfaces Σ_1 and Σ_2. Gears 1 and 2, having surfaces Σ_1 and Σ_2, will transform rotation about axes Oa_1 and Oa_2 with the prescribed gear ratio. The type of spiral bevel gearing depends on the type of generating surface Σ_g.

The generating surface for Gleason's spiral bevel gearing is a cone surface. The head cutter that cuts the gear carries blades with straight-lined profiles. Consider a coordinate system S_c that is rigidly con-

FAYDOR L. LITVIN

is Professor of Mechanical Engineering at the University of Illinois at Chicago. In addition, working in conjunction with NASA, Dr. Litvin has developed many new and important ideas for the mathematical formulation and theoretical understanding of spur, helical, and spiral bevel gears. The material in this article is taken from a much larger work, The Theory of Gearing, by Dr. Litvin, which explains the most general problems of gearing theory.

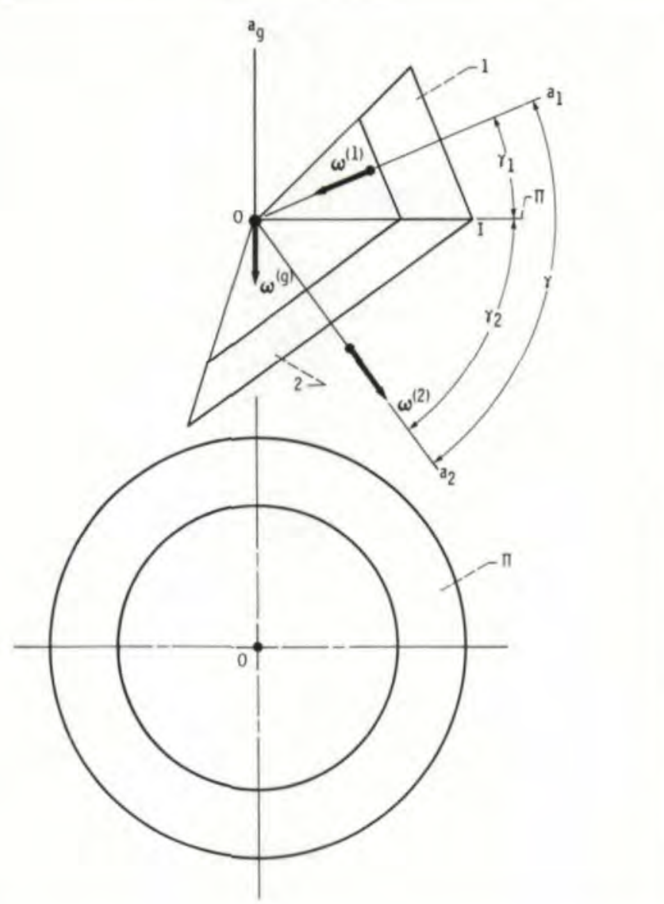

Fig. 1

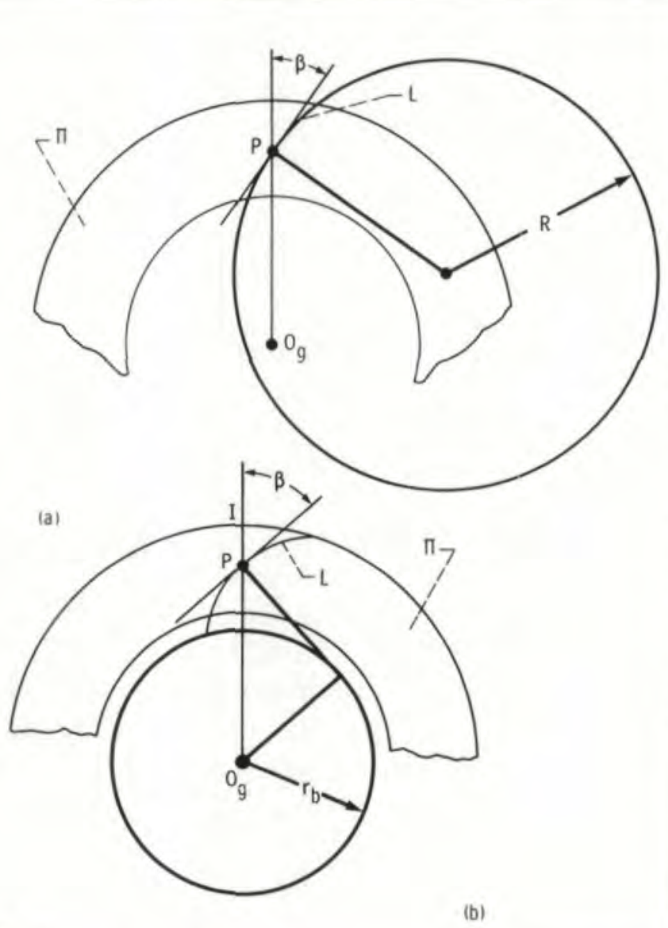

Fig. 2

nected to the head cutter and rotates with it about the *C-C* axis. The head-cutter blades, being rotated about *C-C*, generate a cone in the coordinate system S_c. The angular velocity of rotation about *C-C* does not depend on the generating motions and provides the desired velocity of cutting only.

To generate the gear tooth surface the head cutter has to go through two motions:

(1) Rotation about Oa_g while the generated gear rotates about Oa_{ij}

(2) Rotation about *C-C*.

Rotations of the generating gear and the gear being generated are related since the instantaneous axis of rotation is *OI*. The rotation of the head cutter about *C-C* may be ignored by considering that a generating cone surface is rigidly connected to plane II (with axis *C-C* of the cone) (Fig. 1) and rotates about axis Oa_g. The motion of the generating gear (rotation about Oa_g) is simulated by the rotation of the cradle of the cutting machine which carries the head cutter.

Consider the line of intersection *L* of the generating surface with the pitch plane II. In the case of Gleason's gearing, *L* is a circular arc of radius *R* (Fig. 2a). Line *L* generates a spatial curve on the gear pitch cone that is more like a helix rather than a spiral although the gears are called spiral bevel gears.

The type of spiral bevel gears is related to the type of the longitudinal shape of the gear. We differentiate between the following types of spiral bevel gears.

(1) The Gleason's gearing (Fig. 2a): where the longitudinal shape is a circular arc of radius *R*.

(2) The Palloid System of Klingelnberg (Fig. 2b): where the longitudinal shape is approximately an involute curve for a base circle of radius r_b. The generating surface of the Palloid System of Klingelnberg is generated by a conical worm. The tool is a conical hob which simulates the conical worm.

(3) The Cyclo-Palloid System of Klingelnberg and Oerlikon System (Fig. 3) where the longitudinal shape is an extended epicycloid, traced out by point *P* of the finishing blade of the head cutter. The blade and circle of radius ϱ are rigidly connected and represent a rigid body. The circle of radius ϱ rolls over the gear circle of radius *r*. Thus these circles are centrodes of the head-cutter and of the generating gear. The head cutter rotates about O_c and the generating gear rotates about O_g. Unlike the generation of Gleason's gearing, the rotations of the head-cutter and the generating gear in the case of the Cyclo-Palloid System are related: point *I* is the in-

stanteous center of rotation in the relative motion of the head cutter with respect to the generating gear.

In reality the methods of generation discussed are more complicated because they have to provide a localized contact of gear-tooth surfaces. It is for this reason that two generating surfaces are used instead of one.

Henceforth, we will designate the direction of the tangent to the longitudinal shape at point P by β. Point P is the point of intersection of the instantaneous axis of rotation O_gI and the shape (Figs. 2 and 3). The longitudinal shape (the spiral) can be right-handed or left-handed, similar to the right-handed and left-handed helical gears. Fig. 3 shows right-handed spirals.

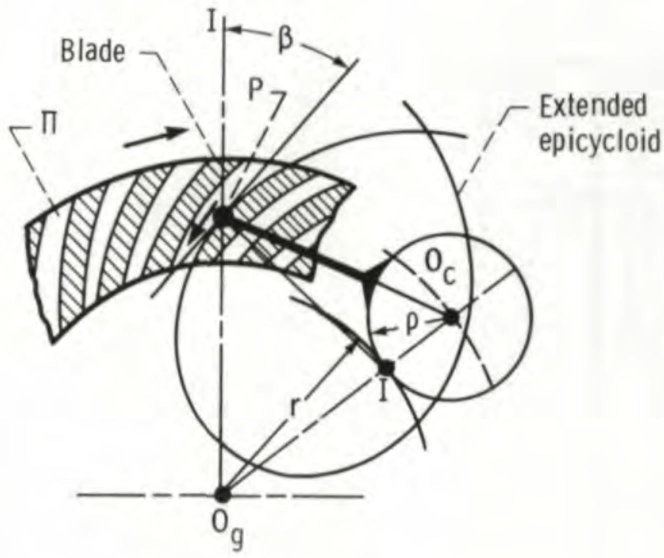

Fig. 3

Tooth Element Proportions

The axial section of the Palloid gearing and the Cyclo Palloid gearing is shown in Fig 4a. This gearing has a constant height of the teeth.

The axial section of the Gleason's gearing is shown in Fig.4b. Tooth height changes proportionally to the distance from the apex and the three cones – the pitch cone, dedendum cone, and addendum cones – have the same apex. In some cases, the gears are designed with different apices for the mentioned cones to provide a constant backlash between the dedendums and addendums of mating gears.

The transverse diametral pitch is given for the back cone. The pitch diameter for the gear is determined by

$$d_i = \frac{N_i}{P} \qquad (i = 1,2) \qquad (6)$$

where P is the diametral pitch and N_i is the tooth number.

The outer cone distance A_o is determined by

$$A_O = \frac{d_i}{2 \sin \gamma_i} \qquad (7)$$

The addendum and dedendum angles are represented by Fig. 4b.

$$\Delta_1 = \tan^{-1} \frac{a}{A_O} \qquad (8)$$

$$\Delta_2 = \tan^{-1} \frac{b}{A_O} \qquad (9)$$

Here a and b are the dimensions of the addendum and dedendum for the back cone expressed in terms of the diametral pitch.

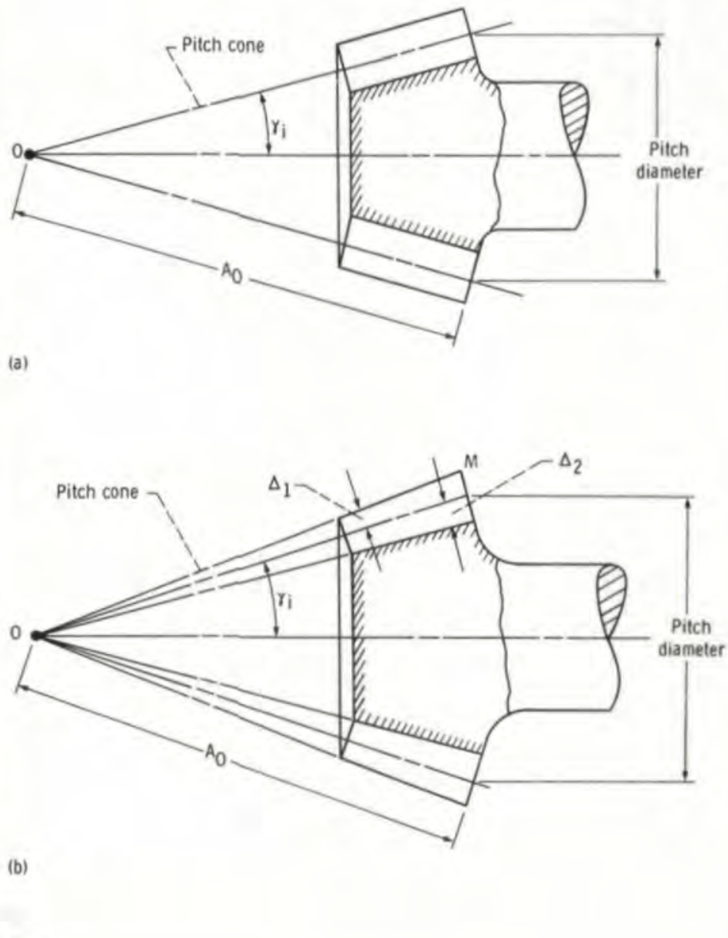

Fig. 4

Acknowledgement: *Reprinted from Theory of Gearing, Faydor L. Litvin. NASA R.P. 1212, AVSCOM T.R. 88-C-085. Available from the Superintendent of Documents, U.S. Govt. Printing Office, Washington, D.C.*