Fatigue Aspects of Case Hardened Gears

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The efficient and reliable transmission of mechanical power continues, as always, to be a central area of concern and study in mechanical engineering. The transmission of power involves the interaction of forces which are transmitted by specially developed components. These components must, in turn, withstand the complex and powerful stresses developed by the forces involved. Gear teeth transmit loads through a complex process of positive sliding, rolling and negative sliding of the contacting surfaces. This contact is responsible for both the development of bending stresses at the root of the gear teeth and the contact stresses at the contacting flanks.

Gear Fatigue

In analyzing the stresses developed in the gear tooth, it is useful to begin with a brief description of the dynamics of tooth contact. Figure 1 depicts the cycle of contact and the contact path of the teeth of a spur gear. The path of contact begins on the tip (the addendum) of the driven gear tooth, goes through the pitch point and finishes on the tip of the driving gear tooth. The gear teeth are in contact along line C_1–C_5. At all points along this line, with the exception of the pitch point, the velocities of the contacting surfaces are different. Because of this, sliding will occur (Ref. 1). At the pitch point the velocities of the contacting surfaces are equal and there is pure rolling. The angular velocity of the driving gear tooth is then transferred to the driven gear by a complex process of positive and negative sliding and rolling. The bending stresses at the root of the gear tooth and the contact stresses on the contacting flanks are therefore generated by the loading conditions of the contacting surfaces. These stresses are responsible for the dual nature of gear fatigue failures (Refs. 2, 3).

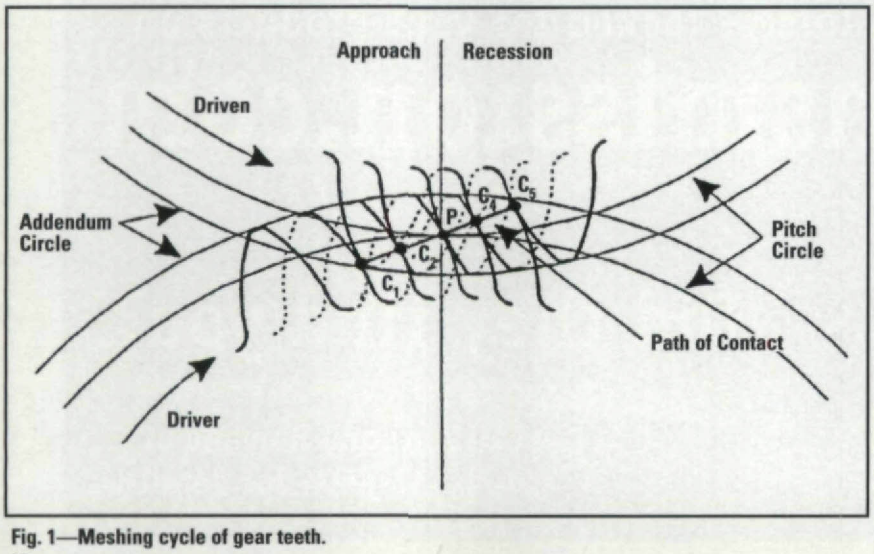

Fig. 1—Meshing cycle of gear teeth.

The bending stresses occurring at the root of the gear tooth arise due to the transfer of torque from one gear to another. The bending stresses, being cyclic in nature, can lead to fatigue crack initiation at the root of the tooth. This region acts as a stress concentrator and the fatigue cracks which develop here are classical fatigue fractures (Ref. 4).

The contact stresses arising in, and on, the contacting surfaces of the gear teeth are a consequence of the forces exerted by one surface on the other (Ref. 5). These forces create pressure distributions that are directly responsible for the development of shear stresses below the surface (Ref. 6). The magnitude and location of these stresses are dependent on the geometry of the gear tooth flank and the dynamic conditions under which the gear is operating (pressure distribution, involute radii, sliding velocities and coefficient of friction). The failures caused by these shear stresses are the typical pitting and spalling types.

The overall design of the gear must take into account these stress systems and minimize their effect on the integrity of the gear. While the material of the body of the gear tooth must display enhanced flexural ductility to counteract the bending stresses developed there (Ref. 7), the flanks of the gear tooth require a hard, wear-resistant surface with enhanced strength to some depth below the surface to resist the orthogonal shear stresses developed in that location.

Tooth Root Stresses. In 1892, Lewis (Ref. 8) made the first documented attempt to calculate the stresses developed in the root of a gear tooth. His approach was based on the analysis of a notched beam in bending mode, approximating the gear tooth shape by a parabola. This basic approach is still accepted as fundamentally correct, with the exception of the effects of what are now known as stress concentrators, investigated by Dolan and Broghamer (Ref. 9). There are a number of ways of calculating the stresses developed in the root of a gear tooth, from the two dimensional analysis adopted by Aida and Terauchi (Ref. 10) to the more sophisticated finite element models which were first introduced by Andrews (Ref. 11).

The bending fatigue of gears is principally governed by the geometry of the gear tooth, the loading conditions and the material properties at the root of the tooth. The authors show, to some extent, the importance of the geometry, but do not consider any of the factors which influence the material properties. These metallurgical factors depend on the material composition and heat treatments adopted, as well as their subsequent transformation products and/or residual stress distributions.

Contact Stresses. From the literature (Ref. 1) it can be seen that contact stresses developed during the contact of gear teeth can be approximated by rollers contacting under a known force, the sliding velocities of which reflect the dynamics of gear contact. From Figure 2 it can be seen that the rollers with a fixed sliding velocity will approximate the contact of gear teeth at certain points along the contact path. Figure 3 displays the semi-elliptical pressure distribution generated on the surface of contacting rollers.

The problems of contact stress, particularly the problems posed by contacting rollers, have been discussed by various authors. The pioneering work of Smith and Liu (Ref. 5), and more recently the work on the elastic shakedown of contacting surfaces by Johnson (Ref. 6) and the finite element modeling of such contacts by Hahn and associates (Refs. 12–14) deserve particular mention. However, the stress state which exists on the contacting surface layers can be significantly influenced by asperity interaction and can include tensile and compressive alternating cycles and shear.

Two different types of plastic zones are produced by rolling and sliding contact, as shown in Figure 4. The first layer is due to macro-contact width, usually between 1–10 mm. The second, which is much shallower, is due to micro asperities contact. This depth h is of the order of $0.5\mu m \leq h \geq 50\mu m$ and is obviously related to surface roughness. Therefore, depending on the friction developed between the contacting sliding surfaces, and the nature and strength of the material of the roller, failure will either initiate at the surface or subsurface or, in some conditions, in both.

Metallurgical Perspectives Relating to Fatigue Performance in Carburized Gears

While metallurgical factors such as oxidation, decarburization, supercarburization, carbide formation, grain boundary segregation, type and density of inclusions present, microcracks and residual stresses all significantly affect the fatigue performance of carburized gears, detailed treatment of these effects is well outside the scope of this discussion. However some of these aspects, such as oxidation and carbide formation, will be discussed in relation to the case studies presented. The role of retained austenite will also be discussed in view of its highly controversial nature.

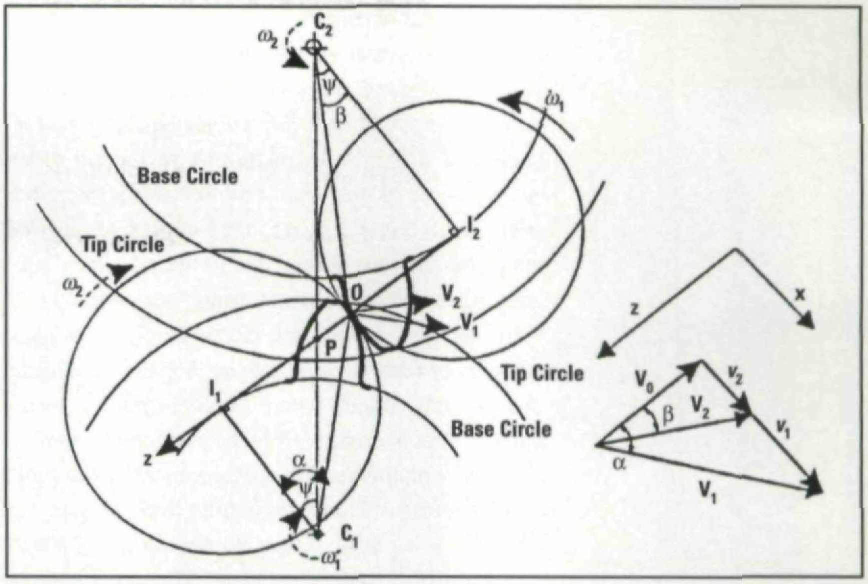

Fig. 2—Contact of involute spur gear teeth.

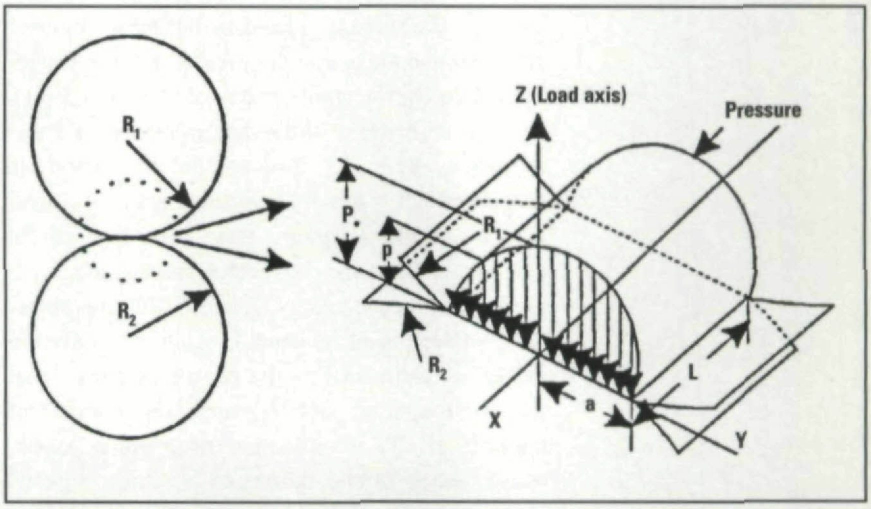

Fig. 3—Pressure distribution in contacting rollers, terminology adopted in Hertzian theory.

Retained Austenite. The mechanisms responsible for the retention of austenite on quenching have been reported in detail in the available literature (Refs. 15–18). It is clear that of all the alloying elements which influence the retention of austenite on cooling, carbon has the greatest effect. Quenching temperature and cooling rates have all been reported to affect the level of austenite retained (Refs. 15, 16).

The effect of the retained austenite on the fatigue resistance of carburized components has also been extensively dealt with in the literature. Nevertheless, some controversy still remains. In the past, the presence of retained austenite has been regarded as detrimental to the extent of adopting cryogenic treatments to reduce the amount present.

A more detailed picture has emerged from the investigations carried out by various authors, notably the work of Krauss (Refs. 19, 20) and Zaccone (Refs. 21–23). These authors have shown that there is a

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possible relationship between fatigue performance and retained austenite, which can account for some of the continuing controversy and explain some of the high fatigue limits being published. The transformation of austenite to martensite at the tip of an advancing crack has been reported in the literature (Ref. 24) to be beneficial to the fatigue resistance of the carburized component due to the associated volume expansion of the transforming austenite.

It has been pointed out (Refs. 22, 23) that large amounts of retained austenite are beneficial in terms of low cycle fatigue where the large plastic strains induce strain hardening from the austenite to martensite transformation and the development of favorable residual stresses. However, in high cycle fatigue, the relatively low plastic strains do not allow any transformation of austenite to occur. The same authors found that the plastic strains needed to transform the austenite are directly related to the prior austenite grain size and the morphology and size of the austenite packets. In fine grain structures (ASTM 9.5–11), much lower levels of strains are necessary for transformation (Refs. 23, 24), so that in high cycle fatigue, which is usually associated with very small amounts of plastic strains, these structures will be *able to transform and resist crack propagation.*

Work by many authors (Refs. 15, 25, 26), points to the reduction of retained austenite by cryogenic means as detrimental to the fatigue properties and fracture toughness of the carburized component because of the development of residual tensile microstresses in the remaining austenite regions. However to this author's knowledge, no systematic investigation has yet been published outlining the nature, or indeed the magnitude, of these microstresses. The effect of cryogenic treatments on the bending and contact fatigue properties of carburized gears can be illustrated by results obtained from gear research conducted at the University of South Australia (Refs. 27–29). Figure 5 shows that the cryogenic treatment was detrimental to the bending fatigue properties of the gears tested.

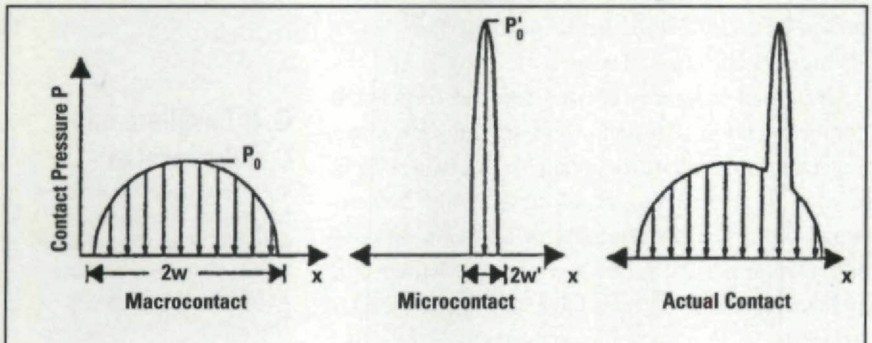

Fig. 4—Illustration of two plastic zones under rolling contact due to macro and micro contact.

The effect of the cryogenic treatment on bending fatigue properties can be explained by only three scenarios:

(a) If it is assumed that the retained austenite present in these gears can transform on straining, and that cryogenic treatment only reduced the levels of retained austenite, then the non-cryogenically treated gears have superior performance due to a higher amount of retained austenite, which can transform ahead of the propagating cracking tip.

(b) If it is assumed that the retained austenite present does not transform on straining, then the non-cryogenically treated gears have superior performance due to the cryogenic treatment imparting detrimental residual tensile microstresses in the remaining regions of untransformed austenite.

(c) If the retained austenite present does not transform on straining, and its presence is detrimental to the bending fatigue properties, then the cryogenic treatment has a greater detrimental effect on the fatigue properties of these carburized gears than the presence of retained austenite in the treated gears.

In all cases, these results point to a detrimental effect derived from the cryogenic treatment of these gears. Figure 6 displays the experimental results of contact fatigue tests carried out to determine the effect of cryogenic treatment. This figure shows that the adoption of cryogenic treatment has proven to be detrimental to the contact fatigue performance of En36A steel.

The role of cryogenic treatment on the contact fatigue behavior of the rollers tested is not as clear as it was in the bending fatigue section. This result follows the trend found in the literature, where the adoption of cryogenic treatment was found to be detrimental to the contact fatigue performance of carburized components. In particular, the experimental results of Kiessling (Ref. 31), Razim (Ref. 26) and Nakamura et al. (Ref. 32) have shown a direct relationship between high levels of retained austenite and high contact fatigue limits.

Case Depth and Core Strength Requirements. The determination of the appropriate case depth in carburized gears must reflect the stresses developed in and on the gear teeth, as discussed in the sections on gear stresses. The aforementioned duality of the stress system in gears places certain limitations, or at least conditions, which must be satisfied in the appropriate selection of the case depth. However, it has been discussed that the role of the hardened layer in the root of the gear tooth is fundamentally different than that required at the contacting flank. The literature, as expected, also displays this duality of role.

It has been known that a case depth selected on the basis of contact fatigue will not necessarily perform at its optimum in bending. This has been acknowledged by some authors. However in practice, the case depth requirements for maximizing bending fatigue resistance are usually overshadowed by the need for contact resistance. A great deal of controversy still surrounds the optimum case depth for bending/contact endurance.

The core strength is of significant importance in the fatigue resistance of the component. This is partly based on the core strength's influence on residual stresses, since the magnitude and indeed the polarity of the residual stresses developed are dependent on the difference in volume expansion of the case and core. The larger the difference, the higher the residual stresses, provided that the core material does not yield (Refs. 33, 34).

This simple criterion is, however, questioned by Ebert et al. (Refs. 35, 36) and McGuire et al. (Ref. 37), who proposed that a carburized structure is basically a two-component composite, and that each of these components will have significantly different elastic limits and plastic properties. At stress levels below the yield stress of both the case and core, the overall stress state will not differ from that anticipated in a homogeneous solid. However, as the load is increased and the stress level becomes higher than the yield stress of the core, the core will flow plastically while the case will still behave elastically. Because of the difference in the Poisson ratio for elastic behaviour (0.3) and plastic flow (0.5), the case and core will have different contracting tendencies. This creates transverse stresses normal to the applied stress in the case-core interface region. This is essentially a tensile biaxial state of stress.

The enhanced ductility of the carburized component, as compared to through hardened components, is derived by the development of compressive transverse stresses in the case, which help it resist the applied stresses. In carburized steels, the presence of retained austenite in the case can provide a further ameliorating effect if it can transform to martensite on straining. The volumetric expansion on transformation can reduce the biaxiality of the stress state in the case. This model highlights the importance of the core structure in the fatigue process, and it also defines the actual role of the hardened case. A deep

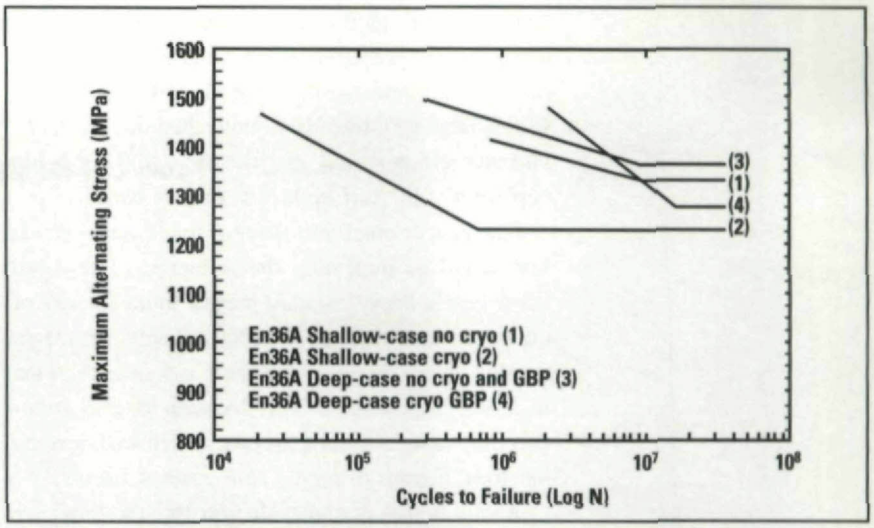

Fig. 5—The effect of cryogenic treatment on the bending fatigue of gears tested.

case is seen as increasing the degree of biaxiality at the case-core interface while restricting the ductility of the case.

The effect of case depth on the fatigue properties of carburized gears is illustrated by the results obtained from tests carried out at the University of South Australia. They show that the shallow case depth consistently out-performed the thicker case depth, irrespective of the post-heat treatment process received (i.e. glass bead peening and/or cryogenic treatment).

The bending fatigue performance in relation to the case depth initially points to the various differences found between deep and thin cased gears (Ref. 30). Some of these differences were:

1. The hardness of the shallow-case samples were lower than the deep-case ones.
2. The retained austenite levels were higher in the shallow-case samples.
3. The microstructures developed in the case of the deep-case gears were different from those found in the shallow-case gears.
4. The crack initiation depth was deeper in the shallow-case samples.

The microstructural differences between the two types of samples were mainly in the degree of non-martensitic phases, which were primarly carbide networks. These carbides account for the higher hardness of the deep-case gears (point 1 above), and the higher amount of retained austenite in the thin-case gears (point 2 above) (Refs. 15, 38–40), since the carbon and the alloying elements, which are among the factors controlling the amount of retained austenite present, are not tied up in carbides in the thin-case gears and hence will increase the levels of austenite retained.

These non-martensitic features found in the case of deep carburized gears are mostly likely due to excessive carbon buildup in the outer layers. This problem, termed supercarburization by Razim (Ref.

26), has been suggested to be due to the adoption of a high carbon potential during carburizing. However, the reader is referred to the work of Goldstein and Moren (Ref. 41), who have shown that this supercarburizing effect could also be due to Chromium depletion of the surface layers (due to oxidation).

The deeper crack initiation of thin-cased gears is interpreted as indicating the influence of resolved stress (applied and residual) present in the interior of the gear tooth. Fatigue cracks can only initiate in regions where the resolved stress is higher than the material fatigue limit. This location is also influenced by defects and inclusions, which will develop localized regions of highly concentrated stress.

The initiation of cracks deep within the thin-case gears is the result of maximum resolved stresses developed in deeper areas below the tooth's surface as compared to deep-case gears. This is also explained by the higher solution carbon content of thin-case gears (Refs. 15, 35, 42), which will allow the development of compressive residual stresses of a higher magnitude. The enhanced performance observed in thin-cased gears can be explained by two separate mechanisms. The first is based on the model proposed by Ebert et al. (Refs. 35, 36) and McGuire et al. (Ref. 37) regarding the rheological reaction between case and core. The second is derived from the microstructural considerations discussed above.

Case Depth Requirements in Contact Conditions. Considering the stress distribution in contacting cylinders as outlined in the preceding sections, it is clear that an element of material in or on the gear flank will experience stresses according to its position, the radius of the roller, the applied load, the respective relative sliding velocity and the coefficient of friction. Failure of the material, irrespective of its nature, whether it is by excessive plastic deformation or fracture, is then a function of the parameters outlined above and the true strength of the material. If the material's strength is a constant throughout its depth (k) (this can include through-hardened components such as bearing races and/or austenitized components), then it is reasonable to conclude that failure will occur in locations below the surface where the shear stresses reach a maximum. If, however, the coefficient of friction is greater than about 0.3, signifying that these shear stresses reach a maximum on the surface, failure will occur on the surface (Ref. 5). However, in conditions where the material exhibits a strength gradient, as is the case for case hardened components, the problem of predicting failure locations becomes significantly more complex.

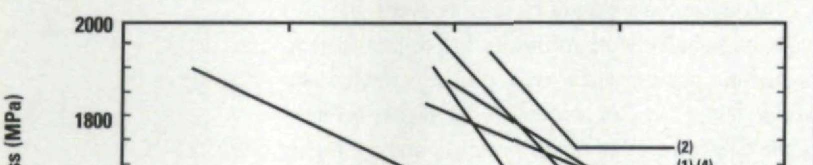

Fig. 6—The effect of cryogenic treatment on the contact fatigue performance of carburized rollers.

The need to accurately predict failure locations in case hardened components is a basic prerequisite in the establishment of case depth requirements. It can, therefore, be stated that the main role of the hardened layer in contact situations is to ensure that the core of the component is not subjected to shear stresses above its cyclic shear strength, and to increase the surface layer's resistance to asperity interaction. In view of the shear stresses developed under the contacting surface, it is clear that the hardened layer has to be of such a depth as to encompass not only the area of maximum shear stress, but also the area further into the interior where, even though the contact stresses are not at their maximum amplitude, they are still significant. The hardened layer should extend to a depth where the shear stresses decrease to a sufficiently low value so as not to pose a threat of crack initiation in the core.

The role of case depth in the contact condition cannot be ignored. Tests conducted at the University of South Australia show that rollers with a shallow case depth display significantly lower fatigue limits than their deep-case counterparts.

The model proposed earlier to account for the material strength in resisting contact damage explains the difference in the performance of rollers with different case depths, this being due to the deep case rollers having a higher contact fatigue resistant layer extending deeper into the roller.

Conclusions

The conclusions to be drawn from the results of the gear tests presented are as follows:

1. Gears with shallow case depths display higher fatigue limits than gears with deeper case depths.
2. Cryogenically treating the gears decreased both the bending and contact fatigue performance of the gears tested.
3. The contact fatigue performance of shallow case rollers was significantly inferior to the deep case rollers.

Recommendations

The bending fatigue strength of a gear tooth limits the amount of load applicable to the flank of the tooth. This, in turn, limits the maximum contact stresses developed. A deep case depth, which is specified to withstand high contact loads, is therefore not necessary if those loads cannot be reached by the bending properties of the gear. At the same time, a thinner case depth, which would increase the allowable bending stresses, does not impart the necessary resistance to contact that these stresses can generate. In the past a compromise was necessary in terms of choosing a case depth which would result in an acceptable fatigue life.

The maximum case depth in the root region should be optimized for maximum bending fatigue, while in the flank of the gear tooth, the case depth should be as deep as possible without running the risk of developing extensive carbides or other non-martensitic phases. One way to achieve this is to use a two step carburizing process, where the component is initially carburized to the case depth desired in the root, taken out, and a copper coating applied to the root region. The component is then recarburized to achieve the desired depth in the flank. This requires careful modeling of the diffusion of carbon in the masked region, as the carbon profile could become very flat due to inward diffusion. A more elegant method would entail a partial oxidation treatment in the root region to reduce the carbon intake in that region. If the controlled formation of Cr_2O_3 or even some type of SiO_2 could be encouraged in the surface of the root region, then the influx of carbon could be controlled to achieve the dual case depth suggested.

Investigation into novel heat treatment technologies should also be considered, especially in the light of the work carried out by Davies and associates at the Westland Helicopter Corp. (Refs. 43–47) dealing with duplex treatments. It should be pointed out that in the manufacturing of case hardened gears, the benefits expected from the best engineering practice and the highest level of accuracy do not materialize if the necessary metallurgical input into gear production is not given the necessary attention and emphasis. ○

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