Gear Design Optimization for Low-Contact Temperature of a High-Speed, Non-Lubricated Spur Gear Pair

Carlos H. Wink and Nandkishor S. Mantri

A gear design optimization approach applied to reduce tooth contact temperature and noise excitation of a high-speed spur gear pair running without lubricant. Optimum gear design search was done using the *Run Many Cases* software program. Thirty-one of over 480,000 possible gear designs were considered, based on low contact temperature and low transmission error. The best gear design was selected considering its manufacturability.

Introduction

Eliminating lubricant in geared systems is both cost-saving and environmentally sound, but does pose some technical challenges. Metal-to-metal contact of tooth surfaces sliding and rolling against each other under contact pressure causes high tooth temperature that may result in material microstructure changes. Tooth surfaces can severely wear away and even deform plastically. Tooth-sliding velocity and contact pressure can be reduced by changing the gear design. However, such design changes may adversely affect gear dynamics and noise—critical parameters of high-speed gears. This paper presents a gear design optimization approach that was applied to reduce both tooth contact temperature and noise excitation of a high-speed spur gear pair running without lubricant. After defining upper and lower boundaries of the main design parameters, and the problem constraints, an exhaustive search within the feasible design region was done using the *RMC* (*Run Many Cases*) software program from The Ohio State University (OSU) GearLab (Ref. 1). Each one of the designs was critically analyzed in terms of manufacturability. The selected optimum gear design was compared to an existing gearset using *LDP* (*Load Distribution Program*) software—also from OSU (Ref. 2)—which was also used to optimize microgeometry modification of profile and lead. Tooth contact temperature was calculated by *LDP* for both the existing and optimum design, and with a dry, steel-on-steel coefficient-of-friction. A favorable correlation between predicted tooth contact temperature of the existing gearset and test results was realized. A 48 percent reduction of tooth contact temperature and a 79 percent reduction of transmission error were achieved with the optimized gear design. The low contact temperature of the optimized design can significantly contribute to preventing tooth surface damage under "no-lubricant" operating conditions.

Tooth Contact Temperature

Conjugate action of gear teeth in mesh consists primarily of sliding and rolling motions. At the pitch line, sliding velocity is zero; however, sliding velocity increases when the conjugated tooth contact line travels away from the pitch line in both directions. Contact pressure of gear teeth in mesh also changes along the line of action (Ref. 3). Heat is generated by sliding friction of tooth surfaces. The temperature distribution is proportional to the distribution of contact pressure and sliding velocity. The instantaneous (or flash) temperature of tooth contact along the line of action is calculated by Blok's contact temperature theory (Ref. 4). The contact temperature is the sum of maximum flash temperature along the line-of-action and the tooth temperature, which is the temperature of the tooth surface before it enters the contact zone (Ref. 4).

The maximum contact temperature is obtained by Equation 1:

$$\theta_{B\,max} = \theta_M + \theta_{fl\,max} \tag{1}$$

where:

θ_M is tooth temperature, °C

$\theta_{fl\,max}$ is maximum flash temperature along the line-of-action which is calculated by Blok's equation:

$$\theta_{fl_{(i)}} = 3162\,K\,\mu_{m(i)}\,\frac{X_{\Gamma_{(i)}}\,w_n}{\left(b_{H_{(i)}}\right)^{0.5}}\,\frac{v_{r1_{(i)}} - v_{r2_{(i)}}}{B_{M1}\left(v_{r1_{(i)}}\right)^{0.5} B_{M2}\left(v_{r2_{(i)}}\right)^{0.5}} \tag{2}$$

where:

K is 0.80—a numerical factor for the Hertzian distribution of frictional heat over the instantaneous contact band width;

$\mu_{m(i)}$ is mean coefficient-of-friction;

$X_{\Gamma_{(i)}}$ is load-sharing factor;

wn is normal-unit-load, N/mm;

$X_{\Gamma_{(i)}}$ is semi-width of the Hertzian contact band, mm;

B_{M1} is thermal contact coefficients of the pinion, and given by $B_M = 10^{-3}\,(\lambda_M\,\rho_M\,C_M)^{0.5}$

B_{M2} is thermal contact coefficients of the gear, and given by $B_M = 10^{-3}\,(\lambda_M\,\rho_M\,C_M)^{0.5}$

λ_M is heat conductivity, $W/(m.K)$

ρ_M is material density in kg/m^3

C_M is specific heat-per-unit-of-mass in $J/(kg.K)$.

$v_{r1_{(i)}}$ is rolling tangential velocities in m/s of the pinion;

$v_{r2_{(i)}}$ is rolling tangential velocities in m/s of the gear;

i is a subscript of line-of-action points.

Gear Dynamics and Noise

Transmission error (TE) is widely accepted in the gear community as one of the major excitation sources of noise and vibration of geared systems (Refs. 3 and 5).

TE can be described as irregularities of the motion transmitted by gear pairs caused by deviations from the ideal tooth contact; these irregularities stem from tooth topological modifications, manufacturing deviations, shaft deflections, tooth deflections and mesh stiffness variation along the line-of-action. Relative accelerations between the gears caused by TE result in vibration of gear masses and dynamic tooth forces (Ref. 6).

Transmission error may be expressed as a linear displacement along the line-of-actions by Equation 3.

$$TE = R_b \left(\delta_2 - \frac{z_2}{z_1}\delta_1\right) \tag{3}$$

where:

TE is transmission error, mm,

R_b is gear base radius

δ_1, δ_2 are the angular rotation of pinion (input) and output gear, respectively,

z_1, z_2 are the number of teeth of pinion (input) and output gear, respectively.

The dynamic response of the geared system to TE excitation is influenced by the mass and stiffness of gears, shaft and other major internal components, and by damping characteristics (Ref. 6).

Gear Design Optimization Approach

The challenge is a multi-objective optimization to minimize tooth contact temperature and transmission error, subject to maximum contact and tooth root stresses below allowable stresses values; and also subject to constraints related to packaging size, such as mounting center distance and maximum face width; and manufacturability such as undercut condition, minimum topland, and root clearance. There are many design variables to be determined as part of the problem solution; these variables refer to the gear geometrical parameters—e.g., module, pressure angle, addendum modifications and outside diameter.

One convenient and robust approach for solving this optimization challenge is to combine the gear design knowledge and computational power of modern computers to completely "sweep" or search the feasible design region to find the optimum design.

Using the *RMC* program developed by the GearLab (Ref. 1), thousands of potential gear design candidates are quickly generated and analyzed based on the gear designer's input. The gearset that best meets the objective functions and constraints can be identified using range reduction methods that select design candidates within defined ranges and parameter prioritization (Ref. 7).

Application Example

An existing spur gear set of an automotive timing gear application that runs at high speed was tested without lubrication; results of this exploratory test were used to create a baseline for comparison with the optimized design.

The gears were made of SAE 4100 (Cr-Mo) series steel and induction-hardened to 56–61 HRC surface hardness. Tooth flanks were ground to achieve AGMA grade A4 (Ref. 8). The gear samples were submitted to a dynamometer cycle of rotational speeds up to 16,000 rpm (pitch line velocity up to 36.3 m/s), and light loads (up to 0.86 N-m/mm of face width). The outlet air temperature recorded during the test was 150°C. Figure 1 shows the baseline gearset after over 100 hours of testing.

The drive-side of the teeth of both gears was severely worn, and material plastically flowed out toward the two faces (Fig. 2). The amount of wear measured on tooth profile of the driver gear (Figs. 1 and 2) was 0.130 mm, and the driven gear was 0.115 mm.

Both gears were metallurgically analyzed. Microhardness-measured results on the gear teeth indicated that the gears were exposed to a temperature range of 450°C and 510°C, which was estimated using the material tempering curve.

In parallel with the metallurgical analysis, the tooth contact temperature was calculated using the *LDP* program (Table 1).

The results of tooth contact temperature are shown (Fig. 3) for the baseline gear set, which is close to the lower limit estimated from the metallurgical analysis. In addition to the tooth contact temperature prediction at nominal design condition, a robustness

Figure 1 Baseline gear set after test.

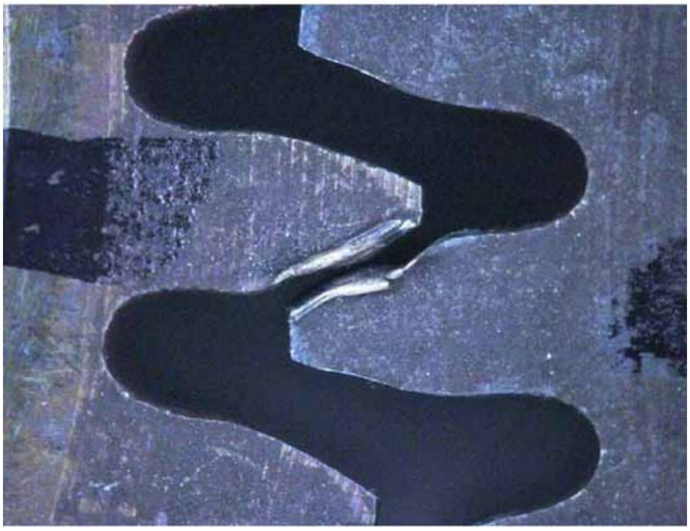

Figure 2 Base gear teeth after test.

analysis was also performed over deviations and tolerances specified on the gear drawings (Fig. 4).

The robustness analysis results indicate that the tooth contact temperature prediction for the baseline gearset falls in a range of 411°C–543°C — with an average of 475°C and standard deviation of 23.78°C. As such, excellent correlation was established for the baseline gear set.

The boundary conditions were then established for the optimum design search in *RMC*, along with upper and lower limits for the design parameters (Table 2). The last column in Table 2 is the number of points between the lower and upper limits. Center distance and gear face width were kept the same as the current design; gear ratio is 1.0.

A total of 483,372 good gear design candidates were generated under those conditions (Fig. 5).

RMC uses a modified equation to calculate tooth contact temperature that is different from Equations 1 and 2; thus *RMC* results were used as directional values only.

The *Range Reduction Method* in *RMC* was used to eliminate candidates with bending and contact stresses that exceed the allowable stresses for the application—despite the acceptable contact temperature and transmission error. The 31 candidates that met the given restrictions are displayed (Fig. 6). The differences among them in terms of tooth contact temperature and transmission errors are minor. (*The point identified with a star was the one selected as the best design because of its manufacturability.*)

Table 1 Temperature parameters used for the calculation

Parameter	Value
Coefficient of friction	0.5
Inlet bulk temperature	150°C
Thermal conductivity	48 W/(m K)
Density	7860 kg/m^3
Specific heat	544 J/(kg K)

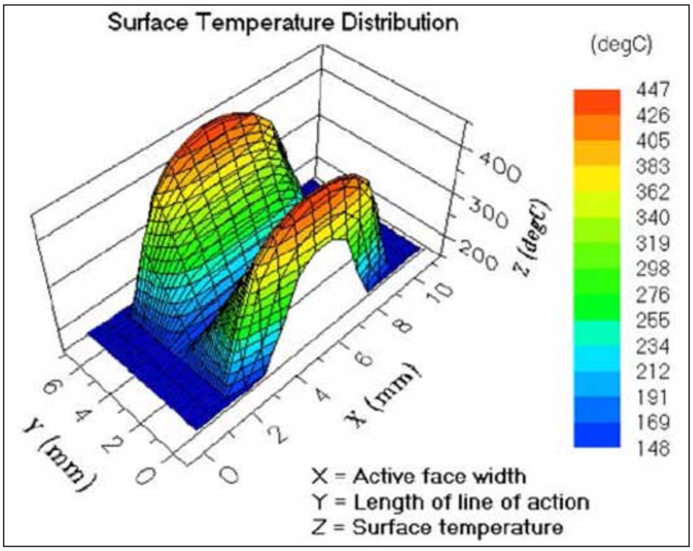

Figure 3 Predicted tooth contact temperature of baseline gear set.

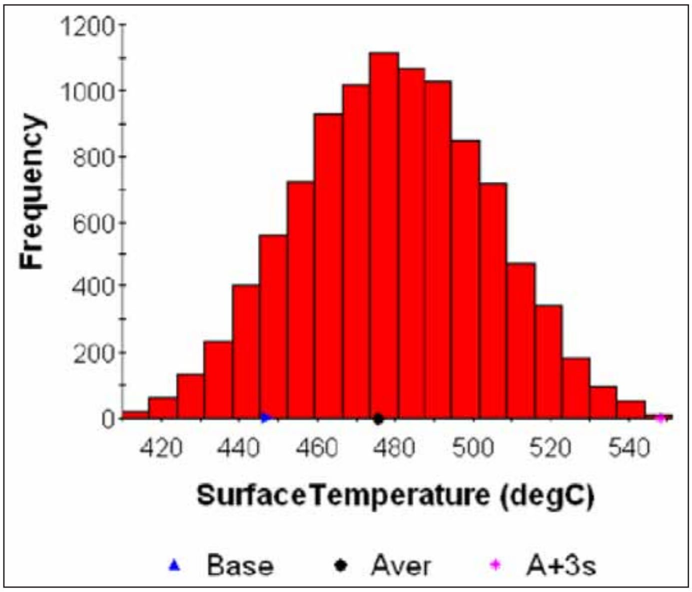

Figure 4 Robustness analysis of baseline gear set.

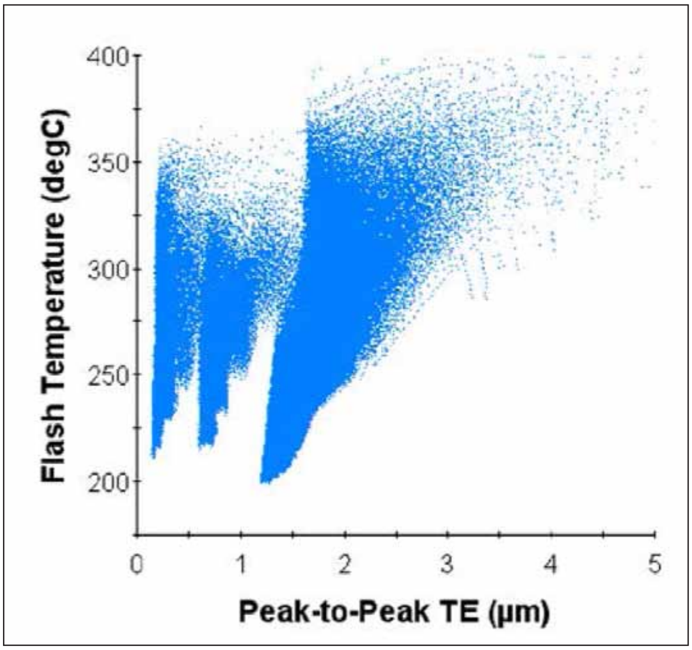

Figure 5 RMC transmission error vs. tooth contact temperature results.

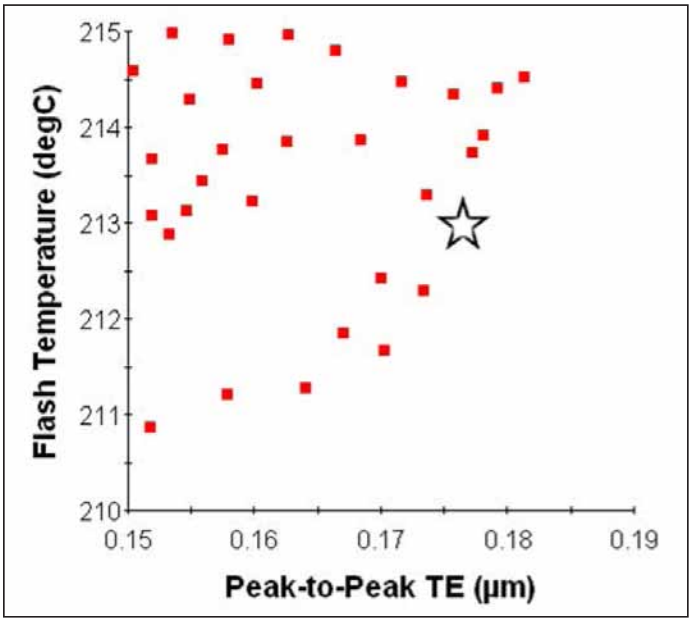

Figure 6 RMC best results (* identifies the selected design).

Table 2 Variable limits for optimization		
Design parameter	**Value**	**Levels**
Number of teeth	10 - 60	51
Pressure angle, degrees	12.5 - 25	20
Tool dedendum coefficient	1 - 1.2	10
Hob shift level	-	10
Backlash coefficient	0.048	-
Minimum topland coefficient	0.20	-
Minimum root clearance	0.15	-
Minimum SAP roll angle, degrees	4	-

Table 3 Results before and after optimization		
Parameter	**Current gear set**	**Optimized gear set**
Number of teeth	41	57
Module, mm	1.06	0.75
Pressure angle, degrees	14	18
Contact ratio	52.24	2.15
Slide to roll ratio	1.32	0.63
Contact stress, MPa	7.97	752
Tooth root stress, MPa	1.31	198
PPTE, μm	0.59	0.12
Contact temperature, °C	447	232

The best design geometry was then transferred into *LDP* to confirm the calculation results, optimize micro-geometry modifications, and do robustness analysis. Microgeometry modifications were defined using the *LDP 3D Microgeometry Analysis* module as 2–6 mm profile crown, and 0–4 mm lead crown. Table 3 shows a comparison of the current design and the optimized design for low contact temperature and transmission error (PPTE).

The robustness analysis also showed that the optimized design is less sensitive to manufacturing deviations than the current one. Predicted tooth contact temperature ranges from 233°C–262°C—with a standard deviation of 6.16°C; compared to the 411°C–543°C—with a standard deviation of 23.78°C for the current design.

Gear samples were made for validation testing (Fig. 7). Two heat treatment processes were considered: 1) induction hardening (current process) and 2) nitriding—which may provide additional wear resistance because of high surface hardness and a white, nitrided layer; in this case the gears were finished before nitriding.

The gear samples of the optimized design were tested with the same dynamometer cycle used for the current gear endurance testing, and results will be compared to the current gear set test results.

Figure 7 Gear samples of optimized design.

Summary

Favorable agreement between the predicted tooth contact temperature using *LDP* and the temperature estimated from microhardness and material tempering curve was obtained to an existing gear set that was tested at high speed and without lubrication. The gear design was then optimized using both the *RMC* and *LDP* programs. The best gear design for low contact temperature and low transmission error was selected from more than 480,000 designs. A 48 percent reduction of tooth contact temperature and a 79 percent reduction of transmission error were achieved with the optimized gear design, which is also more robust to manufacturing deviations than the current design. The main reason for the reduction in contact temperature of the optimized design was the slip-to-roll ratio reduction, which was proportional to the reduction in temperature. The low contact temperature of the optimized design can significantly contribute to prevent tooth surface damage under no-lubricant operating conditions, which will be confirmed through dynamometer endurance testing.

Acknowledgments

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Carlos Wink works for Eaton at their Vehicle Group's headquarters in Galesburg, Michigan as global CoE, gear engineering manager. He has 28 years of experience in gear manufacturing and the design of geared systems for trucks, automotive, hydraulic, and aerospace applications. Wink has a PhD and master degree in mechanical engineering—with a focus on gear design—from the University of Campinas (UNICAMP) in Brazil. Wink also holds a bachelor of science degree in mechanical engineering from the University of Santa Cecilia, also in Brazil.

Nandkisor (Yash) Mantri works for Eaton in Pune-India as a lead gear engineer. He has 14 years of experience in the gear field, six of them specializing in gear design and gear manufacturing, and eight years in gear cutting tool design. Prior to joining Eaton, Mantri worked for Dagger Forsts Ltd., and Jainex Aamcol Ltd. He has a master degree in design from BITS, India.