Prediction of Dynamic Factors for Helical Gears in a High-Speed Multibody Gearbox System

Niranjan Raghuraman, Dr. Sharad Jain and Chad Glinsky

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Introduction

Accurate prediction of gear dynamic factors (also known as K_v factors) is necessary to be able to predict the fatigue life of gears. Standards-based calculations of gear dynamic factors have some limitations. In this paper we use a multibody dynamic model, with all 6 degrees of freedom (DOF) of a high-speed gearbox to calculate gear dynamic factors. We investigate the influence of system dynamics, model fidelity, operating speed, and torque on dynamic factors. We also determine if torsional-only dynamic models that are commonly used to study gear dynamics are adequate to predict gear dynamic factors. The effect of manufacturing errors like shaft runout, tooth spacing error on the dynamic behavior of the system are also studied in this paper to show the importance of tolerance and accuracy in the manufacturing process. The findings from this paper will help engineers to understand numerous factors that influence the prediction of dynamic factors and will help them to design more reliable gears.

Dynamic tooth loads influence the durability of gears, particularly at high speeds. The gear rating standards consider the effect of dynamic tooth loads on gear durability by multiplying the quasi-static stress by a dynamic factor (also known as K_v factor). Accurate calculation of dynamic factor is important to be able to assess the durability of a gear. Standards-based calculation of dynamic factor only considers the influence of operating speed and gear manufacturing quality. The influence of torque and system resonance modes is generally ignored.

In this paper we analyze the influence of operating speed, torque, and system dynamics on the dynamic factors of a high-speed gearbox. We show that the dependence of dynamic factor on torque is significant and must not be ignored. We also show that the system effects are important and that the presence of system resonance modes increases dynamic factors. The dynamic factors calculated in this study are compared with the dynamic factor values suggested by ISO and AGMA standards.

We perform the analysis using a multibody dynamic model of a high-speed gearbox. The model includes shafts, bearings, and helical gears. Traditionally, multibody models with only torsional degrees-of-freedom are used to calculate dynamic gear forces. These models only consider the torsional dynamics of the system and ignore the shaft bending and lateral deflections. In this study, we investigate the influence of shaft bending and lateral deflections on the dynamic factors, particularly at high speeds.

We also look at the effect of manufacturing errors like shaft runout, tooth spacing error on the dynamic behavior of the system to assess the importance of quality grades and accuracy of the gears.

This study will help engineers: (a) to understand the effect of various operational and design parameters on gear dynamic factors; (b) to identify the limitations of standards-based dynamic factor calculations; (c) to create multibody dynamic models which are appropriate for dynamic factor calculations by considering all the relevant physics; (d) to improve gear durability for high-speed applications.

System to Analyze

For this study we chose a high-speed electric-vehicle gearbox with two helical gear stages to perform our investigations. The gearbox consists of an input shaft, intermediate shaft, and output shaft connected by two gear pairs (Fig. 1). All three shafts are supported by rolling element bearings. Input shaft is driven by an electric motor. The rotor of the electric motor is mounted on the input shaft. The output shaft is essentially a differential, but to simplify, the side pinions and side bevel gears are not captured in the model. Since the vehicle is driven by the output shaft, a high inertia ($2\,kgm^2$) is appended to the output shaft to represent the vehicle inertia. The gear geometries of the input and output gear pairs are tabulated in Tables 1 and 2.

To calculate gear dynamic factors, we create a multibody dynamic model of the system described above. In the dynamic model all the shafts are discretized into Timoshenko beam elements. Gear blanks are treated as rigid discs. Gear-mesh compliance is modeled using a linear spring, acting along the line-of-action. Gear mesh compliance is more complicated, as it includes a tooth bending stiffness term that changes with the location of contact line along tooth height and a nonlinear contact stiffness term. The rotor of the electric motor is modeled as a rigid disc directly connected to the input shaft. The electromagnetic interaction between the rotor and stator is ignored in this study. It is important to capture the mass and inertia of the rotor to accurately predict the dynamic behavior of the system. The bearings are represented

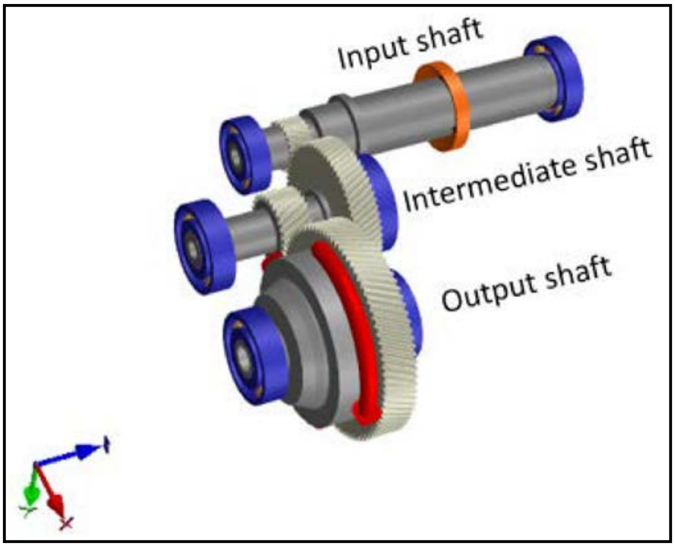

Figure 1 Layout of the gearbox.

Table 1 Input gear mesh geometry

Parameter	Gear 1	Gear 2
No. of teeth	22	65
Module (mm)	1.63	
Pressure angle (deg)	22	
Helix angle (deg)	25	
Outside diameter (mm)	43.15	119.43
Root diameter (mm)	34.29	111.78

Table 2 Output gear mesh geometry

Parameter	Gear 1	Gear 2
No. of teeth	22	65
Module (mm)	1.63	
Pressure angle (deg)	22	
Helix angle (deg)	25	
Outside diameter (mm)	43.15	119.43
Root diameter (mm)	34.29	111.78

as linear springs with constant stiffness matrices. The bearing stiffness matrices and gear mesh stiffness values are obtained from the *RomaxDESIGNER* (Ref. 1) software for different torque levels. The bearing stiffness calculation in *RomaxDESIGNER* is based on a nonlinear contact model that considers internal clearances, the local elastic deformation of the rolling elements and raceways and system-level effects such as raceway misalignments. The gear-mesh stiffness calculation in *RomaxDESIGNER* also uses a detailed mathematical model that includes tooth bending stiffness and nonlinear contact stiffness. The output of this calculation is a mesh stiffness that is the function of gear rotation angle. In this paper, we just take the mean value of this fluctuating mesh stiffness to model the gear-mesh compliance. This is not unreasonable because we do consider the fluctuating nature of mesh stiffness by applying a transmission-error excitation, which is described in the next section. A 5% modal damping is used for all the dynamic simulations presented in this paper.

Modeling Transmission Error Excitation

To calculate the gear dynamic factors from our multibody dynamic model, we apply transmission error (TE) excitations to all the gear meshes. Transmission error is caused by numerous factors, including variation in the tooth compliance as the contact point moves along the tooth height; change in the mesh stiffness as the number of teeth in contact change; any tooth profile modifications; and manufacturing errors. In this study we use the *RomaxDESIGNER* software to calculate the static transmission error for all the gear meshes for various torque levels. These static transmission errors are then used to excite the multibody dynamic model of the system (Fig. 2). The method of using static transmission errors to excite a dynamic model has been widely used in the literature (Ref. 2).

Load Cases to Analyze

An analysis of four different torque levels is conducted, as shown in Table 3. These are torques acting on the input shaft. The speed of the input shaft is varied from 0 rpm to 18,000 rpm, which is maximum operating speed of the electric motor.

For each of the torque levels, the dynamic factors are computed using our multibody dynamic model mentioned above and are compared with the dynamic factors predicted by ANSI/AGMA 2001-D06 (Ref. 3) and ISO 6336 (Ref. 4) standards.

Table 3 Torque levels

Loading side	Torque (Nm)
Drive	50
Drive	80
Drive	120
Drive	160

Gear Dynamic Factor Results and Discussion

The dynamic factor results presented in this section are based on the following definition:

$$\text{Dynamic factor} = \frac{\text{Dynamic mesh force}}{\text{Static mesh force}}$$

Static mesh force is the force acting

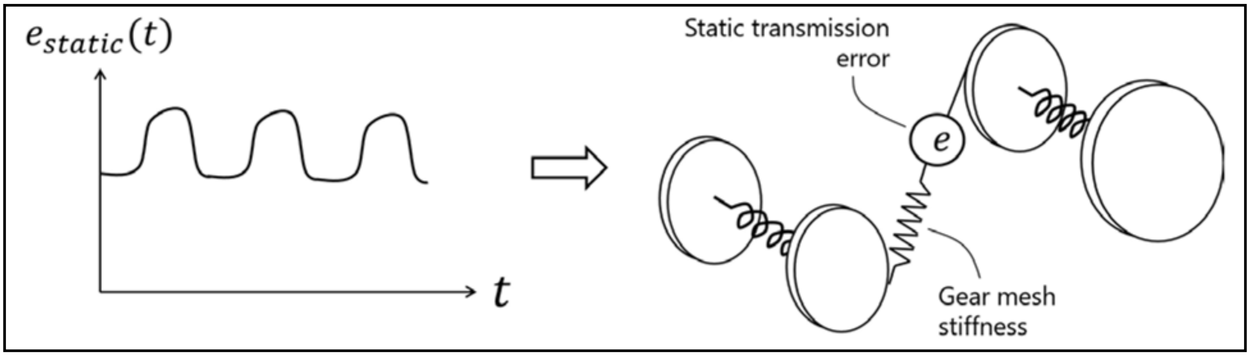

Figure 2 Inclusion of static transmission error as source of excitation of the dynamic model.

in the line-of-action of a gear mesh in a static equilibrium condition. Dynamic mesh force is the sum of static mesh force as well as dynamic fluctuations in the mesh force caused by the transmission-error excitations.

Influence of system effects on dynamic factors. To study the influence of system effects on gear dynamic factors, let us first consider the 50 Nm load case. The static transmission error trace for one mesh cycle for a drive torque of 50 Nm is shown in Figure 3. The peak-to-peak values are 2.86 μm and 0.96 μm for the input and output meshes, respectively.

These static transmission error traces are used to excite the multibody dynamic model of the system. The dynamic model includes 6 DOF for all the components in the system. Figure 4 shows the dynamic factors for the input and output gear meshes for an input torque of 50 Nm in the drive direction predicted by a 6 DOF dynamic model.

The dynamic factors for the input gear mesh are higher than that of the output gear mesh because of the following two reasons:

a) The peak-to-peak variation in the static transmission error is higher for input mesh than output mesh. Since the static TE acts as the source of excitation, a higher TE will produce a higher response.

b) The tooth-passing frequency for the input gear mesh is higher than the output mesh. Therefore the input gear mesh will excite more system resonances than output mesh within a given operating speed envelope.

The dynamic factors predicted from our simulations in Figure 4 show a number of peaks at various operating speeds for both input and output gear meshes. These peaks are caused by the excitation of system resonance modes. For a given speed, if the tooth-passing excitation frequency (or its higher harmonics) of a gear mesh is close to a system resonance mode, then that mode will become excited. The excitation of a resonance mode might increase the dynamic response at gear meshes, which will result in higher dynamic factor.

The red and blue circles in Figure 4 highlight the peaks that occur at input shaft speeds of 5,910 rpm and 13,600 rpm, respectively. The corresponding dynamic factors are 1.08 and 1.13, respectively. Figure 5 shows the mode shape corresponding to the resonance speed of 5,910 rpm.

The peak in the dynamic factor at 5,910 rpm is due to a combination of torsional and bending modes (Fig. 5). At

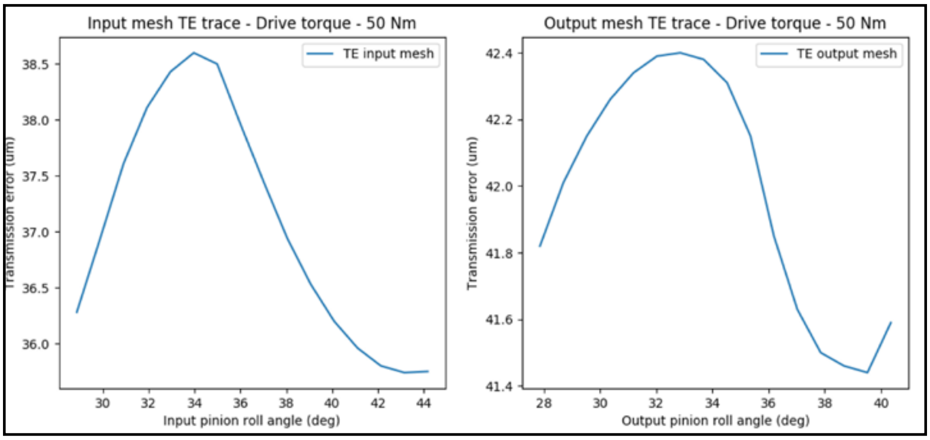

Figure 3 Static transmission error trace for 50 Nm input drive torque for the two gear meshes.

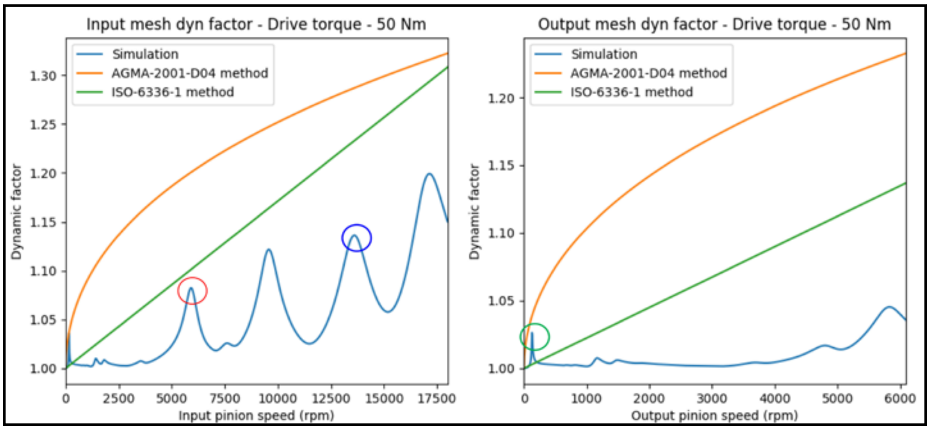

Figure 4 Dynamic factors for input torque of 50 Nm for two gear meshes. The factors are calculated using three methods: (a) multi-body simulation (blue); (b) AGMA standard (orange); (c) ISO standard (green).

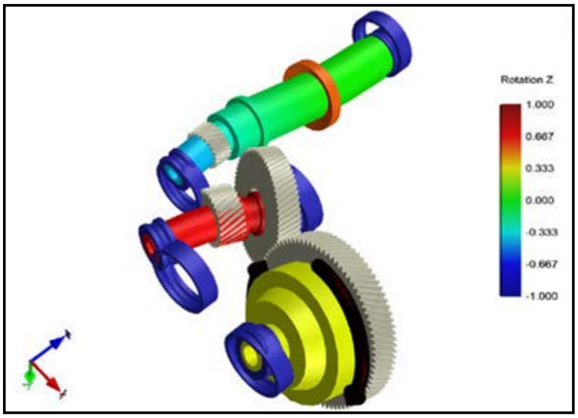

Figure 5 Mode shape corresponding to resonance speed of 5,910 rpm.

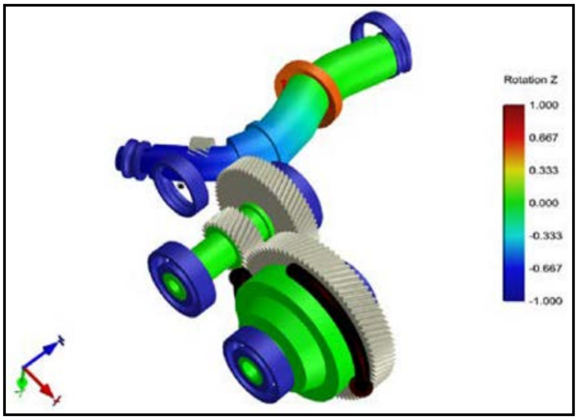

Figure 6 Mode shape corresponding to resonance speed of 13,600 rpm.

the input gear mesh node, the normalized rotational displacements (θ_Z) are in the opposite directions for the input shaft and intermediate shaft, respectively, which causes a reinforcing action followed by a spike in the dynamic force. The peak in dynamic factor is influenced by both bending and torsion modes, with a slightly higher contribution from torsional mode. This illustrates the importance of the inclusion of all degrees of freedom in the dynamic model.

Figure 6 shows the mode shape (normalized displacements in x, y, z, and θ_z directions) at the input resonance speed of 13,600 rpm, highlighted by the blue circle in Figure 4.

Contrary to the previous mode shape at 5,910 rpm (Fig. 5), the contributions from the bending modes to the dynamic factor at 13,600 rpm are low, indicating that the torsional mode is the major contributor to the dynamic factor. Using a similar approach, the modal contributions can be studied for every local maximum in the dynamic factor.

The output gear mesh exhibits a peak in the dynamic factor at the resonance speed of 160 rpm (Fig. 4). Figure 7 shows the mode shape of the three shafts corresponding to this resonance speed. The intermediate shaft exhibits a torsional mode (normalized $\theta_z \sim 1$) while the output shaft shows no torsional mode. The output shaft drives the vehicle with a high polar inertia, making it resistant to any torsional vibration, although few bending modes can be observed at certain natural frequencies. The intermediate shaft also moves in the axial direction, relative to the other two shafts, which also contributes to the dynamic factor at that mesh frequency.

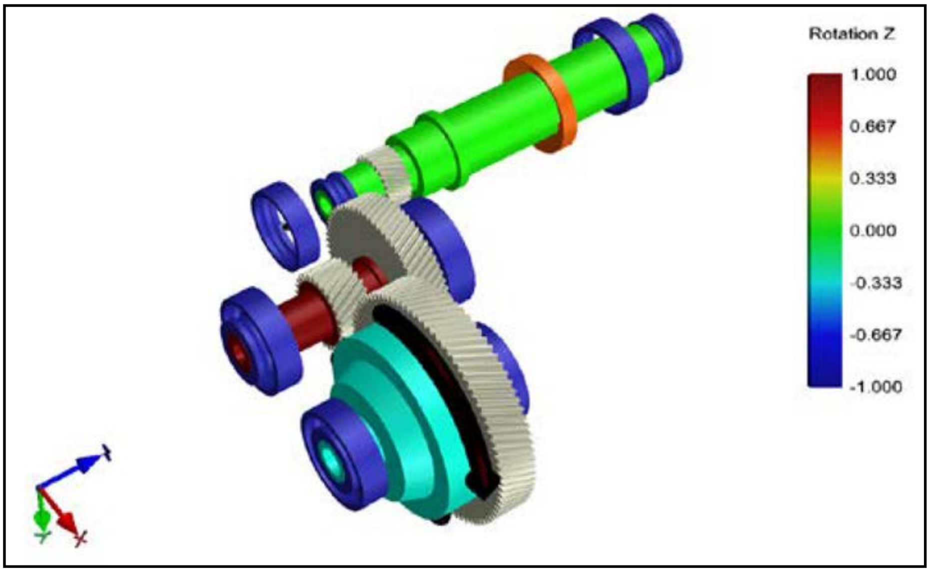

Figure 7 **Mode shape corresponding to resonance speed of 160 rpm.**

Comparison of dynamic factors obtained from dynamic simulations with gear standards. Figure 4 also shows the dynamic factors calculated using the methods prescribed in ISO-6336 and AGMA-2001 standards. The methodology prescribed in the standards is simplistic in nature, intentionally conservative to cater to a wide range of scenarios, and cannot be written to present specific excitations in a gear mesh. The dynamic factors predicted by the standards are much higher than the dynamic factors predicted in the model, which might lead to over-designing a system in some situations.

Torsional-only model vs. 6 DOF models. Torsional-only models are commonly used to study gear dynamics and to calculate gear dynamic factors (Refs. 5-7). In this section we compare the dynamic factors calculated from a torsional-only multibody model with our full 6 DOF models to determine whether a torsional-only representation is good enough or not.

Figure 8 shows the dynamic factors of the input and the output gear meshes predicted by a multibody model with only torsional DOF. There are three resonance modes that get excited within the given operating speed range (0 - 18,000 rpm) of the input shaft. The peak in the dynamic factors, highlighted by the red circle in Figure 8, is investigated by looking at

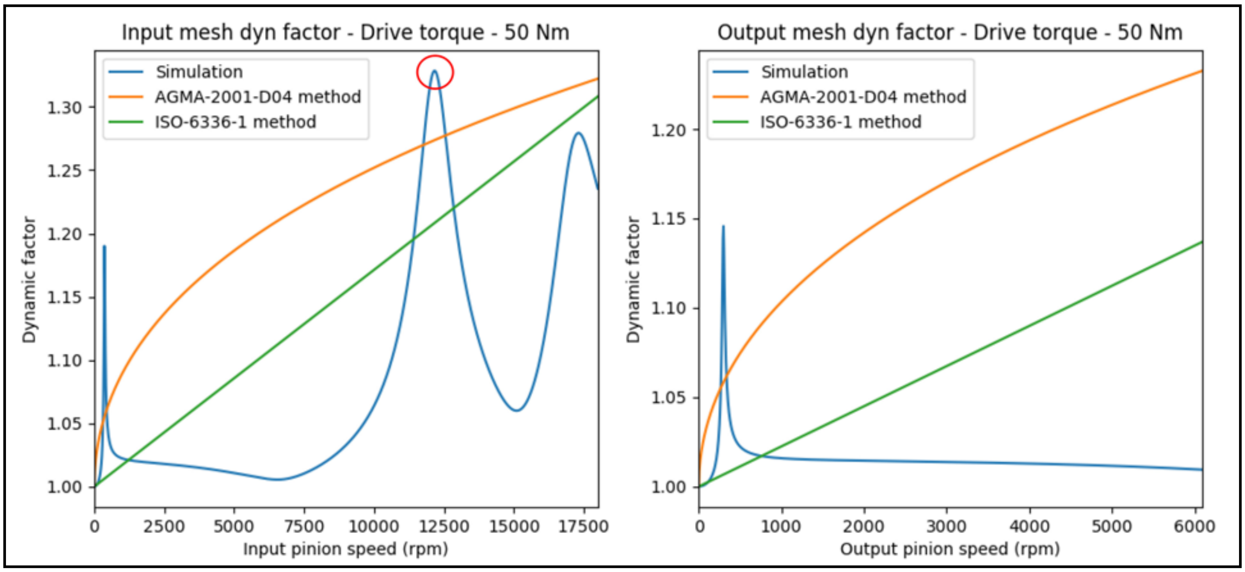

Figure 8 **Dynamic factors calculated using a torsional-only dynamic model.**

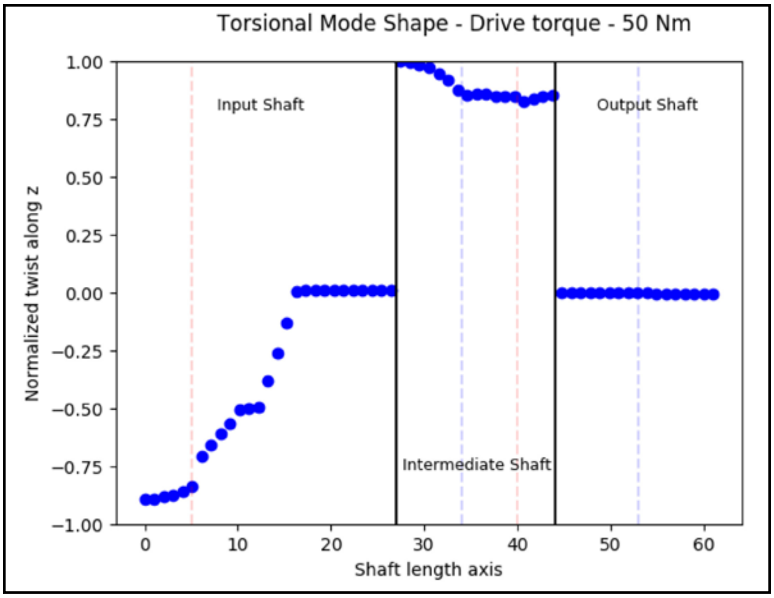

Figure 9 Mode shape of the highlighted section of the resonance mode in Figure 8.

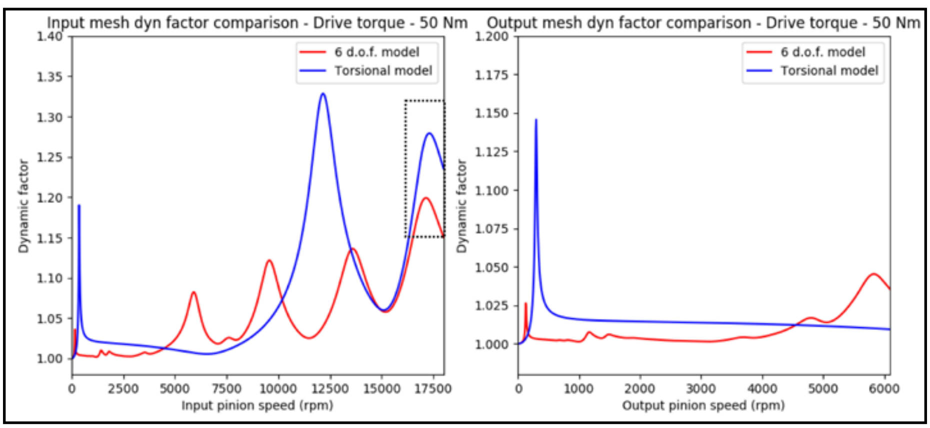

Figure 10 Comparison of dynamic factors calculated using a full 6 DOF model (red) and torsional-only model (blue) for both input and output gear meshes; the mode at 17,000 rpm in the dotted box is dominated by torsional deflections.

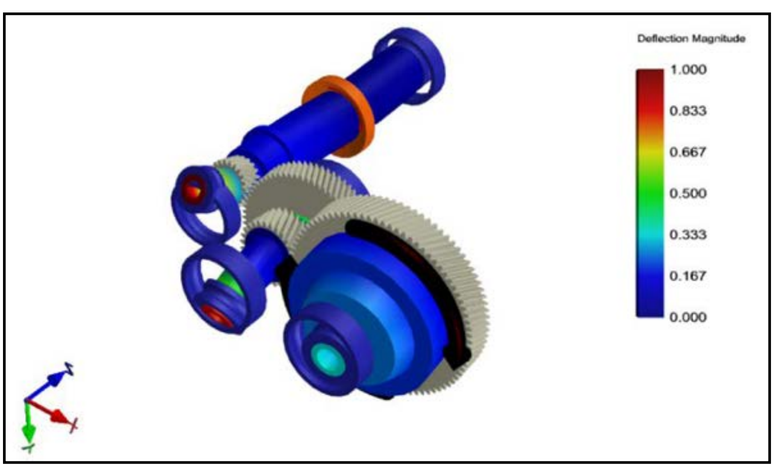

Figure 11 Mode shape for full 6 DOF model corresponding to the 17,000 rpm resonance speed (highlighted by dotted box in Fig. 10).

the mode shapes at that resonance frequency (Fig. 9). The red line represents input mesh node on the shaft and the blue line represents the output mesh node in the shaft.

Figure 10 shows the comparison between the dynamic factors calculated using a full 6 DOF model and torsional-only model. The torsional-only model does not capture any of the bending modes, and all the energy is stored in gear meshes and shaft twisting. In a 6 DOF model, part of the energy is stored in bending the shaft away from the gear mesh, resulting in lower dynamic factors. Hence, the dynamic factors predicted by the torsional only model will be higher than the ones predicted by a full 6 DOF model. Also, the full 6 DOF model has more resonance peaks than the torsional model. This is expected, as the torsional model ignores all the bending modes. However, some modes that are dominated by torsional deflections are present in both models at roughly the same frequency; for example, the mode in Figure 10 that is highlighted by a dotted box.

Figures 11 and 12 show the mode shapes for the highlighted resonance mode (Fig. 10) for the full 6 DOF model and torsional-only model, respectively. Both models predict very similar torsional modes for the three shafts, albeit a slight shift in the peak in dynamic factor curves. In the 6 DOF model the bending modes at the input gear mesh location contribute slightly to the dynamic factor, although the major contributor is the torsional mode. The contribution made by the bending modes is not captured in the torsional-only model, and so it is important to include all six degrees-of-freedom to accurately predict the dynamic behavior of the system.

Effect of torque on dynamic factors. The effect of load on the dynamic factor of the gears is shown in this section. The torque conditions that are simulated to study the effect of load on the dynamic behavior of the system are shown in Table 3.

Figure 13 shows the dynamic factor as a function of torque for the input and output mesh, using the 6 DOF model. The x-axis of the graphs (Fig. 13) represents the gear mesh frequency to illustrate the natural frequencies being shifted to the right as load increases. This is due to the increase in gear meshes and bearings stiffness values with load.

Figure 14 shows the static transmission error traces for the input and output gear meshes for different torques. For input gear mesh, the peak-to-peak TE decreases with torque by a small amount. For output gear mesh, however, the peak-to-peak TE increases substantially with torque.

Figure 13 also shows that as we increase the torque, the dynamic factors decrease. There are two factors that determine the variation in dynamic factors with torque:

a) Change in the static TE trace with torque. Static TE acts as an excitation source for the dynamic model. If the peak-to-peak static TE decreases with torque, this will lead to reduction of the dynamic response at the gear mesh. On the other hand, if the peak-to-peak TE increases with torque, then we will get higher dynamic mesh forces because of increased dynamic response.
b) Dynamic factor is a ratio between total mesh force (static + dynamic) and static mesh force. So, for a fixed dynamic force, if we increase the static force, dynamic factor will reduce.

Figure 15 illustrates the effect of torque on the dynamic factors calculated using a torsional-only model. Similar shifts towards higher frequencies are seen in the dynamic factor peaks, which are reasoned by the increase in system stiffness values at higher loads. The variation in the dynamic factor values with torque is also in agreement with the trends we observed for full 6 DOF model.

Time Domain Simulations

The dynamic factors were calculated at different speeds and torques, as shown in the previous section using a frequency domain approach. Frequency domain solvers are advantageous, as they take less time to solve and inherently assume that the response is caused by one single excitation at a given frequency. In a complex model where there are multiple gear meshes at different mesh frequencies, the net response at a given operating speed can be calculated using the super position principle. This is one of the main limitations of the frequency domain approach; since only one frequency of interest is considered at a time,

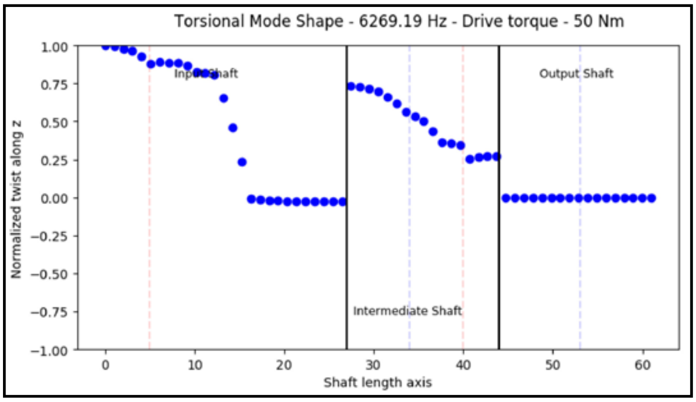

Figure 12 Mode shape for the torsional-only model corresponding to 17,000 rpm resonance speed (highlighted by dotted box in Fig. 10)

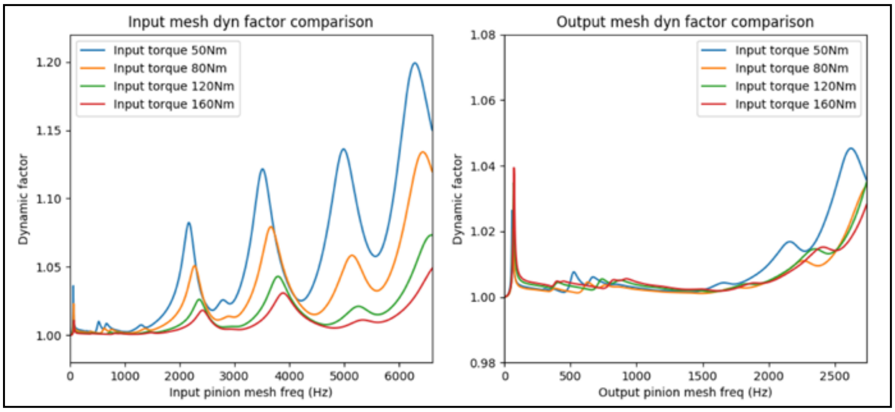

Figure 13 Dynamic factors calculated using a 6 DOF model at four different torque levels for input and output gear meshes.

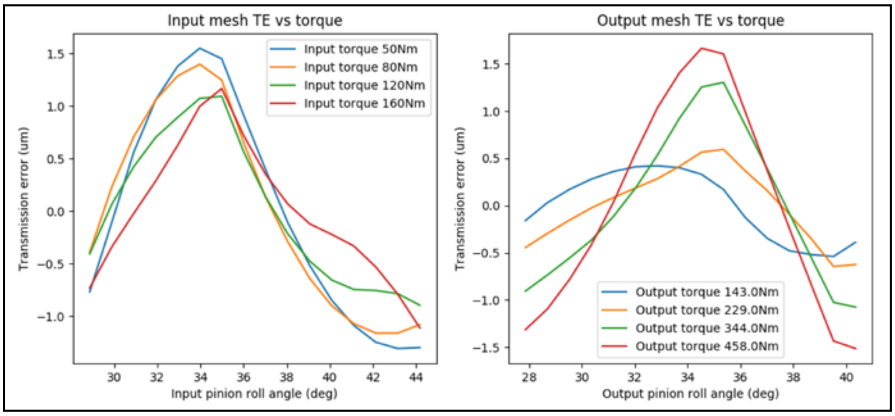

Figure 14 Static transmission error (removing the DC component) traces for different torques.

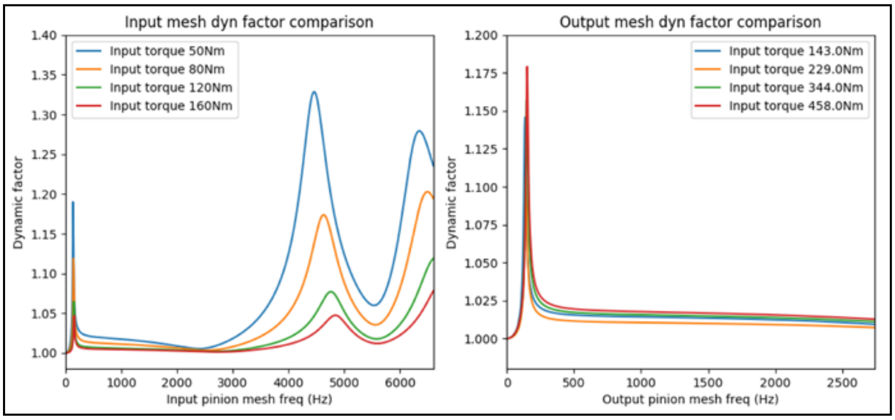

Figure 15 Dynamic factor as a function of torque for input and output mesh using torsional-only model.

the effect of multiple harmonics on gear mesh excitation is hard to capture using this approach.

One other approach is to use time domain simulations where the system response is calculated at every instant of time. The dynamic factors obtained by this method are elaborated in this section. The system is excited by static TE at both the gear meshes simultaneously and the response is calculated for every time step. Comparisons are made between the responses using time domain and frequency domain approach to show the advantages and limitations of one over the other.

Dynamic factors as a function of speed for 50 Nm torque using time domain approach. As shown earlier in this presentation, the system is excited by a static TE at both the input and output gear mesh at a given operating speed. The excitations at the input and output meshes are shown (Fig. 16).

Using the above excitation, the dynamic factors are calculated using time domain approach (Fig. 18) and considering only the torsional mode. The dynamic factor varies with the rotation of the input shaft periodically and the reported dynamic factor (Fig. 17) is the maximum value over 1 rotation of the input shaft. The dynamic factors follow a similar trend between the two approaches. The peak dynamic factor has shifted slightly to a higher speed in the time domain approach due to the contributions of multiple modes of the system and higher harmonics of the gear mesh frequencies.

The dynamic factors predicted using time domain approach are higher than predicted using the frequency domain approach. This is due to the contribution of higher harmonics of the gear mesh excitations, which is not accounted for in the frequency domain method.

A peak in the dynamic factor is observed in the output mesh at around 12,842.9 rpm of input shaft speed. This corresponds to 1,956 Hz output mesh frequency and 4,709 Hz of input mesh frequency. The frequency domain approach considers only one excitation frequency at a time, and the response is calculated at that frequency. Since the excitation frequency for the input and output mesh is different for the same operating input shaft speed, the frequency domain approach fails to capture the response at output mesh due to an input mesh excitation of 4,709 Hz. In time domain simulations the transient response is calculated based on the mesh excitation force at every instant of time. This clearly illustrates the limitations of the frequency domain approach over the time domain approach where the difference in dynamic factors is substantial.

With the trends in dynamic factors for input mesh matching between the time and frequency domain approaches, the system response at a torque of 50 Nm is elaborated in the following sections using the dynamic model by including all degrees of freedom.

System response at 50 Nm torque using time domain simulations. This section

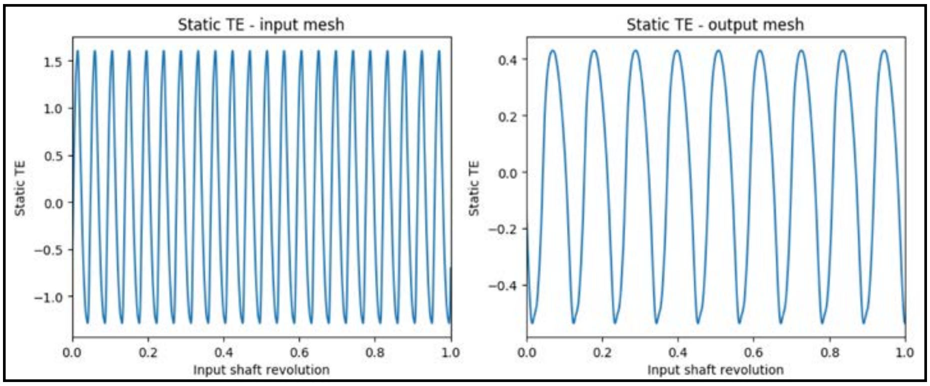

Figure 16 TE excitations in microns at input and output gear meshes.

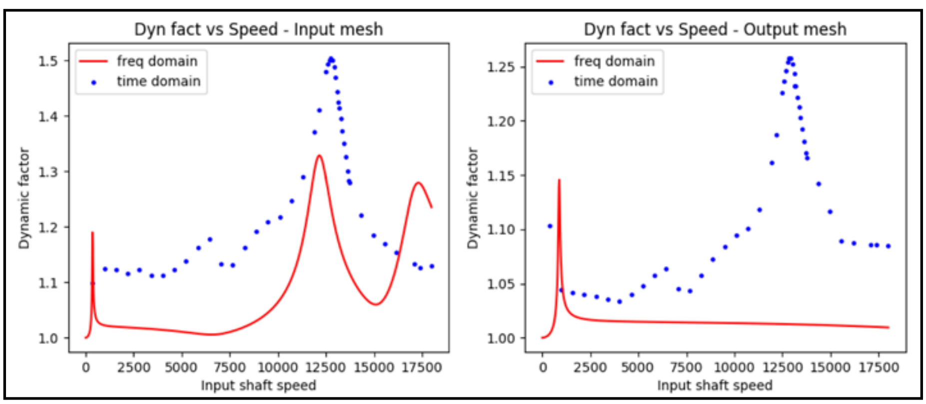

Figure 17 Dynamic factors—time domain vs. frequency domain—torsional only mode.

Table 4 Simulation conditions—input shaft speed	
Input shaft speed (rpm)	**Input mesh dynamic factor**
3,500	1.002
14,100	1.11

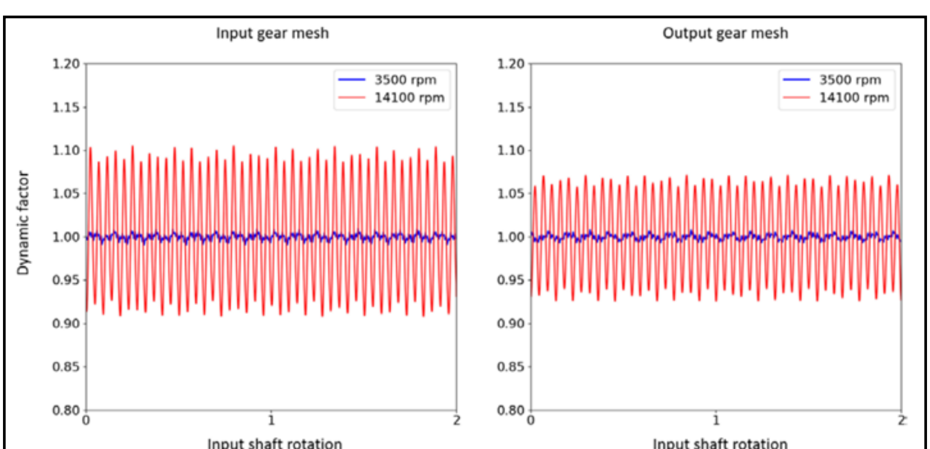

Figure 18 Dynamic factor 50 Nm, torsional DOF.

shows the system response as a function of shaft rotation for an input torque of 50 Nm at two different operating speeds of the input shaft as shown in Table 4. 3,500 rpm corresponds to an input shaft speed where the dynamic factor is lowest from 0 to 18,000 rpm range and 14,100 rpm corresponds to an input shaft speed where the dynamic factor is highest due to resonance.

The system is excited by a static TE force at the two gear meshes, as shown previously. Figure 18 shows the dynamic factors as a function of input shaft revolution for the input and output gear meshes at 3,500 rpm, and 14,100 rpm at 50 Nm, considering all degrees of freedom.

The resonance can be clearly seen in the dynamic factor plots when the system is excited at 14,100 rpm (5,170 Hz) input speed. Figure 19 shows the mode shape at 5,164.5 Hz with the contours representing normalized rotation about the *z* direction (as per the coordinate system in the figure). It is evident that the mode at 5,164.5 Hz is highly torsion-dominated (albeit the slight bending of the input shaft can be seen), which will represent the greatest contribution to dynamic factors than the bending modes.

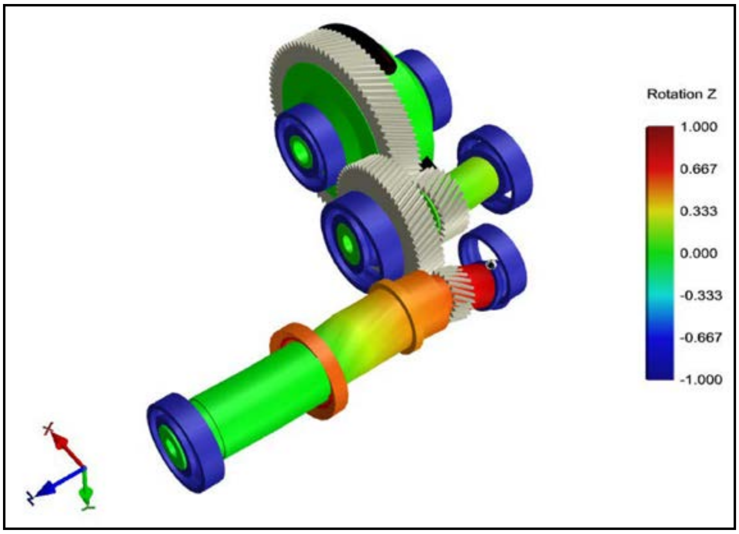

Figure 19 Mode shape at 5,164 Hz showing normalized θz rotation contours.

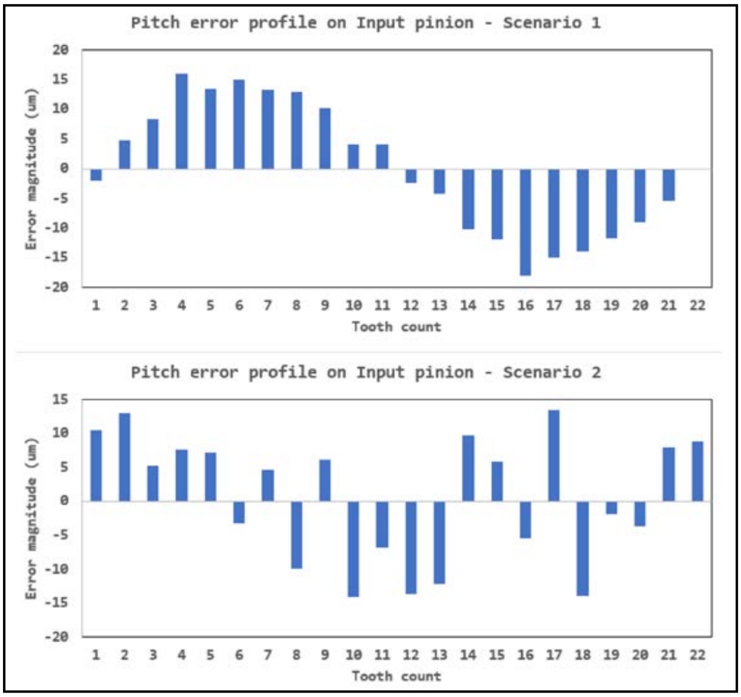

Figure 20 Pitch error profiles.

Table 5 Component orders relative to input and output mesh frequency			
Input shaft frequency	Input mesh frequency	Intermediate shaft frequency	Output mesh frequency
0.045	1	0.015	0.415
0.109	2.407	0.037	1

Effect of Manufacturing Errors on Dynamic Factors

Manufacturing errors like run-out and pitch errors in gears influence the dynamic factors, and this has been studied in the past by Velex, et al (Ref. 8) using analytical models and Kahraman (Refs. 10–11) et al experimentally for spur gear pairs.

In this paper the effect of pitch errors in gear dynamic factors is studied by using the time domain models, as described previously. Pitch error is essentially tooth spacing error and this affects the timing of approach and recess of contact at every tooth cycle, depending on the magnitude of the pitch error. This directly affects the contact parameters (like TE, contact stress) and induces excitations at a frequency different from the mesh frequency, depending on the variation of pitch errors.

Pitch errors are usually related to gear quality and are usually within a band and random in magnitude for every tooth in a gear. Therefore, it will be necessary to look at the system response for 'n' rotations of the gear until the same pair of teeth is in contact again. When hunting ratio is maintained, the number of cycles for the same pair of teeth to come into contact again is very high.

It is impractical to use a frequency domain approach to study the effect of pitch errors, since the harmonics of TE trace is not just an integral multiple of mesh frequency, but also other random frequencies, depending on the pitch error trace which may or may not dominate over the mesh frequency component and its harmonics.

In this study two scenarios of manufacturing errors are analyzed. The error profiles for both the conditions are shown (Fig. 20). In the first scenario a combination of random error and a sinusoidal pitch error is applied to the input gear where the sinusoid can be considered as a representation of the runout,

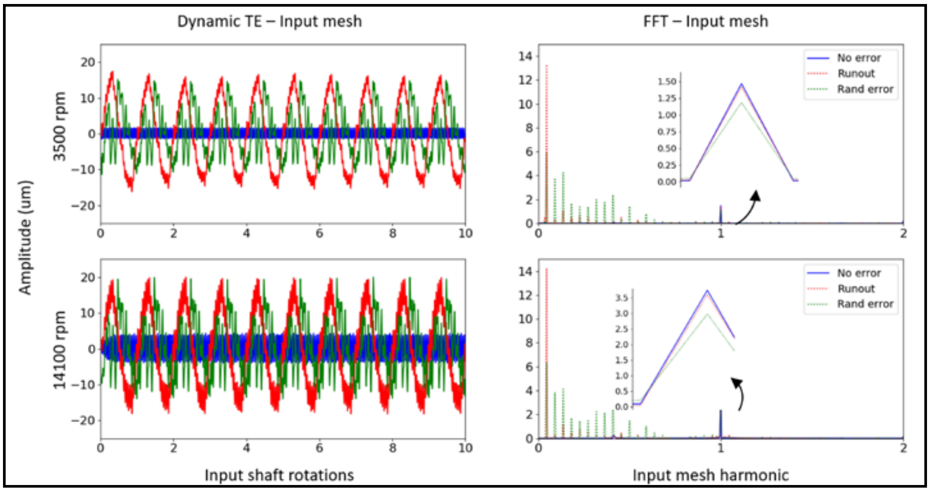

Figure 21 **Dynamic TE — input mesh.**

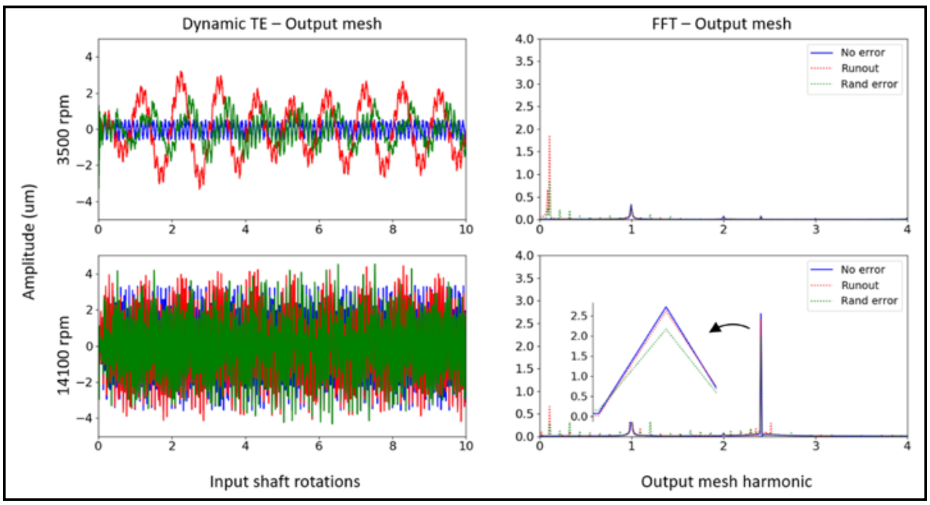

Figure 22 **Dynamic TE — output mesh.**

Table 6 Input mesh order amplitudes of input mesh and output mesh dynamic TE at 14,100 rpm

	Input mesh harmonic of Input mesh dynamic TE (μm)	Input mesh harmonic of Output mesh dynamic TE (μm)
No error	3.74	2.71
Runout	3.6 (3.7 % reduction)	2.61 (3.7 % reduction)
Random error	2.97 (20.58 % reduction)	2.14 (21 % reduction)

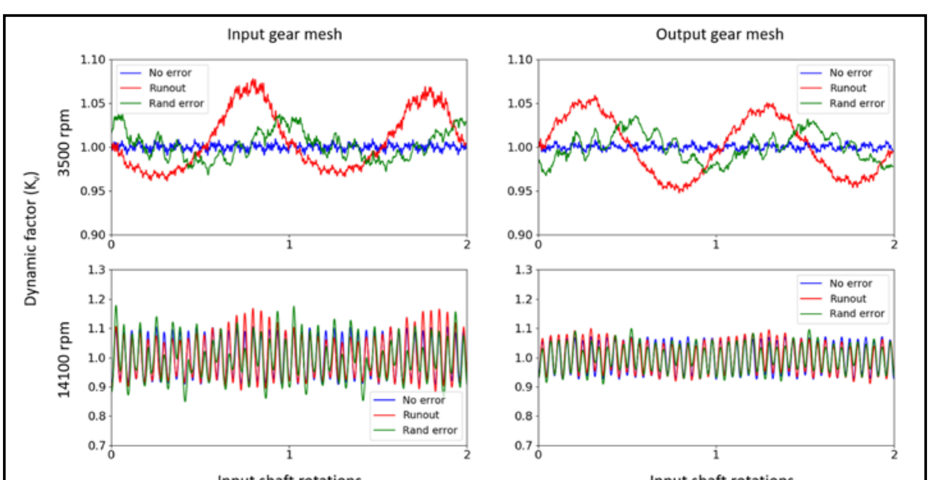

Figure 23 **Dynamic factor as a function of input shaft rotation for three error cases and two speed conditions for input and output mesh.**

i.e. — the amplitude of the sinusoid is 15 μm and the random distribution is between (−5, 5 μm). The second scenario is a purely random pitch error between (−15, 15 μm).

The effect of both scenarios on dynamic behavior is studied and is compared with the case without any errors. Two speed conditions are chosen, shown in Table 4 — 3,500 rpm and 14,100 rpm. Table 5 shows the component frequencies as a function of input and output mesh frequencies. This information is useful in gaining a better understanding of dynamic transmission error plots that follow.

Figures 21 and 22 show the dynamic mesh deflection of the input gear mesh and output gear mesh, respectively, as a function of input shaft rotation for three error conditions and two speeds and corresponding FFT to show the different harmonic content. The speeds are chosen such that 14,100 rpm corresponds to the input mesh frequency, being close to a natural frequency of the system. Without any tooth errors only the mesh orders excite the system. The TE amplitude of the 1st harmonic of input mesh frequency is higher by 2.5 times at 14,100 rpm, compared to 3,500 rpm indicating that the system is operating at resonance. This can be clearly seen in Figure 22, where order 2.4 of the output mesh frequency has a higher amplitude (2.7 μm) for 14,100 rpm than the system at 3,500 rpm (where the amplitude is just 0.1 μm). It is also interesting to note that the dynamic TE of the output mesh has components of the input mesh frequency which is a direct effect of system behavior.

With the introduction of a runout error in the input pinion, dynamic TE has components of input shaft order, which can be expected since runout is an excitation at shaft frequency. At the output mesh the TE amplitude of the input shaft harmonic (abscissa 0.109 in the FFT plot; Fig. 22) is 3 times higher for 3,500 rpm than at 14,100 rpm. This can be explained since the system is excited by the runout in the input gear (at input shaft frequency of 3,500 rpm (58.3 Hz) which is very close to a natural frequency. Due to this near-resonance at 3,500 rpm, a "beating" phenomenon is observed (dynamic TE plot in Fig. 22), which is a classic behavior when a system is excited near resonance.

This "beating" pattern eventually dampens out if the source excitation is consistent at the same frequency near resonance.

When random pitch errors are introduced in the system, multiple shaft orders are excited and the dynamic TE is a combination of the mesh harmonic and multiple harmonics of the input shaft frequency. At 14,100 rpm speed of the input shaft, the 1st harmonic of the input mesh frequency is high compared to 3,500 rpm case indicating resonance at 14,100 rpm.

Table 6 shows the input mesh order amplitudes (abscissa 1 in FFT plot in Fig. 21 and abscissa 2.04 in FFT plot in Fig. 22) of the input and output mesh dynamic TE for the three conditions of pitch error at 14,100 rpm operating speed. Introduction of pitch errors reduces the mesh harmonic component of dynamic TE and the reduction is seen higher for the case of random pitch error than for the case of runout in the input gear. This can be explained from an energy perspective. With the introduction of errors, the energy is spread over multiple shaft harmonic frequencies, thereby reducing the main mesh harmonic component. With a runout error in the input shaft, only the 1st harmonic of shaft frequency is excited, whereas with a random error distribution in the input gear, multiple shaft harmonics are excited, which leads to greater distribution of energy over multiple frequencies and therefore greater reduction in dynamic TE mesh harmonic.

Figure 23 shows the dynamic factors for the three error conditions and two speeds for the input and output gear mesh, respectively. The effect of resonance can be clearly seen in the dynamic factor of both the input and output meshes. Introducing errors to the system increases the maximum dynamic factor, and the increase depends on the nature and magnitude of the manufacturing error in the system.

To completely understand the effects of manufacturing errors in the system, the dynamic factors were calculated using a 6 DOF model solved in time domain and was plotted against operating speed, as shown in Figure 24 for cases with and without manufacturing errors.

A random distribution of error is applied to the input pinion (Fig. 20) to generate the above plots. Each point on the coordinate in Figure 24 is the maximum value of the dynamic factor (which is a function of time) for every speed point. Manufacturing errors increase the dynamic factor of the mesh, and the magnitude of increase depends on the nature and amplitude of the error. Additional peaks in the dynamic factors are revealed when errors are introduced to the system, indicating that the system is excited by shaft orders and its harmonics, which is not the case when there is no error. The additional peaks can be quite significant — depending on the nature of the error, type of gear (spur/helical), etc.

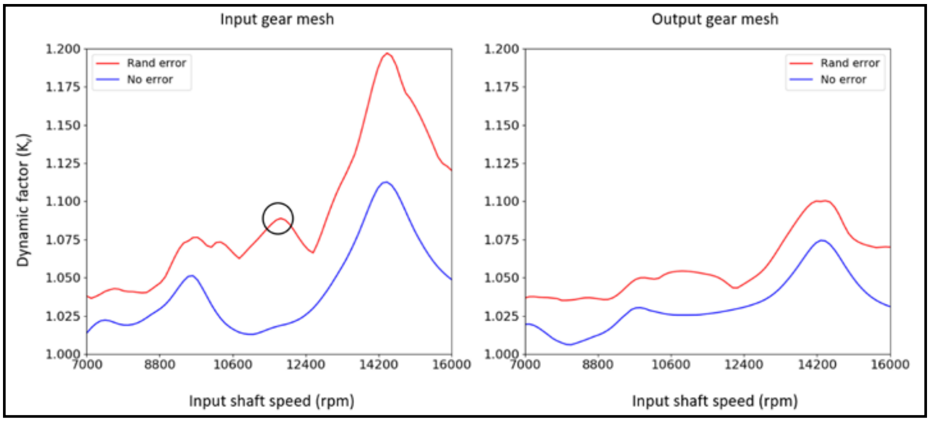

Figure 24 Dynamic factor for input and output mesh as a function of speed — with and without manufacturing errors.

Conclusions

The dynamic behavior was studied for a high-speed, electric-vehicle gearbox using a multibody dynamic model. Based on the dynamic factor results, the following conclusions can be made:

- System effects and system dynamics play a key role in the dynamic factor predictions. System resonances that fall within the operating speed range increase the dynamic factors substantially. Any model that only considers the dynamics of just the gear pair of interest, and ignores the system-level effects, is likely to give incorrect dynamic factors.
- Standards (AGMA or ISO)-based calculations for dynamic factors are simplistic in nature and there can be cases where a detailed system level analysis is required of, for example, manufacturing errors in the system, multiple simultaneous gear mesh excitations, etc.
- Bending modes are important, and a torsional model that ignores the bending deflections is not adequate for the prediction of gear dynamic factors.
- The differences between the time domain and frequency domain approaches were compared and this paper illustrated the limitations of the frequency domain approach in some scenarios.
- Manufacturing errors cause several shaft orders to be excited, which results in higher dynamic factors at system resonances. However, the mesh harmonic component of the dynamic transmission error slightly reduces with the introduction of errors in the system.

Future Work

- Correlation of simulation results with published experimental data (Refs. 10–11)
- Calculate dynamic stress and compute dynamic factors based on the stresses (root/contact), rather than using forces as seen in this study
- Include the effects of flexible gear blanks and housing
- Extend this model to planetary gear sets

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Niranjan Raghuraman is working as an engineering analyst for Romax Technology located in Boulder, CO. His background includes powertrain dynamics, parallel and cross axes gearing, plastic gearing, heat transfer and thermodynamics. At Romax, Niranjan is involved in Solving problems pertinent to gear whine, gear rattle and gear dynamics in transmissions for conventional and electric automotive and other sectors including wind energy, aerospace, etc. Prior to joining Romax, Niranjan worked for Chrysler, as a Powertrain Simulation Engineer, on durability studies for automotive rear axle drive unit and development of energy management models for vehicle powertrain. Niranjan earned his master’s degree from The Ohio State University. He worked on predicting surface temperature on spur and helical gears using Finite Difference Methods as a part of his masters at The Gear and Power Transmission Laboratory with Dr. Don Houser.

Dr Sharad Jain joined Romax in 2007 after graduating from Indian Institute of Technology Roorkee, India in Mechanical Engineering. He also has a Ph.D. in engineering from University of Cambridge, UK. During his Ph.D., Sharad worked on dynamics of high-speed bearings and planetary gearboxes. Sharad has 9 years of experience in mathematical modelling and dynamics and vibration of rotating machines. His areas of interest include bearing dynamics, contact mechanics, automatic discretisation of continuous structures for dynamic analysis, and dynamics of electro-mechanical drivetrains. Currently, Sharad is leading the development of an object-oriented toolbox for multi-physics dynamic analysis at Romax.