Gear Tooth Scoring Design Considerations for Spur and Helical Gearing

by
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Introduction

High speed gearing, operating with low viscosity lubricants, is prone to a failure mode called scoring. In contrast to the classic failure modes, pitting and breakage, which generally take time to develop, scoring occurs early in the operation of a gear set and can be the limiting factor in the gear's power capability.

Scoring is a form of surface damage on the tooth flank, which occurs when overheating causes the lubricant film to become unstable allowing metal to metal contact. Local welding is initiated and the welded junctions are torn apart by the relative motion of the gear teeth resulting in radial score marks. Figs. 1, 2 and 3 illustrate degrees of the scoring phenomenon. Light scoring, which does not progress, may be acceptable, but heavier scoring can destroy the tooth profile and lead to pitting and breakage. Also, scoring can result in noisy operation and the release of metallic paricles into the lubrication system.

The American Gear Manufacturers Association Aerospace Gearing Committee has been investigating the scoring phenomen for many years. AGMA Information Sheet 217.01, "Gear Scoring Design Guide for Aerospace Spur and Helical Power Gears," October, 1965, is a widely used method for predicting scoring based on Professor Blok's flash temperature index concept.[1] The Scoring Information Sheet is currently being updated and this paper is a review of the changes and additions being made.

In addition to the flash temperature approach, the minimum film thickness concept will be covered in the new publication. In the minimum film thickness approach, an elastohydrodynamic film thickness is calculated and compared to the asperity height to determine whether the lubricant film is adequate.

Fig. 2 – Moderate Scoring

Flash Temperature Index

The critical total temperature hypothesis (flash temperature index) appears to be the most reliable method of analysis presently used to predict scoring. It states that scoring will

Fig. 1 – Light Scoring

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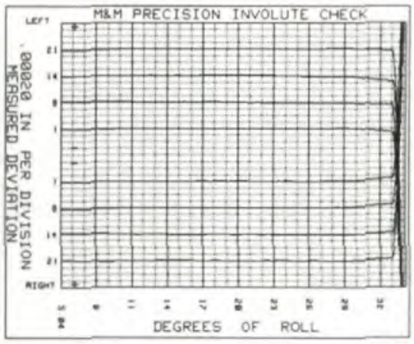

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Fig. 3 – Typical Heavy Scoring

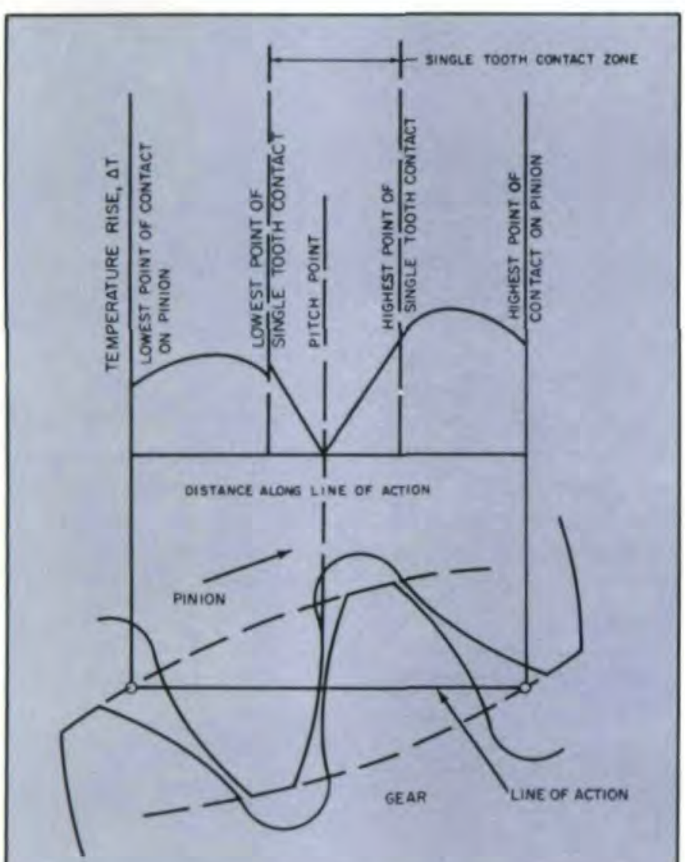

Fig. 4

occur when a critical total temperature, which is characteristic of the particular combination of lubricant and gear material, is reached.

$$T_f = T_b + \Delta T$$

where T_f = Flash temperature index, °F

T_b = Gear blank temperature, °F

ΔT = Maximum rise of instantaneous surface temperature in the tooth mesh above the gear blank's surface temperature.

The gear blank temperature is difficult to estimate. It may be significantly higher than the bulk oil temperature.[2] The heat transfer capability of the gear must be considered in attempting to estimate this parameter.

Often the blank temperature is approximated as the average of the oil temperature entering and leaving the gearbox.

One form of the fundamental flash temperature index formula is:[3]

$$T_f = T_b + \frac{C_f\, f\, W\, (V_1 - V_2)}{(\sqrt{V_1} + \sqrt{V_2})\, \sqrt{B_c/2}}$$

where C_f = Material constant for conductivity, density and specific heat

f = Friction coefficient

V_1 = Rolling velocity of pinion at point of contact, fps

V_2 = Rolling velocity of gear at point of contact, fps

B_c = Width of band of contact

W = Specific loading, normal load divided by face width, lbs./in.

For steel on steel gears, taking C_f as 0.0528, f as a constant 0.06 and adding a term taking surface finish into account[2] the following equation results.[4]

$$T_f = T_b + \left[\frac{W_{te}}{F_e}\right]^{3/4} \left[\frac{50}{50 - S}\right] Z_t\, (n_p)^{1/2}$$

where: W_{te} = Effective tangential load, lbs.

F_e = Effective face width (use minimum contact length for helical gears), in.

S = Surface finish (after running in), RMS

n_p = Pinion RPM

Z_t = Scoring geometry factor

$$Z_t = \frac{.0175\left[\sqrt{e_p} - \sqrt{\dfrac{N_p\, e_g}{N_g}}\right]}{\left[\cos \phi_t\right]^{3/4} \left[\dfrac{e_p \bullet e_g}{e_p + e_g}\right]^{1/4}}$$

Note: Use absolute value of Z_t

where e_p = Pinion radius of curvature, in. } See Appendix I
e_g = Gear radius of curvature, in. }
N_p = Number of pinion teeth (smaller member)
N_g = Number of gear teeth (larger member)
ϕ_t = Pressure angle, transverse operating

The $\frac{50}{50 - s}$ term was developed by Kelly[2] in an experimental program using gears with surface finish in the range of 20-32 RMS. For gears with surface finish rougher than this range, if the computed value exceeds 3, a factor of 3 should be used. For gears with surface finishes finer than 20, the resulting computed factor may be conservative.

The term W_{te} must be adjusted to allow for the sharing of load by more than one pair of teeth. The following analysis, which modifies the tooth load depending on the position of the gear mesh along the line of action, was developed by Dudley using spur gears of standard proportions. If a more accurate prediction of tooth load sharing is available to the reader it would be appropriate to use that analysis.

$$W_{te} = KW_t$$

where W_t = Tangential tooth load, lbs.

1. Unmodified tooth profiles

$$K = 1/3 - 1/3\left[\frac{\theta + \theta_{LD}}{\theta_L + \theta_{LD}}\right]$$

$$\theta_{LD} \leqq \theta < \theta_L$$

$$K = 1.0$$

$$\theta_L \leqq \theta \leqq \theta_H$$

$$K = 1/3 + 1/3\left[\frac{\theta_O + \theta}{\theta_O + \theta_H}\right]$$

$$\theta_H < \theta \leqq \theta_O$$

2. Modified tooth profiles

A. Pinion driving

$$K = \frac{6}{7}\left[\frac{\theta - \theta_{LD}}{\theta_L - \theta_{LD}}\right]$$

$$\theta_{LD} \leqq \theta < \theta_L$$

$$K = 1.0$$

$$\theta_L \leqq \theta \leqq \theta_H$$

$$K = \frac{1}{7} + \frac{6}{7}\left[\frac{\theta_O - \theta}{\theta_O - \theta_H}\right]$$

$$\theta_H < \theta \leqq \theta_O$$

B. Pinion driven by gear:

$$K = \frac{1}{7} + \frac{6}{7}\left[\frac{\theta - \theta_{LD}}{\theta_L - \theta_{LD}}\right]$$

$$\theta_{LD} \leqq \theta < \theta_L$$

$$K = 1.0$$

$$\theta < \theta < \theta_H$$

$$K = \frac{6}{7}\left[\frac{\theta_O - \theta}{\theta_O - \theta_H}\right]$$

$$\theta_H < \theta \leqq \theta_O$$

where θ = Any pinion roll angle (see appendix), radians

θ_{LD} = Roll angle at the pinion limit (form) diameter, radians

θ_L = Roll angle at the lowest point of single tooth contact on the pinion, radians

θ_H = Roll angle at the highest point of single tooth contact on the pinion, radians

θ_o = Roll angle at the pinion outside diameter, radians

A modified tooth profile would be one which has tip and/or flank relief rather than a true involute form.

The flash temperature index should be calculated at five specific points on the line of action and at several additional points of contact. The five specific points are:
1. Outside diameter of pinion
2. Highest point of single tooth contact
3. Pitch point (flash temperature rise will be zero since there is no sliding)
4. Lowest point of single tooth contact
5. Form (contact diameter) of pinion.

Fig. 4 is a typical plot of flash temperature rise along the line of action.

The most convenient way to generate a plot, such as shown in Fig. 4, is by the use of a computer program. By stepping through successive roll angles, the Flash Temperature Index can be calculated at many points. From Fig. 4 it can be seen that there will be two peaks, one during the arc of approach (pinion form diameter to pitch diameter) and one during the arc of recess (pinion pitch diameter to outside diameter). In order to achieve the minimum flash temperature index, the flash temperature rise in the arc of approach should be equal to the rise in the arc of recess. An optimum tooth design can be achieved by the use of long and short addendums. The computer program, starting with standard addendums (1/Diametral Pitch), automatically varies the pinion and gear addendums in defined increments until the optimum flash temperature is obtained. With the resulting long and short addendum designs of this nature, standard tooth thicknesses are no longer applicable. If standard tooth

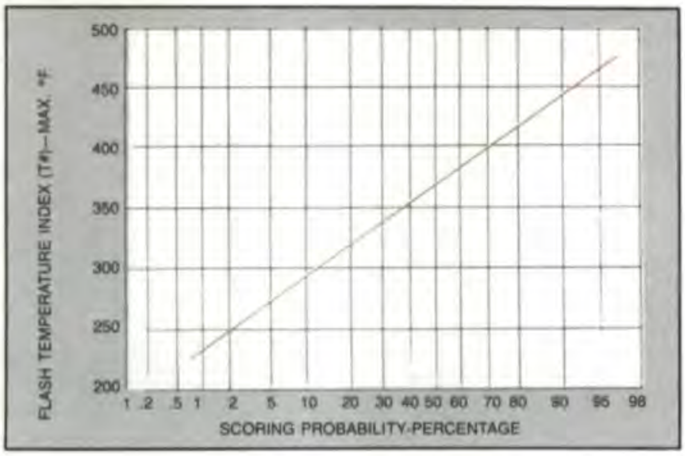

Fig. 5

thicknesses were utilized, an unbalance of bending stresses between pinion and gear would result. To optimize the bending stresses, the program enters a second iteration procedure which varies tooth thickness until bending stresses are balanced.

Fig. 5 presents the results of an aerospace industry survey[4] correlating scoring to flash temperature index. From the data used in the study, it was assumed that a gear set with a calculated index of 276°F, or less, represents a low scoring risk. A calculated index ranging from 277°F to 338°F represents a medium scoring risk where scoring may or may not occur, and a calculated index of 339°F and higher represents a high scoring risk. The data presented in Fig. 5 reflects cases using SAE 9310 steel operating with MIL-L-7808 or MIL-L-23699 synthetic oil. The viscosity of these oils is approximately 4-6 centistokes at 200°F and 18-30 centistokes at 100°F. Mineral oils such as light turbine oils, used in high speed industrial applications, are more viscous and, therefore, can tolerate a higher flash temperature index.

The equation for the flash temperature index assumes a constant coefficient of friction of 0.06. If it is desired to calculate the coefficient of friction at each point on the line of action, the following equation can be used.[5]

$$f = 0.0127 \log_{10} \left[\frac{3.17 \times 10^8}{\mu_o \; V_s \; V_t^2/W} \right]$$

where f = Coefficient of friction

μ_o = Absolute viscosity, in centipoises

V_s = Sliding velocity, i p s (see appendix)

V_t = Sum velocity, i p s (see appendix)

W = Specific loading, lbs./in.

The equation breaks down at the pitch point where the sliding velocity is 0.0 and the friction coefficient goes to infinity.

Using a variable coefficient of friction, the flash temperature index formula becomes:

$$T_f = T_b + f \left[\frac{W_{te}}{F_e} \right]^{3/4} \left[\frac{50}{50 - S} \right] \left[Z_t \, (n_p)^{1/2} \right]$$

$$Z_t = \frac{0.2917 \left[\sqrt{e_p} - \sqrt{\dfrac{N_p \, e_g}{N_g}} \right]}{\cos 0_t^{\;3/4} \left[\dfrac{e_p \bullet e_g}{e_p + e_g} \right]^{1/4}}$$

Scoring Criterion Number

A simplified form of the flash temperature index is presented in the *Gear Handbook*, by Darle Dudley published by McGraw-Hill, 1962. A scoring criterion number is defined.

$$\text{Scoring criterion number} = \left(\frac{W_t}{F_e} \right)^{3/4} \bullet \frac{(n_p)^{\;1/2}}{(P_d)^{\;1/4}}$$

where: W_t = tangential driving load, pounds

F_e = contacting face width inches

n_p = pinion RPM

P_d = diametral pitch

Table 1 gives scoring criterion numbers for various oils at various gear blank temperatures.

If the scoring criterion number is above the values shown in the table, a possibility exists that scoring will be encountered. The gear blank temperature can be taken as the average of the oil in and oil out temperatures.

Minimum Film Thickness Criterion

Scoring is a phenomenon that will occur when gears are operating in the boundary lubrication regime[6] rather than with a hydrodynamic or elastohydrodynamic oil film separating the gear teeth. The film thickness can be calculated[7,8,9] and compared to the combined surface roughness of the contacting elements to determine if metal to metal contact is likely to occur. A criterion used to determine the possibility of surface distress is the ratio of film thickness to composite surface roughness:

$$\lambda = h_{min}/\delta$$

$$\delta = \sqrt{\delta_p^2 + \delta_g^2}$$

where: λ = Film parameter

h_{min} = Minimum oil film thickness, in.

δ_p = Pinion average roughness, RMS

δg = Gear average roughness, RMS

The "Partial Elastohydrodynamic" or "mixed" lubrication regime occurs if the film parameter, λ is between approximately 1 and 4. At higher values, full hydrodynamic lubrication is established and asperity contact is negligible. Below a λ of 1.0 there is a risk of surface distress.

Table 1

Critical Scoring Criterion Numbers

Blank Temp., °F	100°	150°	200°	250°	300°
Kind of Oil	Critical Scoring Index Numbers				
AGMA 1	9000	6000	3000		
AGMA 3	11000	8000	5000	2000	
AGMA 5	13000	10000	7000	4000	
AGMA 7	15000	12000	9000	6000	
AGMA 8A	17000	14000	11000	8000	
Grade 1065 Mil-O-6082B	15000	12000	9000	6000	
Grade 1010 Mil-O-6081B	12000	9000	6000	2000	
Synthetic *(Turbo 35)*	*17000*	*14000*	*11000*	*8000*	*5000*
Synthetic Mil-L-7808D	15000	12000	9000	6000	3000

$$\text{Scoring number} = \left(\frac{W_t}{F_e}\right)^{3/4} \bullet \frac{(n_p)\ 1/2}{(P_d)\ 1/4}$$

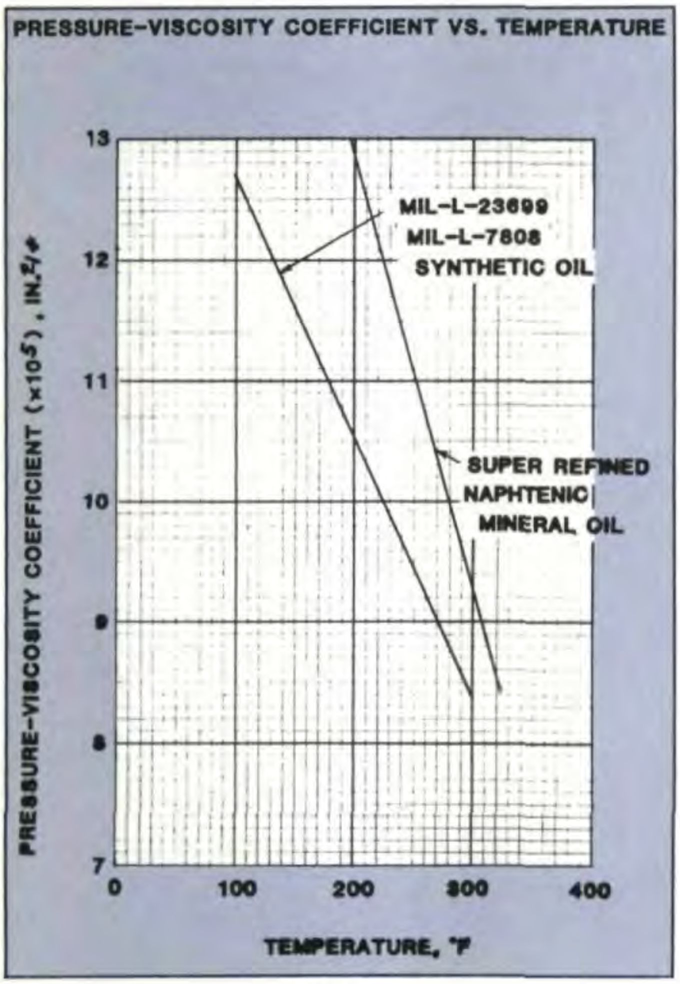

Fig. 6

The minimum elastohydrodynamic film thickness is calculated as follows: (Ref. 7 and 8):

$$H = \frac{2.65(G)^{.54}\ (U)^{.70}}{(W)^{.13}}$$

$$H = \frac{h_o}{R} \text{ (Film thickness parameter)}$$

h_o = Minimum film thickness, in.

R = Equivalent radius, in.

$$R = \frac{e_p \bullet e_g}{e_p \pm e_g}$$

$$\left(\begin{matrix} + \text{ external} \\ - \text{ internal} \end{matrix}\right)$$

e_p = pinion radius of curvature, inches

e_g = gear radius of curvature, inches

G = ∂ E′ (Materials parameter)

∂ = pressure viscosity coefficient, in²/lb (Fig. 6)

$$\frac{1}{E'} = \frac{1}{2}\left[\frac{1-\partial_1^2}{E_1} + \frac{1-\partial_2^2}{E_2}\right]$$

∂_1 = pinion Poisson's ratio

∂_2 = gear Poisson's ratio

E_1 = pinion Young's modulus

E_2 = gear Young's modulus

W = W′/E.R (Load parameter)

W' = Specific loading, lbs/in.

$$U = \frac{\tfrac{1}{2}\ (v_1 + v_2)\ \mu_o}{E'\ R}$$

v_1 = $w_p e_p$ = rolling velocity of pinion at point of contact, in/sec.

v_2 = $w_g e_g$ = rolling velocity of gear at point of contact, in/sec.

w_p = pinion angular velocity, rad/sec.

w_g = gear angular velocity, rad/sec.

μ_o = absolute viscosity, Reyns (lb sec/in²)

μ_o = $\varrho Z_k /\ 6.9\ (10)^6$

ϱ = specific gravity (Fig. 7)

Z_k = kinematic viscosity, centistokes (Fig. 8)

Closure

An extensive survey of aerospace power gears operating with synthetic lubricants at high temperature revealed that calculated oil films were in the order of 0.000010 to 0.000020 inches. With surface roughness in the order of 20 RMS it can be seen that these gears are operating with λ less than 1.0; are in the boundary lubrication regime and, therefore, prone to scoring problems.

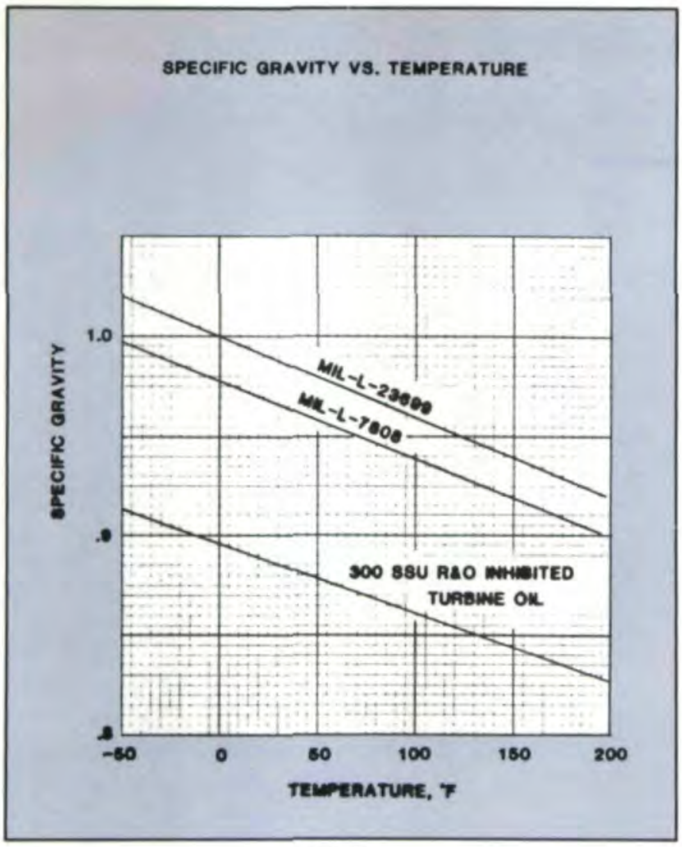

Fig. 7

Table 2

Scoring Analysis of a High-Speed Gearset

Design Parameters

Pinion teeth: 25	Gear teeth: 85
Pinion speed: 12,223 rpm	Horsepower: 281
Face width: 1.0 in.	Helix angle: 0
Diametral pitch: 10	Pressure angle: 25°
Lubricant type: MIL-L-23699 oil	Gear blank temperature: 160°F
Oil viscosity: 1.22 (10^{-6}) lb-sec/in.^{2}	Pressure viscosity coefficient: 11.4 (10^{-5}) in.^{2}/lb
Gear material: SAE 9310 steel	Surface finish: 20 μin. rms

Roll Angle, θ (deg)	Friction Coefficient, f	Flash Temperature Rise, ΔT(°F)*	Flash Temperature Rise, ΔT(°F)†	EHD Film Thickness, h_{min} (μin.)	Film Parameter, λ
15.32	0.0	0	0	0	0.0
16.41	0.014	0	40	22	0.77
17.50	0.018	18	58	21	0.73
18.60	0.021	23	68	20	0.72
19.69	0.023	28	71	20	0.71
20.78	0.025	30	70	20	0.71
21.87	0.027	30	64	20	0.72
22.96	0.030	30	6[illegible]	20	0.71
24.05	0.032	22	42	21	0.73
25.14	0.035	13	25	21	0.75
26.23	0.041	5	7	22	0.76
27.32	0.040	7	9	22	0.78
28.41	0.034	13	25	23	0.80
29.50	0.031	22	41	23	0.81
30.59	0.028	25	52	24	0.84
31.68	0.026	25	59	25	0.88
32.77	0.024	25	62	26	0.91
33.86	0.021	22	61	27	0.96
34.95	0.019	18	57	29	1.01
36.04	0.016	13	48	30	1.08
37.13	0.012	7	33	34	1.19

*f is variable
†f held constant at 0.06

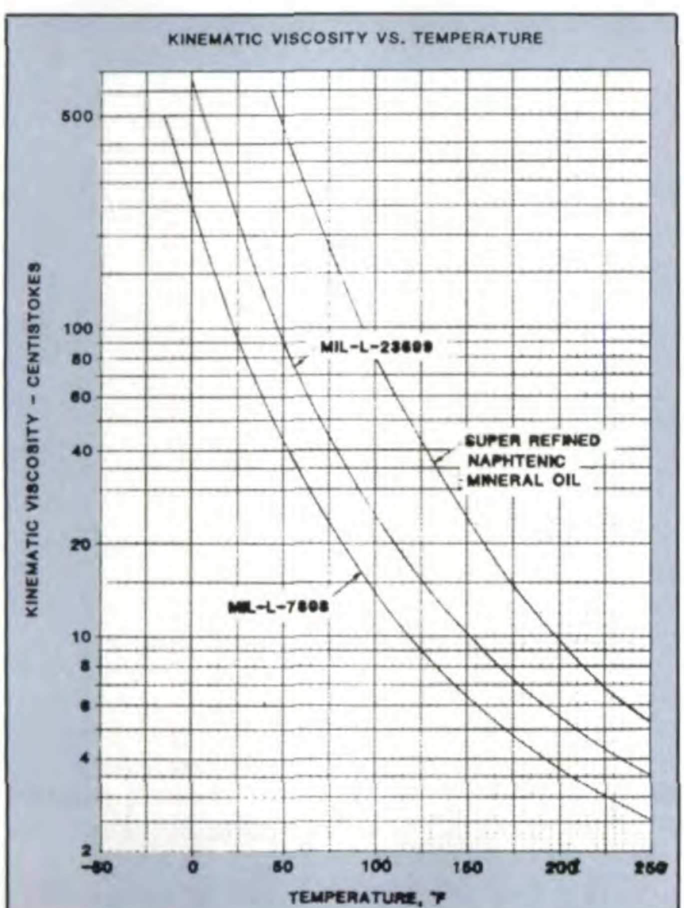

Fig. 8

(continued on page 48)

GEAR TOOTH SCORING . . .
(continued from page 14)

APPENDIX 1

Gearset Geometry for Calculating Flash Temperature Index

Parameters for Calculating Friction Coefficient and Scoring Geometry Factor

$\theta = \tan \phi$

$e_p = r_{bp} \tan \phi$

$e_g = C \sin \phi_t - e_p$

$V_s = \left| \omega_p e_p - \omega_g e_g \right|$

$V_t = \omega_p e_p + \omega_g e_g$

Table 2 shows the results of a computer analysis of a high speed gear set with standard addendums. The flash temperature index is the maximum flash temperature rise, 71°F, plus the gear bulk temperature 160°F. The index of 231°F presents a low scoring risk (Fig. 5) which could be slightly reduced by optimizing tooth proportions. The calculated coefficient of friction is significantly lower than the assumed valued of 0.06 with a corresponding lower flash temperature rise. The calculated minimum film thickness is 0.000020 with a λ term of .71 indicating operation in the boundary lubrication regime.

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This article was reprinted from AGMA Technical Conference 1981, P219, and Gear Drive Systems, by Peter Lynwander, published by Marcel Dekker, 270 Madison Ave., N.Y.

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