Influence of Geometrical Parameters on the Gear Scuffing Criterion – Part 2

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ABSTRACT

In Part 1 several scuffing* criteria were shown ultimately to converge into one criterion, the original flash temperature criterion according to Blok.

In Part 2 it will be shown that all geometric influences may be concentrated in one factor dependent on only four independent parameters, of which the gear ratio u, the number of teeth of the pinion z_1, and the addendum modification coefficient of the pinion x_1 are significant; whereas, the addendum modification coefficient of the wheel x_2 is of minor importance. Again, Blok's flash temperature criterion is confirmed. The low number of significant geometric parameters allows an examination of the influence of different shapes and values of the load sharing factor.

*Scuffing and scoring are synonyms for the same phenomenon. Since scoring may also have another meaning, the ISO Technical Committee 60 decided to apply the word scuffing in the ISO standards.

Determination of the Maximum Contact Temperature

For routine calculations and for the optimization of parameters the maximum contact temperature has to be determined by an iteration process or by a direct approximative expression.

The Equations (1) and (2), shown in Part 1, are rewritten for the maximum value

$$\Theta_{Bmax} = \Theta_M + \Theta_{flmax} \quad (1)$$

$$\Theta_{flmax} = \mu_{mC} X_{top} \frac{W_{Bt}^{3/4} v^{1/2}}{a^{1/4}} \quad (2)$$

where

$$X_{top} = X_M \{X_B X_\Gamma\}_{max} \quad (3)$$

For representative steels the thermal contact coefficient is

$$X_M = 50\ K.N^{-3/4}.s^{1/2}.m^{-1/2}.mm \quad (4)$$

The geometry factor is

$$X_B = 0.51\ (u+1)^{1/2} \frac{\text{abs}\ (\sqrt{1+\Gamma} - \sqrt{1-\Gamma/u})}{(1+\Gamma)^{1/4}(u-\Gamma)^{1/4}} \quad (5)$$

where

$$\Gamma = \frac{\tan \alpha_y}{\tan \alpha'_t} - 1 \quad (6)$$

The load sharing factor X_Γ is one of the trapezoid functions represented in Figs. 1 to 4.

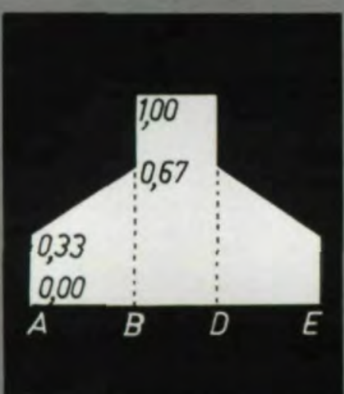

Fig. 1 – Traditional load sharing factor for a gear pair with unmodified tooth profiles.

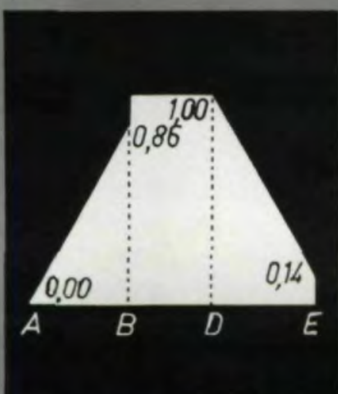

Fig. 2 – Traditional load sharing factor for a gear pair with modified tooth profile, designed for high load capacity if the pinion is driver.

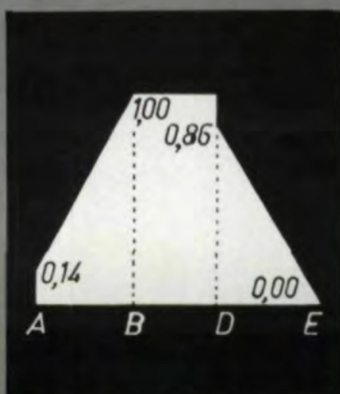

Fig. 3 – Traditional load sharing factor for a gear pair with modified tooth profile, designed for high load capacity if pinion is follower.

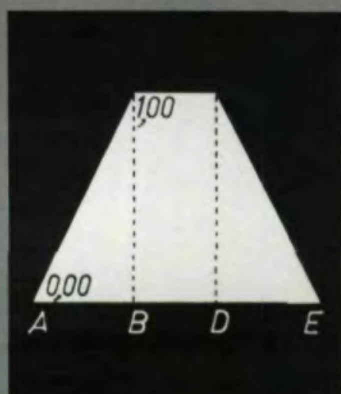

Fig. 4 – Traditional load sharing factor for a gear pair with modified tooth profile designed for smooth meshing.

The bulk temperature may be roughly approximated [1] by

$$\Theta_M \approx \Theta_{oil} + 0.47\ \Theta_{flmax} \text{ for sump lubrication} \quad (7)$$

$$\Theta_M \approx 1.2\ \Theta_{oil} + 0.56\ \Theta_{flmax} \text{ for jet lubrication} \quad (8)$$

The safety factor may be defined [1] by

$$S_B = \frac{\Theta_S - \Theta_{oil}}{\Theta_{Bmax} - \Theta_{oil}} \quad (9)$$

The iteration process for the maximum flash temperature depends solely on geometric influence factors which are concentrated in one single form factor. See Equation (3). That form factor depends on four parameters, u, z_1, x_1, x_2, of the seven independent parameters mentioned in Table 2. To examine the dependency on the four independent parameters, full calculations were made for 968 combinations. See Table 1.

Table 1 Parameters selected to cover the geometric field of application.

u	z_1	x_1	x_2
1	40 50 63 80 100 125 160	0,00 0,10 0,20 (z_1>16)	−0,20
1,6	32 40 50 63 80 100 125	------------------------	0,00
2,5	25 32 40 50 63 80 100	0,30 0,40 0,50 0,60	0,20
4	20 25 32 40 50 63 80	------------------------	0,40
6,3	16 20 25 32 40 50 63	------------------------	--------

(Symbols, terms and units chosen in accordance with the international standard)

- a center distance (mm)
- A point of path of contact at tip of wheel
- b facewidth (mm)
- B lower point of transverse single contact
- C pitch point
- C_2 weight factor (value 1,5)
- D upper point of transverse single contact
- E point of path of contact at tip of pinion
- F_t tangential force at reference circle (N)
- GAM parameter on the line of action
- GAMA parameter on the line of action at point A
- GAMAB parameter on the line of action between A and B
- GAMB parameter on the line of action at point B
- GAMD parameter on the line of action at point D
- GAME parameter on the line of action at point E
- GAMED parameter on the line of action between E and D
- S_B safety factor, Equation (10)

NOMENCLATURE

- TAA1 tangents of transverse tip pressure angle of pinion
- TAA2 tangents of transverse tip pressure angle of wheel
- TAT tangents of transverse working pressure angle
- trapez number corresponding with figure number of
- v pitch line velocity (m/s)
- W_{Bt} specific tooth load[1]
- x_1 addendum modification coefficient of pinion
- x_2 addendum modification coefficient of wheel
- X_B geometry factor, Equation (6)
- X_{BE} geometry factor at point E
- X_{Ca} tip relief factor [1]
- XGAM load sharing factor
- X_M thermal contact coefficient, Equation (4) ($K.N^{-3/4}.s^{1/2}.m^{-1/2}.mm$)
- X_Q approach factor [1]
- X_{top} form factor, Equation (3), (1) ($K.N^{-3/4}.s^{1/2}.m^{-1/2}.mm$)
- X_ϵ contact ratio factor, [1]
- X_Γ load sharing factor, Figs. 1 to 4
- z_1 number of teeth of pinion
- α_t transverse working pressure angle
- α_y pressure angle of arbitrary point
- β helix angle
- Γ linear parameter on line of action, Equation (6)
- Θ_B contact temperature, Equation (1) (°C), Part 1
- Θ_{Bmax} maximum contact temperature, Equation (1), Part 2 (°C)
- Θ_{fl} flash temperature, Equation (2) (°C), Part 1
- Θ_{flaint} approximated mean value of the flash temperature, Equation (4) (°C), Part 1
- Θ_{flmax} maximum flash temperature, Equation (2), Part 2 (°C)
- Θ_{int} integral temperature, Equation (3) (°C), Part 1
- Θ_M bulk temperature, Equation (8), (9) (°C)
- Θ_{oil} oil temperature (°C)
- Θ_S scoring temperature[1] (°C)
- μ_{mC} mean coefficient of friction at pitch point[1]
- μ_{my} mean local coefficient of friction[1]
- π product of factors in comparison, Equation (7), Part 1

The essential part of the program, consisting of four procedures written in PASCAL, is presented in Table 2. The form factor X_{top} is the largest of four possible extreme values situated in point B, point D, located between A and B (marked by the parameter GAMAB), and located between D and E (marked by the parameter GAMED). (See Fig. 5)

$$X_{top} = \text{largest of} \begin{cases} \text{FMAX (GAMA, GAMB, GAMAB)} \\ \text{FLASHFACTOR (GAMB,trapez)} \\ \text{FLASHFACTOR (GAMD,trapez)} \\ \text{FMAX (GAME, GAMD, GAMED)} \end{cases} \quad (10)$$

The examination of the form factor produced two different conclusions; one about the confirmation of the flash temperature concept and the other about the influence of the load sharing factor.

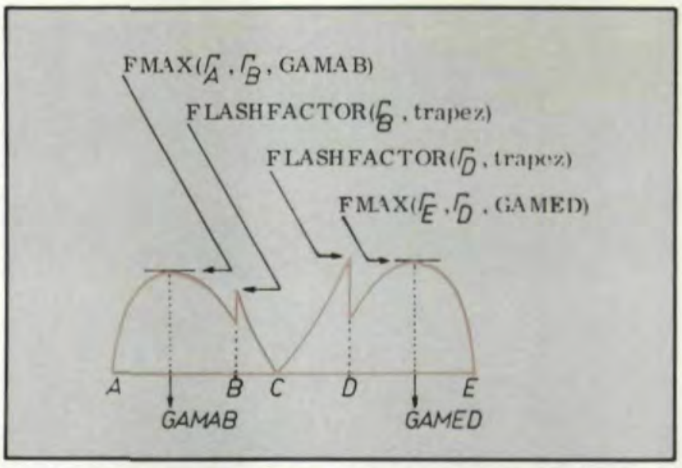

Fig. 5 – The form factor is the largest of four possible extreme values of the product $X_M X_B X_\Gamma$.

Table 2 Procedures for the determination of the maximum value of the product of thermal contact factor, geometric factor and load sharing factor.

```
procedure GAMMAPARAMETERS;
begin TAA1:=sqrt (sqr((1+2*(1+X1)/Z1)/0.93969262)-1);
      TAA2:=sqrt(sqr((1+2*(1+X2)/Z2)/0.93969262) -1);
      GAMA:=-Z2*(TAA2/TAW-1)/Z1; GAME:=TAA1/TAW-1;
      GAMD:=GAMA+6.283185/Z1/TAW; GAMB:=GAME-6.283185/Z1/TAW
end;

function FLASHFACTOR(GAM:real; trapez:integer):real;
var Q1,Q2:real;
begin U:=Z2/Z1; Q1:=1+GAM; Q2:=1-GAM/U; XGAM:=1;
  XB:=0.51*sqrt((U+1)/sqrt(Q1*Q2*U))*abs(sqrt(Q1) -sqrt (Q2));
  case trapez of
      1:begin Q1:=1/3; Q2:=1/3 end;
      2:begin if GAM<0 then Q1:=0 else Q1:=1/7; Q2:=6/7 end;
      3:begin if GAM>0 the Q1:=0 else Q1:=1/7; Q2:=6/7 end;
      4:begin Q1:=0; Q2:=1 end end;
  if GAM<GAMB then XGAM:=Q1+Q2*(GAM-GAMA)/(GAMB- GAMA);
  if GAM>GAMD then XGAM:=Q1+Q2*(GAME-GAM)/(GAME/-GAMD);
  FLASHFACTOR:=50*XB*XGAM
end;

function F(GAM:real):real;
begin F:=FLASHFACTOR(GAM,trapez)
end;

function FMAX(GA, GB:real; var G:real):real;
var A,B,C,D,FA,FB,FC,FD:real;
begin A:=GA; if GA<GB then B:=GB-0.01 else B:=GB+0.01; C:=A;
      FA:=F(A); FB:=F(B); FC:=FA;
      if FA>FB then begin C:=B; FC:=FB; B:=A; FB:=FA end;
      repeat A:=C; C:=B+0.382*(A-B); G:=B;
             FA:=FC; FC:=F(C); FMAX:=FB
      until (abs(B-A)<0.01) or (FB<FC); if FB<FC then
      repeat D:=C+0.382*(A-C); FD:=F(D);
         if FC>FD then begin A:=B; FA:=FB; B:=D; FB:=FD end
                   else begin B:=C; FB:=FC; C:=D; FC:=FD end;
         G:=C; FMAX:=FC
      until abs(B-A)<=0.01
end;
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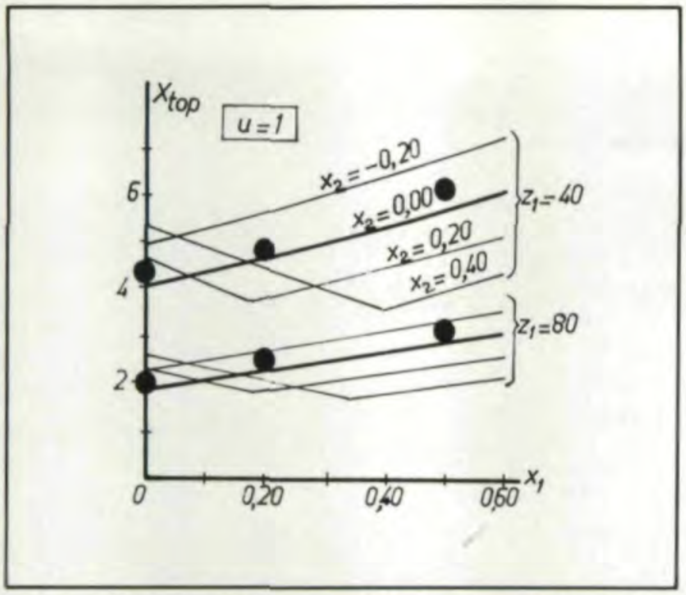

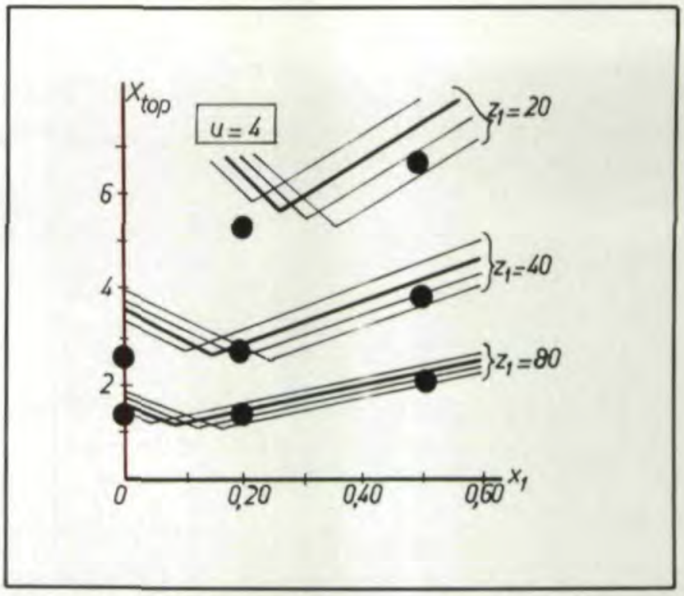

Figs. 6, 7 – Comparison of the form factor for the flash temperature with a similar one for the integral temperature (heavy dots).

Confirmation of the Flash Temperature Forumla

The curves of the form factor , Equation (3), for the determination of the maximum flash temperature coincide very well with similar curves for a quick approximation of the integral temperature, proposed by Hirt.[2] See Figs. 6 and 7. This confirms again that the integral temperature constitutes an approximation of the maximum contact temperature, in spite of all attempts to create a criterion with a different physical interpretation.

The empirical values characteristic of the integral temperature formula were determined by numerous tests. Hence, the integral temperature formula may be understood as a concise representation of gear tests, confirming the flash temperature criterion and, in addition, replacing the original tests of Blok, which were lost in May, 1940.

It is gratifying that the applicability of the flash temperature criterion for gears is confirmed in an unintentional way, leading to one criterion based upon the flash temperature concept, enriched with several practical influence factors gained in numerous gear tests and by field experience. See Fig. 8.

Field of Application

Without exception, the results of tests expressed in integral temperature can be applied to the flash temperature criterion. Similarly, if the flash temperature criterion is not applicable to a certain case, then the integral temperature criterion is not valid either.

The field of application of the flash temperature criterion covers straight and mild-extreme-pressure mineral oils, and, to a certain extent, extreme-pressure oils and perhaps synthetic oils. The theoretical basis of the flash temperature concept provides a boundary for its field of application, which prevents a dangerous situation similar to predicting pitting with a bending strength formula. Most likely, there are different physical causes for scuffing or related phenomena, and in the future additional criteria may be needed.

Diagrams of the Form Factor

Among the four parameters determining the shape and value of the form factor, the addendum modification coefficient of the wheel x_2 has been shown to be of less importance than the three other parameters u, z_1, and x_1. The curves shift to approximately the same curve if the ordinate x_1 is replaced by $x_1 + (1{,}4/u)x_2$. See Figs. 6 and 7. Hence, the diagrams in Figs. 9 to 13, being exact for $x_2 = 0$, may be applied for any x_2. Moreover, the diagrams in Figs. 9 to 14 include an indication of the location of the maximum contact temperature on the path of contact.

The drastic reduction of several complicated geometric influence factors to only three parameters presents a convenient view on the consequences of the selection of geometric quantities. It may be helpful for the designer to achieve optimization.[3]

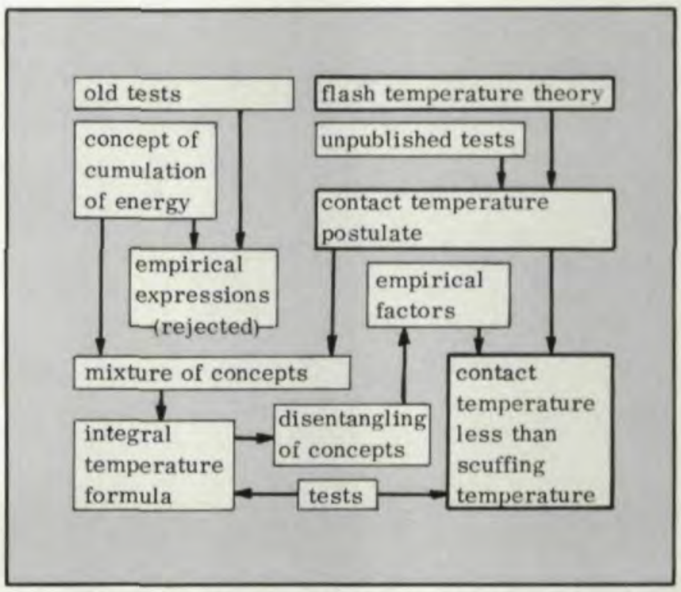

Fig. 8 – Development of the gear scuffing criterion.

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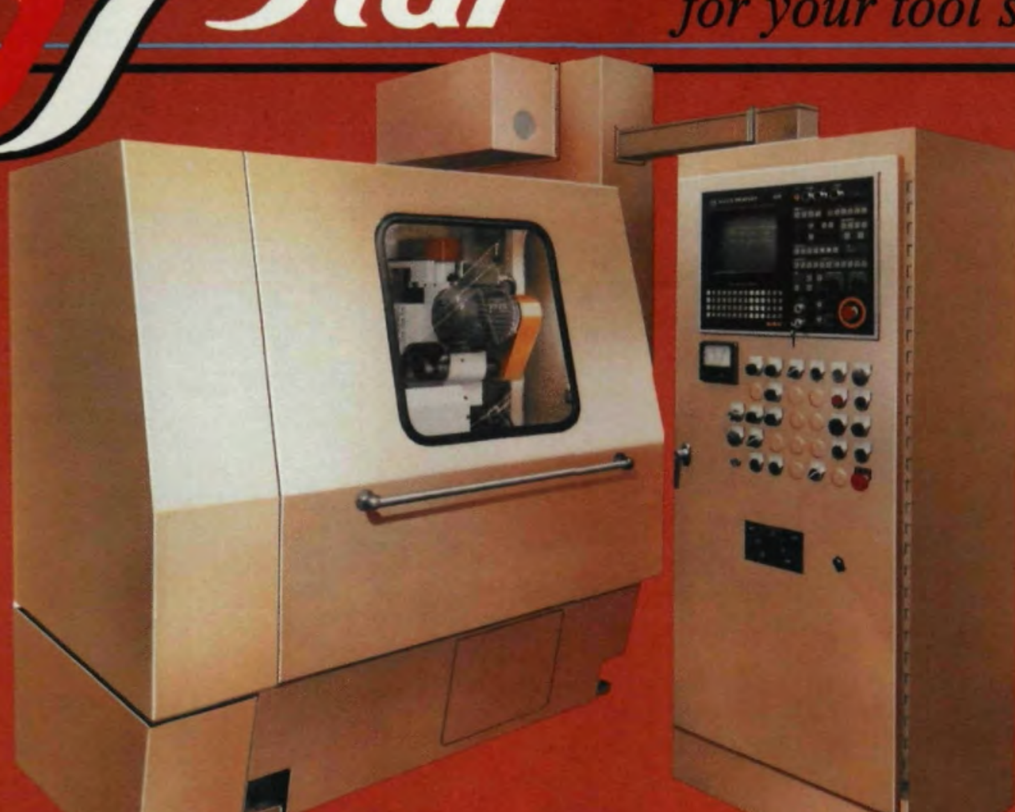

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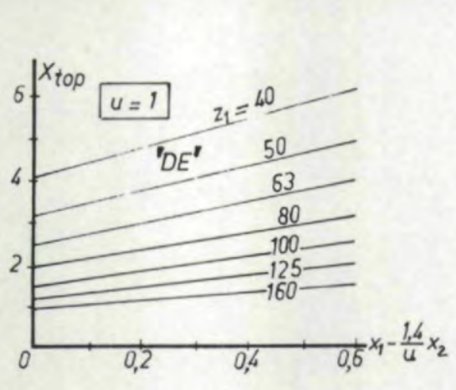


Fig. 9

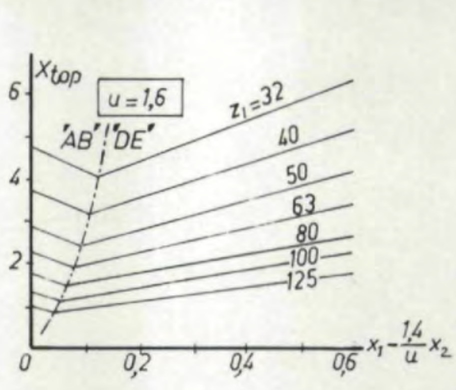


Fig. 12

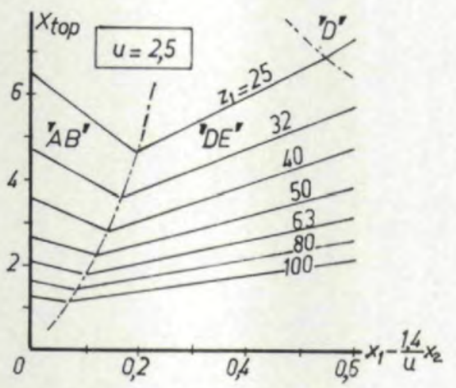


Fig. 10

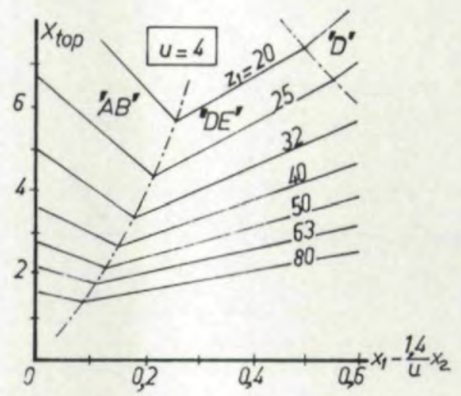


Fig. 13

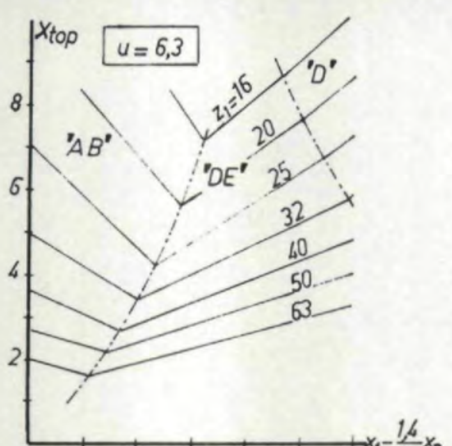
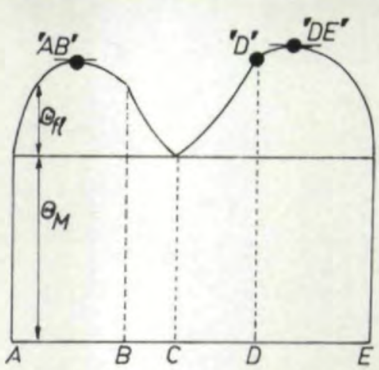


Fig. 11



Maximum contact temperature. For low values of x_1, the greatest maximum is between A an B, and X_{top} decreases with x_1. For larger values of x_1, the greatest maximum is between D and E, and X_{top} increases with x_1. The greatest maximum is at point D for largest values of x_1.

Fig. 14

Figs. 9-13 – Form factor X_{top} for the maximum flash temperature, valid for the load sharing factors for smooth meshing.

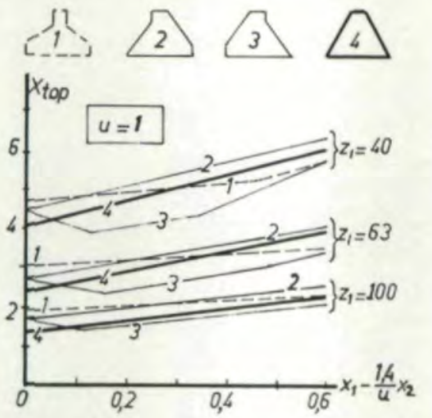


Fig. 15

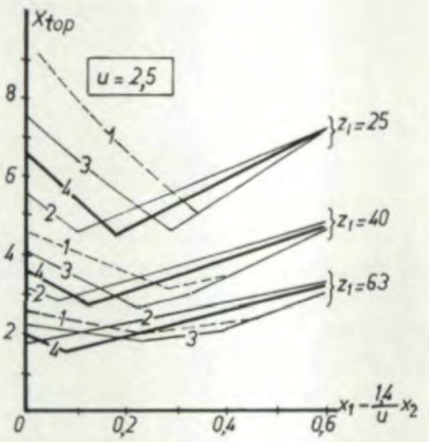


Fig. 16

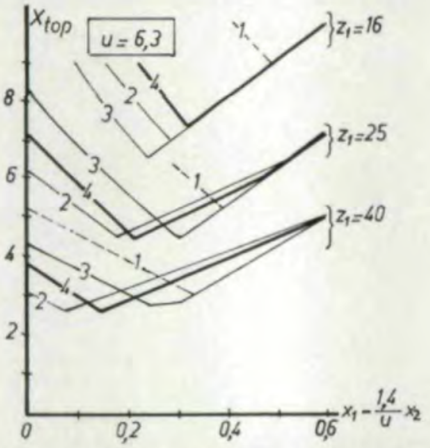


Fig. 17

Figs. 15-17 – Form factor X_{top} for the maximum flash temperature, valid for different load sharing factors, labeled with their figure numbers 1 to 4.

Another more or less unexpected result is shown in Figs. 15 to 17. The traditional load sharing factors, with distinct differences in shape and value, produce approximately the same form factors. See Figs. 1 to 4.

Conclusions

1. The scuffing (scoring) criterion is the flash temperature criterion according to Blok. It is based on the maximum contact temperature, being the sum of the bulk temperature and the maximum flash temperature. Enriched with some influence factors, it is presented in an international standard.
2. Several attempts to find a different criterion ultimately confirmed the applicability of the flash temperature criterion for engineering practice. The integral temperature, also presented in the same international standard, proved to be an approximation of the contact temperature.
3. For engineering practice the postulate of the maximum contact temperature (flash temperature criterion) was confirmed by numerous tests on gears, the results of which were expressed as a single value called integral temperature. All test results expressed in integral temperature data are fully applicable to the flash temperature criterion.
4. All geometrical influences can be concentrated in one form factor of the flash temperature formula. Diagrams of the form factor include an indication of the location of the maximum contact temperature on the path of contact. Such diagrams can be helpful for optimization of the design.
5. Different variants of traditional load sharing factors show nearly the same influence on the value of the flash temperature.
6. The theoretical basis of the flash temperature concept provides a basis for its field of application, preventing a misuse. To cover different physical causes for scuffing or related phenomena, additional criteria may be needed.

References:

1. ISO 6336/1: Introduction and general influence factors.
   ISO 6336/2: Calculation of surface durability (pitting).
   ISO 6336/3: Calculation of root stress (tooth breakage).
   ISO 6336/4: Calculation of scuffing load capacity.
   ISO 6336/5: Material qualities and endurance limits.
   In May, 1985, the above parts were approved by ISO/TC 60 to be published as Draft International Standards. These lengthy basic documents will be followed by simplified parts for several fields of application.
2. **HIRT, M.** "An Improved Method to Determine the Scoring Resistance of High Power Speed Gearing." Proc. 13th Turbomachinery Symposium.
3. **MENG, Huirong, LIU, Chong, CHEN, Qitai.** "Research Into the Increase of Scuffing Load Capacity by Optimization of Surface Flash Temperature of Gear Teeth." 2nd World Congress on Gearing, Paris, 1986.

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