Computer-Aided Design of the Stress Analysis of an Internal Spur Gear

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Abstract:

Internal gearing is an important element of epicyclic gear trains. However, specific design methodology has not been adopted and the procedure for designing external gears is often employed for internal gear design. Unfortunately, internal gears differ from external gears in tooth shape and rim support, thereby complicating the stresses at critical points. The intent of the present article is to present design concepts that will accurately model the stresses in internal gears. The article employs a finite element method to investigate the effects of different parameters on the stresses of the gear. Finally, a simulation of the stress fluctuations when two gears mesh is presented.

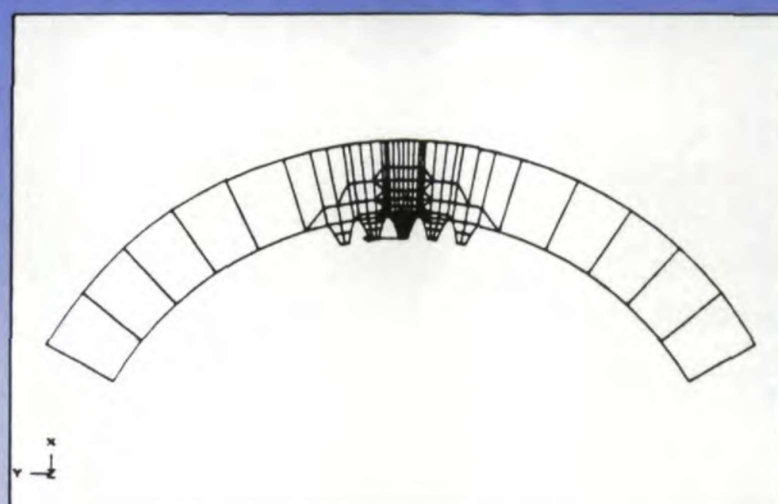

Fig. 1 – FEM model of the internal gear.

Introduction

Although there is plenty of information and data on the determination of geometry factors and bending strength of external gear teeth, the computation methods regarding internal gear design are less accessible. Most of today's designs adopt the formulas for external gears and incorporate some kind of correction factors for internal gears. However, this design method is only an approximation because of the differences between internal gears and external gears. Indeed, the tooth shape of internal gears is different from that of external gears. One has a concave curve, while the other has a convex curve. Usually, internal gear teeth are thick at their base, and tooth height is short. These two characteristics make the use of the Lewis beam model for the calculation of the stresses improper, because the beam model assumes the gear tooth is a long cantilever beam, and the bending stress at the root is the dominant factor. However, for internal gears the stress distribution around the root fillets is very complex. Factors such as rim thickness, rim support conditions, root fillet radius and loading positions, must all be taken into account when accurate stress estimation of internal gears is required. Therefore, in order to meet the demand of today's high performance gear design, more information on the effects of those parameters and a more useful design concept with better numerical results should be investigated.

The work presented here provides more information about the behavior of internal spur gears under specific situations. Those areas examined were:

a. The effects of rim thickness on the root fillet stress and the whole stress distribution,
b. The effects of fillet radius on root fillet stresses,
c. The effects of loading position,

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DR. DENNIS A. GUENTHER *is an Associate Professor of Mechanical Engineering at The Ohio State University where he has served since receiving his Ph.D. in 1974. He has done extensive work in the area of failure analysis and identification of failure mode, having investigated and analyzed virtually all*

d. The effects of support condition,
e. The effects of different pressure angles on the root fillet stresses.

Analysis

In the past decade, the finite element method (FEM) has been heavily used in the stress analysis of gears and has shown promising results. Drago and Pizzigati[1] have done extensive research of gear stress problems using finite element methods and have found that FEM is a good approach to investigating the rim effect on tooth fillet stresses. Moreover, it can also be used to establish stress history by subsequently changing tooth contact loading conditions for a number of points along the line of action. In this way, a quasidynamic simulation can be obtained; also the mean and alternating stresses can be calculated for each section and the critical section in fatigue can be determined. The two dimensional FEM is relatively simple to apply and is satisfactory for thin and flat gears. Drago[1] showed that his FEM results were about 12% higher than the calculations reported by AGMA.[2-5]

In the present study, the internal spur gear has been transformed into a finite element model as shown in Figs. 1 and 2.

To reduce computational costs, only a section of the gear is modeled and only the teeth in the vicinity of the loaded tooth are included. This model employed two kinds of elements, the first being a thin shell parabolic triangle, and the second, a thin shell parabolic quadratic element with the uniform thickness of 2.54 cm (1"). Basically, the models of various cases have about 800 nodes and 250 elements. In all, a total of 79 cases were tested with 162 to 261 nodes and 559 to 869 elements for the different cases. Additional detail is given by Hwang.[6]

The finite element analysis was carried out through the computer facility at the Advanced Design Method Laboratory (ADML), in the Department of Mechanical Engineering of The Ohio State University. The establishment of the FEM model and finite element calculation of this research were conducted through the software package, I-DEAS, running on DEC VAX 11/750 computers.

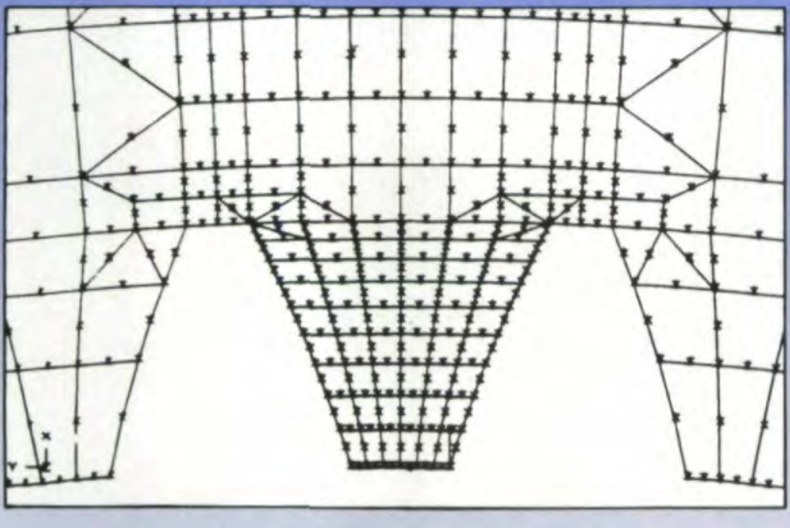

Fig. 2—FEM model of the internal gear.

phases of machinery failures. His research and teaching efforts in recent years have centered on system dynamics, design of machine elements, computer aided design, vehicle dynamics, and accident reconstruction. His interest in gear design has grown out of involvement with teaching machine design and evaluation of gear failures.

DR. JOHN F. WIECHEL *is a project engineer with S.E.A., Inc. in Columbus, Ohio. He received his B.S. and M.S. degrees in mechanical engineering from Purdue University and a Ph.D., from The Ohio State University. Reconstruction of vehicular and machinery accidents constitutes the majority of his current work with a focus on locating, identifying, and analyzing failed machine components and evaluation of the potential of the failure to produce injury.*

Because of the nature of the finite element method, the accuracy of the results depends very much on the mesh design of the model. Based on the study of Drago,[1] very coarse meshes will give poor results around the tooth fillet area. The best results can be derived only through several experiments of the element size and the aspect ratio of the elements used around the fillet area.

In order to assure that the mesh design and element type used can yield accurate results, verification of the model must be conducted. This model verification was carried out through the comparison of the FEM calculation in this research and the FEM and experimental results of Aida.[7] Aida's FEM model employed 1251 nodes and 2122 elements based upon a thin shell linear triangular element. He also conducted photoelasticity experiments for the verification of his model. The finite element models of this study were then established and based upon the same gear dimensions specified by Aida.

It should be noted that since the load is applied at the tooth tip and the tip is far away from the tooth root, the effect of the point load singularity upon the root fillets is minimal and can be neglected. Therefore, in the FEM model, the mesh surrounding the point load region was not modified. At the root fillet areas, the element density has significant influence on the accuracy of the results. Type, size and aspect ratio of the elements in these areas should be carefully examined. Within this study, the FEM model was established to reflect the general trends of the stress responses corresponding to different parameters of internal spur gears. The mesh and the elements used are shown in Fig. 2. This model derived the fillet stress data in the average sense over the root regions similar to the data based upon strain gage measurements. The boundary condition imposed upon this model is that the section is fixed at both ends.

The results of two FEM analyses and experimental data are presented in Fig. 3. They indicate that the results of the tensile root fillet were better than Aida's model, and the error is about 13%. However, the compressive root fillet gave worse results than that of Aida's model, but the average error was still

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13%. Therefore, the overall performance of the model can be regarded as satisfactory, and the same mesh design as well as the element types can be applied to other cases. This comparison has laid the foundation of the FEM model employed for the parametric study presented here.

Effects of Rim Thickness. Usually internal gears have their rim thickness relatively thinner than that of external gears and, unfortunately, the rim thickness does affect the total stress distribution. Therefore, it is worthwhile to study its influences. Fig. 4 presents the FEM results of the minimum and maximum principle stresses at root fillets of the internal spur gear under the influence of different rim thickness for a pressure angle of 20°. As the rim thickness becomes thinner, the stresses at the tensile root stay nearly the same or go up a little; however, the compressive root increases its stress significantly. This effect is especially apparent when the rim thickness is less than 4/P (P=5 in the current study). If the rim is thick, the stresses at the root fillet converge to specific values.

The explanation is that when the rim is thin, the whole rim is not rigid enough. Therefore, the rim deforms as shown in Fig. 5, and this creates high tensile stresses at the outside fiber of the rim and high compressive stress at the inner fiber of the rim (which is around the root fillet areas of teeth). At the tensile root, the tensile stress due to tooth bending is cancelled by the compressive stress generated by rim bending. However, if the rim is thick, it is very hard to bend because of its high rigidity. The whole model can be described as similar to a cantilever beam, and the root fillet stresses approach constant values. Fig. 6 shows the corresponding Von Mises stresses of the case presented in Fig. 4. Figs. 7 through 9 are the maximum principle stresss distribution within gears. Figs. 10 through 12 shows the minimum principle stress distribution. Little difference was found in stress distributions for rim thicknesses of 8/P and 6/P. The location of maximum principle stress at root fillets will tend to move to the bottom of the tooth space apart from the loaded tooth as the rim becomes thinner.

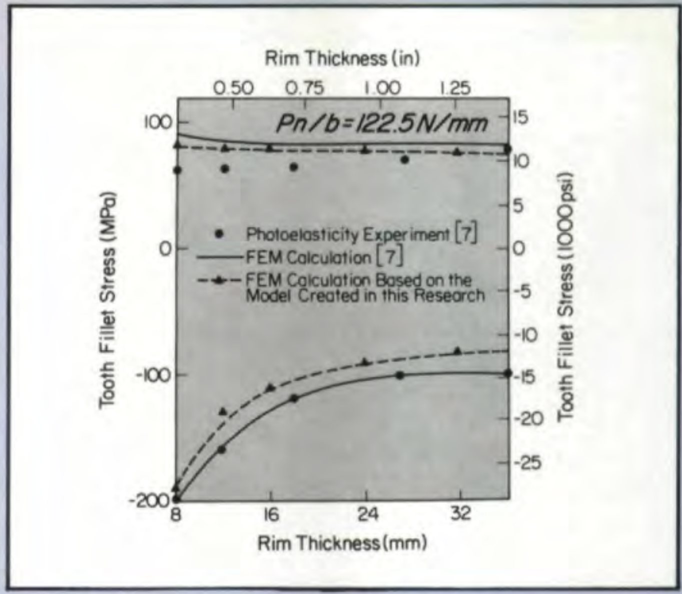


Fig. 3—Comparison of the results based on FEM calculations of different mesh designs and experiment.

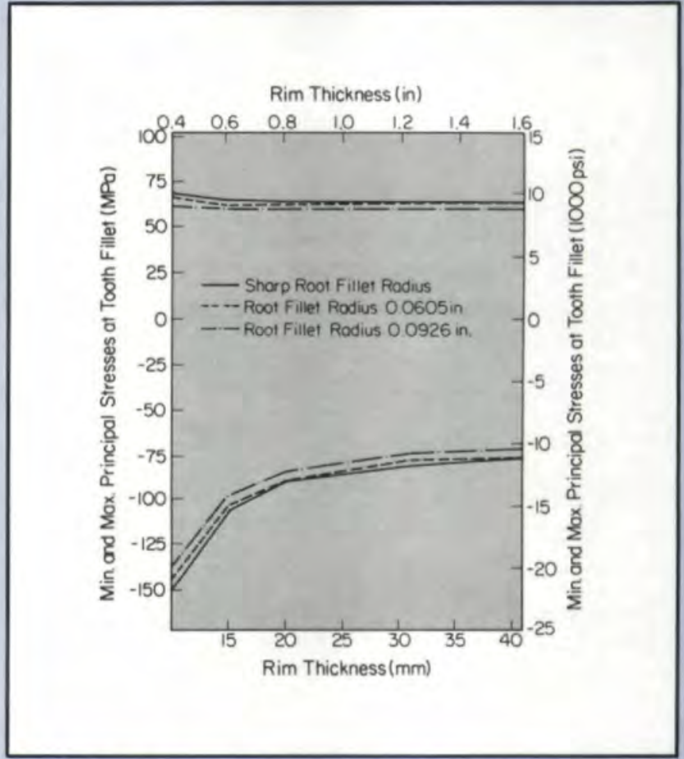


Fig. 4—Root fillet principal stress of internal gears ($\phi = 20$) with different rim thickness and fillet radii.

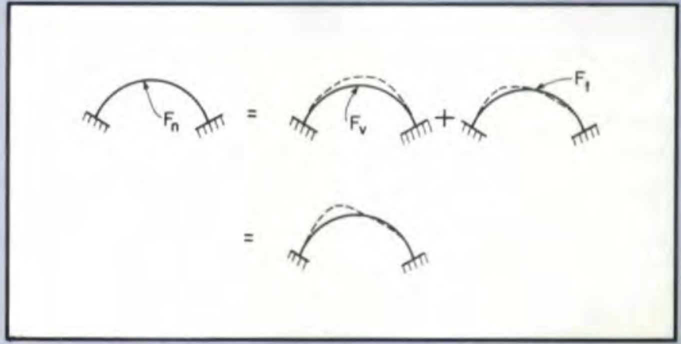


Fig. 5—Schematic illustration of rim bending model.

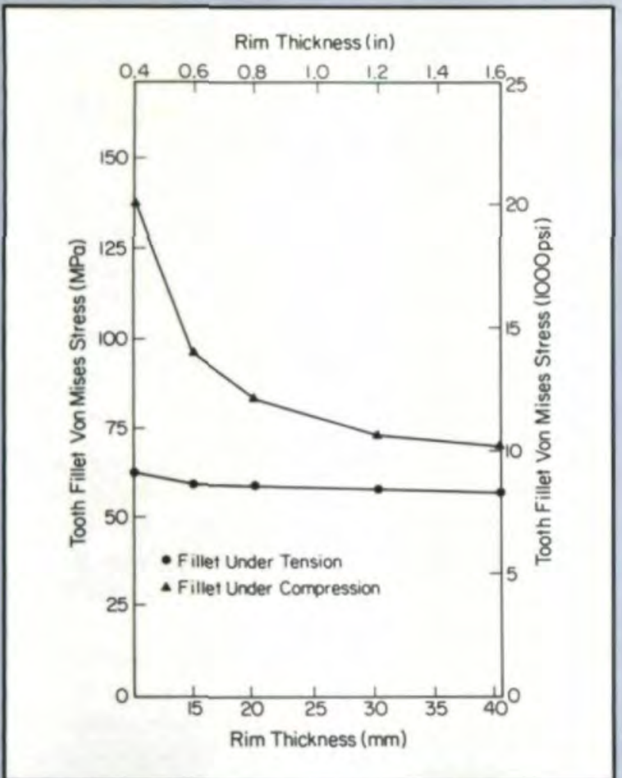


Fig. 6 – Root fillet Von Mises stresses of internal gears (ϕ = 20) with different rim thickness and fillet radii.

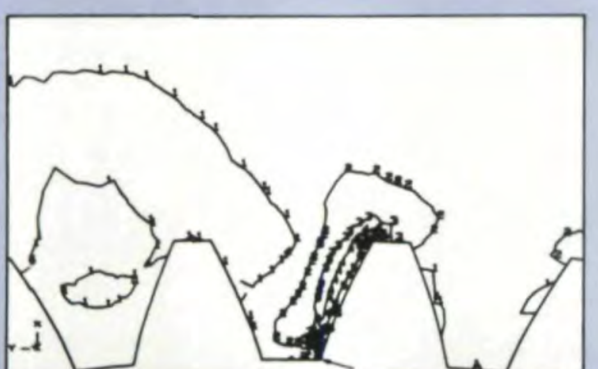


Fig. 7 – Maximum principal stress distribution of the internal gear (ϕ = 20, rim thickness = 6/P).

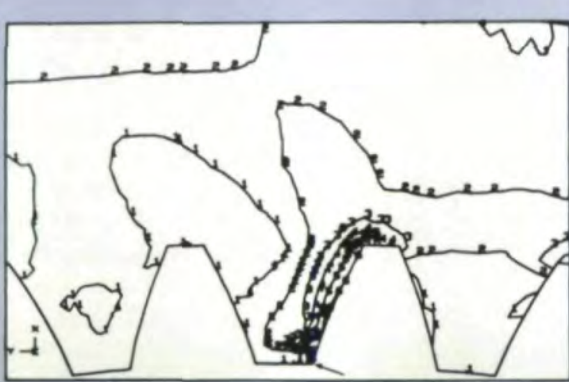


Fig. 8 – Maximum principal stress distribution of the internal gear (ϕ = 20, rim thickness = 4/P).

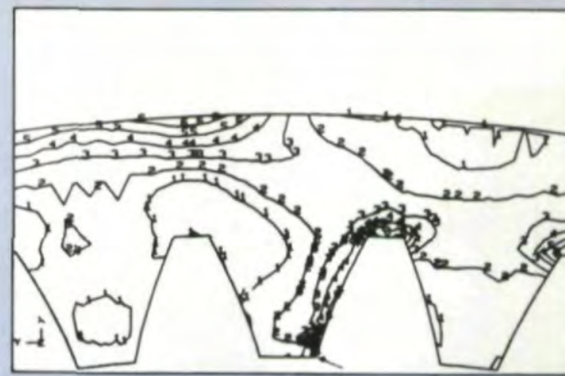


Fig. 9 – Maximum principal stress distribution of the internal gear (ϕ = 20), rim thickness = 2/P).

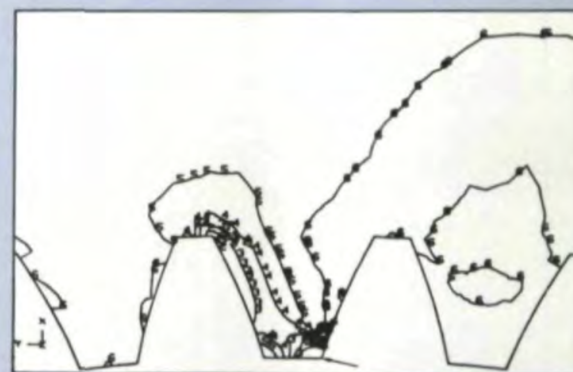


Fig. 10 – Minimum principal stress distribution of the internal gear (ϕ = 20, rim thickness = 6/P).

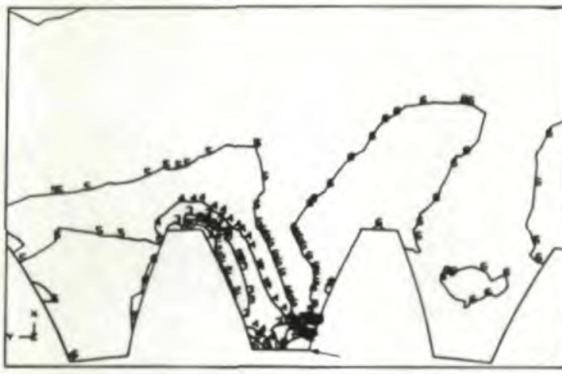
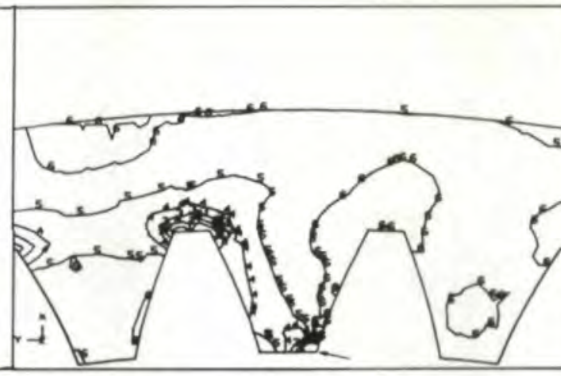


Fig. 11 (Left) – Minimum principal stress distribution of the internal gear (ϕ = 20, rim thickness = 4/P).

Fig. 12 (Right) – Minimum principal stress distribution of the internal gear (ϕ = 20, rim thickness = 2/P).



As the rim becomes thinner, the neighboring teeth are also subject to higher stresses. In other words, due to the severe bending of the rim, high stresses are built up within the rim near the loaded tooth. Under such situations, the maximum stress or the critical area may be located in the rim. The deformation of the loaded tooth and the whole rim is shown in Figs. 13 and 14 (rim thickness = 8/P), and Figs. 15 and 16 (rim thickness = 2/P). The model should contain more teeth in order to get better results for the case of a very thin rim.

Effects of Root Fillet Radius. Root fillet radius is the major factor that determines the extent of local stress concentration and is the deciding factor for the local stress magnitude. For external gears, a larger fillet radius has a local stress level smaller than that of a small fillet radius. The effect is very significant. However, in this research, the effect of fillet radius showed the same trend, but was not as pronounced because of the mesh design at the fillet areas. More accurate and detailed information should be gathered based upon comprehensive FEM modelling together with experimental verification. Aspect ratio and element density will be the key factors in this effort.

Effects of Loading Positions. When two gears mesh, the contact point will follow the path of the line of action and for each tooth in mesh, the force transmitted will be normal to the tooth surface and subsequently move along the involute profile. A simulation of the tooth stress variation can be made if subsequent loads at different positions on the tooth profile are applied to the model, and the fillet stress is calculated by FEM. This study has selected five positions along the profile to carry out this simulation as indicated in Fig. 17. At all positions, the tangential component of the

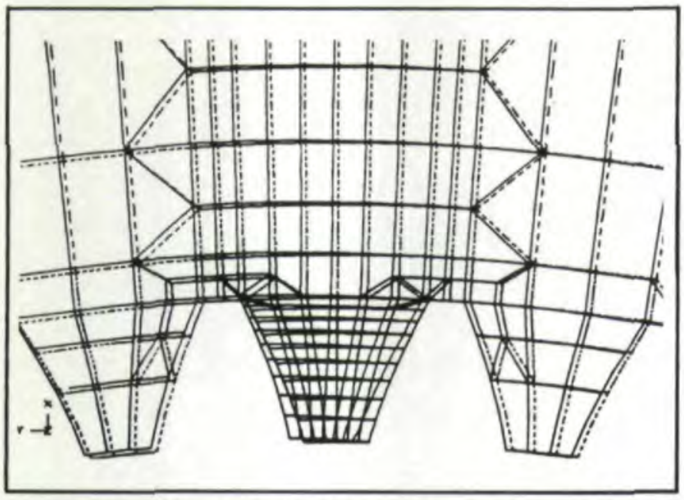


Fig. 13 – Deformation of teeth ($\phi=20$, rim thickness = 8/P).

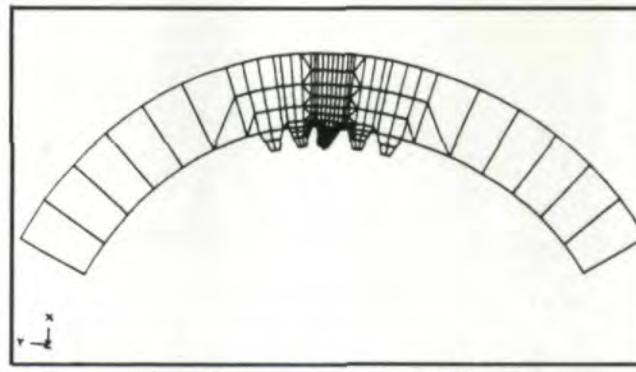


Fig. 14 – Deformation of rim (ϕ = 20, rim thickness = 8/P).

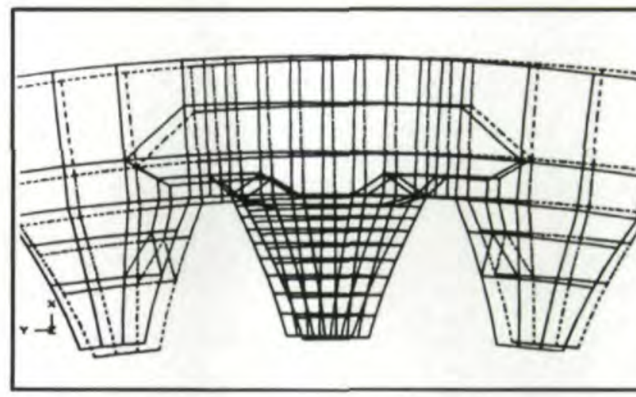


Fig. 15 – Deformation of teeth ($\phi=20$, rim thickness = 2/P).

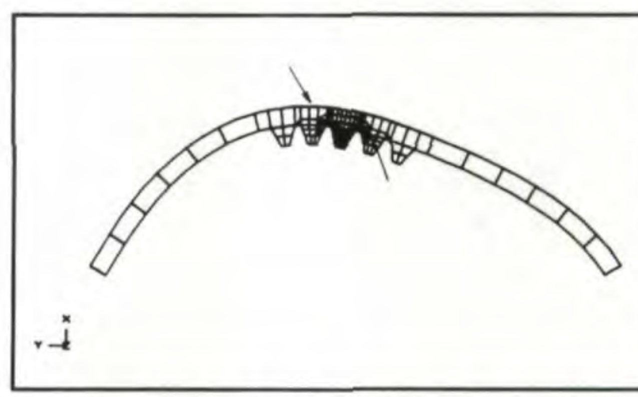


Fig. 16 – Deformation of rim (ϕ = 20, rim thickness = 2/P).

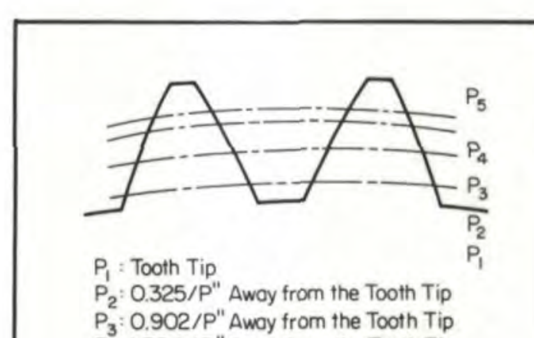


Fig. 17 – Different loading position on a tooth.

load remains constant. Fig. 18 demonstrates the effect of different loading positions. As those curves show, the variation of the stresses at both tensile and compressive fillets is essentially linear unless the rim thickness is very thin. The explanation follows.

The stress due to tooth bending is linearly proportional to the bending moment. In this study, the tangential force component, which is the main factor determining the bending moment, stays the same; therefore, the bending stress is directly proportional to the moment arm. The moment arm is the distance between the tooth root and the location where load is applied. A linear relationship is thus expected.

Rim bending also makes a contribution to resultant stresses, but unless the rim thickness is thin, the effect is very limited. For thin rim cases, the deformation of rim and the rim bending stresses are very sensitive to the applied load. Some nonlinear variation due to the complicated rim deformation occurs, and it also nonlinearizes the variation of the resultant stresses. Figs. 19 and 20 show the fillet Von Mises stress for the cases in Fig. 18. Figs. 21 and 22 show the maximum principle stress distribution of the gear loaded at different positions. The stress patterns in the rim and neighboring teeth are the same except for the difference in magnitude. However, the stress distribution within the loaded tooth changes acording to the loading positions. The location of the maximum stress behaves the same way as the rim thickness effect does; namely, as the loading position approaches the root fillet, the point of maximum stress shifts its location to the bottom of the tooth space apart from the loaded tooth. The loading position effect also shows its influence the same way for those cases with different support conditions.

Effects of Support Condition. The support condition for a section of an internal gear will be dependent upon how many bolts are used in mounting the gear. The model used within this research is a section spanning two neighboring bolts cut out from the whole gear. At both ends, the boundary condition is assumed totally fixed. Four sections spanned by 60°, 80°, 100°, and 120° were analyzed. The variation of the support condition showed no obvious effect on the fillet stress. However, the variation of the support condition does have significant influence on thin rim gears.

If the span is small, the rim is rigid and, consequently, the fillet stress will be small. As the span increases, the rim becomes more flexible and the fillet stress will then increase accordingly. Especially for the compressive fillet as shown in Figs. 23 and 24, the stress magnitude may increase by 30%. The reason is that if the rim is thick enough, the whole rim is very rigid and the loaded tooth is very similar to a cantilever beam. Even though the support condition changes, the stress does not increase appreciably unless the total span between two ends is very large. For thin rim cases, the rim deformation is important and the change of the support condition will easily affect the rigidity of the rim as well as the fillet stresses.

Effects of Pressure Angle. Three different pressure angles were investigated in this study, specifically 14.5°, 20°, and

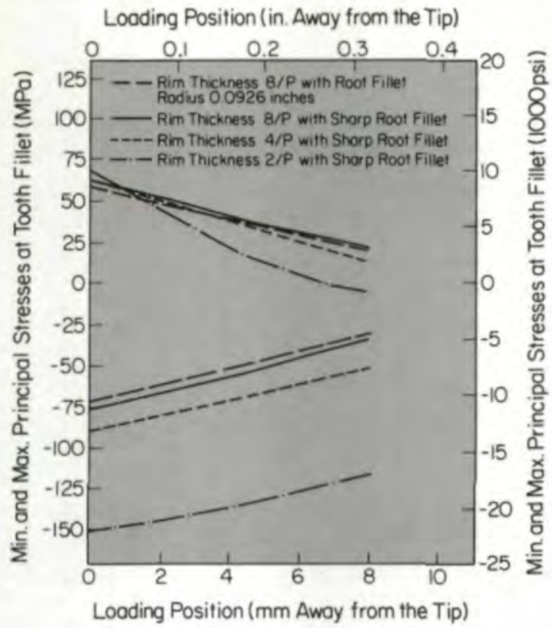


Fig. 18 – Root fillet principal stresses of internal gears ($\phi = 20$) with different rim thickness and loading positions.

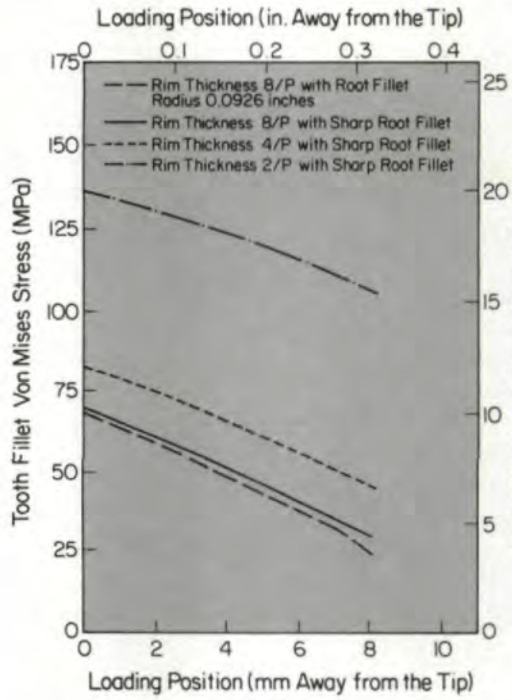


Fig. 20 – Compressive root fillet Von Mises stresses of internal gears (ϕ = 20) with different rim thickness and loading positions.

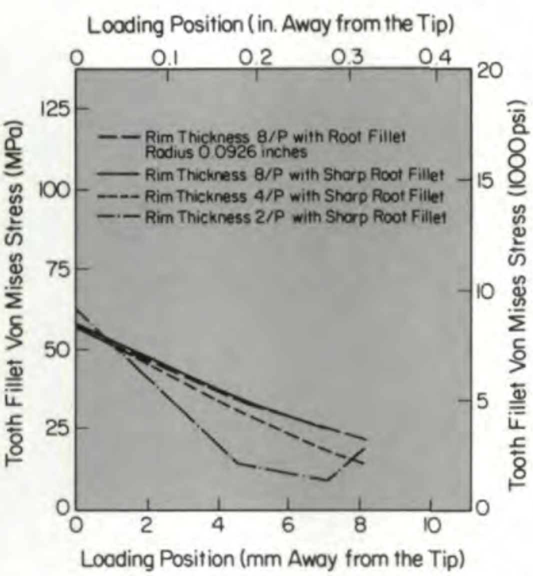


Fig. 19 – Tensile root fillet Von Mises stresses of internal gears (ϕ = 20) with different rim thickness and loading positions.

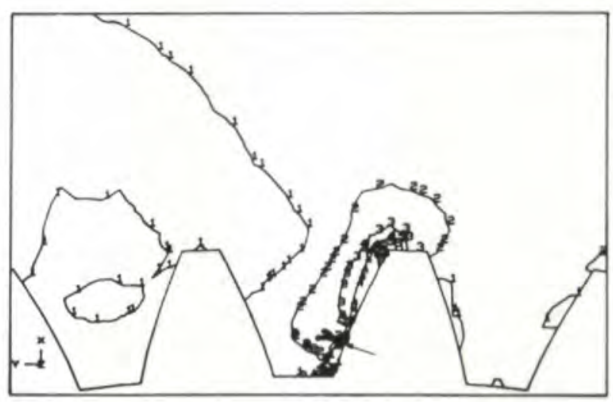


Fig. 21 – Maximum principal stress distribution of internal gear (ϕ = 20, rim thickness = 8/P) loaded at 0.325/P from the tip.

25° angles. Usually, the large pressure angle gear can sustain a larger load transmitted because a larger cross sectional area exists at the root section. Therefore, for the same loading, smaller stresses are expected. Figs. 25 and 26 substantiate this phenomenon. When the rim thickness is larger than 4/P, the larger the pressure angle, the smaller the root fillet stresses. However, if the rim thickness is less than 4/P, then, as the pressure angle increases, the stresses also increase. Rim bending is the probable cause of this phenomonen. For a rigid rim, the loaded tooth is similar to a cantilever beam, and the stress is then determined by the root cross sectional area.

As discussed previously, there are many parameters that affect the stress distribution of internal gears. Although these effects vary, each parameter has the same influence on internal gears regardless of pressure angle.

Conclusions

Other Approaches To Static Stress Analysis of Gears. Although the finite element method has frequently been applied to gear stress analysis, it usually is expensive in terms of CPU time and man-hours required to establish or edit the model. Besides, FEM can only give approximate results rather than exact

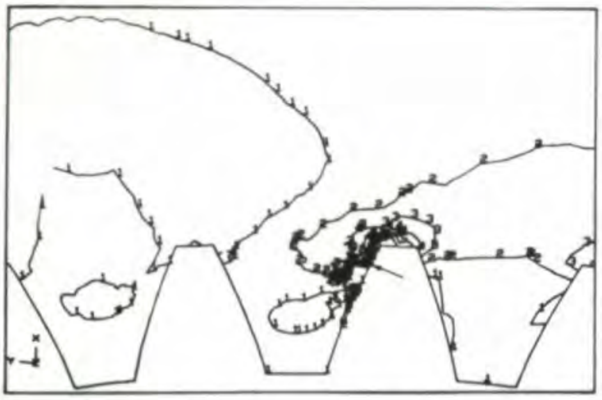


Fig. 22 – Maximum principal stress distribution of internal gear (ϕ = 20, rim thickness = 8/P) loaded at 1.593/P from the tip.

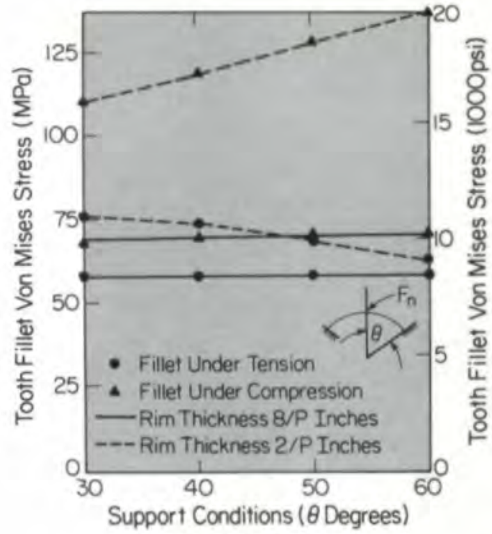


Fig. 24 – Root fillet Von Mises stresses of internal gears (ϕ = 20) with different rim thickness under various supports.

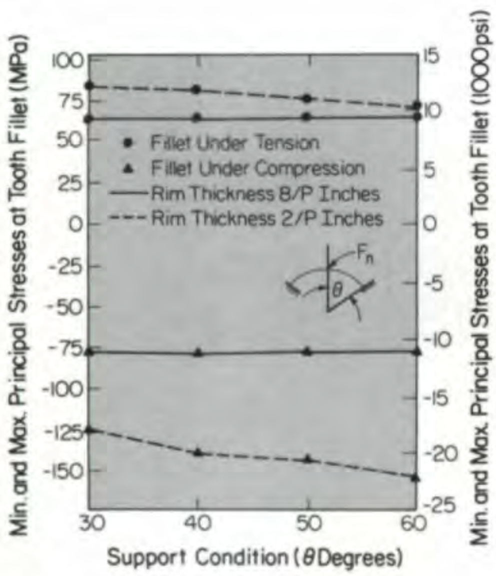


Fig. 23 – Root fillet principal stresses of internal gears (ϕ = 20) with different rim thickness under various supports.

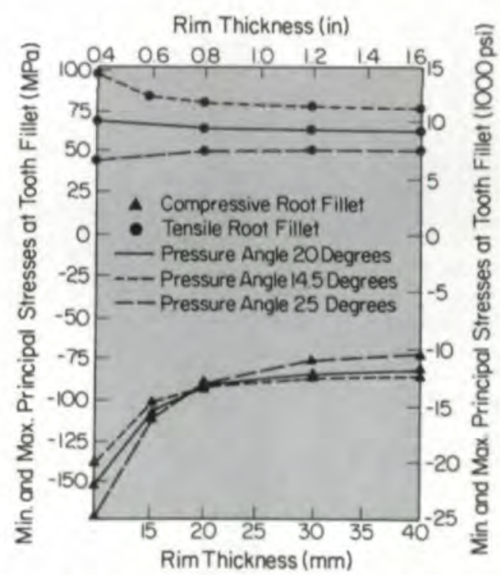


Fig. 25 – Root fillet principal stresses of internal gears with different pressure angles.

answers. There is a relatively new approach to this problem. This approach is called the Boundary Integral Method, which is based on two dimensional theory of elasticity and appropriate transform functions. The solution is exact for the boundary curve chosen and as close as possible to the given one which represents the shape of the stressed piece. Several researchers[8,9] have done investigations on external spur gears, but no research has been done on the internal spur gears. It is believed that the application of this method to the internal gear stress problems can yield satisfactory results, especially in the investigation of the stress concentration around the root fillets. This method is more efficient as far as the CPU time is concerned, and the stress values have better accuracy.

Optimization of Internal Gear Design. The traditional approach to gear design is to utilize design formulas and gear ratings found in standards and codes in order to meet the requirements of the expected operating conditions. This approach does give a feasible solution for a given design; however, it does not guarantee this design is the best possible solution. Nowadays, it is possible to take advantage of high speed digital computers to determine the optimal solution from among many feasible answers. Optimization of the tooth profile geometry is an important area within this field.[10] It is possible to synthesize a noninvolute spur gear pair with optimal load carrying ability based on the bending and contact strength. All these tooth profiles can be synthesized by using man-computer interactive design procedures on CAD/CAM systems.

Summarizing the results and discussion presented here, the following conclusions have been reached:

1. Rim thickness has a dramatic effect on the fillet stresses and stress distribution. When the thickness is less than 4/P, the stress at the tensile fillet changes, and the stress at the compressive fillet increases sharply.
2. As the rim thickness decreases, the position of the maximum stress will tend to move to the bottom of the tooth space apart from the loaded tooth.
3. Rim thickness has a global effect on the stress distribution of the whole gear, and it is recommended that the thickness should be at least 4/P. If the rim is too thin, rim bending may be a severe problem, and the most critical area may shift to the rim.
4. Root fillet radius has a local effect only, and, in order to accurately determine its influences, different element types and meshes should be used. Experimental methods are also suggested.
5. The root fillet should be carefully modelled in order to derive accurate results. Element density and the aspect ratio of elements should be chosen based upon comprehensive study. Experimental data must be used to justify the validity of the models.
6. The fillet stresses vary linearly as the loading position changes, but for thin rim cases, nonlinear relationships are present due to the influence of the deformation of the flexible rim. Usually, if rim thickness is less than 3/P, the rim bending effect will appear.
7. When the loading position approaches the tooth root, the loca-

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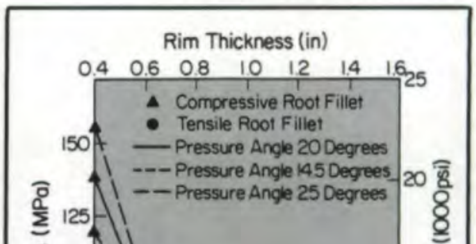

Fig. 26 – Root fillet Von Mises stresses of internal gears with different pressure angles.

TECHNICAL CALENDAR

JUNE 22-24. University of Wisconsin-Milwaukee, Microcomputer Applications in Worm Gear Design & Analysis. A workshop enabling students to develop customized computer-aided gear design systems. For further information, contact: John M. Leaman, Center for Continuing Engineering Education, UW-M, 929 N. 6th St., Milwaukee, WI 53203. (414) 227-3110.

AUGUST 10-12. Ohio State University, Gear Noise Course. Material covered includes noise measurement and analysis, causes, reduction techniques, modeling and modal analysis of gear boxes. For further information, contact: Mr. Richard D. Frasher, College of Engineering, OSU, 2070 Neil Ave., Columbus, OH 43210. (614) 292-8143.

SEPTEMBER 27-29. American Society for Metals 11th Annual Heat Treating Conference, McCormick Place, Chicago, IL. Presentations on subjects including heat treating, statistical process control, new energy applications, quenching and cooling improvements. For further information, contact: ASM International, Metals Park, OH 44073. (216) 338-5151.

NOVEMBER 5-10. International Conference on Gearing, Zhengzhou, China. ASME-GRI and several international gear organizations are sponsoring this meeting. For more information contact: Inter—Gear '88 Secretariat, Zhengzhou Research Institute of Mechanical Engineering, Zhongyuan Rd, Zhengzhou, Henan, China. Tel: 47102. Cable 3000. Telex 46033 HSTEC CN.

NOVEMBER 8-10. American Society for Metals Near Net Shape Manufacturing Conference, Hyatt Regency, Columbus, OH. Program will cover precision casting, powder metallurgy, design of dies and molds, forging technology and inspection of precision parts. For further information contact: Technical Department Marketing, ASM International, Metals Park, OH 44073.

CALL FOR PAPERS—The Society of anufacturing Engineers for its **1988 Gear Processing & Manufacturing Clinic, Nov. 15-17, Indianapolis, IN.** Some suggested topics include gear basics, shaper cutter applications, automotive applications, aerospace gears, heat treating and workholding devices. For further information, contact: Joseph A. Franchini, SME, One SME Drive, P.O. Box 930, Dearborn, MI 48121. (313) 271-1500 x394.

CALL FOR PAPERS — Tennessee Technological University for its **1st Internat'l Applied Mechanical Systems Design Conference, March 19-22, 1989, Nashville, TN.** Papers are invited on general mechanical systems subjects including strength, fatigue life, kinematics, vibration, robotics, CAD/CAM, and tribology. Deadline for first drafts is Oct. 1, 1988. For further information, contact: Dr. Cemil Bagci, Dept. of Mech. Eng., TTU, Cookeville, TN 38505. (615) 372-3265.

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tion of the maximum stress will tend to move to the bottom of the tooth space apart from the loaded tooth.

8. Support conditions do not change the stress distribution or fillet stresses very much for a rigid rim. For thin rim gears, support conditions become an important issue. It may affect the rigidity of the rim and thus increase the stresses considerably.
9. For a rigid rim, the larger the pressure angle, the smaller the root fillet stress. But for thin rims, the larger the pressure angle, the larger the fillet stress.
10. Internal spur gears with different pressure angles respond the same way to the influences of rim thickness, support conditions, loading positions, etc.

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