Practical Optimization of Helical Gears Using Computer Software

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Summary

The aim of this article is to show a practical procedure for designing optimum helical gears. The optimization procedure is adapted to technical limitations, and it is focused on real-world cases. To emphasize the applicability of the procedure presented here, the most common optimization techniques are described. Afterwards, a description of some of the functions to be optimized is given, limiting parameters and restrictions are defined, and, finally, a graphic method is described.

Introduction

Before defining optimization techniques and optimum gear design, it is necessary to introduce certain concepts. Any mechanical system, in this case a gear set, can be represented by a model where all the physical properties are approximately reproduced. And in most cases, the system model can be expressed as a mathematical model.

A mathematical model is a model that represents a system by mathematical relationships, and it can be divided into system variables, system parameters, system constraints, and mathematical relations.

A mechanical system can be modelled for two reasons. The first is to evaluate or analyze its behavior. The second reason is to obtain a design. A design is defined by its geometric configuration, the materials used, and the task it performs.

In most cases, there is more than one solution when designing a mechanical system. Therefore, a criterion for selecting the "best solution" must be established.

A design can be modified to generate different alternatives, and the purpose of the study is to define a criterion for evaluating alternatives and choosing the best one. Cost has to be related to another quantity easier to evaluate. An evaluation model that includes an evaluation criterion is a decision-making model, called an optimization model.

The design procedure has four steps:

1. Recognition of a need,
2. Statement of the problem,
3. Creation of alternative solutions,
4. Selection of alternatives.

Searching for the optimum solution is a technique that can become very cumbersome, but basically it can be described as follows:

1. The selection of a set of variables to describe the design alternatives;

2. The selection of an objective expressed in terms of the design variables, which should be minimized or maximized;

3. The determination of a set of constraints, expressed in terms of the design variables, which must be satisfied by any design.

A summary of the formal mathematical treatment of the optimization procedure is related next.

Mathematical Definition of Optimization

Assume that the design variables are named $x_1, x_2, x_3, ... x_n$, and that they can be arranged into a vector **x**. It is also assumed that the design variables are real numbers. The objective of the optimization has to be expressed as a function $f(\mathbf{x})$ of the design variables. The constraints are classified as equality and inequality constraints. They also have to be functions of **x**. Therefore, the constraints of the design must be expressed as:

$h(\mathbf{x}) = 0$ for equality constraints

$g(\mathbf{x}) < 0$ for inequality constraints

Resuming the optimization problem, it can be stated as:

Min $f(\mathbf{x})$ over **x**

subject to

$\boldsymbol{h}(\mathbf{x}) = 0$, and $\mathbf{g}(\mathbf{x}) < 0$

where **h** and **g** are vectors representing several constraint functions.

There are some cases where the designer wants to satisfy more than one optimization function. One alternative is to combine the individual optimization function into a global function if possible. The other alternative is to formulate the optimization problem as

$$\min F(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}) + \ldots + w_n f_n(\mathbf{x})$$

with

$$w_1 + w_2 + \ldots + w_n = 1$$

But this alternative may lead to an erroneous solution, since the weighting values are selected in a subjective manner.

Depending on each particular optimization problem, there will be a mathematical solution for the proposed model. To prove that the model has a mathematical solution, several aspects can be formulated. These concepts will be extended when explaining the graphical optimization method.

1. *Solution Domain*. It is the isolated region within the space solution defined by the **x** variables. The boundaries of the solution domain are the inequality constraints. Any point inside the isolated region represents a solution for the design problem, but only one must be selected as the optimum.

2. *Boundaries*. They are represented by the equality values of the inequality constraints $g(\mathbf{x})$. The absence of proper bounds may cause a serious problem. In many cases, the solution is found at the boundaries of the solution domain.

3. *Monotony*. This is a property of certain functions that for an increment of the independent variables, **x** produces an increment or decrement of the function. This property can be exploited because it can be proved that in a monotonic function bounded by a constraint, the optimum is always at the boundaries.

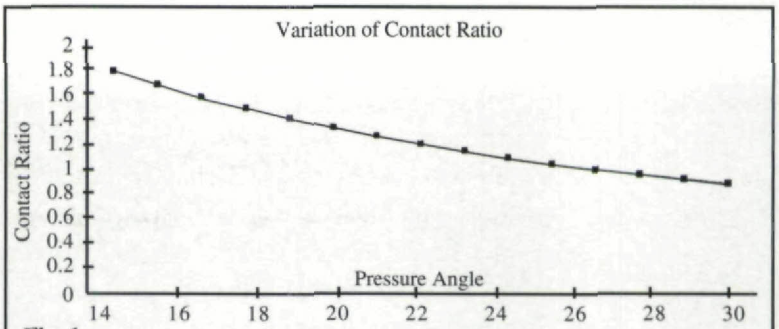

Fig. 1

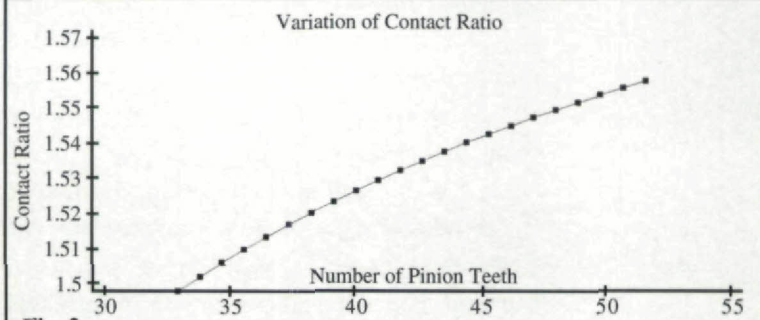

Fig. 2

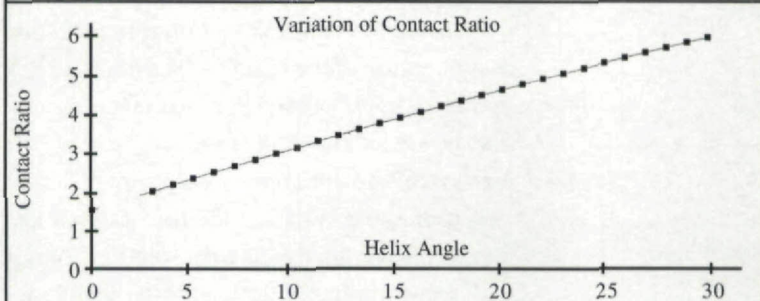

Fig. 3

It is very important to point out that the design functions of gear sets behave monotonically. Examples of this behavior are shown in Figs. 1-3 where the contact ratio is plotted against pressure angle, pinion teeth number, and helix angle. If the objective function is the contact ratio, and the independent variable is the helix angle, from Fig. 3 it can be said that the optimum is at the upper value of the helix angle.

Graphical Optimization Technique for Helical Gears

In previous sections, the optimum design of mechanical systems has been briefly defined, but little has been said regarding optimum gear design. Optimum gear design is a subject that has awakened the interest of engineers around the world, and many papers and articles have been published regarding this subject. Optimum gear design has the particularity that each problem has different objective functions, constraints, and parameters; thus, it is not possible to define a unique procedure for designing optimum gears.

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A more general way of searching for the optimum was studied, and a simpler optimization technique was developed. The algorithms specially developed for optimum design, such as the Reduced Gradient Method, the Gradient Projection Method, Box's Method, Johnson's Method,[1-2] etc., can be very difficult to apply because of the complexity of the mathematical model for gear design. The technique presented here is based on

1. Definition of the objective function based on real needs,

2. Definition of the gear design variables and parameters,

3. Identification of the design constraints,

4. Construction of the solution domain with gear design software instead of by constructing the mathematical model with an optimization program.

5. Graphical representation of the solution design region and conduct of a search for the optimum point. Once the graphical representation of the solution domain is obtained, the location of the optimum is very simple.

The idea of a graphical solution is simple and easy to handle. First of all, the solution domain is all known, and the selection of the boundaries (constraints) and objective functions can be reordinated using a general purpose graphics program. The optimization can be conducted with one or two independent variables (optimization with more independent variables can be obtained by grouping the variables in pairs, leaving the rest of them as parameters).

Objective Function. In optimum gear design, the objective function can be a single function or several functions, depending on each application and each particular case. For instance, the objective function that seems to be most logical is cost. But cost is affected by different parameters, and the engineering definition of cost can be expressed in different ways. For example, an objective function will be to reduce the manufacturing cost. This can be achieved by designing a gear set modifying only the helix angle: therefore, the manufacturing cost will depend only on the settings of the cutting machine. Or the solution will depend not only on the helix angle, but on the teeth number, speed ratio, materials, heat treatment, etc., and the optimization will be more complex.

The definition of the objective function is the starting point of the optimization, and it has to be identified as precisely as possible, in order to reduce time-consuming calculations and problem statements. In the automobile industry, for example, objective functions might be described as noise and perhaps cost. Therefore, the mathematical model for the optimization problem will be more complex.

The relation between the objective function and the mathematical model is determined based on the designer's judgment and experience. Some of these relationships are:

Cost > Teeth Number, Face Width, Cutting Machine Settings, Surface Finishing, etc.

Noise > Contact Ratio, Teeth Number, etc.

Independent Variables and Parameters. The list of variables for designing a gear set is very large. In the examples presented in this article, the independent variables are pinion teeth number, helix angle, and pressure angle. The dependent variables are pinion and gear pitting and bending stresses, contact ratio, length of action, tangential velocity, critical scoring number, and/or speed ratio and center distance. The parameters are material properties, transmitted power, design factors (stress multipliers) AGMA quality level, and/or speed ratio and center distance.

Constraints. Constraints are variables that define the boundaries of the solution domain, and almost any solution of an optimum gear design lies in a boundary. Typical constraints are the minimum life due to bending or pitting stresses, the maximum allowable scoring number, the minimum contact ratio, the AGMA quality level, etc.

Solution Domain. Once the independent and dependent variables and the parameters are determined, and the constraints are defined, the solution domain can be constructed by storing all the calculations performed with a design software into a database. The data can be arranged for producing plots of the solution domain. Fig. 4 shows an example of a solution domain. Then a 3D plot of the behavior of the objective function can be obtained, as shown in Fig. 5. In this example, the objective function is the maximization of the pinion bending life, and the optimum is located at the intersection of the maximum pressure angle and the minimum contact ratio limit. Fig. 6 shows another example where the independent variables are the pinion teeth number and the pressure angle, and the objective function is the contact ratio.

Searching the Optimum. After the solution domain is defined and the database is filled in, the identification of the optimum is a quite simple task. First, define the objective function within the dependent variables. Second, select up to two independent variables for generating the plots. Plot the optimization function with respect to the independent variables. The optimum can be located directly from the plot. If the database is large, the data can be analyzed by blocks of information; in other words, by isolating small regions of the solution domain.

If the gear design problem was defined with more than two independent variables, the former procedure can be used by isolating two of the independent variables, keeping the rest of them as parameters, and locating the optimum for the reduced solution domain. Then with the two independent variables that define the optimum as parameters, repeat the search using another two variables, and so on This procedure may seem quite complicated, but with a general purpose database program, it is simplified.

Example 1. The definition of the problem is as follows:

Objective function — Maximize the contact ratio for a minimum cost.

In this case, cost is related to those parameters that can be modified without affecting the production cost. The helix angle, for instance, has to be set up on the cutting machine, therefore, a modification on its value affects the characteristics of the design without modifying the cost of the gear. Therefore, only two independent variables are selected: helix and pressure angles.

According to the designer's criterion and application of the design, the parameters must be established. For this example, the parameters are

Speed ratio	2
Normal Diametral Pitch	10
Pinion Teeth Number	40
Face Width	2.6
Addendum Proportions	Normal
AGMA Quality Level	10
Material Properties	BHN = 180
	Sat = 25,000 psi
	Sac = 85,000 psi
	$E = 30 \times 10^6$

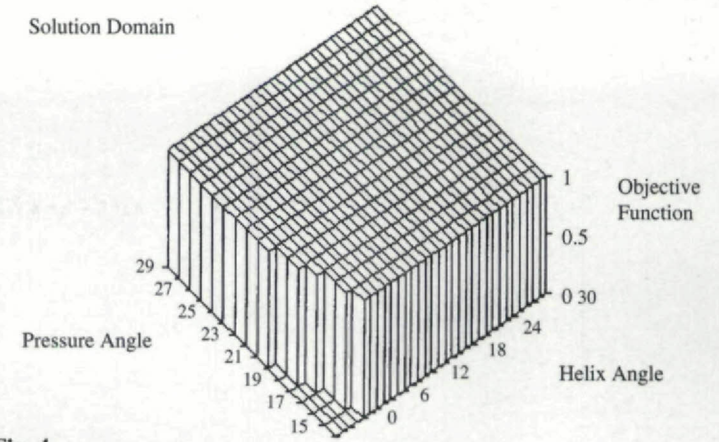

Fig. 4

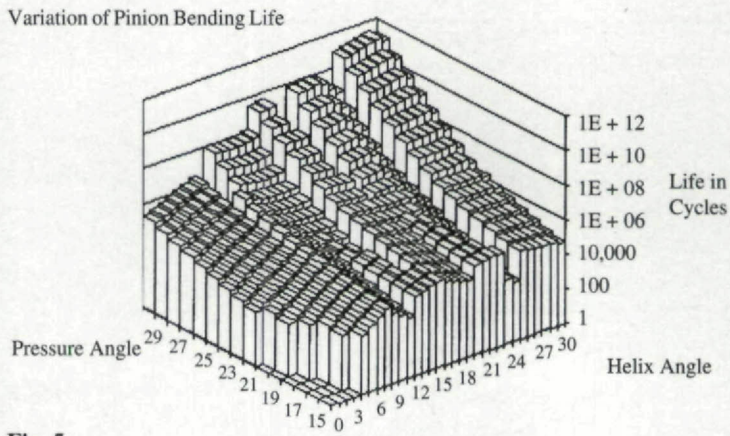

Fig. 5

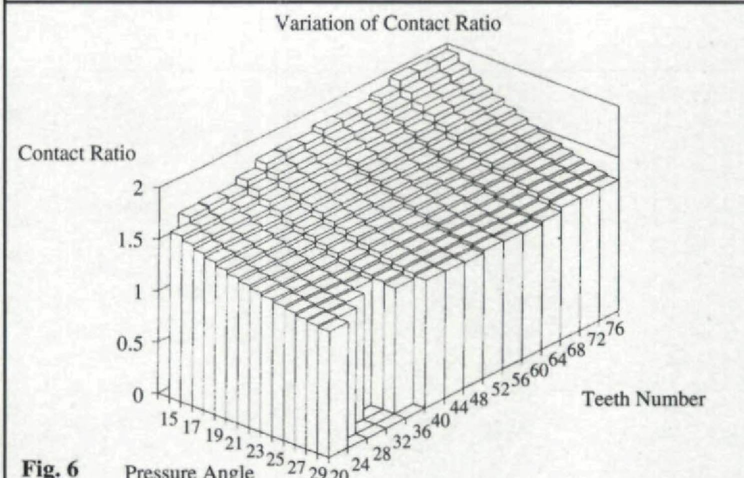

Fig. 6

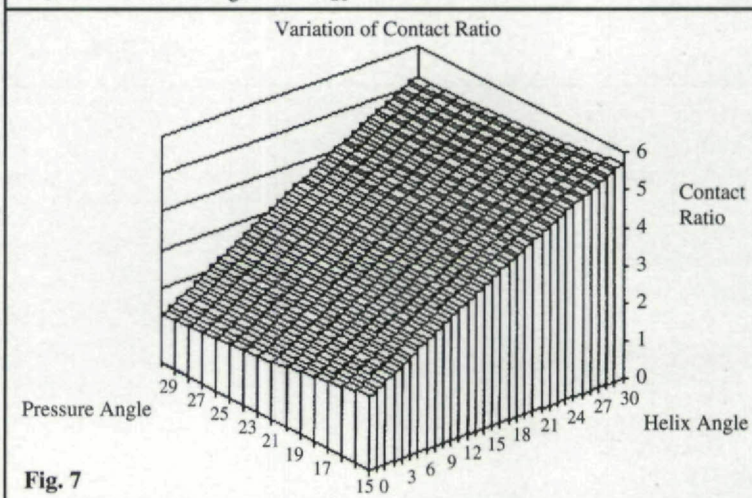

Fig. 7

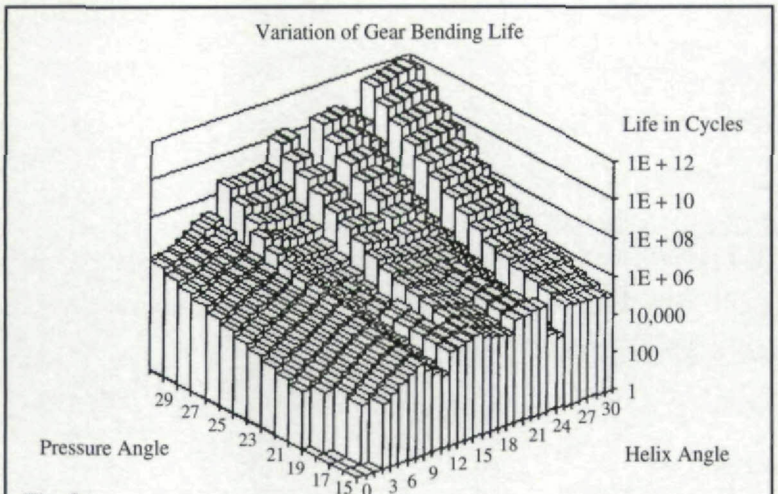

Fig. 8

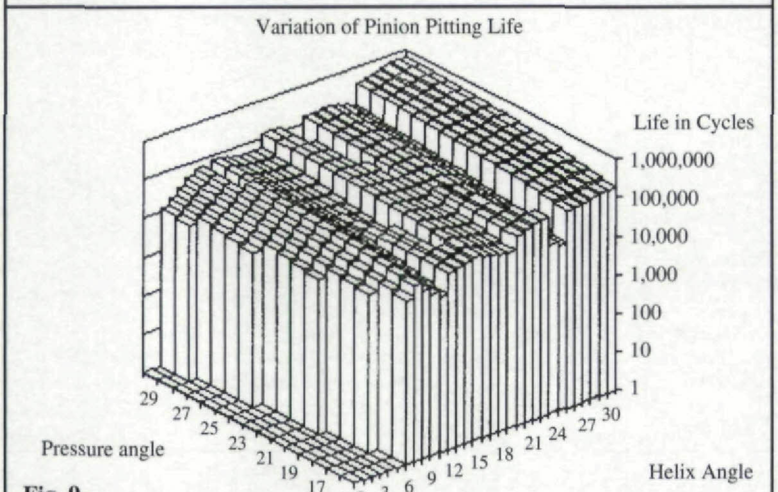

Fig. 9

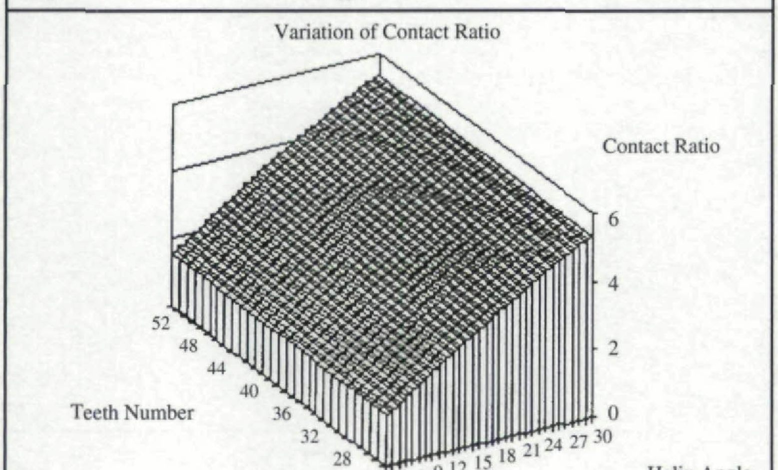

Fig. 10

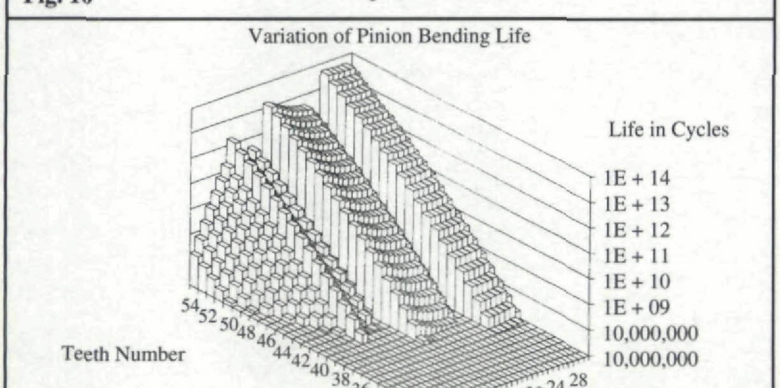

Fig. 11

Eliminating the parameters from the design functions, the dependent variables are obtained: Center distance, gear teeth number, pitting stress, bending stress, scoring number, pitting life, bending life.

The constraints are defined by the designer, and the limiting values are imposed from each particular application. In this case, the inequality constraints are

Critical Scoring Number < 20,000
Contact Ratio 1.2
Bending Life > 1 X 10^7
Pitting Life > 1 X 10^7
Helix Angle 45°
14.5° < Pressure Angle < 28°

The equality constraints are all the application factors for calculating pitting and bending stresses. It is important to point out that the stress is not limited to the allowable material stresses. Instead, the life is limited to a minimum value.

All the possible solutions were calculated with gear design software, and they were stored into a general purpose graphics program. The program generates plots of the calculated variables. The results were plotted as shown in Figs. 7, 8, and 9. Fig. 7 shows the variation of the contact ratio as a function of the independent variables. This can be seen the solution domain and the location of the optimum. From this figure, it can be stated that the optimum is found at the point where the helix angle equals 45° and the pressure angle equals 15°.

Figs. 8 and 9 show the behavior of gear bending life and pinion pitting life. At this point, the designer must take into consideration the particular application of the gear set. If the failure criterion is pitting, then the optimum will be the point with maximum contact ratio. If the failure criterion is bending, then the optimum will be at the point the helix angle equals 30°, and the pressure angle equals 28°. If the design must satisfy both criteria, the designer should select the most restrictive solution.

Example 2. The definition of the problem is as follows:

Objective function — Maximize the contact ratio and bending life. In this case, instead of the pressure angle, the influence of the pinion teeth number is studied.

Independent Variables — helix angle and teeth number. The design definition is about the same as in Example 1.

The parameters are:

Speed ratio	2
Normal Diametral Pitch	10
Pressure Angle	20°
Face Width	2.6
Addendum Proportions	Normal
AGMA Quality Level	10
Material Properties	BHN = 180
	Sat = 25,000 psi
	Sac = 85,000 psi
	E = 30 x 10^6

Eliminating the parameters from the design functions, the dependent variables are obtained:

Dependent Variables

Center Distance
Gear Teeth Number
Pitting Stress
Bending Stress
Scoring Number
Pitting Life
Bending Life

The constraints are defined by the designer, and the limiting values are imposed from each particular application. In this case, the inequality constraints are:

Constraints

Critical Scoring Number < 20,000
Contact Ratio > 1.2
Bending Life > 1 x 10^7
Pitting Life 1 x 10^7
Helix Angle < 30°
Pinion Teeth Number > 6

The equality constraints are the same as in Example 1.

The solution domain was calculated with a gear design software program, and all the data were stored into a general purpose graphics program. The results were plotted as shown in Figs. 10-12. Fig. 10 shows the variation of the contact ratio as a function of the independent variables. The solution domain is seen, and the optimum is located along the line for the value of helix angle equal to 30° The objective function was the contact ratio and the bending life. Therefore, the behavior of bending life is plotted in Fig. 11, and the optimum value is located at the point the helix angle equals 30° and 34 teeth. To verify that this solution is in agreement with the pitting life, Fig. 12 shows the behavior of pitting life. From this figure, it is verified that the optimum lies at an allowable solution.

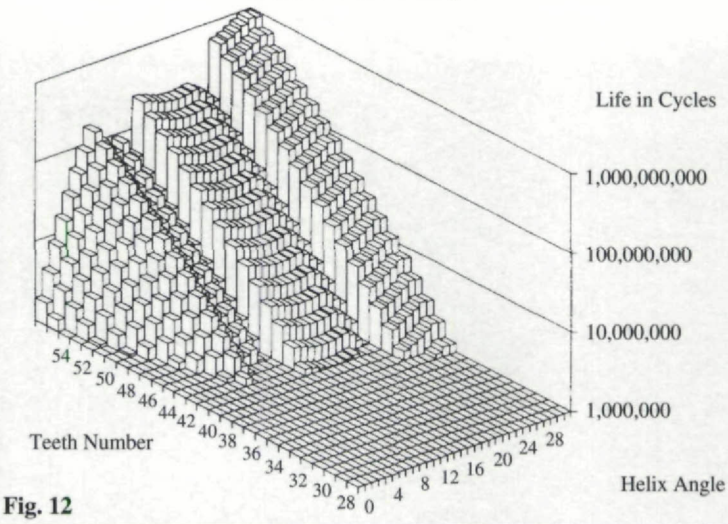

Fig. 12

Conclusions

A simple procedure for optimum gear design was presented. The procedure is adjustable for the optimization of any combination of objective functions, and it allows the designer to impose actual restrictions. Besides, it is not necessary to have a deep understanding of complicted optimization techniques. Also, this procedure does not require special optimization programs. Any gear design optimization problem can be solved by generating with a gear design software plots of the solution domain. The location of the optimum is simple, and it can be determined visually from the plot or reviewing the data.

Graphical optimization gives an overview of the entire problem and allows the designer to identify the optimum solution without complicated interpretation of the results. ■

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