

Drive Line Analysis for Tooth Contact Optimization of High-Power Spiral Bevel Gears

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Management Summary

It is common practice in high-power gear design to apply reliefs to tooth flanks; they are meant to prevent stress concentration near the tooth edges. Gears with crowning have point contact without load, and when load is applied, instantaneous contact turns from point into a Hertzian contact ellipse. The contact area grows and changes location as load increases. To prevent edge contact, the gear designer has to choose suitable reliefs considering contact indentations as well as relative displacements of gear members.

In the majority of spiral bevel gears, spherical crowning is used. The contact pattern is set to the center of the active tooth flank and the extent of the crowning is determined by experience. Feedback from service, as well as from full-torque bench tests of complete gear drives, has shown that this conventional design practice leads to loaded contact patterns, which are rarely optimal in location and extent. Oversized reliefs lead to small contact area, increased stresses and noise, whereas undersized reliefs result in an overly sensitive tooth contact.

Today it is possible to use calculative methods to predict the relative displacements of gears under operating load and conditions. Displacements and deformations originating from shafts, bearings and housing are considered. Shafts are modeled based on beam theory. Bearings are modeled as 5-degree-of-freedom supports with non-linear stiffness in all directions. Housing deformations are determined by FEM analysis and taken into account as translations and rotations of bearing outer rings. The effect of temperature differences, bearing preload and clearances are also incorporated.

With the help of loaded tooth contact analysis (LTCA), it is possible to compensate for these displacements and determine a special initial contact position that will lead to well-centered, full-torque contact utilizing a reasonably large portion of the available tooth flank area. At the same time, crowning can be scaled to the minimum necessary amount. This systematic approach leads to minimum tooth stressing, lower noise excitation, increased reliability and/or power density as compared to conventional contact design method.

Introduction

In a majority of spiral bevel gears produced, the tooth contact is initially placed at the center of the tooth flank during manufacturing. Sufficient crowning is applied to prevent the contact from reaching tooth edges under load. However, the use of large crowning also has a downside of increasing

contact stresses because the area in contact at a particular moment is reduced. With constantly growing demands for higher power density and lower noise generation, there are pressures for decreased crowning. In a larger sense, there is often a great need to optimize tooth flank topography for a certain application. This requires accurate knowledge about the behavior of

tooth contact under load. Regardless of the optimization goal, the change of relative position of bevel gears under load is an important factor.

When loads and temperature differences are applied to a gear drive, the relative position of pinion and wheel changes due to deformations and displacements related to bearings, shafts and housing. This causes changes in the tooth contact—the significance of which is dependent on magnitude and mutual relations of the displacements as well as characteristics of the tooth geometry. One of the main concerns is the spreading and movement of contact pattern that, ideally, should be located at the center of the tooth flank under load and cover as much of the flank area as possible. If the behavior of the contact pattern is known, pre-compensation can be applied in the finish machining of tooth flanks to ensure good running properties under load. This usually means that the tooth flank topography is modified so that the initial contact pattern (without load) is moved from the center of the flank by a certain amount.

Traditionally, the knowledge of tooth contact behavior has been attained through practical experience, but that requires time-consuming and expensive prototype testing. An alternative approach is one based on computer simulation, by which significant cost savings are possible. In recent years, tooth contact optimization based on loaded tooth contact analysis (LTCA) has been applied with good success in numerous customer projects involving marine, industrial and automotive bevel gear applications. Using LTCA, the mesh of spiral bevel gears is simulated using 3-D tooth geometry, taking into account the actual, relative position of gears under load. This paper describes a computational process used to determine how the relative position of bevel gears changes when load and temperature differences are applied on a gear drive. The process is a combination of different calculation methods and is hereafter referred to as drive line analysis (DLA). In addition to the methods usually used in DLA, some alternative approaches are also mentioned to provide a more general overview of applicable methods.

Relative Position of Bevel Gears

In nominal position, the pitch cone apexes of bevel pinion and wheel (if not a hypoid gear pair) coincide. Deviation from this position (location + orientation) can be fully defined by four displacement values, hereafter referred to as relative displacements. As shown in Figure 1, they consist of deviation of shaft angle (*S*); offset (*E*); pinion axial location (*P*); and wheel axial location (*G*).

Driveline Analysis

To fully understand tooth contact behavior in a certain application, the chain of events from assembly (tooth contact adjustment) to operating conditions (loads and temperature differences applied) must be traced. To accurately determine displacement of bevel gears, a detailed analysis of the whole drive line—consisting of shafts, bearings and housing—is required. The “core” of analysis is comprised of separate calculation models for pinion shaft and wheel shaft, hereafter referred to as shaft-calculation models. These models are used to simulate deformations and displacements of shafts and bearings. Commercial software with ranging levels of capabilities is available for this purpose. Beam theory is practi-

cally always used to calculate shaft deflection, but there are significant differences in the way bearings are modeled. At the simplest level, bearings are considered as radially stiff “hinges” that do not represent reality very well. In the more advanced software, such as used in DLA described in this paper, bearing internal geometry and stiffness nonlinearity are considered in order to accurately model real behavior.

Bearing Stiffness

Gears are usually supported by a gear unit housing with rolling bearings. Bearing stiffness varies significantly, depending on the type of rolling element (Fig. 2) affecting the displacement behavior of the shaft-bearing system. Another significant factor is the internal alignment capability (Fig. 3).

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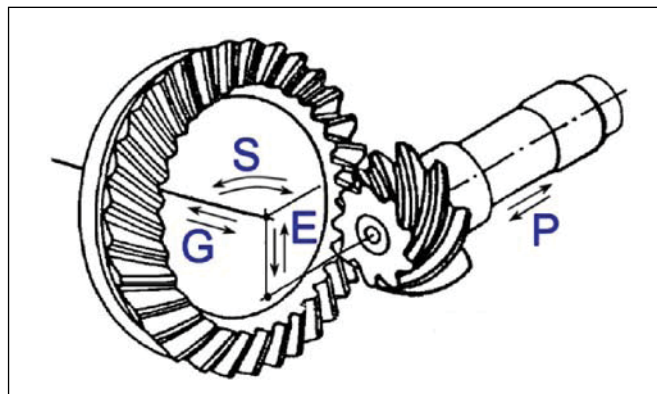


Figure 1—Relative displacements of bevel gears.

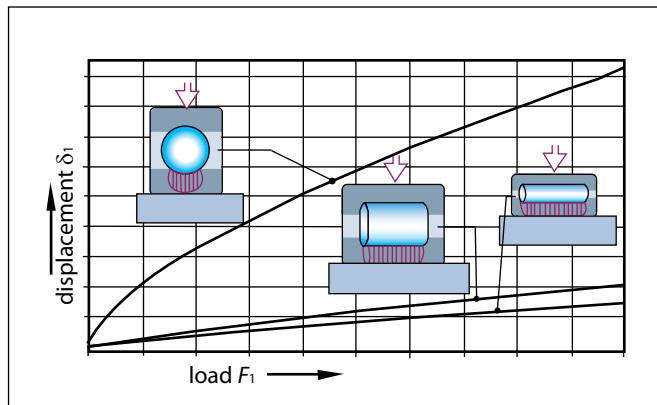


Figure 2—Radial stiffness of different bearing types (Ref. 2).

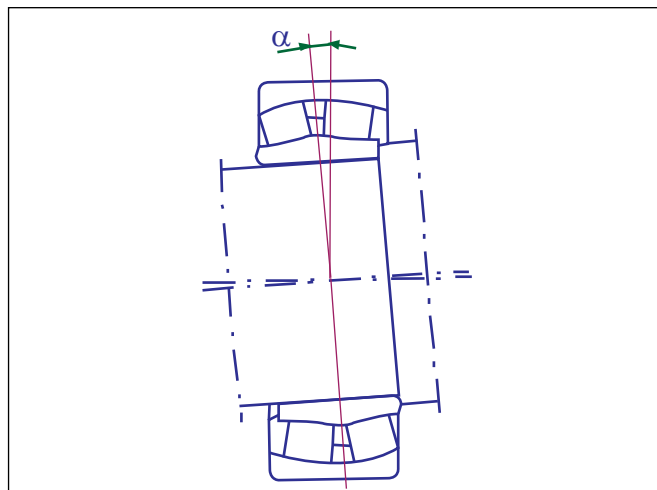


Figure 3—Bearing internal alignment capability.

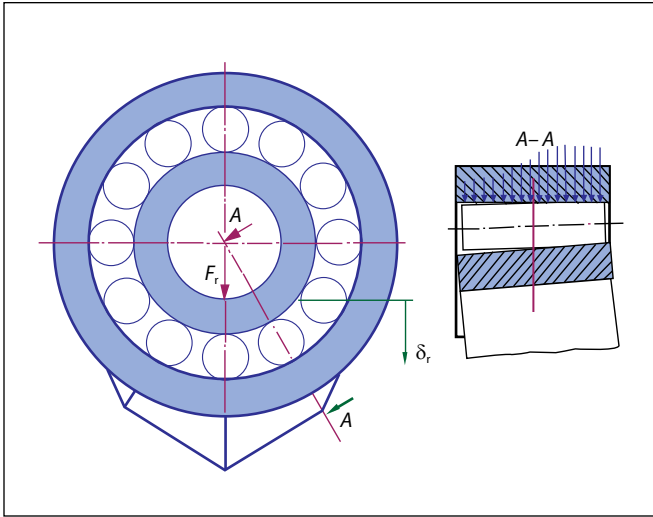


Figure 4—Deformation of individual element contact.

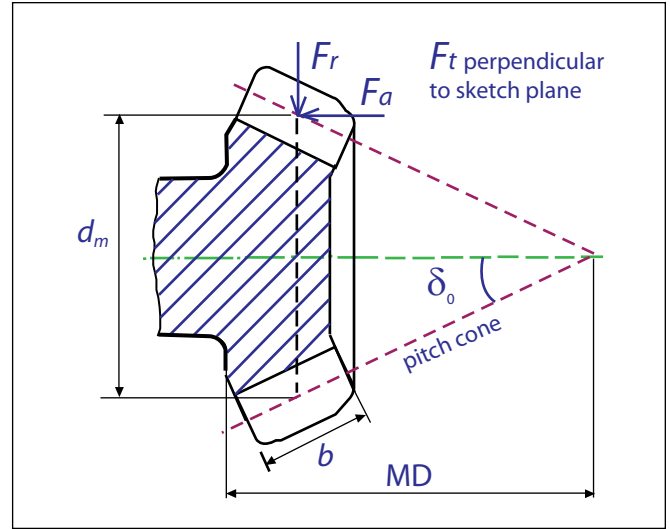


Figure 5—Application point of tooth forces.

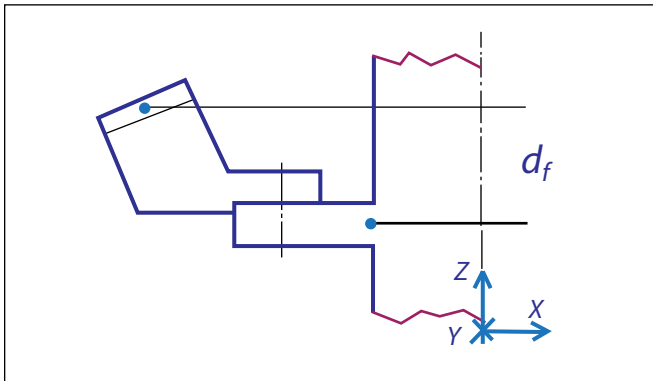


Figure 6—Application point used in shaft calculation.

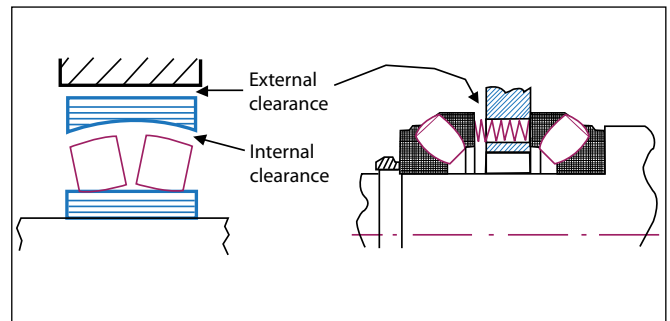


Figure 7—Examples of internal and external bearing clearances.

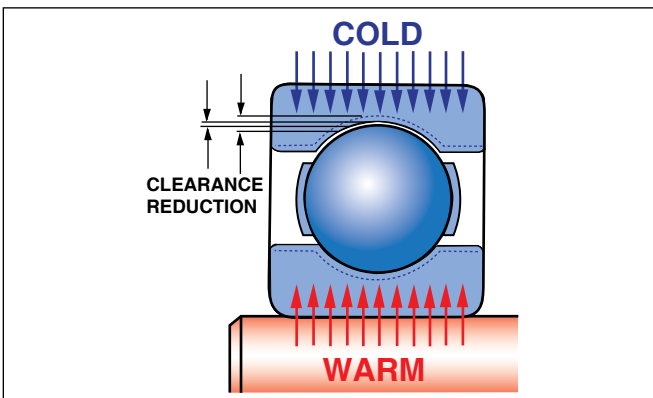


Figure 8—Clearance reduction due to temperature difference (Ref. 3).

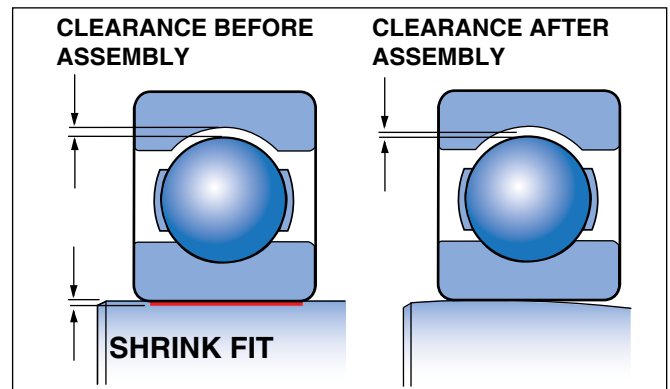


Figure 9—Clearance reduction due to shrink fit (Ref. 3).

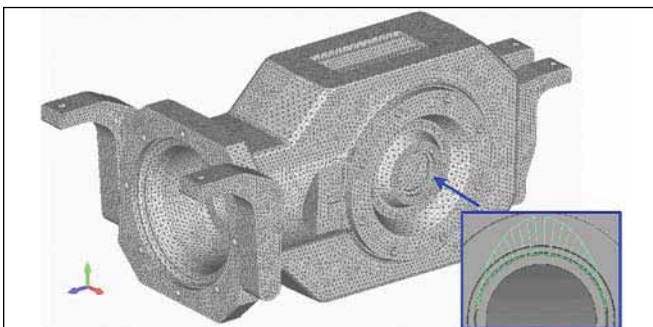


Figure 10—Example of radial bearing load on FE model of housing.

In shaft calculation models used in DLA, bearings are modeled as supports with five degrees of freedom—two radial, two tilt and one axial direction (Fig. 11). The “missing sixth degree of freedom” is the bearing rotation, which is typically of no interest.

Nonlinear bearing stiffness in every direction is modeled starting from deformations of individual contacts between rolling elements and raceways (Fig. 4), also taking into account the internal clearance and operational contact angle. With this modeling method it is possible to accurately predict the distribution of loads and, subsequently, the displacements. Although nonlinear bearing stiffness leads to iterative calculation, the calculation times are minimal due to the analytical

theories applied. A more detailed description of the modeling theory can be found in DIN ISO 281, Supplement 4 (Ref. 1).

Loads

Tooth forces are considered as point loads acting on the center of the tooth flank at mean pitch diameter d_m (Fig. 5). Tooth force components F_r , F_t , and F_a are calculated based on mean spiral angle β_m , normal pressure angle α_n and pitch cone angle δ . Especially for the wheel, the axial location of the acting point of the tooth forces does not always represent the axial location where the forces are actually conveyed to the shaft. An example of such a situation is presented in Figure 6. In shaft calculation models this is taken into account by transferring the tooth forces axially by a distance of d_f and correspondingly adding two bending moments $My = F_r d_f$ and $Mx = F_t d_f$. The same principle is used to correctly model bearing reactions in cases with bearings with a non-zero pressure angle (e.g., taper roller bearings).

In addition to tooth forces there are usually external forces that also need to be included, such as propeller thrust force in marine thrusters. They are applied to their appropriate location on the shaft using the same principles as with the tooth forces. The weight of components is seldom important from deformations' point of view, but might instead be significant for other reasons discussed later in this paper.

Bearing Clearances and Pre-Load

Depending on the arrangement, bearing clearance and pre-tension can have significant influence on gear displacements. In shaft calculation models, values in operating conditions are used, which often differ significantly from assembly values due to temperature differences. Clearances can be divided into *internal* and *external* clearances (Fig. 7).

Internal radial clearances in operating conditions are calculated based on clearance class (e.g., CN, C3, etc.), shrink-fits of bearing rings and temperature difference between inner and outer ring (Figs. 8–9). Both internal and external radial clearances cause shaft displacement, but internal clearance also affects bearing stiffness. Therefore, precise, external radial clearance should be modeled as movement of the outer ring, not as increased internal clearance. However, the significance of this matter is minor. In axial direction, internal and external clearances are basically the same thing. Axial pre-load/external axial clearance in operating conditions is calculated based on initial setting (assembly), temperature difference between shaft and housing, distance of bearings and bearing pressure angle.

External radial clearances are used when bearings need to be free in the axial direction. External axial clearances are sometimes applied to bearings in *O* or *X* arrangement to prevent excessive preloading due to temperature differences. Ideally, these clearances should be reduced to very small values in operating conditions, in which case their influence on displacements would be negligible. However, because temperature differences are usually not exactly known in the design phase (with clearances chosen preferably “on the safe side,” i.e.—too large rather than too small) and gear drives are often loaded in different operating temperatures, consideration of external clearances is also a part of DLA.

Deformation of Gear Housing

Deformation of gear housing is considered through FE

analysis, performed with commercial software. Bearing reactions obtained from preliminary shaft calculation models are used as loadings for the FE model. Loads are applied to the radial and axial support surfaces of bearings as pressure distributions with resultant forces corresponding to the bearing reactions (Fig. 10). External loads are applied if such exist.

After the FE model is solved, translation and rotation values of bearing bores are extracted from the displacement results. With displacements as the main result, a relatively coarse FE mesh (compared to, for example, stress analysis) is sufficient. In shaft calculation models, bearing bore displacements are described with the same five degrees of freedom as bearing stiffness (Fig. 11)—i.e., three translational and two rotational displacements are used to describe movement of one bearing bore. These values can be extracted from the FE node displacements in different ways. One way is to choose representative nodes with 90° spacing from the support surfaces and calculate the five displacement values from them. A more sophisticated method is to place a node in the middle of the bearing bore and connect it to the cylindrical surface by beam elements with very small axial stiffness and ball joint-type connections at the ends. In this way the displacement of the center node directly represents the sought after values. Similar results can also be achieved by fitting an un-deformed cylinder to the displacement field using a best-fit procedure. All of the mentioned methods have been used successfully as a part of DLA.

After the shaft calculation models are re-run using the bearing bore displacements, changes in bearing reactions are checked. If considerable change is observed, the FE model is

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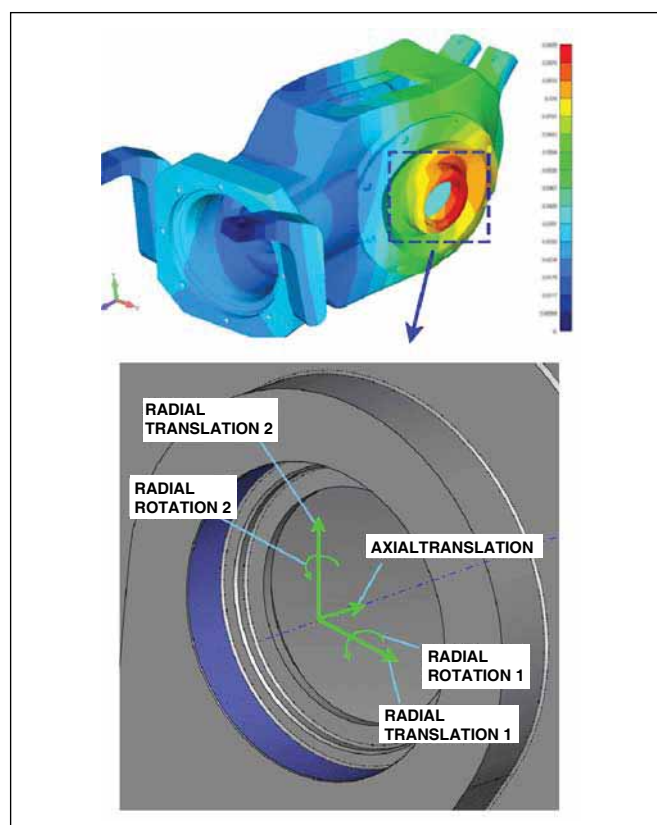


Figure 11—Determination of bearing displacement from FE results.

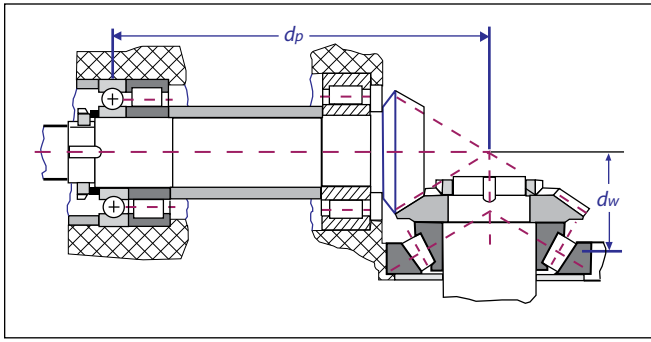


Figure 12—Effective distance for temperature difference between housing and shafts.

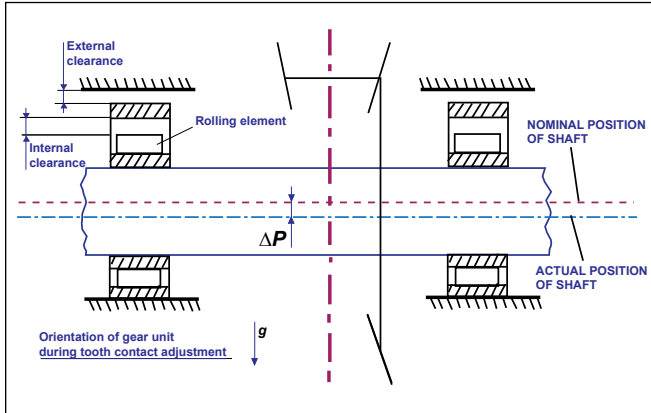


Figure 13—Example of additional displacement (P) due to gear unit orientation during tooth contact adjustment.

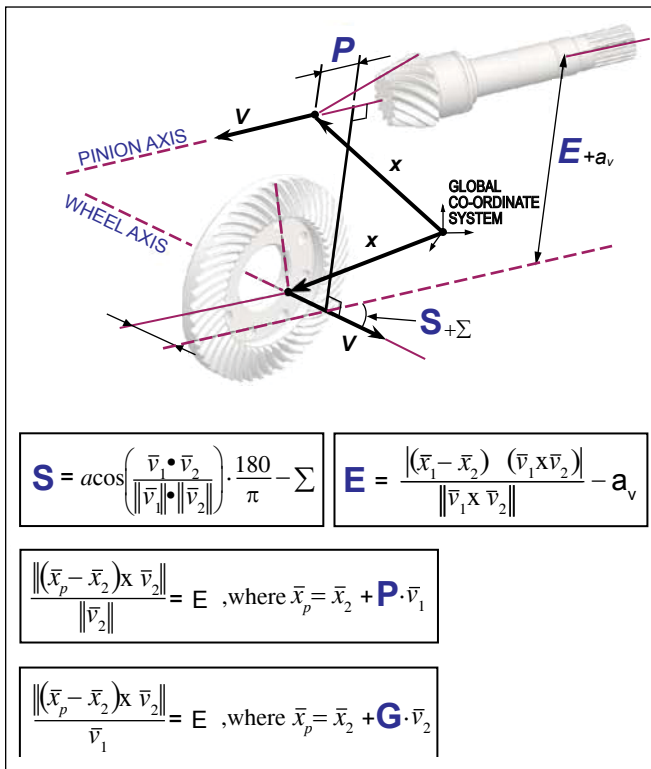


Figure 14—Arbitrary position of gear member described with vectors.

no longer valid and the process is repeated. Typically, only one iteration is required.

Effect of Temperature Differences to Axial Location of Bevel Gears

In addition to bearing clearances and pretension, temperature differences also affect the axial location of bevel gears. The significance is strongly dependent on the material of the housing and the distance between the bevel gear centerlines and axial bearing location (Fig. 12).

Effect of Gear Drive Orientation during Tooth Contact Adjustment

Usually during the assembly of gear drives no thermal differences exist. Therefore, bearing clearances (internal and external) have significantly larger values compared to operating conditions. In shaft calculation models it is assumed that without load, shafts are initially in their nominal position—i.e., “hovering” in the middle of bearing clearances in the radial direction. However, during tooth contact adjustment, the position of shafts might differ significantly from this assumption due to the clearances and weight of components. This is especially true in cases where enlarged clearances are used due to expected high-temperature differences. The matter is illustrated with an example in Figure 13 where the horizontal shaft is displaced from its nominal position due to gravity and internal/external bearing clearances. This corresponds to an additional change in relative displacement P . Generally, also E , G and S displacements can be affected depending on the bearing arrangement and orientation of the gear unit. In addition to clearances in radial direction, axial clearances can also cause additional displacements. For example, in situations with spring-loaded axial clearance (Fig. 7), the clearance during assembly might lie on the opposite side as compared to operating conditions. If significant displacements from the nominal position of shafts are to be expected during tooth contact adjustment, they are taken into account in the calculations. This requires knowledge of the orientation of the gear drive during tooth contact adjustment.

In addition to clearances, bearing deformation can also contribute to the gravity-induced additional displacements. One example is a taper roller bearing (small pressure angle) without preload that has a small axial stiffness that can lead to axial movement of the shaft if it has been subjected to large axial force from heavyweight components.

Calculation of Relative Displacements of Gear Members

After the relative displacements have occurred, pinion and wheel are considered to be in arbitrary position in three dimensional space, which can be described by location and direction vectors x_1 , x_2 , v_1 and v_2 (Fig. 14). These vectors are extracted from the results of shaft calculation models—i.e., deflections and inclinations of the neutral axis at the locations of the bevel gear teeth (Fig. 15). Relative displacements E , P , G and S are then calculated from the vectors using basic vector algebra.

When determining the vectors, care has to be taken so that displacements from correct axial location on the shaft are used. As Figure 6 demonstrates, the axial location of tooth forces does not always coincide with the location that determines displacements of the bevel gear. The significance of

this is magnified in cases where the shaft inclination changes rapidly near the location of the bevel gear (Fig. 16).

Application of Analysis Results

Relative displacements are used in tooth geometry optimization—which is based on LTCA. In most cases, the goals of optimization are related to noise, power density, robustness and efficiency. Regardless of the goal, knowledge of the relative position of gears under load is a valuable piece of information. For instance, in optimization for stresses the goal is to distribute load evenly on the tooth and to utilize as much of the tooth flank area as possible. When the relative displacements become known, a centralized location of tooth contact under load can be assured. Therefore the portion of crowning that was previously intended to prevent edge contact—due to unknown movement of tooth contact—can be reduced.

The simplest case of optimization is the one with constant load and operating conditions. The situation becomes more complex when multiple load levels and different temperature conditions have to be considered. In such cases drive line analysis is repeated several times, with different input. The resulting, optimal tooth geometry might be a compromise between several different load cases. Figure 17 is an example of such a situation in an automotive application. With a 50% load, the contact approaches the toe; and with a 100% load, the heel. In this case the crowning and initial contact pattern were chosen so that a satisfactory compromise between different loaded conditions was obtained.

In reality the actual relative position of gears under load will vary in a certain range—even in cases with single load. To some extent, the tooth geometry should be designed so that it can withstand these variations; the essential sources of variations include:

- Variation in tooth finish machining
- Variation in tooth contact adjustment during assembly
- Variation of housing machining and assembly clearances between components
- Variation of temperature and loading conditions

The first two of the mentioned sources of variations are strongly operator-specific, especially when the correctness of tooth contact is judged by applying a marking color on the tooth flanks and observing the contact pattern after rotation of the gears. In tooth finish machining, variation is also affected by repeatability related to machine kinematics and tool settings. In cases where gears are assembled to certain mounting distances without checking the contact pattern, variation is determined by a tolerance stack-up of related components.

Variation related to manufacturing deviations and assembly clearances usually increases with the number of mounting surfaces between bevel gears and housing (Fig. 18). These deviations are distinguished from the gear unit’s orientation-dependent deviations discussed earlier by the fact that they are “locked” during assembly and will not change after the bolts are tightened. In shaft calculation models it is assumed that relative positions (say, perpendicularity and concentricity) of all mounting surfaces are free from deviations. Furthermore, the clearances between housing components (such as bearing carriers and main housing) are not considered. Indeed, all components are assumed to be situated in the middle of their

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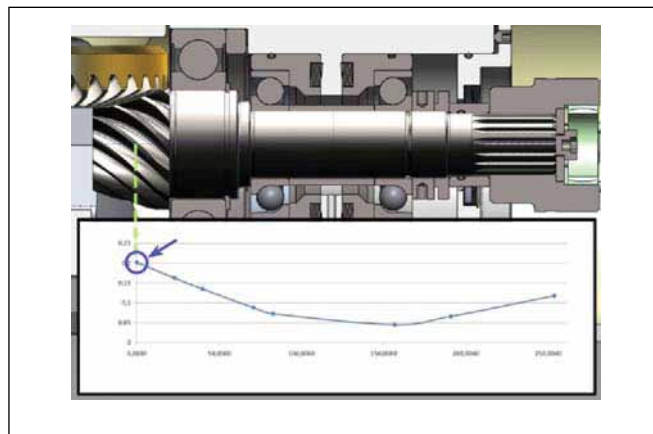


Figure 15—Location on neutral axis where shaft displacements are taken.

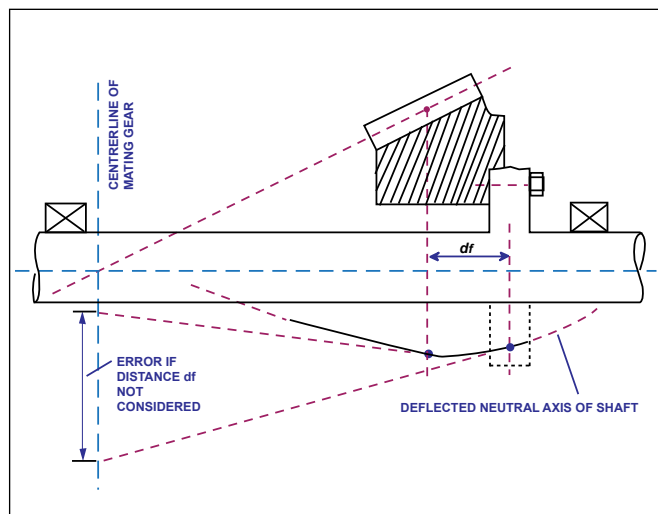


Figure 16—Shaft inclination changes.

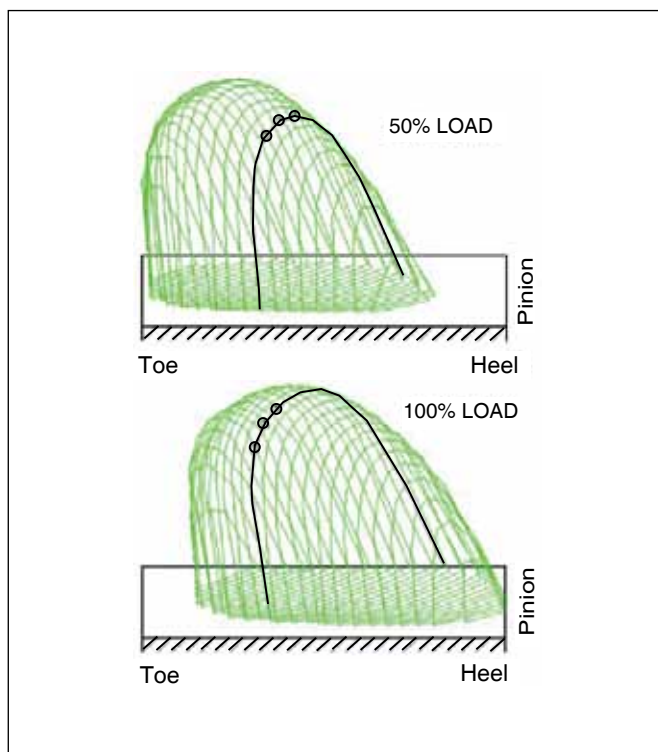


Figure 17—Tooth contact optimization in different load cases.

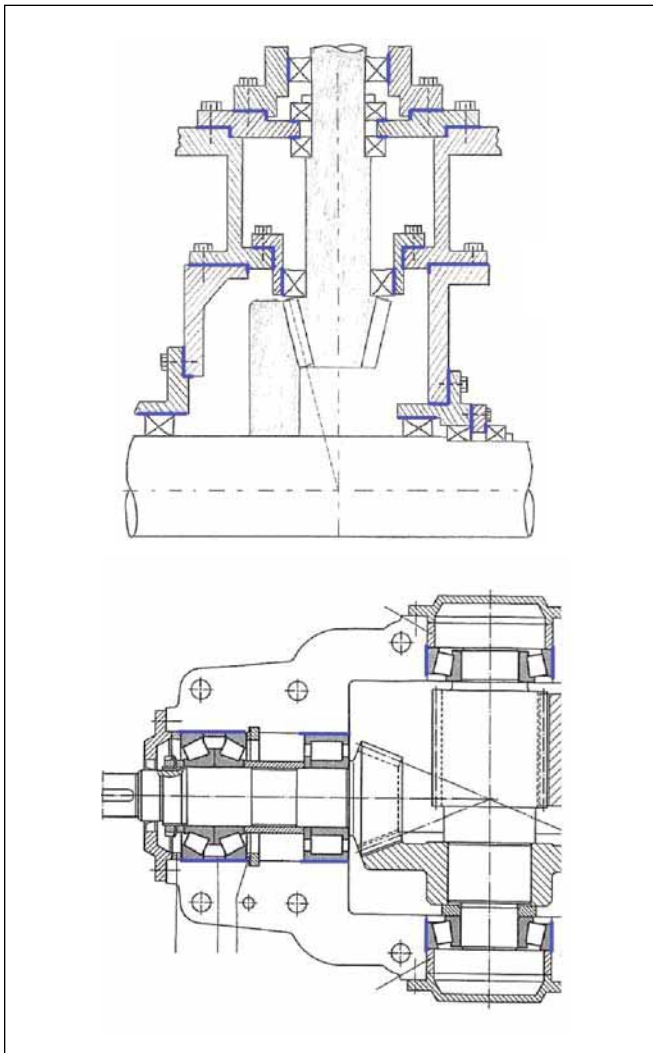


Figure 18—Examples of constructions with large and small number of mounting surfaces (marked with blue).

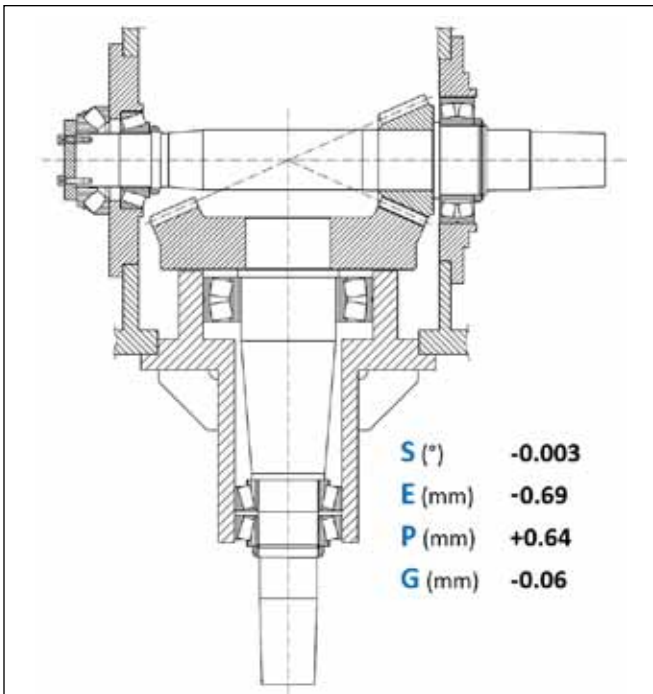


Figure 19 (Example 1)—Basic drive line construction and results of drive line analysis.

assembly clearances.

For each application, probable ranges of the abovementioned variations are determined and tooth contact's sensitivity to them is analyzed. The results of this kind of sensitivity study can either be used to specify appropriate tolerances for the mentioned variations or to modify the tooth geometry to accommodate for known tolerances.

Validation of Analysis Methods

To demonstrate the validity of the described analysis procedures, two actual example cases are presented. The demonstration is done by comparing actual, documented loaded contact patterns to ones determined by LTCA using the calculated, relative displacements. LTCA is performed using *Becal* software (Ref. 4). In *Becal* the simulation of mesh under load is based on a combination of analytical and numerical methods that have been calibrated against experimental data as well as numerical reference calculations (Ref. 5).

In both example cases the relative displacements of gears were determined by the analysis procedures described in this paper and used in LTCA. Actual topography of tooth flanks was measured with a coordinate measuring machine and used in LTCA. The calculation methods of *Becal* require that the measured topography deviations are approximated by a second-order surface. Accuracy of the approximation declines when the deviations form a complex surface. In all of the examples, the deviations were such that the approximation was able to represent reality with reasonable accuracy. To improve the accuracy, areas of the tooth flanks that were concluded to be free of contact were excluded from the surface fitting procedure.

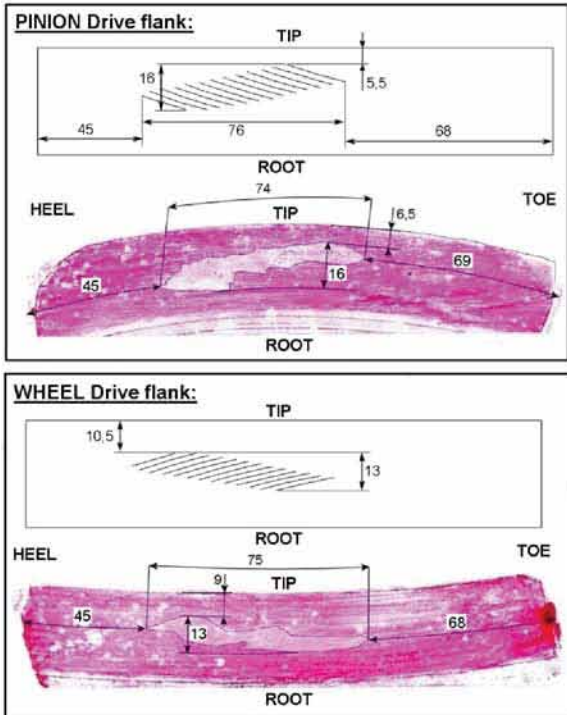
Example 1: Full-torque test of a marine thruster upper gear unit. Actual loaded contact pattern was documented in a full-torque test under quasi-static conditions (slow roll). Temperature differences were nonexistent and therefore omitted from analysis. Basic construction of the gear unit and results of DLA are presented in Figure 19. Comparison of actual and calculated contact patterns showed good agreement (Fig. 20). Note that the contact patterns calculated by *Becal* are presented in radial projection, but all dimensions are given along tooth arc.

The results of DLA were also verified by directly measuring shaft displacements during full-torque testing of another gear drive of the same type. Figure 21 shows that there was good agreement between calculation and reality. The effect of housing deformation on measurements (translation and rotation of the surface to which the dial indicator was attached) was taken into account to enable valid comparison.

Example 2: Endurance test of bevel gears of an industrial gear unit. In this case actual loaded contact patterns were documented during an endurance test of an industrial gear unit (Fig. 22). Comparison to calculated results showed good agreement (Fig. 23). It should be noted that in this case the tooth contact pattern was not yet optimized.

Without consideration of relative displacements, the calculated contact pattern for the drive side would have looked

UNLOADED



LOADED

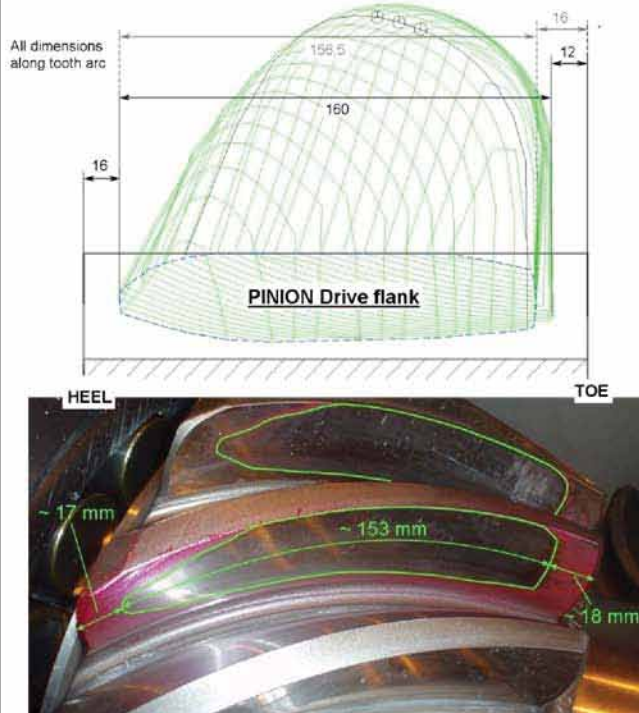


Figure 20 (Example 1)—Comparison of actual and calculated contact patterns.

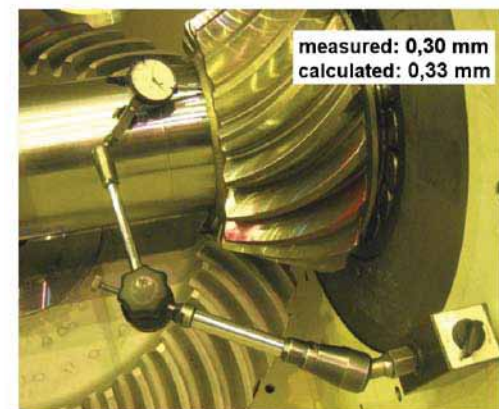
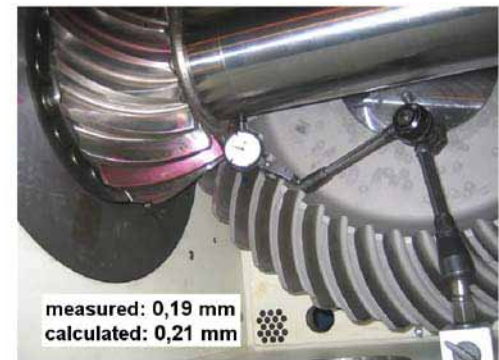


Figure 21—Comparison of measured and calculated shaft displacements.

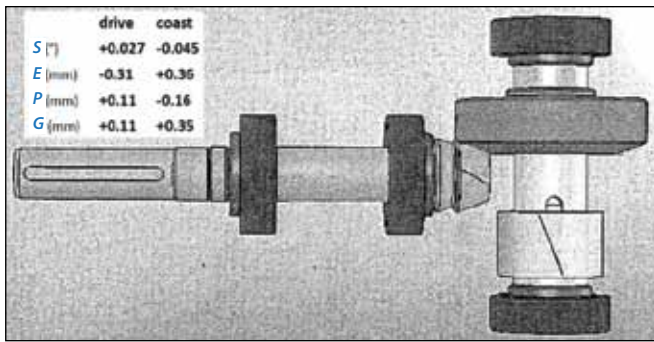


Figure 22 (Example 2)—Basic drive line construction and results of drive line analysis.

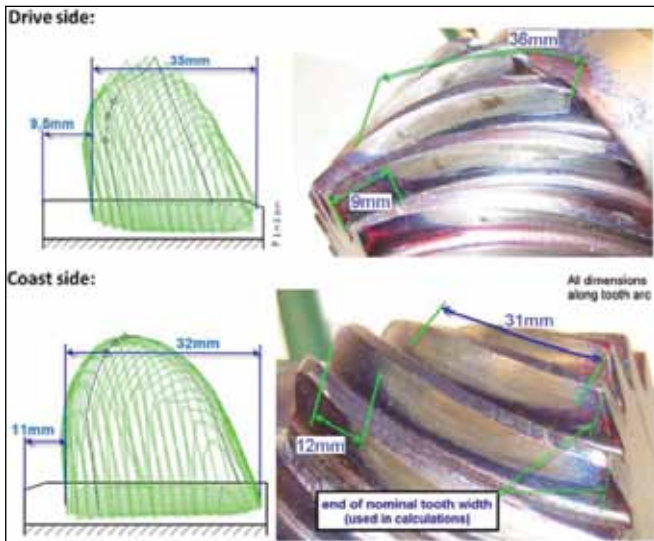


Figure 23 (Example 2)—Comparison of actual and calculated contact patterns under load.

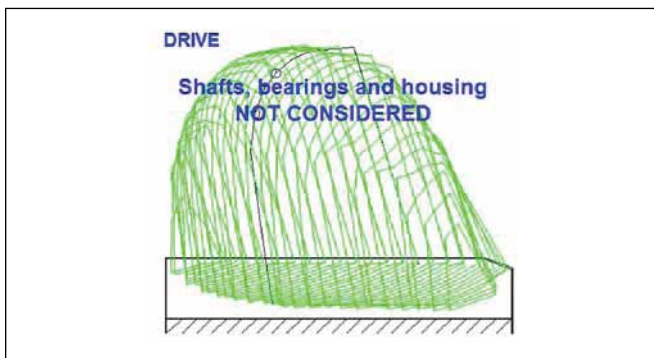


Figure 24—Calculated contact pattern without consideration of relative displacements.

as presented in Figure 24. This illustrates the significance of drive line analysis in this particular case.

Conclusions

In this paper computational analysis procedures for determining relative displacements of spiral bevel gears under load have been presented. The method has been verified by comparison with actual test data.

Significance of the presented factors (e.g., bearing clearances) greatly varies by application. The described analysis method is used to analyze gear drive constructions of different designs. Therefore the goal has been to make the analysis process generally applicable, containing as many of the influencing factors as possible and regardless of their significance.

It is the gear designer's responsibility to assess which factors are relevant, but it has also been seen that many factors with little effect can add up to a significant one. ⚙

Outlook: Subjects for Future Study

1. **Effect of dynamic loading.** The following questions are yet to be answered: Is the quasi-static approach presented in this paper sufficient to represent dynamic situations? How does the relative position of gears vary during vibration? How should application factor K_A and dynamic factor K_v be dealt with in drive line analysis?
2. **Variation of tooth forces.** Currently the tooth forces are considered as components F_t , F_r and F_a that are calculated at d_m using β_m , α_n and δ . Friction is not considered and the tooth forces are assumed to act on the same point at all times. In reality the resultant of tooth forces is comprised of multiple pressure distributions acting on different tooth flanks. Location, direction and magnitude of the resultant vary during mesh; the effect of this variation to gear displacements should be assessed.
3. **Comparison to FEM-based LTCA.** Thus far the validation based on contact patterns has been limited by the accuracy of the *Becal* program. *Becal's* semi-analytical calculation approach results in short calculation times—but also limits accuracy. Another limitation is the second-order surface approximation method used to model actual tooth flank topography. By using more accurate methods—such as a nonlinear FE model—more accurate data could possibly be obtained. In near future, an in-house-developed FE code (Ref. 6) will be used for this task.

Acknowledgements

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