Tooth Root Optimization of Powder Metal Gears: Reducing Stress from Bending and Transient Loads

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This paper will provide examples of stress levels from conventional root design using a hob and stress levels using an optimized root design that is now possible with PM manufacturing. The paper will also investigate how PM can reduce stresses in the root from transient loads generated by abusive driving.

Introduction

By 2020, some 100 million cars with geared transmissions are estimated to be manufactured yearly. If the amount of transmission gears is estimated at 6 kg per vehicle, the total mass for gears will be 600 million kilograms. This amount represents a huge opportunity for powder metal (PM) as a replacement technology with improved system performance. The manufacturing technology inherent to PM offers design freedom and performance advantages not available with wrought machined steel. But introducing the PM-specific geometrical features using traditional machining is very costly, and sometimes impossible on a mass-production scale.

The automotive gear industry is very conservative and often unaware of what can be achieved with PM technology, so this paper is aimed at just one part of the gear — the root — where PM gear design and manufacture can reduce stress levels by introducing design features difficult to obtain using traditional gear manufacturing technologies.

Traditional Gear Hobbing and its Limitations of Root Geometry

The root of the gear — when hobbed — often goes unspecified in the gear drawing; it is indirectly given in the tool drawing and data. The root is a function of the trochoid movements of the hob flutes, gear rotation, and the geometry of the tip of the hob. There are also limitations as to what hob radius might possibly be used, that is given by Equation 1 (Ref. 1):

$$r_{c(max)} = \frac{0.785398\cos\Phi - b\sin\Phi}{1-\sin\Phi} \tag{1}$$

where:

r_c is the hob tip radius
b is the dedendum constant
Φ is the pressure angle

In an effort to quiet the gear mesh, the teeth tend to become more slender and the pressure angle decreases. In order to hob such a gear, short-pitched hobs are used. Figure 1 shows two gear spaces cut with different pressure angles on hobs. The root on the left gear is more favorable from a stress point of view than the root cut on the right gear, where a short-pitched hob has been used. The short-pitched hob has longer service life and is sometimes necessary for cutting gears with smaller pressure angles.

Figure 2 displays the change in root stress for a gear cut with 20° (left) and 11° (right) pressure angle hobs. The peak stress for the gear cut with a 20° pressure angle is 352 MPa versus 405 MPa for the gear cut with an 11° hob.

PM gear technology does not suffer from these limitations in root shape, and the root can therefore be actively designed in coordination with the tool manufacturer.

Root Geometry of Powder Metal Gears

Since no hobbing action is required when making powder metal gears, some of the limitations as well as a number of ISO recommendations regarding root shape can be ignored, and a more

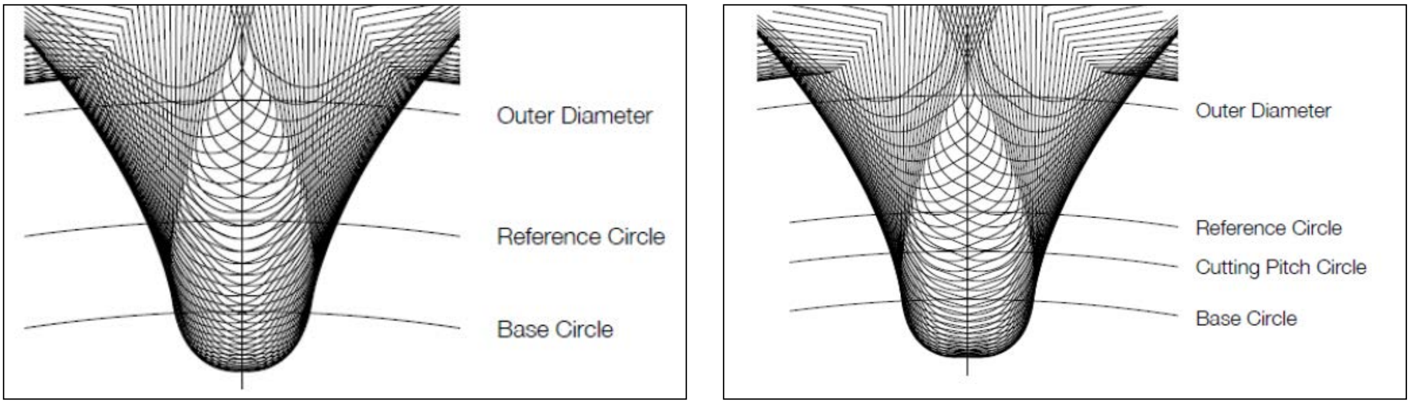

Figure 1 Left: Tooth space cut with 20° pressure angle on hob; Right: Tooth space cut with 11° pressure angle on hob.

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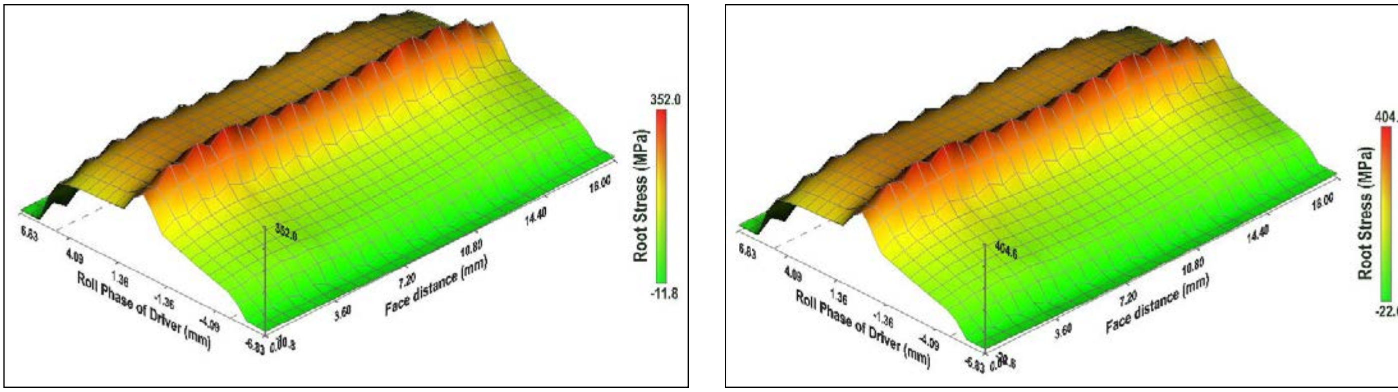

Figure 2 Right: Root stress for gear cut with 20° pressure angle hob; Left: Root stress for gear cut with 11° pressure angle hob (Image courtesy Dontyne Systems).

active design philosophy adopted. In Figure 3 there are 4 different roots depicted — all based on the FZG gear geometry. They are:

- Original 1.99 mm (given by hob tip radius $\rho a_0 = 0.8$ mm)
- Full radius
- Optimized curve shape
- Asymmetric gear tooth

The original 1.99 mm root design is one adopted for tooth root bending fatigue testing internally at Höganäs AB. The first modification is to introduce a large root radius to decrease tooth root stresses. This is done using *KISSsoft's* suggestion for the largest possible root radius that may be manufactured with a hob. The final optimization step is done by hand, applying a progressive transition using a spline in *Pro Engineer*. The result is similar to what is presented by Kapelevich (Ref. 2), with the difference that the latter has an analytical optimization routine. Sanders (Ref. 3) and several others have also investigated this, but since the favorable geometry calculated has been impossible to hob in mass production, it has not gained widespread use. What is of interest here is the manufacturability inherent in PM that allows for efficient mass production of an optimized root.

To investigate the influences of the modifications, both static and dynamic finite element analyses are performed using *Calculix*. For tooth roots the critical stress is very often found to be caused by impacts and abuse, which begs the question of whether the root — when modified for lower static stress — will show a reduction in dynamic stress and how it scales.

Finite Element Models

Figure 4 shows the model for the calculations in which a section of the gear — consisting of five teeth — is cut. On the middle tooth a force is applied on the tip, tangent to the base circle. On the inner hub and cut boundaries, displacements are locked. The calculations are performed under plane strain conditions, with second-order triangular elements (Fig. 5).

Calculations are done with an elastic material model; corresponding material properties are given in Table 1.

Table 1 Material properties for the simulations

Property
Young's modulus, E=160 GPa
Poisson's ratio, ν=0.28
Density, ρ=7.30 g/cm3

In the static calculations, a force is applied (Fig. 4) and stresses are calculated. The force is chosen, arbitrarily, to give a peak stress of 500 MPa for the original gear design. This corresponds to $F = 7.128$ kN; the same force is used for the dynamic calculations. Since the models are linear, all results will scale proportionally with the force. In the dynamic calculation the force is applied as a step corresponding to a sudden impact. The speed of the wave propagation in the gear compared to a typical rotational speed justifies the constant load applied to the tip. This will be discussed further below. The time step in the dynamic calculations is $\Delta t = 0.1\ \mu s$. The asymmetric gear uses a different base radius, so for that gear the loading force has been increased so that the transmitted torque is equivalent to the other gears.

Results

The results from the calculations (Table 2; Fig. 4) demonstrate that an optimized root can reduce even the most optimized machined root by another 5 percent. The authors have inves-

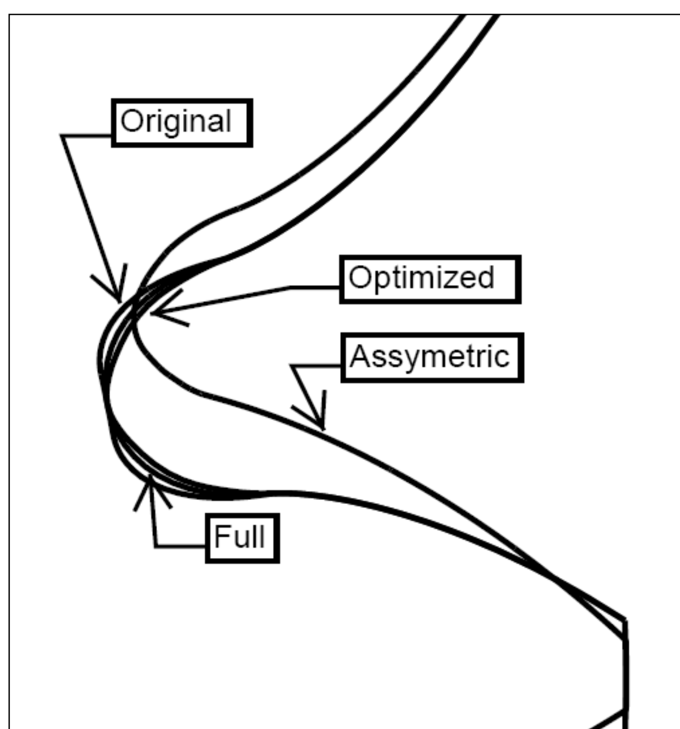

Figure 3 Root geometries compared.

Table 2 Peak stress in tooth root					
Geometry	**Static stress [MPa]**	**Difference**	**Dynamic stress [MPa]**	**Difference**	**Dynamic amplification**
Original	500	-	1010	-	2.02
Max radius	434	-13%	865	-14%	1.99
Optimized	408	-18%	792	-22%	1.94
Asymmetric	405	-19%	719	-29%	1.78

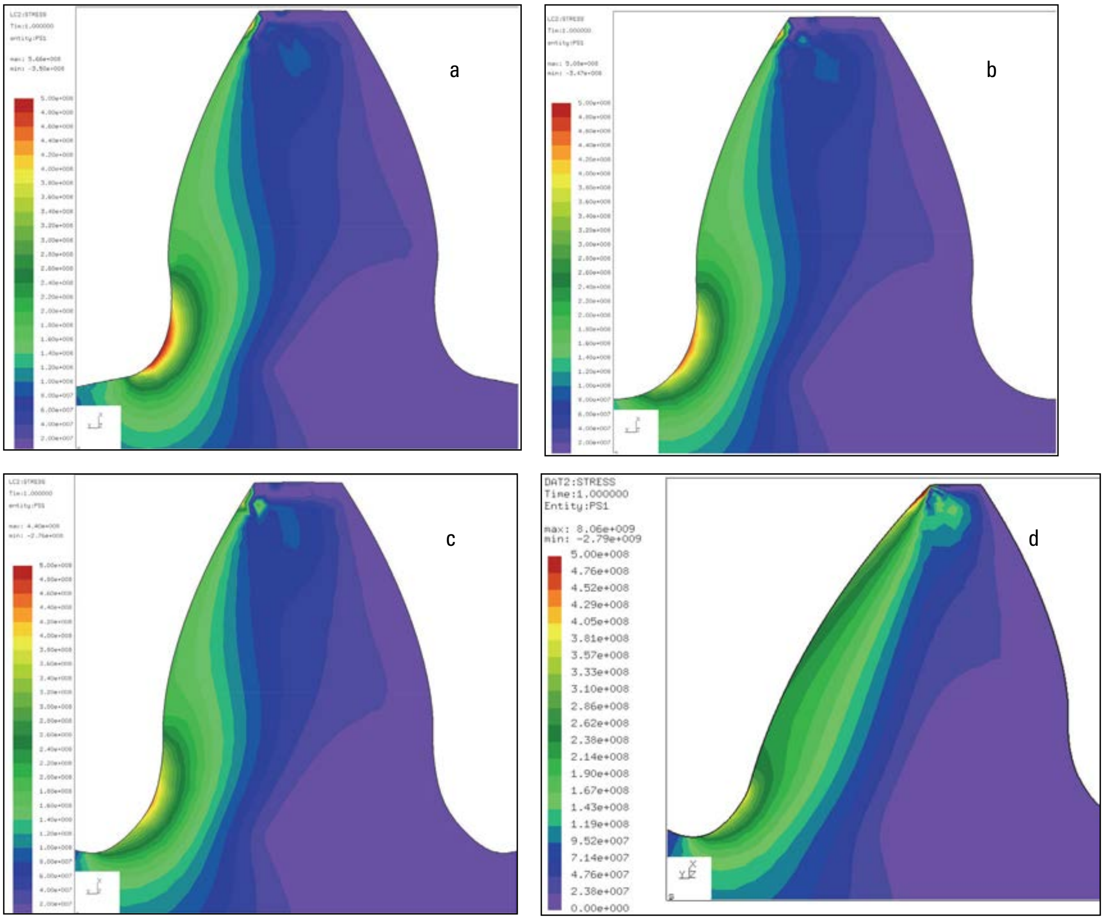

Figure 4 Tooth root stresses/static analysis: a) original; b) maximum radius; c) optimized; and d) asymmetric. (Note: same stress scale for all geometries.)

tigated several other in-production root geometries and found 5–25 percent improvements in root stress reduction. Also, the compression side of the symmetric gears is improved to the same extent as the tension side.

The asymmetric root shows the highest stress reduction; this is not typical for this type of gear teeth but may occasionally be the result. There is always an accuracy error and simplifications made in the model — such as omitting residual stresses and surface roughness — that have to be taken into account when drawing conclusions from the results.

The true benefit of the asymmetric tooth is the *contact stress reduction*, but one has to be careful and balance root stress and contact stress on the coast side if the gear is being used in, for example, an automotive transmission.

The reason that the dynamic amplification factor is improved with reduced stress

Table 3 Gear data			
GEAR		**OPTIMIZED**	**MATING**
NUMBER OF TEETH		16	24
MODULE		4.50	4.50
PRESSURE ANGLE		20°	20°
PITCH DIAMETER (PD)		72	108
BASE DIAMETER		67.6579	101.4868
OUTER DIAMETER		82.46	118.36
ROOT DIAMETER	Current	61.34	98.98
	Optimized	61.11	-
TOOTH TIP RADIUS		0.06	0.06
TOOTH THICKNESS AT PD		7.664	7.630
FACE WIDTH		14	14
MATING GEAR NUMBER OF TEETH		24	16
CENTER DISTANCE		91.500	

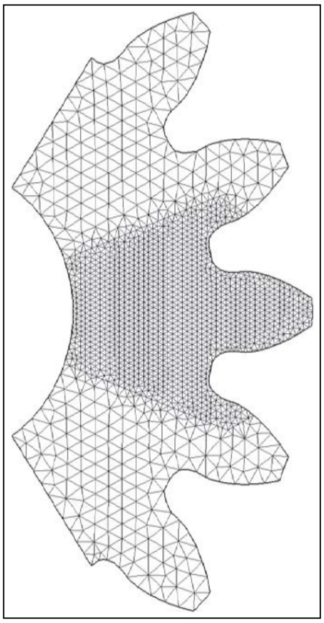

Figure 5 Mesh in FE model.

levels at the root can be found in the dynamics of the gear tooth itself. The Eigen frequency of the gear tooth is elevated when bending stiffness increases, resulting in less sensitivity towards sudden loads.

Conclusions

It has been demonstrated that it is possible to reduce root stress in a gear tooth by replacing the cut trochoid root shape with a curve shape defined by a spline that is designed, iteratively, to reduce root stress. It is also possible to manufacture a gear wheel with this root shape in mass production using PM manufacturing technology. It may not be possible to manufacture this root shape using conventional cutting technology at the same speed as is possible with PM technology.

It was also demonstrated that the dynamic stress levels benefit significantly from a non-trochoid root, and that the amplification factor is reduced as a result of increased tooth stiffness.

Asymmetric gear teeth designed for reduced contact pressure may be designed in such a way that their root stress is reduced to levels where they can operate at similar stress levels as cut gears. Their increased stiffness will improve their dynamic properties.

Discussion

By employing a more active design of the root in particular, and the gear shape in general, a PM transmission gear can be subjected to lower stress than a conventionally cut gear. This can be achieved without sacrificing productivity, provided that powder metal manufacturing technology is used. As a matter of fact, PM is normally the more cost-efficient and significantly less resource-demanding alternative to conventional gear manufacturing.

The next step will be to utilize the PM design advantage in the rebuild of an automotive six-speed manual transmission. The possibility to reduce the stresses in the root will enable a different modulus to be used in the design; i.e., reducing contact pressure by increasing contact ratio — and thus expanding the range of gears that PM can replace.

Where contact stress cannot be kept within the allowable stress levels, asymmetric gearing will be introduced. The challenge will be to balance stresses and NVH when driving on the coast-side of the gear.

Also, the effects tooth stiffness have on dynamic stress amplification can be utilized in the design of other non-involute gear types such as Convoloid gear teeth, asymmetric gear teeth or internal gear teeth.

With these techniques, a PM substitution of the solid steel gears in this particular automotive gearbox will be possible. This work will be presented in another article.

Acknowledgement. *The authors wish to thank* Dr. Alex Kapelevich *for his valuable input on the root optimization and asymmetric gear tooth design. His contributions include his asymmetric gear tooth by Direct Gear Design methodology and corresponding stress calculations. He has also analytically optimized the gear root using this method. The authors used these results to create a similar root design that produced the same stress results that are presented in this paper (Ed.'s Note: Kapelevich's new book,* Direct Gear Design, published by CRC Press, *is now available for online purchase at both CRC Press ($129.95) and Amazon.com ($117.58).*

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Correction

The technical paper in the March/April issue, "Optimization of a Process Chain for Gear Shaft Manufacturing," by Fritz Klocke, Markus Brumm, Bastian Nau and Arne Stuckenberg, was incorrectly credited as being an American Gear Manufacturers Association (AGMA) paper. The work is in fact owned solely by the authors.