Gear Backlash Analysis of Unloaded Gear Pairs in Transmissions

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A best practice in gear design is to limit the amount of backlash to a minimum value needed to accommodate manufacturing tolerances, misalignments, and deflections, in order to prevent the non-driving side of the teeth to make contact and rattle. Industry standards, such as ANSI/AGMA 2002 and DIN3967, provide reference values of minimum backlash to be used in the gear design. However, increased customers' expectations in vehicle noise reduction have pushed backlash and allowable manufacturing tolerances to even lower limits. This is especially true in the truck market, where engines are quieter because they run at lower speeds to improve fuel economy, but they quite often run at high torsional vibration levels. Furthermore, gear and shaft arrangements in truck transmissions have become more complex due to increased number of speeds and to improve efficiency. Determining the minimum amount of backlash is quite a challenge. This paper presents an investigation of minimum backlash values of helical gear teeth applied to a light-duty pickup truck transmission. An analytical model was developed to calculate backlash limits of each gear pair when not transmitting load, and thus susceptible to generate rattle noise, through different transmission power paths. A statistical approach (Monte Carlo) was used since a significant number of factors affect backlash, such as tooth thickness variation; center distance variation; lead; runout and pitch variations; bearing clearances; spline clearances; and shaft deflections and misalignments. Analytical results identified the critical gear pair, and power path, which was confirmed experimentally on a transmission. The approach presented in this paper can be useful to design gear pairs with a minimum amount of backlash, to prevent double flank contact and to help reduce rattle noise to lowest levels.

Introduction

Many components inside a truck transmission, such as gears, synchronizers, and sliding sleeves, can move to a certain extent in the axial, radial, and circumferential directions. These components may rattle as excited by time-varying torsional vibration that comes from internal combustion engines, such as diesel-powered engines. Gears are susceptible to rattle and are a major contributor to the overall rattle noise level in transmissions. The meshing gears that are unloaded may often leave contact, moving freely back and forth within the backlash as torque fluctuates and causing repetitive impacts. The vibro-impact dynamics induced by gear backlash leads to vibration, rattle noise, and dynamic loads (Ref. 1). An effective approach to reduce rattle noise on transmissions is to lower the amount of gear backlash (Refs. 2–3).

Gear backlash is the clearance between a pair of teeth in mesh (Fig. 1), and can be described as the freedom of one gear to move around its axis while the mating gear is held stationary (Ref. 4). Backlash is required to ensure no contact of the non-driving side of the teeth, accommodate lubricating film on the teeth, and prevent tooth binding under running conditions (Ref. 5). When designing gear pairs, backlash is incorporated into the gear mesh by changing center distance between gears, or most commonly, by reducing tooth thickness of one or both gear members; this results in a tooth space width that exceeds the thickness of the engaging tooth on the operating pitch circles (Refs. 4 and 6). Industry standards, such as ANSI/AGMA 2002 and DIN 3967, and gear handbooks (Refs. 4–5), provide minimum values of design backlash as a function of gear module (or diametral pitch) and center distance. However, the actual backlash depends on other system elements and operating conditions. Manufacturing tolerances of gears, housings, shafts, bearing clearances, and other components cause the amount of backlash to vary considerably. Also, thermal expansions at operating temperatures, and elastic deflections caused by transmitted loads, result in additional backlash variation. This is difficult to predict in multispeed transmissions of numerous gear pairs, shafts, and bearings. Center distance of gears carrying load increases due to the radial component of force, resulting in larger backlash. However, a more complex shaft deflection shape and magnitude may cause tooth binding

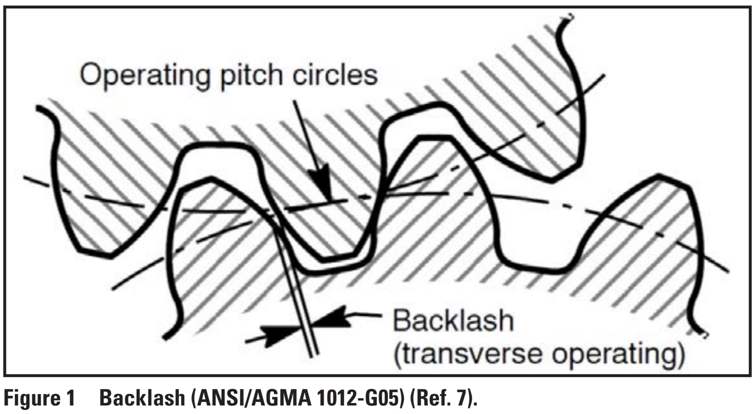

Figure 1 Backlash (ANSI/AGMA 1012-G05) (Ref. 7).

of meshing gear pairs that are unloaded. This condition may be easily overlooked in the gear design and transmission development process.

Determining the dimensions and tolerances of gears and other system components that result in minimum amount of backlash and gears performing properly, while keeping reasonable cost in a production setting, is quite a challenge. This paper presents an investigation of actual backlash of unloaded meshing gear pairs of a pickup truck transmission with six forward speeds. An analytical model was developed to determine the actual backlash of each meshing gear pair not when transmitting torque, but when subject to shaft deflection effects of other gears in the power path. A statistical approach Monte Carlo was used, considering key factors that affect backlash, such as tooth thickness variation, center distance variation, runout, pitch and lead variations, bearing clearances, spline clearances, and shaft deflections. Analytical results were verified experimentally on a transmission for the critical gear pairs and 3 power paths. The approach presented in this paper can be useful to design gear pairs with minimum amount of backlash in order to prevent double flank contact and help reduce rattle noise to lowest levels.

Gear Backlash Calculation

Backlash in an assembled gear pair may be calculated in the transverse plane by taking the difference between the width of the tooth space and the tooth thickness of the engaging tooth on the operating pitch circles. Hence, backlash is governed by the center distance at which the gears are operated and by the effective tooth thickness of each of the gears (Ref. 6).

Operating pitch diameters are determined from the numbers of teeth, and the center distance at which gears operate and can be calculated as (Ref. 7):

$$d_{w1} = \frac{2a}{u+1}, d_{w1} = 2a - d_{w1} \tag{1}$$

where, d_{w1} and d_{w2} are the operating pitch diameter of the pinion and gear respectively, a is the center distance, and $u = z_2/z_1$ is the gear ratio.

Circular tooth thickness in the transverse plane at the operating pitch diameter is calculated from a given tooth thickness at pitch circle diameter, as follows:

$$s_{wt1,2} = d_{w1,2}\left[\frac{s_{t1,2}}{d_{1,2}} + (\text{inv}\,\alpha_t - \text{inv}\,\alpha_{wt})\right] \tag{2}$$

where s_{wt} is the circular tooth thickness in the transverse plane at the operating pitch diameter, subscripts 1 and 2 designate pinion and gear, respectively, s_t is the circular tooth thickness in the transverse plane at the generating pitch diameter, d, and are the involute angle at the generating pitch diameter and operating pitch diameter, respectively, and are calculated as $inv\ \alpha = tan\ (\alpha)\ \text{-}\alpha$. The transverse pressure angle at the operating pitch diameter is calculated as $cos(\alpha_{wt}) = d_b/d_w$, where d_b is the base circle diameter.

Transverse backlash at the operating pitch diameter can be calculated by Equation 3:

$$J_t = \frac{\pi d_{w1}}{z_1} - s_{wt1} - s_{wt2} \tag{3}$$

This same set of equations can be used to calculate the actual backlash when the center distance, a, and tooth thickness, s_t, are replaced by an effective center distance, a_e, and effective tooth thicknesses, S_{et}.

The effective center distance provides for:

- Deflections of housing, shafts, and bearings under load
- Displacements of shafts and bearings due to manufacturing deviations and bearing clearances
- Displacements of gear axes due to radial clearance of splined joints
- Differential expansion due to changes in temperature

In addition to the tooth thickness tolerances, the effective tooth thickness also provides for:

- Mesh misalignment of gears due to elastic deflections of housing, shafts, and bearings
- Skew of gear axes due to housing deviations and bearing clearances
- Gear tooth deviations, such as runout, profile, lead, and pitch deviations

The factors listed above are based on Appendix A of ANSI/AGMA 2002, which underlines the importance of good judgment and experience to assess minimum expected backlash requirements, since the worst-case tolerances are not likely to coincide (Ref. 6). Thus, a statistical analysis is a sound approach to determine the actual backlash range considering the significant number of factors involved.

Statistical Analysis

The backlash range can be statistically determined using the Monte Carlo method, which simulates thousands of variable combinations, and then averages the results and draws statistical conclusions. The combinations are generated from random numbers selected within the design allowable variation and a characteristic statistical distribution of each factor.

The simulation procedure can be broken down into these steps (Ref. 8):

i. Fit gear production data to an appropriate distribution function; goodness-of-fit statistics are used to determine an acceptable model
ii. Assign a distribution function to each variable in the backlash equation and upper and lower limits of each of the factors of interest
iii. Generate random variables from each of the distributions
iv. Calculate backlash using the random variables for each factor
v. Predict the estimation error and adjust sampling size, if needed

The acceptance criteria of the Monte Carlo simulation result are given by:

$$\frac{3\sigma}{\mu\sqrt{N}} \le \varepsilon \tag{4}$$

where ε is an acceptable error, μ and σ are the mean and standard deviation of the backlash results, respectively, and N is the number of iterations.

Case Study: pick-up truck 6-speed transmission. As a case study the Monte Carlo model of backlash calculation was implemented into an *Excel* spreadsheet to determine the actual backlash range of a pick-up truck transmission with six forward speeds (Fig. 2).

This 6-speed transmission is rated to 440Nm input torque for 5.5-ton vehicles. The main shaft and countershaft are both supported by three bearings. All gears are helical, with helix angles from 19 to 33 degrees. Gear flanks are hard-finished, except for reverse gears.

A complete transmission model was built into *RomaxDESIGNER* software to calculate elastic deflections and mesh misalignments under load for each transmission power path. These include housing, shafts, and bearing deflections. The deflections and misalignments under different torque levels were transferred to the Monte Carlo model in the *Excel* spreadsheet.

Random numbers of each variable were generated, assuming normal distributions, in order to create 25,000 iterations. The estimation error of the Monte Carlo simulation was set to less than 1%.

The statistical backlash calculation was done at three torque levels and in two driving conditions — drive and coast. The latter simulates a truck going downhill using its engine brake.

The following factors were included in the backlash analysis:

- Circular tooth thickness tolerance
- Gear runout
- Lead variation (*fHb*)
- Index deviation (*fp*)
- Needle bearing clearance variation
- Spline radial clearance variation
- Gear mesh misalignment due to deflections
- Shaft deflections
- Housing deflections

Bearing stiffness

Housing manufacturing variations, such as true position of housing bores, were assumed to have a minor effect on backlash because of their small magnitude over a large span between bearing supports; as such, they were not included in the analysis. Tapered roller bearing clearance variations were not considered, as countershaft bearings are pre-load-mounted. Gear profile slope variations were assumed as negligible compared to other factors. Thermal expansion effects were not included in the analysis.

After running the statistical analysis to each gear pair in all transmission gear speeds (power paths), results identified the reverse gear pair (when running in the sixth speed) and drive torque condition to be among the lowest-expected backlash results.

A sensitivity study was then carried out for that specific gear pair and condition in order to delineate the contribution of each individual parameter variance on backlash variability within the expected ranges of the parameters. The study was done by allowing one variable to vary according to its specified distribution, while holding all other variables constant.

The sensitivity study results are shown in Figure 3, using a tornado plot. The results are normalized to the mean value of expected backlash, which is shown at the top of the chart for reference.

The deflections of shafts, housings, and bearings play a significant role in the overall backlash result. For that particular gear pair, when in 6th speed and drive condition, the deflections bring reverse gears closer to each other, resulting in smaller effective center distance and, consequently — less backlash. As torque goes up, deflections increase, and backlash of reverse gears goes down.

The tooth thickness variation, followed by spline radial clearance and pitch circle diameter runout, also significantly affects the actual backlash for that particular condition.

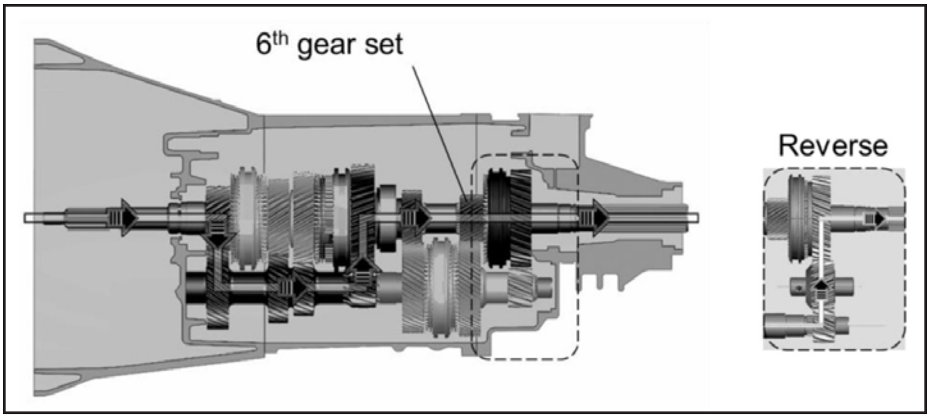

Figure 2 Pick-up truck transmission.

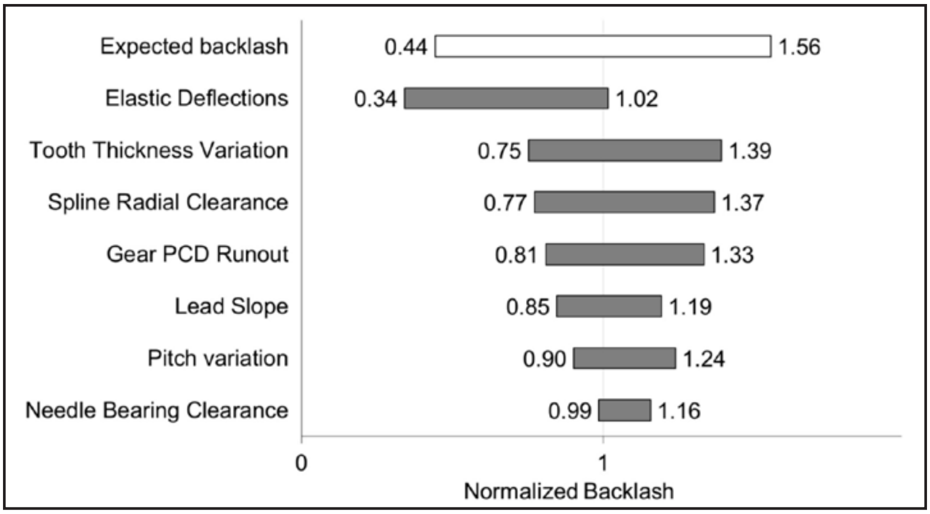

Figure 3 Sensitivity study results.

Experimental Verification

To verify the calculation results a transmission unit was built with components made to the design dimensions. A special housing was prepared in which a small window was opened around the reverse idler gear location (Fig. 4). It was assumed that changes of housing stiffness due to the small windows are negligible.

The transmission unit was fixed to a test rig, where two specific static tests were performed in order to verify the critical gear pair and condition identified on the backlash study.

The transmission was shifted into sixth

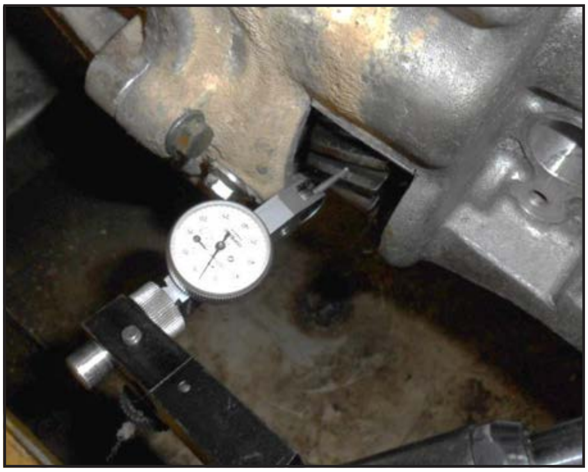

Figure 4 Backlash measurement of reverse idler gear.

gear speed, and torque was applied to the transmission input shaft. While holding the torque, gear backlash was measured using a dial test indicator with its tip positioned on a reverse idler gear tooth. This procedure was repeated at no load, half design load, and design load conditions.

Figure 5 shows a comparison of measured backlash results and expected results, using the Monte Carlo approach. All three data points fall within the expected distribution. Measured backlash at half design load and at the design load are close to the expected mean, with 69% and 68% of cumulative distribution, respectively. However, under no load, the measured backlash is closer to the upper limit of the normal distribution curve. The difference may be caused by clearances of other components in the sixth-speed power path, which may have allowed the reverse geartrain to slightly rotate during the backlash measurement process, causing that rotation to be captured as gear tooth backlash. Under torsional load condition those clearances are eliminated, thus making the backlash measurement process more consistent.

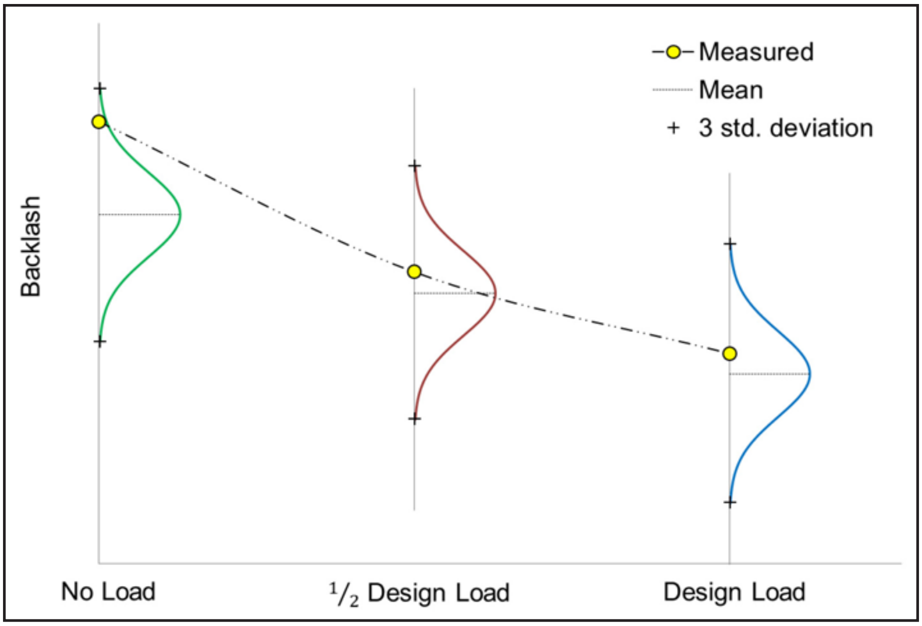

Figure 5 Measured backlash compared to expected values.

Discussion

The actual backlash of gear pairs is a result of the effective center distance at which the gears operate and the effective tooth thickness of each gear member. The effective center distance and the effective tooth thickness should be carefully determined at the system level, including important factors such as deflections of shafts; housing and bearings; misalignments and clearances of bearings and spline couplings; and tooth deviations, such as lead, runout and pitch variation. This is important when defining the right amount of tooth thickness reduction in the gear design that will attain reduced rattle noise levels while preventing the non-driving side of the teeth to make contact. Regarding adverse operating conditions, special attention should be given to the backlash of unloaded gear pairs when transferring power through other gear pairs, since the resulting deflections may lead to critical backlash results of gears that are unloaded, as shown in the case study. The approach using Monte Carlo simulation worked well to determine the expected gear tooth backlash and delineate the contribution of each individual parameter on backlash variability, which can be helpful to choose suitable manufacturing tolerances for gears and other parts in the assembly. The approach presented in this paper can be used to design gear pairs with minimum amounts of backlash to prevent double flank contact, and help reduce rattle noise to lowest levels.

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Carlos Wink (Ph.D., Master's degree mechanical engineering with gear design focus — University of Campinas, Brazil; B.S. mechanical engineering — Santa Cecilia, Brazil.) is a recognized expert in gear technology and power transfer, with extensive and global experience in developing mechanical power transfer products, such as automotive and commercial vehicle transmissions, gearboxes, rotary actuators for aerospace applications, progressive differentials, and gear pumps. As an engineering manager for Eaton Corp.'s Center of Excellence for components, he leads two teams of experienced engineers — one based here in the states, the other internationally. During his more than 25 years with Eaton, Wink has held positions in both manufacturing and product engineering, and has been instrumental in successfully bringing products from a blank sheet of paper into production. His technical expertise has helped to significantly elevate Eaton's technical capability globally by introducing key advanced modeling techniques and tools and standardized engineering processes. A founder and leader of Eaton's Gear Community of Practice, Wink also guides Eaton's participation in The Ohio State University GearLab, AGMA membership, and SAE L. Ray Buckendale Lecture Committee. The author of a dozen technical papers, Wink is also the inventor of two patents and serves as a reviewer for three internationally renowned engineering journals.