Designing Very Strong Gear Teeth by Means of High Pressure Angles

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The purpose of this paper is to present a method of designing and specifying gear teeth with much higher bending and surface contact strength (reduced bending and surface contact stresses). The primary means of achieving this is by specifying gear teeth with significantly higher pressure angles. This paper will show calculation procedures, mathematical solutions, and the theoretical background and equations to do this. The required user input factors for the method described in this paper are: numbers of teeth, pinion and gear; diametral pitch; center distance; desired minimum top land, pinion and gear; desired contact ratio; and maximum backlash. The desired pressure angle can then be entered and another value re-entered to make comparisons. The output factors would be the outside diameters of the pinion and gear, and other gear data, based on the entered pressure angle. In the past, higher pressure angle gears have not been commonly designed and specified because of the relative difficulties involved in designing them and the lack of appropriate and easy-to-use tools to evaluate them. This paper contributes to making it easier to accomplish this task.

(The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.)

Introduction and Background

By convention, commonality of cutting tools, and familiarity, several pressure angle choices have become industry standards for gear designs. These include 14.5, 20, 22.5 and 25° pressure angles. Twenty and 25° pressure angle gears are typically the most commonly used today, although pressure angles as high as 28° to 30° have been used on occasion for maximum-strength applications.

Higher pressure angle designs were typically not considered or used because of the limitations listed above, the most significant being reduced gear teeth top lands and fillet radius. Standard gear tooth proportions and universal rack geometry are based on the pressure angles listed above and do not lend themselves to designing gears with pressure angles above around 30°.

For higher pressure angle gear designs, numbers of teeth and other gear tooth macro-geometry items must be considered carefully so that the natural limitations of high pressure angle gearing are avoided while exploiting their benefits.

If not carefully designed, manufacturing the gears can be difficult and more expensive. Gear cutting tools will usually be non-standard. Gear noise will usually be higher due to the decreased contact ratio and reduced root/tip clearance, which can entrap oil.

Separating force from the gear teeth increases directly with the tangent of the pressure angle. In parallel shaft assemblies, this needs to be considered, as the increased forces arising from high pressure angle gears may unacceptably shorten shaft or bearing life. In planetary gearboxes, the net separating force is theoretically zero: the sun/planet separating force is canceled out by the planet/ring separating force. Therefore, planetary applications offer an excellent opportunity to utilize higher pressure angle gear teeth.

The advantage of the methods described in this paper is that gear teeth and gear tooth data of various different pressure angles can be calculated, compared and evaluated quickly and easily, and high pressure angle gears can be designed and specified. Gears can be designed based on desired top lands, contact ratio, and hob tip radius.

Discussion

As stated above, two of the primary attributes of high pressure angle gears that are potentially limiting factors and therefore must be considered, calculated, and accounted for are: 1) circular tooth thickness at the outside diameter of the gear (top land thickness); and 2) hob tip radius and the resulting generated tooth root fillet radius; both of these are considered — each in a separate section.

In this report four separate gear tooth pairs are evaluated and described:

1. Traditionally designed gearset with 25° pressure angle serving as reference design
2. High pressure angle gearset with hob tip radius of zero and 36° pressure angle
3. High pressure angle gearset with a 0.007 inch hob tip radius and 35° pressure angle
4. High pressure angle gearset with a 0.020 inch hob tip radius and 33.5° pressure angle

To ease in the evaluation and comparison, and to isolate the pressure angle influence on the results so that pressure angle is the main variable, the following inputs remain the same for both the traditional 25° pressure angle design and the high (maximized) pressure angle design:

Numbers of gear and pinion teeth; diametral pitch; center distance (standard center distance); the circular tooth thickness of pinion and gear for the high pressure angle gears is equal and is set by the backlash. The 25° pressure angle reference design uses 15% long addendum pinion and 15% short addendum gear tooth proportions and the resulting gear tooth circular tooth thicknesses to provide close to equal bending stresses for the pinion and gear. This represents a typical relatively standard gear design.

Program user inputs

- Number of teeth, pinion and gear: z_1, z_2
- Diametral pitch: P_d
- Center distance (default = standard): a
- Desired minimum top land, pinion and gear: T_{oP}, T_{oG}
- Desired contact ratio: m_c
- Maximum backlash: B

Summary of user inputs for example of high pressure angle gear set run

- 16 tooth pinion, 29 tooth gear
- 6 diametral pitch
- 3.75 inch center distance (standard)
- Desired minimum top land, pinion and gear: 0.030 inch
- Desired contact ratio: 1.15
- Maximum backlash: 0.0120 inch

Definition of terms (Note: Subscripts P = pinion; G = gear, where necessary.)

z_1, z_2 = Number of teeth
D_{bP} = Base diameter, pinion
D_{bG} = Base diameter, gear
P_d = Diametral pitch
D_{oP} = Outside diameter, pinion
D_{oG} = Outside diameter, gear
d = Pitch diameter
t_o = Circular tooth thickness at outside diameter (or any known diameter)
a = Center distance
p_b = Base pitch
D_R = Root diameter
Z = Length of line of action
m_c = Contact ratio
ϕ = Pressure angle
ϕ_o = Pressure angle at outside diameter (or any known diameter)
s = Circular tooth thickness at standard pitch diameter

Calculations for pressure angle corresponding to input top land (tooth thickness at O.D.) and contact ratio, m_c

Mathematical solution:

$$D_o = \frac{d\cos\phi}{\cos\left(\operatorname{arcinv}\left(\frac{s}{d} - \frac{t_o}{D_o} + \operatorname{inv}\phi\right)\right)} \tag{1}$$

$$M_c = \frac{P_d\left(\sqrt{\left(\frac{D_{oP}}{2}\right)^2 - \left(\frac{D_{bP}}{2}\right)^2} + \sqrt{\left(\frac{D_{oG}}{2}\right)^2 - \left(\frac{D_{bG}}{2}\right)^2} - a\sin\phi\right)}{\pi\cos\phi} \tag{2}$$

Standard gear equations used:

$$d = \frac{z}{P_d}$$

$$D_b = d\cos\phi$$

$$p_b = \frac{(\pi\cos\phi)}{P_d}$$

$$s = \frac{\pi}{2P_d} - \frac{B}{2}$$

Calculation procedure. Using input information and standard gear equations above, calculate d, D_b, and s for pinion and gear. Since pressure angle is the unknown, create an initial trial value and substitute Equation 1 (above). Because the D_o term appears in the denominator as well as being the unknown variable solved for, the denominator value is set to the standard D_o value. Because the contribution of this term is small, the potential "error" that this causes is negligible to the end calculation. Calculate the outside diameter D_o for both pinion and gear from Equation 1.

Next, substitute the D_o values calculated from Equation 1 into Equation 2, and calculate the contact ratio at that pressure angle. Iterate the pressure angle and calculate using Equations 1 and 2 to converge on the desired contact ratio that was input. The result is the pressure angle necessary to achieve the desired contact ratio, top land, and also meet other input conditions.

Derivation of Equations

Outside diameter. Since top land is known by user input and circular tooth thickness s is known by calculation from inputs or as a separate input, Equation 3 can be solved for D_o:

$$t_o = D_o\left(\frac{s}{d} + \operatorname{inv}\phi - \operatorname{inv}\phi_o\right) \tag{3}$$

Solve Equation 3 for $\operatorname{inv}\phi_o$:

$$\operatorname{inv}\phi = \frac{s}{d} - \frac{t_o}{D_o} + \operatorname{inv}\phi \tag{4}$$

The involute function is $\operatorname{inv}\phi = \tan\phi - \phi$ radians. If the numeric value of $\operatorname{inv}\phi$ is known and the angle is desired, this contains a transcendental term, and the solution must be iterated using the Newton method; the result is called "ARCINV." Look up or solve for the angle of the ARCINV.

$$\cos\phi_o = \left(\frac{d\cos\phi}{D_o}\right) \tag{5}$$

$$\therefore D_o = \left(\frac{d\cos\phi}{\cos\phi_o}\right) \tag{6}$$

Substituting Equation 4 into Equation 6:

$$D_o = \frac{d\cos\phi}{\cos\left(\operatorname{arcinv}\left(\frac{s}{d} - \frac{t_o}{D_o} + \operatorname{inv}\phi\right)\right)} \tag{7}$$

Contact ratio:

$$m_c = \left(\frac{z}{P_b}\right) \tag{8}$$

But Equation 9:

$$p_b = \left(\frac{\pi\cos\phi}{P_d}\right) \tag{9}$$

and Equation 10:

$$Z = \sqrt{\left(\frac{D_{oP}}{2}\right)^2 - \left(\frac{D_{bP}}{2}\right)^2} + \sqrt{\left(\frac{D_{oG}}{2}\right)^2 - \left(\frac{D_{bG}}{2}\right)^2} - a\sin\phi \tag{10}$$

Combining Equations 8, 9 and 10 gives:

$$m_c = \frac{P_d\left(\sqrt{\left(\frac{D_{oP}}{2}\right)^2 - \left(\frac{D_{bP}}{2}\right)^2} + \sqrt{\left(\frac{D_{oG}}{2}\right)^2 - \left(\frac{D_{bG}}{2}\right)^2} - a\sin\phi\right)}{\pi\cos\phi} \tag{11}$$

Any other pressure angle can be entered and another value re-entered to make comparisons.

A spreadsheet or computer program can be created from the equations and formulas included in this paper to automate the mathematical calculations.

As stated above, the result of this convergence routine shows the pressure angle for the gearset above with the design input values, considering the input value for top land thickness.

However, as also stated above, the second important consideration is the hob tip radius and the root fillet radius it generates for both the pinion and gear. The pressure angle for this consideration is almost certainly different from the pressure angle considering the pinion and gear top land only. So, in order for a gear design to meet both design criteria (matching the top lands and hob tip radius/tooth root fillet radius), the smaller of the two pressure angle values must be used in the design of the gear teeth.

All gear data and stress calculations were performed using the *AGMA GRS 3.1.7* gear rating suite program. Due to the length of the program data outputs, the data was summarized and shown in a more abbreviated format.

Result of calculation for pressure angle from example from top land consideration:

Pressure angle from top land input = 35°

Calculations for Pressure Angle Corresponding to Input Hob Tip Radius Consideration

Mathematical solution. Program inputs in addition to the inputs from the example above:

CSW = (circular space width of gear) which is also the circular tooth thickness of the hob tooth

b = dedendum of gear (addendum of hob tooth)

r = (hob tip radius)

Summary of User Inputs for Example of High Pressure

Angle Gearset Run for Hob Tip Radius Calculations

CSW = 0.2663 inch

b = 0.1833 inch

r = 0.020 inch

$$2\left(\left(\frac{r}{\sin\phi}\right) - r + b\right)\tan\phi = \mathrm{CSW} \tag{12}$$

$$r\left[\frac{1-\sin\phi}{\cos\phi}\right] = \left(\frac{\mathrm{CSW}}{2}\right) - b\tan\phi \tag{13}$$

$$r = \frac{\left[\left(\frac{\mathrm{CSW}}{2}\right) - b\tan\phi\right]\cos\phi}{1-\sin\phi} \tag{14}$$

$$r = \frac{\left[\left(\frac{\mathrm{CSW}\cos\phi}{2}\right) - b\sin\phi\right.}{1-\sin\phi} \tag{15}$$

When $r = 0$:

$$\text{numerator} = \left(\frac{\mathrm{CSW}\cos\phi}{2}\right) - b\sin\phi = 0$$

$$\frac{\mathrm{CSW}\cos\phi}{2} = b\sin\phi \tag{16}$$

$$\tan\phi = \frac{\mathrm{CSW}}{2b} \tag{17}$$

$$\phi = \tan^{-1}\left(\frac{\mathrm{CSW}}{2b}\right) \tag{18}$$

This is the pressure angle for a hob tip radius of 0 (sharp corner).

Calculation method: using the values for r, b, and CSW, iterate the pressure angle and find the pressure angle that matches the input hob tip radius using Equation 15. Also, Equation 18 can be used to find the pressure angle that matches a hob tip radius of zero.

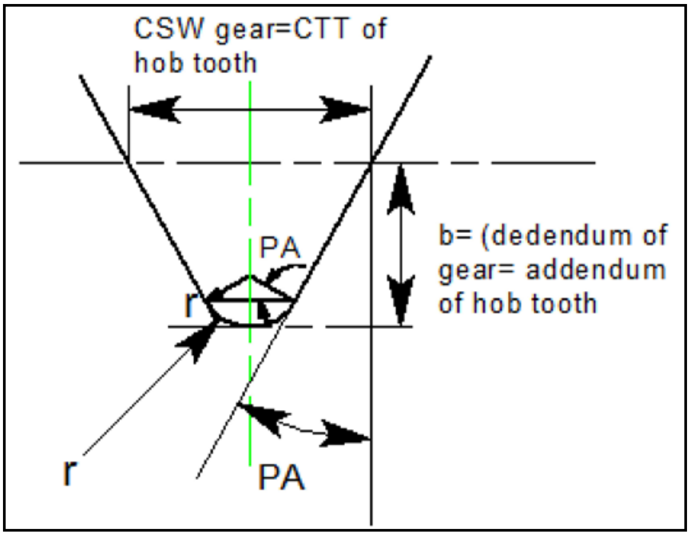

Figure 1 **Hob tooth details.**

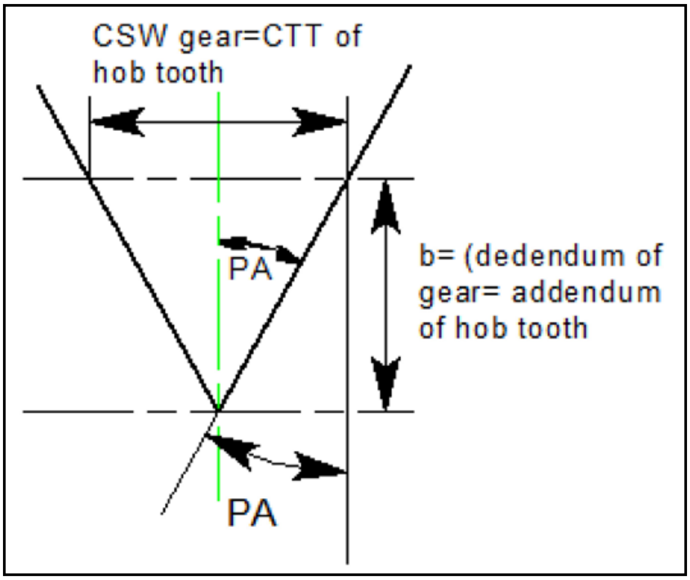

Figure 2 **Hob tooth details for sharp corner (zero hob tip radius).**

Once a pressure angle is calculated for the desired hob tip radius, enter this pressure angle into Equation 1 to determine the outside diameters for the pinion and gear, and into Equation 2 to determine and calculate the new contact ratio — which most likely will differ from the earlier input value. The remaining gear data and stress analysis calculations can be performed using these new values of pressure angle and outside diameters.

Result of calculation for pressure angle from example for hob tip radius:

For this example we want to avoid a zero hob tip radius, and we will set the hob tip radius to 0.020 inch and calculate the pressure angle of the gear teeth that will produce this 0.020 inch radius. A pressure angle of 33.5° is the calculated number, so a 33.5° pressure angle will be used for the further calculations of the remaining gear data and stress and rating analysis.

For a hob tip radius of zero (sharp corner), the pressure angle for the gear teeth is 36°.

SPUR GEAR SPECIFICATIONS

oerlikon fairfield

PART NUMBER	25 PA Pinion		25 PA Gear
---BASIC DATA ---			
NUMBER OF TEETH	16		29
RATIO		1.813 -- Hunting Tooth Ratio	
DIAMETRAL PITCH NORMAL		6.000	
CIRCULAR PITCH NORMAL		0.5236	
PRESSURE ANGLE NORMAL		25.000°	
HELIX ANGLE AND HAND	SPUR	0.000°	SPUR
LEAD	∞		∞
DIAMETRAL PITCH TRANSVERSE		6.000	
CIRCULAR PITCH TRANSVERSE		0.5236	
PRESSURE ANGLE TRANVERSE		25.000°	
PITCH DIAMETER (CUT)	2.6667		4.8333
BASE CIRCLE DIAMETER	2.4168		4.3805
OUTSIDE DIAMETER	3.0500		5.1167
ANGLE OUTSIDE DIAMETER	44.1071°		34.5851°
ROOT DIAMETER	2.2667		4.3333
ROOT CLEARANCE	0.0583		0.0583
ADDENDUM MODIFICATION	15.000%		-15.000%
ADDENDUM	0.1917		0.1417
WHOLE DEPTH (STANDARD)	0.3917		0.3917
FACE WIDTH	2.0000		2.0000
----TOOTH SIZE----			
CIR TKS OPERATING NORMAL	0.2821 -- 0.2791		0.2355 -- 0.2325
CIR TKS CUT TRANSVERSE	0.2821 -- 0.2791		0.2355 -- 0.2325
CIRCULAR TKS CUT NORMAL	0.2821 -- 0.2791		0.2355 -- 0.2325
CIR TKS AT OD NORMAL	0.0670 -- 0.0640		0.0930 -- 0.0900
MEASUREMENT OVER PINS	3.1018 -- 3.0965		5.1760 -- 5.1699
FMC STANDARD PIN/BALL	0.28800 PINS		0.28800 PINS
PIN/BALL CONTACT DIAMETER	2.6777 -- 2.6733		4.7736 -- 4.7682
CIR SPACE WIDTH NORMAL	0.2445 -- 0.2415		0.2911 -- 0.2881
----OPERATING DIMENSIONS----			
CENTER DISTANCE OPERATING		3.7500	
CENTER DISTANCE STANDARD		3.7500	
PRESSURE ANGLE TRANS. OPERATING		25.0000°	
ANGLE OPERATING PITCH DIA		26.7175°	
PITCH DIAMETER OPERATING	2.6667		4.8333
DIAMETER SAP	2.4733		4.5719
ANGLE SAP	12.4574°		17.1232°
DIAMETER TIF	2.4639		4.5589
ANGLE TIF	11.3616°		16.5186°
DIAMETER HPSTC	2.8311		4.9283
ANGLE HPSTC	34.9574°		29.5369°
DIAMETER LPSTC	2.5830		4.6970
ANGLE LPSTC	21.6071°		22.1713°
BACKLASH NORMAL OPERATING		0.0120 -- 0.0060	
INVOLUTE CONTACT RATIO		1.4067	
HELICAL CONTACT RATIO		0.0000	
TOTAL CONTACT RATIO		1.4067	
SLIP RATIO SAP	-1.7763		-1.5759
SLIP RATIO OD	0.6118		0.6398
APPROACH / RECESS RATIO		45.06% / 54.94%	

Figure 3 Spur gear specifications: 25° pressure angle.

SPUR GEAR STRESS ANALYSIS

oerlikon fairfield

PART NUMBER	25 PA Pinion		25 PA Gear
NUMBER OF TEETH	16		29
DIAMETRAL PITCH NORMAL		6.000	
PRESSURE ANGLE		25.000°	
HELIX ANGLE	SPUR	0.000°	SPUR
CENTER DISTANCE OPERATING		3.7500	
FACE WIDTH	2.0000		2.0000
ADDENDUM	0.1917		0.1417
WHOLE DEPTH (STANDARD)	0.3917		0.3917
HOB EDGE RADIUS	0.0429		0.0429
CIRCULAR TKS CUT NORMAL	0.2821		0.2355
--- FACTORS CALCULATION & APPLICATION ---			
J FACTOR (AGMA 908)	0.4051		0.3948
I FACTOR (AGMA 908)		0.1103	
AGMA 2000 QUALITY		8	
APPLICATION FACTORS		1.0000	
Km MOUNTING FACTOR		1.0000	
Kv DYNAMIC FACTOR		1.0074	
PITCHLINE VELOCITY (fpm)		1	
--- STRESS ALLOWABLES ---			
BENDING (psi)	65,000		65,000
SURFACE (psi)	225,000		225,000
--- STATIC TORQUE RATING ---			
BENDING (in-lbs)	11,704		20,670
SURFACE (in-lbs)	7,571		13,723
--- DYNAMIC TORQUE RATING AT PINION 1 RPM ---			
BENDING (in-lbs)	11,617		20,518
SURFACE (in-lbs)	7,515		13,621
--- STRESS AT PINION 16000 in-lbs TORQUE, 1 RPM ---			
BENDING (psi)	89,522		91,873
SURFACE (psi)		328,302	
--- LIFE IN CYCLES TO 1% FAILURE ---			
BENDING	283,613		228,177
SURFACE	11,742		11,742
--- LIFE IN HOURS TO 1% FAILURE ---			
BENDING	4,727		6,893
SURFACE	196		355
--- GEAR GEOMETRY ERRORS ---			

-- NO ERRORS --

Figure 4 **Spur gear stress analysis: 25° pressure angle.**

SPUR GEAR SPECIFICATIONS

oerlikon fairfield

PART NUMBER	33.5 PA Pinion		33.5 PA Gear
---BASIC DATA ---			
NUMBER OF TEETH	16		29
RATIO		1.813 -- Hunting Tooth Ratio	
DIAMETRAL PITCH NORMAL		6.000	
CIRCULAR PITCH NORMAL		0.5236	
PRESSURE ANGLE NORMAL		33.500°	
HELIX ANGLE AND HAND	SPUR	0.000°	SPUR
LEAD	∞		∞
DIAMETRAL PITCH TRANSVERSE		6.000	
CIRCULAR PITCH TRANSVERSE		0.5236	
PRESSURE ANGLE TRANVERSE		33.500°	
PITCH DIAMETER (CUT)	2.6667		4.8333
BASE CIRCLE DIAMETER	2.2237		4.0304
OUTSIDE DIAMETER	2.9820		5.1592
ANGLE OUTSIDE DIAMETER	51.1928°		45.7844°
ROOT DIAMETER	2.3000		4.4800
ROOT CLEARANCE	0.0204		0.0190
ADDENDUM MODIFICATION	-5.400%		-2.240%
ADDENDUM	0.1577		0.1629
WHOLE DEPTH	0.3410		0.3396
FACE WIDTH	2.0000		2.0000
----TOOTH SIZE----			
CIR TKS OPERATING NORMAL	0.2588 -- 0.2558		0.2588 -- 0.2558
CIR TKS CUT TRANSVERSE	0.2588 -- 0.2558		0.2588 -- 0.2558
CIRCULAR TKS CUT NORMAL	0.2588 -- 0.2558		0.2588 -- 0.2558
CIR TKS AT OD NORMAL	0.0300 -- 0.0260		0.0300 -- 0.0270
MEASUREMENT OVER PINS	3.1554 -- 3.1513		5.3190 -- 5.3148
FMC STANDARD PIN/BALL	0.32000 PINS		0.32000 PINS
PIN/BALL CONTACT DIAMETER	2.6488 -- 2.6452		4.8234 -- 4.8195
CIR SPACE WIDTH NORMAL	0.2678 -- 0.2648		0.2678 -- 0.2648
----OPERATING DIMENSIONS----			
CENTER DISTANCE OPERATING		3.7500	
CENTER DISTANCE STANDARD		3.7500	
PRESSURE ANGLE TRANS. OPERATING		33.5000°	
ANGLE OPERATING PITCH DIA		37.9232°	
PITCH DIAMETER OPERATING	2.6667		4.8333
DIAMETER SAP	2.4061		4.5693
ANGLE SAP	23.6748°		30.6021°
DIAMETER TIF	2.3967		4.5577
ANGLE TIF	23.0375°		30.2505°
DIAMETER HPSTC	2.8559		5.0399
ANGLE HPSTC	46.1748°		43.0159°
DIAMETER LPSTC	2.4869		4.6642
ANGLE LPSTC	28.6928°		33.3707°
BACKLASH NORMAL OPERATING		0.0120 -- 0.0060	
INVOLUTE CONTACT RATIO		1.2230	
HELICAL CONTACT RATIO		0.0000	
TOTAL CONTACT RATIO		1.2230	
SLIP RATIO SAP	-0.9339		-0.6728
SLIP RATIO OD	0.4022		0.4829
APPROACH / RECESS RATIO		51.78% / 48.22%	

Figure 5 Spur gear specifications: 33.5° pressure angle.

SPUR GEAR STRESS ANALYSIS

oerlikon fairfield

PART NUMBER	33.5 PA Pinion		33.5 PA Gear
NUMBER OF TEETH	16		29
DIAMETRAL PITCH NORMAL		6.000	
PRESSURE ANGLE		33.500°	
HELIX ANGLE	SPUR	0.000°	SPUR
CENTER DISTANCE OPERATING		3.7500	
FACE WIDTH	2.0000		2.0000
ADDENDUM	0.1577		0.1629
WHOLE DEPTH	0.3410		0.3396
HOB EDGE RADIUS	0.0203		0.0285
CIRCULAR TKS CUT NORMAL	0.2588		0.2588
--- FACTORS CALCULATION & APPLICATION ---			
J FACTOR (AGMA 908)	0.5487		0.6168
I FACTOR (AGMA 908)		0.1273	
AGMA 2000 QUALITY		8	
APPLICATION FACTORS		1.0000	
Km MOUNTING FACTOR		1.0000	
Kv DYNAMIC FACTOR		1.0053	
PITCHLINE VELOCITY (fpm)		0	
--- STRESS ALLOWABLES ---			
BENDING (psi)	65,000		65,000
SURFACE (psi)	225,000		225,000
--- STATIC TORQUE RATING ---			
BENDING (in-lbs)	15,853		32,297
SURFACE (in-lbs)	8,733		15,828
--- DYNAMIC TORQUE RATING AT PINION 1 RPM ---			
BENDING (in-lbs)	15,770		32,128
SURFACE (in-lbs)	8,687		15,745
--- STRESS AT PINION 16000 in-lbs TORQUE, 1 RPM ---			
BENDING (psi)	65,949		58,672
SURFACE (psi)		305,358	
--- LIFE IN CYCLES TO 1% FAILURE ---			
BENDING	1.184E+07		8.437E+09
SURFACE	42,816		42,816
--- LIFE IN HOURS TO 1% FAILURE ---			
BENDING	394,752		5.097E+08
SURFACE	1,427		2,587
--- GEAR GEOMETRY ERRORS ---			

-- NO ERRORS --

Figure 6 Spur gear stress analysis: 33.5° pressure angle.

SPUR GEAR SPECIFICATIONS		oerlikon fairfield
PART NUMBER	35 PA Pinion		35 PA Gear
---BASIC DATA ---			
NUMBER OF TEETH	16		29
RATIO		1.813 -- Hunting Tooth Ratio	
DIAMETRAL PITCH NORMAL		6.000	
CIRCULAR PITCH NORMAL		0.5236	
PRESSURE ANGLE NORMAL		35.000°	
HELIX ANGLE AND HAND	SPUR	0.000°	SPUR
LEAD	∞		∞
DIAMETRAL PITCH TRANSVERSE		6.000	
CIRCULAR PITCH TRANSVERSE		0.5236	
PRESSURE ANGLE TRANVERSE		35.000°	
PITCH DIAMETER (CUT)	2.6667		4.8333
BASE CIRCLE DIAMETER	2.1844		3.9592
OUTSIDE DIAMETER	2.9651		5.1405
ANGLE OUTSIDE DIAMETER	52.5898°		47.4469°
ROOT DIAMETER	2.3000		4.4800
ROOT CLEARANCE	0.0297		0.0275
ADDENDUM MODIFICATION	-10.480%		-7.838%
ADDENDUM	0.1492		0.1536
WHOLE DEPTH	0.3325		0.3303
FACE WIDTH	2.0000		2.0000
----TOOTH SIZE----			
CIR TKS OPERATING NORMAL	0.2588 -- 0.2558		0.2588 -- 0.2558
CIR TKS CUT TRANSVERSE	0.2588 -- 0.2558		0.2588 -- 0.2558
CIRCULAR TKS CUT NORMAL	0.2588 -- 0.2558		0.2588 -- 0.2558
CIR TKS AT OD NORMAL	0.0330 -- 0.0300		0.0330 -- 0.0300
MEASUREMENT OVER PINS	3.1564 -- 3.1526		5.3197 -- 5.3157
FMC STANDARD PIN/BALL	0.32000 PINS		0.32000 PINS
PIN/BALL CONTACT DIAMETER	2.6438 -- 2.6404		4.8178 -- 4.8141
CIR SPACE WIDTH NORMAL	0.2678 -- 0.2648		0.2678 -- 0.2648
----OPERATING DIMENSIONS----			
CENTER DISTANCE OPERATING		3.7500	
CENTER DISTANCE STANDARD		3.7500	
PRESSURE ANGLE TRANS. OPERATING		35.0000°	
ANGLE OPERATING PITCH DIA		40.1189°	
PITCH DIAMETER OPERATING	2.6667		4.8333
DIAMETER SAP	2.4122		4.5772
ANGLE SAP	26.8370°		33.2385°
DIAMETER TIF	2.4027		4.5660
ANGLE TIF	26.2473°		32.9131°
DIAMETER HPSTC	2.8827		5.0623
ANGLE HPSTC	49.3370°		45.6523°
DIAMETER LPSTC	2.4673		4.6407
ANGLE LPSTC	30.0898°		35.0331°
BACKLASH NORMAL OPERATING		0.0120 -- 0.0060	
INVOLUTE CONTACT RATIO		1.1446	
HELICAL CONTACT RATIO		0.0000	
TOTAL CONTACT RATIO		1.1446	
SLIP RATIO SAP	-0.7680		-0.5822
SLIP RATIO OD	0.3680		0.4344
APPROACH / RECESS RATIO		51.57% / 48.43%	

Figure 7 Spur gear specifications: 35° pressure angle.

SPUR GEAR STRESS ANALYSIS

PART NUMBER	35 PA Pinion		35 PA Gear
NUMBER OF TEETH	16		29
DIAMETRAL PITCH NORMAL		6.000	
PRESSURE ANGLE		35.000°	
HELIX ANGLE	SPUR	0.000°	SPUR
CENTER DISTANCE OPERATING		3.7500	
FACE WIDTH	2.0000		2.0000
ADDENDUM	0.1492		0.1536
WHOLE DEPTH	0.3325		0.3303
HOB EDGE RADIUS	0.0074		0.0164
CIRCULAR TKS CUT NORMAL	0.2588		0.2588
--- FACTORS CALCULATION & APPLICATION ---			
J FACTOR (AGMA 908)	0.5846		0.6581
I FACTOR (AGMA 908)		0.1292	
AGMA 2000 QUALITY		8	
APPLICATION FACTORS		1.0000	
Km MOUNTING FACTOR		1.0000	
Kv DYNAMIC FACTOR		1.0053	
PITCHLINE VELOCITY (fpm)		0	
--- STRESS ALLOWABLES ---			
BENDING (psi)	65,000		65,000
SURFACE (psi)	225,000		225,000
--- STATIC TORQUE RATING ---			
BENDING (in-lbs)	16,889		34,460
SURFACE (in-lbs)	8,866		16,069
--- DYNAMIC TORQUE RATING AT PINION 1 RPM ---			
BENDING (in-lbs)	16,801		34,279
SURFACE (in-lbs)	8,819		15,985
--- STRESS AT PINION 16000 in-lbs TORQUE, 1 RPM ---			
BENDING (psi)	61,902		54,990
SURFACE (psi)		303,063	
--- LIFE IN CYCLES TO 1% FAILURE ---			
BENDING	4.156E+08		3.216E+11
SURFACE	48,990		48,990
--- LIFE IN HOURS TO 1% FAILURE ---			
BENDING	1.385E+07		1.943E+10
SURFACE	1,633		2,960
--- GEAR GEOMETRY ERRORS ---			

-- NO ERRORS --

Figure 8 Spur gear stress analysis: 35° pressure angle.

SPUR GEAR SPECIFICATIONS

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---BASIC DATA ---			
NUMBER OF TEETH	16		29
RATIO		1.813 -- Hunting Tooth Ratio	
DIAMETRAL PITCH NORMAL		6.000	
CIRCULAR PITCH NORMAL		0.5236	
PRESSURE ANGLE NORMAL		36.000°	
HELIX ANGLE AND HAND	SPUR	0.000°	SPUR
LEAD	∞		∞
DIAMETRAL PITCH TRANSVERSE		6.000	
CIRCULAR PITCH TRANSVERSE		0.5236	
PRESSURE ANGLE TRANVERSE		36.000°	
PITCH DIAMETER (CUT)	2.6667		4.8333
BASE CIRCLE DIAMETER	2.1574		3.9102
OUTSIDE DIAMETER	2.9602		5.1350
ANGLE OUTSIDE DIAMETER	53.8317°		48.7700°
ROOT DIAMETER	2.3000		4.4800
ROOT CLEARANCE	0.0325		0.0299
ADDENDUM MODIFICATION	-11.940%		-9.500%
ADDENDUM	0.1468		0.1508
WHOLE DEPTH	0.3301		0.3275
FACE WIDTH	2.0000		2.0000
----TOOTH SIZE----			
CIR TKS OPERATING NORMAL	0.2588 -- 0.2558		0.2588 -- 0.2558
CIR TKS CUT TRANSVERSE	0.2588 -- 0.2558		0.2588 -- 0.2558
CIRCULAR TKS CUT NORMAL	0.2588 -- 0.2558		0.2588 -- 0.2558
CIR TKS AT OD NORMAL	0.0290 -- 0.0260		0.0300 -- 0.0260
MEASUREMENT OVER PINS	3.1572 -- 3.1535		5.3203 -- 5.3164
FMC STANDARD PIN/BALL	0.32000 PINS		0.32000 PINS
PIN/BALL CONTACT DIAMETER	2.6406 -- 2.6373		4.8142 -- 4.8106
CIR SPACE WIDTH NORMAL	0.2678 -- 0.2648		0.2678 -- 0.2648
----OPERATING DIMENSIONS----			
CENTER DISTANCE OPERATING		3.7500	
CENTER DISTANCE STANDARD		3.7500	
PRESSURE ANGLE TRANS. OPERATING		36.0000°	
ANGLE OPERATING PITCH DIA		41.6278°	
PITCH DIAMETER OPERATING	2.6667		4.8333
DIAMETER SAP	2.4126		4.5784
ANGLE SAP	28.6827°		34.8947°
DIAMETER TIF	2.4032		4.5674
ANGLE TIF	28.1213°		34.5849°
DIAMETER HPSTC	2.8928		5.0709
ANGLE HPSTC	51.1827°		47.3084°
DIAMETER LPSTC	2.4589		4.6310
ANGLE LPSTC	31.3317°		36.3562°
BACKLASH NORMAL OPERATING		0.0120 -- 0.0060	
INVOLUTE CONTACT RATIO		1.1177	
HELICAL CONTACT RATIO		0.0000	
TOTAL CONTACT RATIO		1.1177	
SLIP RATIO SAP	-0.7003		-0.5427
SLIP RATIO OD	0.3518		0.4119
APPROACH / RECESS RATIO		51.47% / 48.53%	

Figure 9 **Spur gear specifications: 36° pressure angle.**

SPUR GEAR STRESS ANALYSIS

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NUMBER OF TEETH	16		29
DIAMETRAL PITCH NORMAL		6.000	
PRESSURE ANGLE		36.000°	
HELIX ANGLE	SPUR	0.000°	SPUR
CENTER DISTANCE OPERATING		3.7500	
FACE WIDTH	2.0000		2.0000
ADDENDUM	0.1468		0.1508
WHOLE DEPTH	0.3301		0.3275
HOB EDGE RADIUS	-0.0019		0.0076
CIRCULAR TKS CUT NORMAL	0.2588		0.2588
--- FACTORS CALCULATION & APPLICATION ---			
J FACTOR (AGMA 908)	0.6096		0.6876
I FACTOR (AGMA 908)		0.1311	
AGMA 2000 QUALITY		8	
APPLICATION FACTORS		1.0000	
Km MOUNTING FACTOR		1.0000	
Kv DYNAMIC FACTOR		1.0053	
PITCHLINE VELOCITY (fpm)		0	
--- STRESS ALLOWABLES ---			
BENDING (psi)	65,000		65,000
SURFACE (psi)	225,000		225,000
--- STATIC TORQUE RATING ---			
BENDING (in-lbs)	17,612		36,005
SURFACE (in-lbs)	8,993		16,300
--- DYNAMIC TORQUE RATING AT PINION 1 RPM ---			
BENDING (in-lbs)	17,520		35,817
SURFACE (in-lbs)	8,946		16,214
--- STRESS AT PINION 16000 in-lbs TORQUE, 1 RPM ---			
BENDING (psi)	59,363		52,630
SURFACE (psi)		300,910	
--- LIFE IN CYCLES TO 1% FAILURE ---			
BENDING	4.369E+09		3.783E+12
SURFACE	55,642		55,642
--- LIFE IN HOURS TO 1% FAILURE ---			
BENDING	1.456E+08		2.285E+11
SURFACE	1,855		3,362
--- GEAR GEOMETRY ERRORS ---			

-- NO ERRORS --

Figure 10 Spur gear stress analysis: 36° pressure angle.

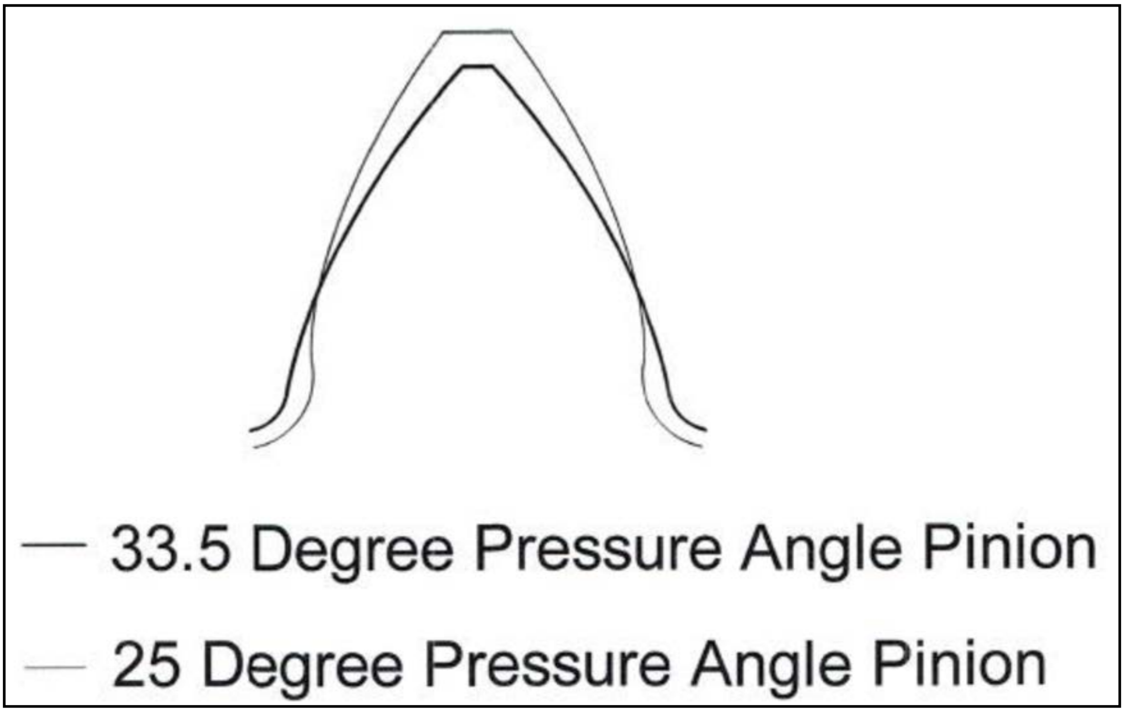
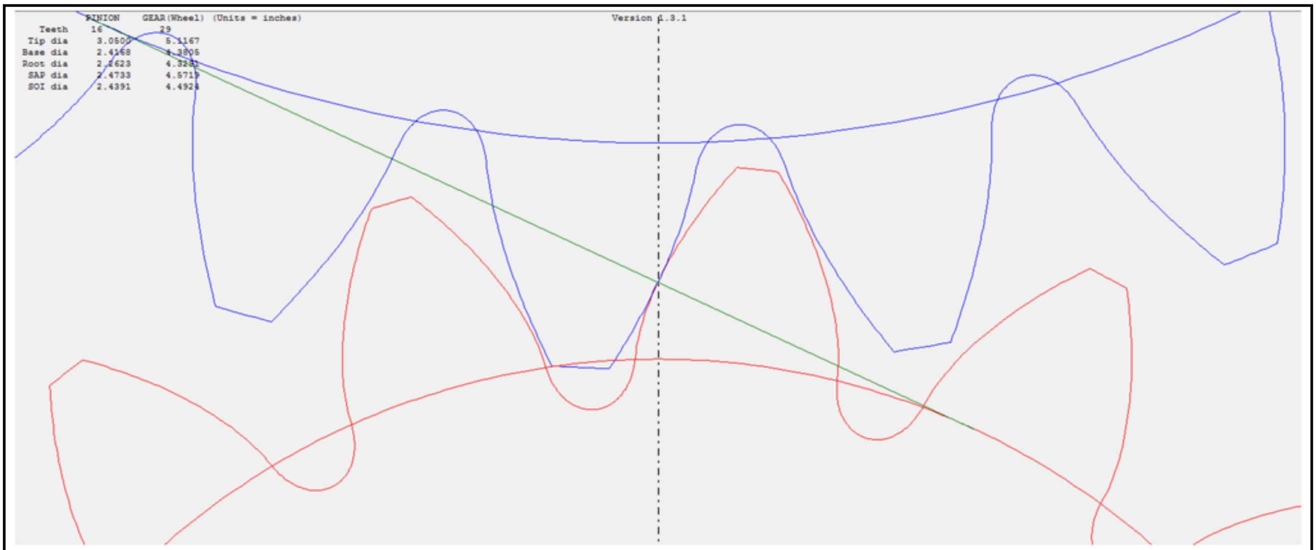


Figure 11 Reference 25 PA and 33.5 PA gear tooth profiles.



	PINION	GEAR (Wheel)	(Units = inches)
Teeth	16	29	
Tip dia.	3.0500	5.1167	
Base dia.	2.4168	4.3805	
Root dia.	2.2623	4.3291	
SAP dia.	2.4733	4.5719	
SOI dia.	2.4391	4.4924	

Figure 12 25° pressure angle gear teeth.

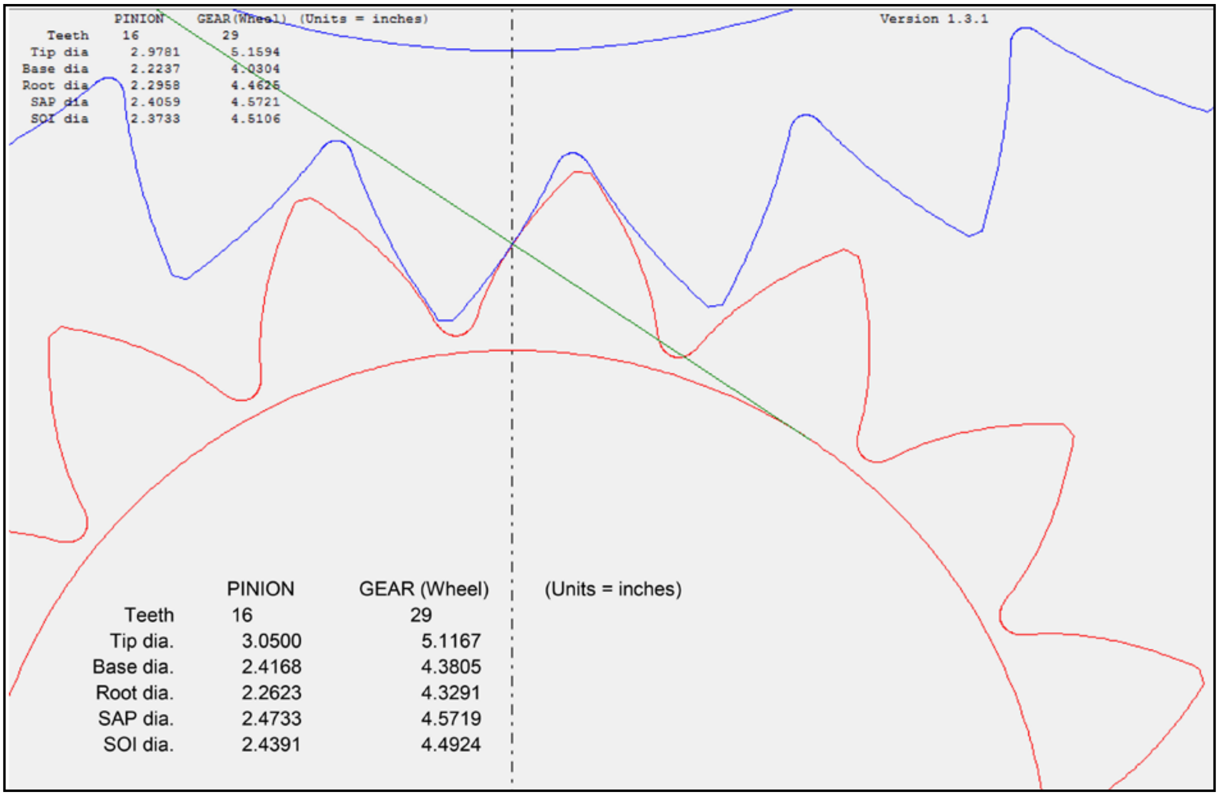


Figure 13 **33.5° pressure angle gear teeth.**

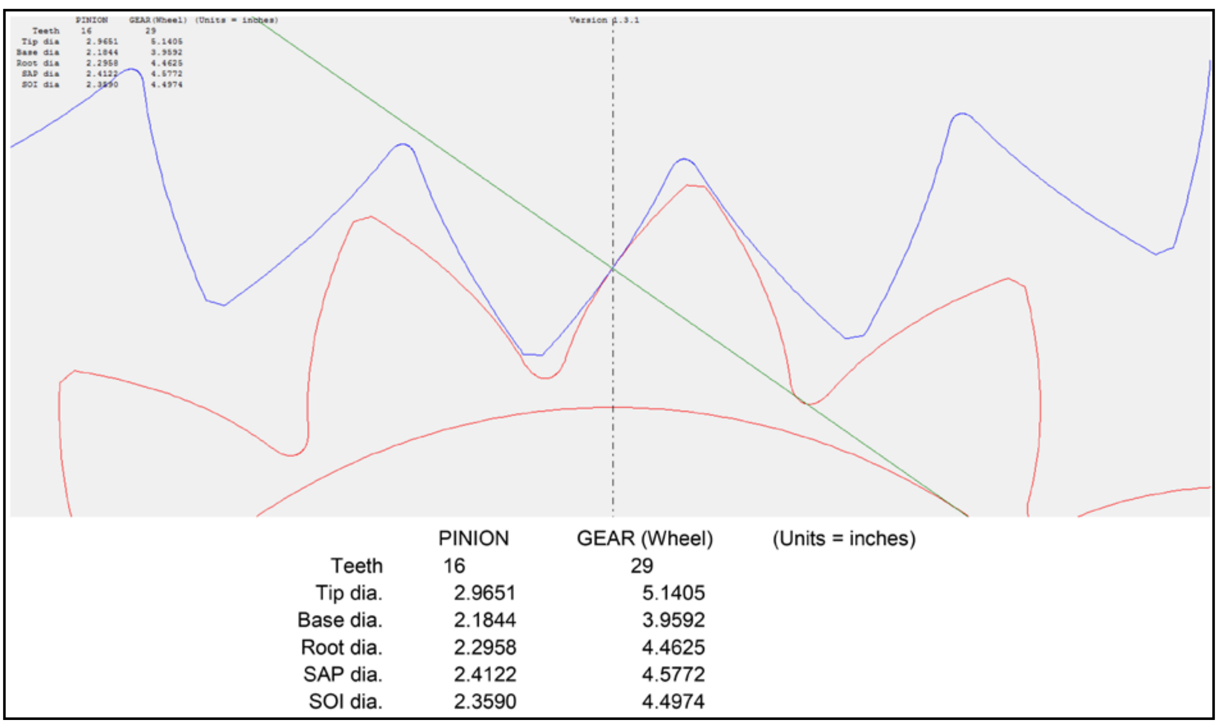


Figure 14 **35° pressure angle gear teeth.**

Review of Results and Commentary

All gear trials were run with the following common gear data attributes:

- 6 DP, 3.75 inch center distance,
- 0.012 inch maximum backlash,
- 16,000 in-lb pinion torque,
- 1 RPM pinion speed.

As stated above, the 25° pressure angle gearset uses 15% long addendum pinion and 15% short addendum gear tooth proportions, and the resulting circular tooth thicknesses.

***The first trial was for the 25° pressure angle design; this serves as the benchmark. Following are the relevant results*:**

- Bending stress, PSI, pinion/gear is 89,522/91,873; static torque rating bending, (in-lb): 11,704/20,670
- Surface contact stress, PSI, pinion and gear: 328,302; static torque rating surface P/G: 7,571/13,723

***The second trial was to determine the pressure angle at the following conditions*:**

0.030 inch top land; 2.1/DP whole depth, 1.15 contact ratio.

- The calculated outside diameters (Eq. 1) are 2.965 inches pinion and 5.141 inches gear
- The calculated pressure angle to match these conditions is 35°
- The hob tip radius at these conditions is 0.007 inches pinion / .016 inches gear.
- Bending stress, PSI, pinion/gear is 61,902/54,990; static torque rating bending, (in-lb): 16,889/34,460
- Surface contact stress, PSI, pinion and gear: 303,063; static torque rating surface P/G: 8,866/16,069
- This represents a bending stress reduction from the 25 PA gearset of 31% pinion and 40% for the gear
- The increase in static torque rating for bending strength is 44% pinion and 67% for the gear
- The increase in static torque rating for surface contact strength is 17% for both pinion and gear

***The third trial was to determine the pressure angle at the above conditions for the second trial: 0.030 top land, 2.1/DP whole depth, 1.15 contact ratio — but with the following additional requirements*:**

Hob tip radius equal to 0.020 inches

- Calculated outside diameters are 2.982 inches pinion and 5.159 inches gear
- Calculated contact ratio is 1.22
- Calculated pressure angle to match these conditions is 33.5°
- Bending stress, PSI, pinion/gear: 65,949 / 58,672; static torque rating bending, (in-lb): 15,853 / 32,297
- Surface contact stress, PSI, pinion and gear: 305,358; static torque rating surface P/G: 8,733 / 15,828
- This represents a bending stress reduction from the 25 PA gearset of 26% pinion and 36% for the gear
- The increase in static torque rating for bending strength is 35% pinion and 56% for the gear
- The increase in static torque rating for surface contact strength is 15% for both pinion and gear

Summary and Conclusion

A method has been presented to calculate the pressure angle for gear teeth when the top land, contact ratio, and backlash are specified as inputs. This method does not consider hob tip radius, which may be a small value or even zero for this calculation. A zero or very small hob tip radius is probably not a good or acceptable design. So another example was shown where a given hob tip radius was used as an input, and another set of equations and calculations were shown separately to evaluate the pressure angle that matches the given hob tip radius in addition to the other input variables as before. This pressure angle would be smaller than either the zero hob tip radius value or the 0.007 inch hob tip radius value in the proposed example and is a preferred design, and the resultant gear data was calculated.

Gear tooth circular tooth thicknesses were standard in all of these high pressure angle examples but could have been specified as non-standard values if desired and used in the existing calculations. Likewise, the top land values for the pinion and the gear were the same (0.030 inch), but could have been different from each other.

All mathematical formulas and their derivation were shown. The results for the examples shown are a much higher pressure angle than is currently used in gearing and produces significantly reduced bending and surface contact stress for the higher pressure angle gearing. Note that there are other items to be considered, such as ease of manufacturing, and that pressure angles this high have other considerations to take into account. The purpose of this paper is to determine and quantify the use of high pressure angles for gear teeth and to calculate them and not to suggest good design practice.

A typical application for this type of high pressure angle gearing would be one with slow speed, large, coarse pitch teeth, and lower quality level, although other conditions could also be considered for these gears. Applications where a high power density and high gear tooth strength are required should consider this type of gearing.

Note that these methods can also be used for more typical pressure angle designs where the user simply wants to design gear teeth and gearsets based on top lands, contact ratio and hob tip radius as inputs.

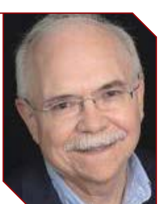
All gear data and stress calculations were performed using the *AGMA GRS 3.1.7* gear rating suite. Due to the length of the program data outputs, the data was summarized and shown in more abbreviated format.

In order to ensure further validation of the gear tooth stresses, it is suggested that alternative FEA (finite element analysis) be performed on the gear teeth and the results of this be compared to the results from the above analysis.

Dolan and Broghamer (Ref.6) discuss the applicability of their work and stress correction factors to different pressure angles.

References

1. Townsend, Dennis B. *Dudley's Gear Handbook: The Design, Manufacture, and Application of Gears*, Second Edition, 1992.
2. Dudley, Darle W. *Handbook of Practical Gear Design*, 1984.
3. Drago, R. J. *Fundamentals of Gear Design*, 1984.
4. Vogel, W. F. *Involuometry and Trigonometry,* 1945.
5. National Broach. "Modern Methods of Gear Manufacture."
6. Dolan, T. J. and E.L. Broghamer. "A Photoelastic Study of Stresses in Gear Tooth Fillets," *University of Illinois Bulletin,* 1942.



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