Double Differential for Electric Vehicle and Hybrid Transmissions — Sophisticated Simplicity

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What Are the Common High Reduction Transmissions?

The duty of high reduction transmissions is reducing high input rpm into lower rpms, for example to propel the wheels of a vehicle or the rotor of a helicopter. The output rpm of such a transmission is in the range between 0 and 1,000 rpm. The input rpm can be 20,000 rpm and higher if the prime mover is an electric motor or a jet engine.

The conventional transmissions which can be operated with high input speeds and can accomplish high reductions are:

- Multistage transmissions employing cylindrical gears
- Planetary transmissions
- Bevel worm gear reductions with ratios of 20 in one stage
- Pericyclic transmissions with nutating bevel gears
- Cycloidal transmissions

Multistage Transmissions Employing Cylindrical Gears

Multi-stage transmissions with cylindrical gears require a multitude of shafts with bearings and gears. For a reduction ratio of 20, at least four stages are required. Four reduction stages require four shafts, eight bearings and four gear meshes. Only the observation of four gear meshes indicates an overall efficiency of 92.2% if the efficiency of one single stage is 98% ($0.98^4 = 0.922$). Four stage cylindrical transmissions require a rather large transmission housing envelope (Ref. 1).

Planetary Transmissions

Depending on the design, planetary transmission, in connection with conventional cylindrical gear reduction stages, can achieve high ratios and obtain high efficiency.

Bevel Worm Gear Reductions with Ratios of 20 in One Stage

Bevel worm gear drives are, for example, called High Reduction Hypoids (HRH) or Super Reduction Hypoids (SRH). The worm shaped pinions have one to five teeth and the ring gears have typically 27 to 75 teeth. The maximal achievable ratios are in the range of 75. Ratios above 15 only have a reduced back driving capability. Gear sets without back driving capability are selflocking. Self-locking gear sets cannot be used in a vehicle drive train or in a helicopter main rotor drive. Bevel worm gear drives also create high sliding velocities due to the large component in face width direction. A five-tooth SRH pinion, meshing with a 60-tooth ring gear creates 617 m/min relative sliding between the flank surfaces with a pinion speed of 10,000 rpm (equal transmission input speed). This is higher than the maximum sliding expected in a hypoid axle drive of a sports car while driving faster than 200 km/h (125 mph) with a pinion speed of 4,000 rpm. The example explains that a doubling of the transmission input will not only reduce the efficiency but also has the risk of surface damage and premature failure (Refs. 2, 3).

Pericyclic Transmissions with Nutating Bevel Gears

Pericyclic transmissions as introduced in (Refs. 4, 5, 6) can achieve very high reductions in the range of 20 to 100 without generating high relative surface sliding. As the shaft angle between two bevel gears approaches 180°, the relative sliding velocity drops down to zero. Because of shaft angles higher than 160° in the most common pericyclic transmissions, the relative sliding velocities are uncritical, even if the input speeds are 20,000 rpm or higher. Pericyclic transmissions have angled bearing seats of the nutating members and the high forces which are applied to the bearing at the angled seat have to be supported with pre-loaded tapered roller bearings. Another possible area of attention in pericyclic transmissions are the fluctuating axial mass forces the nutating members generate. High speed pericyclic transmissions require a mirror image arrangement of an even number of nutating members as well as precise timing of the gears and precise balancing.

Cycloidal Transmissions

Cycloidal transmissions are the two-dimensional analog to pericyclic transmissions. One revolution of the eccentric input shaft will rotate the output shaft by one to two tooth pitches. The radial mass forces of cycloidal transmissions cannot be compensated by a second cycloidal disk arrangement side by side. As a result, high reduction cycloidal transmissions are only used when low input speeds are reduced to very low output speeds.

If high ratios between 10 and 100 should be achieved, designers prefer multi-stage cylindrical transmissions often combined with planetary reductions. Multistage transmissions are often applied in the industry and deliver a reasonable power density.

For future high reduction transmissions, it is desirable to create a very compact high reduction transmission with easy to manufacture components and predictable operating conditions. If all involved parts are well known as standard machine design components, then the prediction of durability

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and endurance life is possible by applying the calculation algorithms provided by the standards of the AGMA (American Gear Manufacturers Association), ISO (International Standardization Organization) and other national standards. Those algorithms rely on tenths of thousands of fatigue life testing as well as many application factors which have been evaluated for many decades. In safety engineering, those proven algorithms and application factors are the engineer's most valuable tools.

Bevel Gear Based High Ratio Transmissions

Three interesting solutions of high-speed reducers which are based on bevel gears are shown in Figure 1. Automotive transmissions cannot be self-locking and must provide a good efficiency. The super reduction hypoid (SRH) on the top, left is limited to a ratio of about 15 to fulfill these two requirements. The SRH arrangement of input shaft and the two drive shaft output flanges is ideal for the adaptation between an electric motor and the driving wheels. Only two shafts and four bearings are required, which makes the SRH solution very cost effective and compact. The differential unit can be placed inside of the ring gear, similar as it is in case of a hypoid axle drive unit.

A high ratio solution realized with straight

bevel gears is shown on the top right in Figure 1. This pericyclic reducer requires 6 straight bevel gears to perform two opposite nutating motions. Each revolution of the nutating (light blue) gears will rotate the green output gears by one or two pitches, depending on the number of teeth of the dark blue reaction gears. The contact ratio of this unit is very high, such that 6 to 10 pairs of teeth are always in contact.

The latest, bevel gear based high reduction e-drive is the double differential in the bottom photo in Figure 1. The double differential is very compact and can realize ratios from 5 to 80 and even higher. The following chapters will explain its functionality and its advantages as electric vehicle transmission.



Figure 1 High ratio bevel gear transmissions.



Figure 2 Double differential transmission.

What is a Double Differential?

The new developed solution for a low, medium, or high reduction transmission with high power density and the application of standard design elements is the double differential, shown in Figure 2.

The double differential transmission is symmetric and has a high-power density. The input rotation 21 from shaft 20 is transmitted to gears 15 and 17 and causes a rotation 22 of gear 15, and a rotation 24 of gear 17. Gears 15 and 17 are both in mesh with gear 16. Gear 16 is rigidly connected to the housing 18. The fact that gear 16 cannot rotate will cause a rotation 23 of the carrier 19. Gears 15 and 11 as well as gears 17 and 13 are rotationally constrained with each other, for example via a spline connection. The carrier rotation 23 gives a first component of rotation to output gear 12. The rotations 22 and 24 add a second component of rotation to output gear 12. If all eight involved bevel gears have the same number of teeth, then the output rotation 25 would be zero. The explanation is that, for example a 90° rotation ϕ_2 of the carrier 19 would rotate gears 15 and 17 by 90° in the directions 22 and 24. The output gear 12 therefore receives a 90° rotation ϕ_2 from the carrier and a 90° rotation ϕ_3 (in the opposite direction) from the gears 11 and 13 and as a result will not rotate, independent from the input rotation 21.

While this example seems not of any obvious practical interest, the example was merely used to demonstrate the interesting functionality of double differential transmissions. In the example, the ratio is $\phi_1/\phi_4 = \infty$.

A derivation of the equation for the ratio by using individual numbers of teeth provides the ability to find the variety of possible ratios by variation of the tooth numbers of the gears 14/16 versus 15/17 and 10/12 versus 11/13.

	$\phi_2/\phi_3 = z_2/z_1$	(1)
or:	$\phi_3 = \phi_2 \cdot z_1 / z_2$	(2)
	$\phi_4 = \phi_2 - \phi_3 \cdot z_4 / z_3$	(3)
	$\phi_1 = \phi_2 + \phi_3 \cdot z_2 / z_1$	(4)
plug (2) in (4):	$\phi_1 = \phi_2 + \phi_2 = 2 \cdot \phi_2$	(5)
or:	$\phi_2 = \phi_1/2$	(6)
plug (6) in (3):	$\phi_4 = \phi_1/2 - \phi_3 \cdot z_4/z_3$	(7)
plug (6) in (2):	$\phi_3 = \phi_1/2 \cdot z_1/z_2$	(8)
plug (8) in (7):	$\phi_4 = \phi_1/2 \cdot [1 - z_1/z_2 \cdot z_4/z_3]$	(9)
re-arranged:	$R = \phi_1 / \phi_4 = 2 / [1 - (z_1 \cdot z_4) / (z_2 \cdot z_3)]$	(10)

whereas:

- z_1 ... Number of teeth gear 14 and gear 16
- z_2 ... Number of teeth gear 15 and gear 17
- z_3 ... Number of teeth gear 10 and gear 12
- $z_4 \hdots$ Number of teeth gear 11 and gear 13
- ϕ_1 ... Angle of rotation gear 14
- ϕ_2 ... Angle of rotation carrier 19
- $\varphi_3...$ Angle of rotation gear 15 (and gear 17 in negative φ_3 direction)
- $\varphi_4...$ Angle of rotation gear 12 (and output shaft 26)
- R... Ratio of input speed divided by output speed

In the following four examples, different numbers of teeth combinations are used to demonstrate the extremely high range of ratios that can be realized with the double differential without a significant change of the transmission size:

- Example 1: z_1 =40; z_2 =39; z_3 =40; z_4 =40; Ratio R=-78.000
- Example 2: z₁=40; z₂=41; z₃=40; z₄=40; Ratio R=82.000
- Example 3: $z_1 = 45$; $z_2 = 50$; $z_3 = 40$; $z_4 = 40$; Ratio R = 20.000
- Example 4: $z_1 = 30$; $z_2 = 50$; $z_3 = 40$; $z_4 = 40$; Ratio R = 5.000

Extended Double Differential with Two Inputs

A possible extension of the function of the double differential transmission is shown in Figure 3. In addition to the graphic in Figure 2, in Figure 3 the gears 30, 31 and shaft 32 have

been added. Gear 16 is connected to a cylindrical gear 30, which is arranged rotatable to the housing 18, and in mesh with pinion 31, which is connected to a second input shaft 32. This possibility of a second input allows a variety of interesting input speed combinations with two different prime movers, e.g., electrical motors, which have different speed and torque characteristics. One motor, for example, can be a high torque and low speed motor which runs on a constant speed signal without speed regulation. The second motor would then, for example, rotate backwards if an output rpm of 0 is required. In case of quick acceleration up to a vehicle cruising speed, the second motor is first slowed down to 0 rpm and the stored kinetic energy of the differential gears and the fast-rotating carrier is used for the vehicle acceleration. Several seconds later, when the vehicle reaches half of its cruising speed, the second motor starts to rotate in positive rotational direction to accelerate the vehicle further to the desired speed. In a conventional electric vehicle drive system, high amounts of energy are drawn from the battery during this acceleration. The extended double differential allows storing kinetic energy during gentle driving periods and during deceleration and breaking actions which can be used as described before.

In the case of two inputs, there is not one particular number for the ratio which leads to the following relationship between the output rotation and the two input rotations:

	$\phi_2 = \phi_3 \cdot z_2 / z_1$	(11)
or:	$\phi_3 = (\phi_2 - \phi_5) \cdot z_1 / z_2$	(12)
	$\phi_4 = \phi_2 - \phi_3 \cdot z_4 / z_3$	(13)
	$\phi_1 = \phi_2 + \phi_3 \cdot z_1 / z_2$	(14)
insert (12) in (14):	$\phi_1 = \phi_{2+}(\phi_2 - \phi_5) \cdot z_1 / z_2 \cdot z_2 / z_1 = 2 \cdot \phi_{2-} \phi_5$	(15)
or:	$\phi_2 = (\phi_1 + \phi_5)/2$	(16)
insert (16) in (13):	$\phi_4 = (\phi_1 + \phi_5)/2 - \phi_3 \cdot z_4/z_3$	(17)
insert (16) in (12):	$\phi_3 = [(\phi_{1+}\phi_5)/2 - \phi_5] \cdot z_1/z_2$	(18)
insert (18) in (17):	$\phi_4 = (\phi_1 + \phi_5)/2 \cdot [1 - z_1/z_2 \cdot z_4/z_3] + \phi_5 \cdot z_1/z_2 \cdot z_4/z_3$	(19)
second input rotation	$\phi_6 = -\phi_5 \cdot z_5 / z_6$	(20)

whereas:

 z_5 ... Number of teeth gear 30

 $z_6...$ Number of teeth gear 31



Figure 3 Extended double differential with two inputs.

ϕ_5 ... Rotation angle of gears 16 and 30

Two special cases can be encountered by applying Equation 19 for different input rotations ϕ_6 . In case 1, the output speed (rotation angle ϕ_4) is equal to the speed of gear 16 (rotation angle ϕ_5). In this case, the output rotation ϕ_4 is equal to the input rotation ϕ_1 , which results in a ratio of R = 1.00:

$\phi_5 = \phi_4$ inserted in (19):	$\phi_4 = (\phi_1 + \phi_4)/2 \cdot (1 - z_1/z_2 \cdot z_4/z_3) + \phi_4 \cdot z_1/z_2 \cdot z_4/z_3$	Z3(21)
(21) solved for ϕ_4 :	$\phi_4/2 \cdot (1-z_1/z_2 \cdot z_4/z_3) = \phi_1/2 \cdot (1-z_1/z_2 \cdot z_4/z_3)$	(22)
or reduced:	$\phi_4 = \phi_1$	(23)
resulting in:	R = 1.00	(24)

In case 2, the input rotation ϕ_5 is zero, which simplifies Equation 19, and it becomes equal to Equation 9:

 $\phi_5 = 0 \text{ plugged in (19): } \phi_4 = (\phi_1 + 0) \cdot (1 - z_1/z_2 \cdot z_4/z_3) + 0 \cdot z_1/z_2 \cdot z_4/z_3 (25)$ elimination of zero terms: $\phi_4 = \phi_1/2 \cdot [1 - z_1/z_2 \cdot z_4/z_3]$ (26)

Equation 26 is equal to Equation 9. Equation 9 is based on the fact that gear 16 is rigidly connected to the transmission housing which presents the case $\phi_5 = 0$, which in turn proves that Equation 19 is conclusive.

The gears in a double differential can be straight bevel gears, spiral bevel gears, or face gears with additional helical gears

for the second input. In case of high input speeds, ground spiral bevel gears will deliver the highest efficiency and the lowest noise emission in connection with a high load carrying capacity. The axial forces, which are the result of the normal flank forces, can be minimized in a double differential by using reversed spiral angles and adjusting the values of the spiral angles for the outer and the inner planets differently for the outer and inner side gears. For the bearing dimensioning of the planets, the expected centrifugal forces have to be considered.

Due to the fact that no hypoid offsets are used, the relative surface sliding has no component in face width direction but consists only of profile sliding. The relative profile sliding of a spiral bevel gear set with a ratio

which is close to 1.0 and an outer diameter of 120 mm (typical for automotive double differential transmissions) with a speed of 1,000 rpm amounts to a maximum of about 84 m/min. The relative speed between the two fastest gears (14 and 15) in a double differential transmission is only about 50% of the input speed. Equation 8, $\phi_3 = \phi_1/2 \cdot z_1/z_2$ delivers a speed of gear 15 which is only 48.8% of the input speed, if $z_1 = 40$ and $z_2 = 41$ ($\phi_3 = \phi_1/2 \cdot 40/41 = 0.488 \cdot \phi_1$) The relative speed between gear 14 and gear 15 is therefore in this case $\phi_1 - \phi_3 = 0.512 \cdot \phi_1$. This means the relative speed between the fastest gears in a double differential transmission is typically only about half of the input speed. If the input speed is 10,000 rpm, then the double differential has only 10.84 m/min $\cdot 0.512 = 430.08$ m/min. Compared to a standard spiral bevel gear transmission, the double differential transmission has in this case only 51.2% of the sliding velocity.

The expanded double differential allows a variety of interesting applications due to the second input (input 2). If, for example, input 2 is connected to a low-speed high torque motor and input 1 is connected to a high-speed low torque motor which rotates, then it is possible to choose the speed of input 1 (e.g., 25,333 rpm) and of input 2 (e.g., 4,000 rpm) such that the output speed is 0 rpm. This example is based on the following number of teeth:

$$z_1 = 45; z_2 = 50; z_3 = 40; z_4 = 40; z_5 = 60; z_6 = 20$$

with a speed of input 2 (shaft 32) of $n_6 = 4,000$ rpm, and the first reduction $-z_6/z_5 = -20/60$ the speed of gear 30 is equal to $n_5 = -1,333$ rpm. The speed of the output shaft is $n_4 = 0$.

Equation 19 is also valid if instead of the angles ϕ , the rotational speeds n in rpm are used:

$$n_4 = (n_1 + n_5)/2 \cdot [1 - z_1/z_2 \cdot z_4/z_3] + n_5 \cdot z_1/z_2 \cdot z_4/z_3$$

becomes: $0 = (n_1 - 1, 333)/2 \cdot [1 - 45/50 \cdot 40/40] - 1,333 \cdot 45/50 \cdot 40/40$

or: $0 = (n_1/2 - 666.7) \cdot 0.1 - 1,200$

resulting in: $n_1 = 25,333$ rpm

The practical application of this example can be a vehicle which slows down from cruising speed to a full stop in front of an intersection traffic light (Figure 4, left to center). During this deceleration the kinetic energy "moves" from the vehicle body to the carrier of the double differential. When the vehicle stands



Figure 4 Energy balance—vehicle with mechanical energy storage.

still at the red light, the high-speed motor rotates at 25,333 rpm (and the carrier with the planets with 12,666 rpm). All the kinetic energy from the moving vehicle body (less friction losses) is now stored in the fast-rotating carrier and the planets.

After the traffic light changes to green, n_6 is reduced by the vehicle control electronics to down to zero rpm. This will create a resistance from the output shaft, which reduces n_1 from 25,333 rpm to 8,840 rpm, while n₆ reduces to zero rpm, which will accelerate the vehicle from 0 to 35 mph, less the friction losses (Figure 4, center to right). During the acceleration period, the kinetic energy of the double differential assembly with gears 10, 11, 12, 13, 14, 15, 16 and 17 as well as the carrier 19 and the motor connected to input 1 is utilized to deliver most of the acceleration energy. Driving faster than 35 mph will simply require rotating the input 1 faster. At a vehicle speed of 70 mph, the speed of input 1 will reach $n_1 = 17,680$ rpm. Depending on the duty cycle of a vehicle (highway or city driving), the lowspeed motor can be turned off like in the example above and a not shown clutch can be applied in order to lock input 2. In this case, the motor connected to input 1 will deliver all the energy

required, for example, for a light duty city driving. The two graphs in Figure 4 show that in the energy balance, a friction loss has been considered.

When attempting to constantly back-charge bursts of recuperative energy to a battery, the electrical efficiency becomes very low and the battery's chemical capacity to accept large amounts of energy within only several seconds is limited. A medium size sedan that drives at 56 km/h (35 mph) has about 0.4 kWh kinetic energy. Reducing the speed rather quickly in front of a traffic light that just turned red would require recuperating the 0.4 kWh within about two to three seconds. As a result, it is likely that not more than 0.10 kWh can be back charged to the battery and 0.35 kWh are converted to heat, either in the brake disks or in the electronic vehicle control modules. The double differential including the motor on input 1 can store about 0.24 kWh with an efficiency of about 86%, which means that 0.21 kWh are available in form of a rotation of the double differential when the vehicle comes to a full stop before the red light of energy, due to the continuous combustions and the internal resistance, mainly from piston ring friction and from the constant acceleration and deceleration of the pistons and the rods.

The double differential with two inputs can also be utilized to collect and transmit the energy from an electric motor and a combustion engine to the driving wheels of a hybrid vehicle. With such an arrangement, optimal speed combinations for each of the two prime movers can be found, which also allows eliminating any additional transmission in the hybrid vehicle.

Gear 10 in Figure 3 is not required for the function of the double differential. It was used to make the transmission symmetric, and it was anticipated that in case of large tooth and transmission housing deformation (under high load), gear 10 would help to keep the torque on gears 11 and 13 equal. If symmetry and balance is not an issue, then gear 10, and in addition gears 13 and 17, can be eliminated in order to simplify the double differential transmission and reduce manufacturing cost.



Figure 5 Energy loss per tooth mesh and efficiency calculation.

(0.19 kWh are converted to heat). This energy will be used only several minutes later to accelerate the vehicle after the traffic light turns green. Short term energy storage cannot be done efficiently with today's battery technology. The double differential concept allows a size reduction of the battery by maintaining the same mileage capacity.

The combination of two input speeds is allowing a wide variety of possibilities to adopt the double differential transmission to different driving conditions by achieving an optimal motor and transmission efficiency. The additional aspect of easy energy storage in a fast-rotating differential carrier unit will support the vehicle batteries especially when high energy bursts are required, for example, to accelerate a heavy truck from zero to 48 km/h (30 mph). The fact that both motors must rotate with high speeds while the vehicle is not moving requires very little energy while the external resistance is zero. In contrast, idling internal combustion engines do require considerable amounts

Efficiency Estimations

An efficiency calculation for 20,000 rpm input speed and 3.8 Nm input torque was conducted. The calculation was performed with the efficiency module of the Gleason Engineering and Manufacturing System (GEMS). This software considers the precise macro and micro geometry of the spiral bevel gears and uses a complex elastohydrodynamic approach to obtain good efficiency predictions (Ref. 7).

The input power of 7.955 kW actuates the outer planets with the first tooth mesh between input gear 14 and planet 15, the second tooth mesh between planet 15 and reaction gear 16, and a third tooth mesh between inner planet 11 and output gear 12. The input parameters for the efficiency calculations as well as the energy loss have been recorded in the graphic in Figure 5. The first mesh has a relative speed of 9,756 rpm and a torque of 3.8 Nm. The energy loss in the first tooth mesh as calculated is only 4 W. Also, the second tooth mesh shows a relative speed of 9,756 rpm

but a high torque of 304 Nm. The energy loss in the second tooth mesh was calculated as 302 W. To establish equilibrium between the three tooth meshes, the torque of the third mesh is 304 Nm-3.8 Nm= 300.2 Nm. The efficiency calculation resulted in an energy loss of the third tooth mesh of 330 W.

In order to obtain the transmission efficiency, the sum of the lost power was divided by the input power (times 100) and sub-tracted from 100%, which results in a gear efficiency of 92%.

A second way of analyzing the tooth mesh losses of mesh two and mesh three appears a very realistic representation of the transmission physics in a double differential. In the discussed designs, the outer and inner planets are connected. Their respective teeth have similar normal forces while the outer planet meshes with reaction gear and the inner planet meshes with the output gear. If the outer and inner planet shown in Figure 5 were replaced with one planet with a double wide face width and if the tooth contact was doubled in length, then the efficiency calculation program shows an energy loss in the double wide mesh of 517 W. The total energy loss is then 521 W, which is equivalent to an efficiency of 100-521 W / 7,958 W $\cdot 100 = 93.5\%$.

The efficiency of a four-stage cylindrical gear transmission, with an approximated efficiency of 98% per stage, results in $0.98 \cdot 0.9$ $8 \cdot 0.98 \cdot 0.98 = 0.922 \rightarrow 92.2\%$. In conclusion, the efficiency of the double differential is very similar to cylindrical gearing; however, a four-stage helical gear transmission requires four shafts and builds in a larger space than a double differential. The compact design of the double differential may present several physical advantages, leading to an efficiency advantage compared to other high reduction concepts.

Double Differential Inline Solution

In order to allow placing the double differential transmission between the wheels of a drive axle in a vehicle, a proposal of an additional configuration is shown in Figure 6.

The transmission in Figure 6 has an additional differential function between the two output shafts 26 and 41. Output shaft 26 remains on the right side of the transmission housing and the added output shaft 41 exits the transmission housing at the left side. Gear 10, which is not required for the correct function of the double differential, has been eliminated, and shaft 41 acts now as

main transmission shaft, which was the function of shaft 26 in Figure 2. Gear 12 in Figure 2 was replaced in Figure 6 by gear 40. Gear 40 is hollow inside in order to create a space for the placement of 4 differential gears 42, 43, 44 and 45. Gears 42 and 43 are the planets which are held in position relative to gear 40 with pin 46. Pin 46 is connected to gear 40, which is the gear with the final output speed. Gears 44 and 45 are the side gears. Output shaft 26 is connected to side gear 44 and output shaft 41 is connected to side gear 45. The differential unit in Figure 6 will accomplish the differential function between the two output shafts 7 and 6 with conventional straight bevel gears. The end cap 47 closes the differential which is inside of gear 40 and acts as a radial sleeve bearing of shaft 26 and as a thrust sleeve bearing for gear 44. The walls of the hollow space in gear 40 are utilized as thrust sleeve bearings of gears 42 and 43.

The additional differential function accommodates different wheel speeds while the vehicle is, for example, driving through a curve. A differential, similar as found in conventional axle drives, has been integrated in gear 40. The transmission in Figure 6 has an output shaft 26 which could be connected to the right wheel and an output shaft 41 which could be connected to the left wheel. The input shaft 20 is still located at the left side of the transmission. If input shaft 20 is connected to an electric motor with a hollow shaft, then the transmission in Figure 6 as well as the electric motor can be in-line with the drive axle of a



Figure 6 Double differential between the driving wheels—Solution 1..



Figure 7 Double differential between the driving wheels—Solution 2.

vehicle. This means that output shaft 26 can be connected via a first drive shaft and CV joints to the right-side driving wheel and the second output shaft can be connected via a second drive shaft and CV joints to the left-side driving wheel.

A second possibility of a double differential oriented between the driving wheels, with an integrated differential function, is shown in Figure 7. This proposed design does not require a hollow motor shaft, and the differential function is realized with cylindrical gears. The input speed and torque is transmitted via the planets to the reaction gear, which is connected to the power take off gear (speed ϕ_5). The rotation ϕ_5 is transmitted to an intermediate pinion (idler), which is in mesh with a second cylindrical pinion. The second pinion is connected to a shaft that crosses over to the left side of the motor, where it is connected to a third cylindrical pinion that is in mesh with a second cylindrical gear. Cylindrical pinions two and three require the same number of teeth as do cylindrical gears one and two. This arrangement allows for a smaller size motor (compared to the solution shown in Figure 6).

Double Differential KISSsoft Animations and First Prototype

Although the functionality of the double differential development is explained in great detail, it is difficult to visualize the kinematic of this design. In order to make the high reduction function easy to understand, KISSsoft AG provided several animated designs (Ref. 8).

A screen shot of the one of the animations is shown in Figure 8. The input on the left side rotates the large, dark blue side gear on the left side of the model. The left side gear actuates the light blue outer planets which are rigidly connected to the inner planets. The planets roll on the large dark blue reaction gear on the right side, which is connected to the transmission housing and therefore cannot rotate. This kinematic arrangement puts the carrier in rotation and the inner light blue planets roll on the light gray output gear. Each rotation of the planets causes one rotation of the carrier in the same direction the gear mesh between the outer planets, and the reaction gear advances. This is the reason why the rpm of the planets is only 50% of the input gear rpm. If the tooth count of all the gears was identical, then no output rotation would occur. As the outer planets roll on the fixed reaction gear, the inner planets roll on the output gear, which in this case (like the reaction gear) would also not



Figure 8 KISSsoft animation—Double differential with ratio 18.



Figure 9 Prototype transmission with motor—Ratio 81.

rotate. If the number of teeth is changed, for example, by adding one tooth to the inner planets, then each full revolution of the planets (and the carrier) will rotate the light gray output gear by one pitch in a direction, reverse to the carrier rotation.

The ratio of the transmission in Figure 8 is +18. This was realized by using 40 teeth on all involved gears, except the two outer planets received 45 teeth. If the two outer planets had 35 teeth, then the ratio would be -14 (refer to equation 10). This example shows how flexible this transmission concept can be used to design a wide range of different ratios without a considerable change of the transmission size.

The first real size prototype of the double differential transmission is shown in Figure 9. This prototype achieves a ratio of 81. All eight gears are ground spiral bevel gears. Only two design calculations were required in order to manufacture the eight spiral bevel gears. It is interesting to mention that due to the similarity of all eight gears, only two different blade geom-

etries (one left hand and one right hand) were required for the soft cutting of all members. The prototype has an electric motor attached, which serves to demonstrate the interesting three-dimensional motion of the planet gears and their high reduction ratio. It was very simple to assemble the transmission unit with the correct backlash and tooth contact. The gears were all ground to low flank form deviations and the shaft shoulders and axial housing dimension have been manufactured to customary tolerances. The assembly did not require any shimming, and the tooth contact resembled the prediction from the design and analysis software. Because of the symmetric design and the fact that always two opposite gears transmit the torque, no bending and warping of the transmission is expected. This fact will prevent large contact position changes under load. The symmetry makes the double differential design insensitive to contact movement and edge contact.

The axial forces, especially of the planets, can be minimized by choosing opposite spiral angle direction for the outer and the inner planets, as shown in the model in Figure 9. In the arrangement in Figure 9, all gear meshes put load on the drive side, if the input gear rotates clockwise. The input gear rotates the outer planets (teeth of the planet on top move to the right), which puts load on the convex flanks of the planets (drive side). The planets try to rotate the reaction gear counterclockwise, which is not possible, but puts the load on the convex flanks of the reaction gear (drive side). If the ratio is positive, then also the output gear rotates clockwise, which puts the load on the convex flanks of the output gear (drive side). This is an interesting phenomenon, which improves efficiency and NVH behavior of double differential transmissions.

In order to make the transmission motion of the eight gears better visible, always two opposite pairs have been black oxidized, and the two opposite mating members were chrome plated.

Summary

The fascination of the automotive differential has led to the idea to build a second differential unit around a first center unit. Both units have the same axes around which they rotate with different speeds.

The potential of double differentials as ultrahigh reduction speed reducers is significant. Only the tooth-count of the gears in the outer differential unit must be changed in order to achieve ratios between 5 and 80 without a noticeable change of the transmission size.

Double differentials are well suited for high input speeds. The planets rotate with only about half of the input speed. This fact is attributed to the carrier, which rotates nearly with the same speed as the planets. The relative motion between the outer planets and the input gear, as well as the sliding velocity, are therefore only 50% of the value of two conventionally meshing bevel gears that roll with the same input speed, which is an ideal condition for a transmission with high input speeds.

Ground spiral bevel gears are recommended for the double differential application. Due to the load sharing of the two opposite planets, the torque of each gear is only 50% compared to a conventional bevel gear mesh. This effect results in very high-power density of this already very compact unit.

The efficiency of the double differential is comparable to multiple stage cylindrical gear transmissions comprising the same ratio, in contrast to the fact that always two pairs of gears are transmitting the rotation and torque. The compact size of the double differential can be translated to additional efficiency advantages compared to other transmission concepts, which qualifies this new transmission type very well for the speed reduction and transmission in electric vehicles and hybrids.

Although this paper concentrates on the application of double differentials to electric vehicles and hybrid cars, there are many other applications in the industry which require high ratios. Double differentials could be used in helicopters, wind turbines, agricultural equipment, and many other industrial applications.

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manufacturing machines. Under his leadership the world of bevel gear cutting has converted to environmentally friendly, dry machining of gears with significantly increased power density due to non-linear machine motions and new processes. Those developments also lower noise emission level and reduce energy consumption. For 35 years, Dr. Stadtfeld has had a remarkable career within the field of bevel gear technology. Having received his Ph.D. with summa cum laude in 1987 at the Technical University in Aachen, Germany, he became the Head of Development & Engineering at Oerlikon-Bührle in Switzerland. He held a professor position at the Rochester Institute of Technology in Rochester, New York From 1992 to 1994. In 2000 as Vice President R&D he received in the name of The Gleason Works two Automotive Pace Awards-one for his high-speed dry cutting development and one for the successful development and implementation of the Universal Motion Concept (UMC). The UMC brought the conventional bevel gear geometry and its physical properties to a new level. In 2015, the Rochester Intellectual property Law Association elected Dr. Stadtfeld the "Distinguished Inventor of the Year." Between 2015–2016 CNN featured him as "Tech Hero" on a Website dedicated to technical innovators for his accomplishments regarding environmentally friendly gear manufacturing and technical advancements in gear efficiency. Stadtfeld continues, along with his senior management position at Gleason Corporation, to mentor and advise graduate level Gleason employees, and he supervises Gleason-sponsored Master Thesis programs as professor of the Technical University of Ilmenau-thus helping to shape and ensure the future of gear technology.

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