

# Performance Analysis of Hypoid Gears by Tooth Flank Form Measurement

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## Introduction

The traditional way of controlling the quality of hypoid gears' tooth flank form is to check the tooth flank contact patterns. But it is not easy to exactly judge the tooth flank form quality by the contact pattern. In recent years, it has become possible to accurately measure the tooth flank form of hypoid gears by the point-to-point measuring method and the scanning measuring method. But the uses of measured data of the tooth flank form for hypoid gears have not yet

been well developed in comparison with cylindrical involute gears. In this paper, the tooth flank form measurement of generated face-milled gears, face-hobbed gears and Formate®/generated gears are reported. The authors discuss the advantages and disadvantages of scanning and point-to-point measuring of 3-D tooth flank forms of hypoid gears and introduce some examples of uses of measured data for high-quality production and performance prediction.

## Point-To-Point or Scanning Measurement

The traditional measuring method for the tooth flank form of hypoid gears is carried out by point-to-point measurement using coordinate measuring machines (CMMs). That method is used to avoid the effect of friction between the 3-D sensor probe and tooth flank surface on the measured result.

Recently, a reliable measuring method by scanning the objective tooth flank has been realized. The scanning measuring machine, as shown in Figure 1, uses a 2-D sensor. The negative influence of tooth flank friction on the measured results is avoided by control of rotation of the z-axis and by parallel movement of the sensor head position considering the sensitivity direction of the sensor probe and the scanning direction of the tooth flank (Ref. 1).

Figure 2 shows how those two kinds of measurement are performed. A typical point-to-point measuring pattern is a 5 x 9 grid of the tooth flank. These measuring points usually exist well inside the tip and side edges of the objective tooth flank. Using the scanning measuring method, edge-to-edge measurement is possible. That measurement covers all of the tooth flank area, including the tooth tip and the area near the toe and heel. The scanning measuring machine that the authors have developed can incorporate various measuring patterns. For example, to simulate gear performance, a pattern of 29 lines of profile and one line of lead, or a pattern of three lines of profile and three lines



Figure 1—The scanning measuring method of hypoid gears.

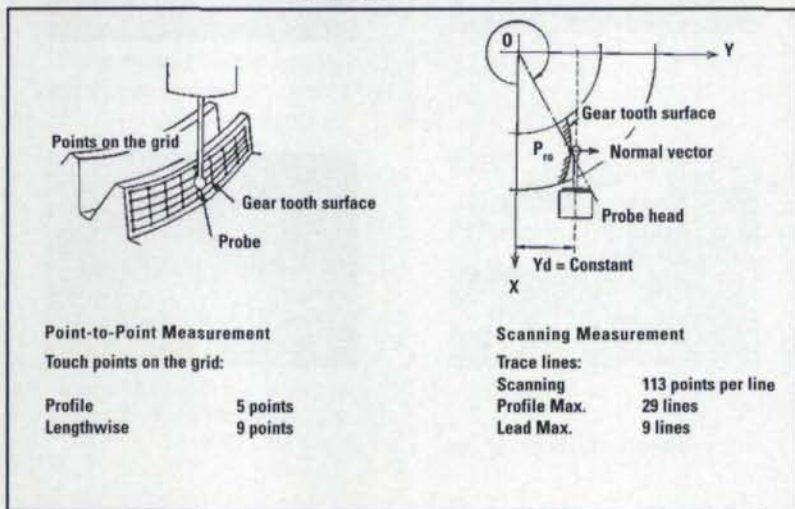


Figure 2—Two kinds of measuring methods for hypoid gears.

of lead are usually incorporated. Each line has 113 measuring points.

Figure 3 compares data of a tooth flank form measured by both the scanning and point-to-point methods. The objective tooth flank form has some waviness (high-frequency components). That waviness was found to be the main cause of unpleasant gear noise. The scanning method can detect the detail of a tooth flank form, but the point-to-point method could not do so.

### Two Kinds of Methods to Define Reference Tooth Surface

#### Direct approach (Machine setting base).

Existing CMMs normally incorporate the nominal surface calculated by cutting machine setting parameters and cutter geometry for both ring gear and pinion as the reference surface to define tooth flank form deviation. It is good in practice to check whether the tooth cutting is performed correctly or not. But hypoid gears are heat treated, usually case hardened, and then lapped. After heat treatment or lapping, tooth surfaces are distorted or deformed from that of the cutting stage. That means gears have lost their datum and it is therefore often not useful to measure the tooth flank form deviation of hypoid gears according to the reference based on the machine setting.

**Conjugate approach.** In the case of the scanning method, the reference tooth surface of ring gears is calculated by cutting machine setting parameters just like the former case. However, for the pinion, the conjugate surface to the ring gear surface is used. In this case, two reference surfaces can rotate theoretically without motion error.

The conjugate surface of the pinion is a virtual one. If the pinion and gear teeth are measured using such a conjugate approach, the measured values of the flank form deviation would show how far the form is from its true surface, which would realize conjugate motion. The same is true of cylindrical involute gears. That means we can use the experience of cylindrical involute gears for designing and manufacturing hypoid gears, and the measured data of the gear set can be used at any stage of designing or manufacturing.

**Direct prediction of contact pattern from measured results.** Figure 4 shows an example of the output forms of measured results from the conjugate approach for a Formate® hypoid gear set. Here, the ring gear's measured curve is shown by setting it to the position of contact corresponding to that of the measured pinion. Because the conjugate pinion tooth flank form is incorporated for reference in defining the tooth

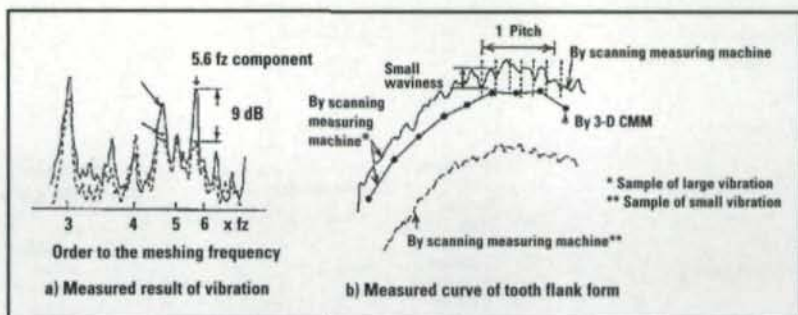


Figure 3—Difference in vibrational state of gears, when tooth flank form deviations have different high frequency components, and the comparison of tooth flank form measured by point-to-point and scanning methods for that case.

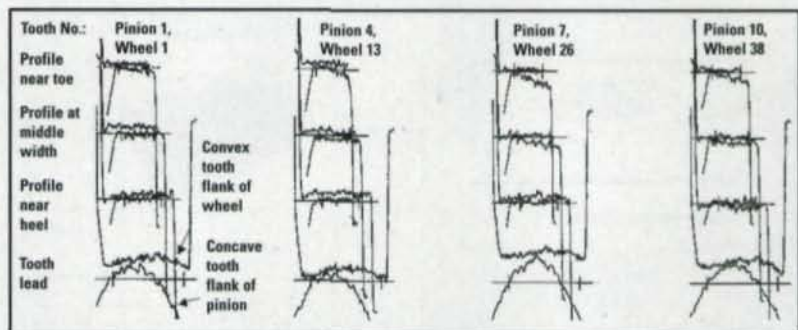


Figure 4—Measured tooth profile and lead form of Gleason face-milled, Formate®/generated gear set expressed in mating state.

flank form deviation, every point on both curves corresponds to contact. From this presentation of measured results, we therefore can directly see the contact position of tooth flank of actual gears on the measured curves—that is, we can directly see the outline of the tooth flank contact pattern.

### Measurement of Gears from Different Tooth Cutting Methods

The scanning method is capable of measuring tooth flank form deviation of hypoid gears being cut by different cutting methods. Figure 5(a) shows a sample of measured results from a generated face-milled gear set, and Figure 5(b) shows that of a face-hobbed spiral bevel gear set.

#### Tooth Contact Analysis Under No Load

Conventionally, tooth contact analysis (TCA) of hypoid gears is performed using tooth surface geometry calculated by the tooth cutting machine setting parameters for both ring gear and pinion. Figure 6(a) shows an example of tooth contact analysis of Gleason Helixform® hypoid gears (gear set 1 shown in Table 1) by direct approach (Gleason TCA). Using the concept of the conjugate tooth flank forms of mating gears and composite error surface (Ref. 2), a simulation program for predicting the gear performance was developed. Figure 6(b) is an example of the output of simulated results by conjugate approach for the same gears under the same conditions. The results of (a) and (b) agree well, when the same mounting position (V/H value) is given.

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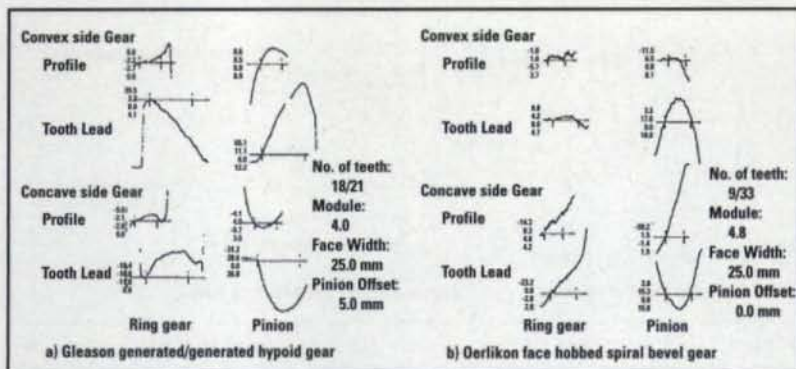


Figure 5—Measured tooth flank form of gears of different cutting methods.

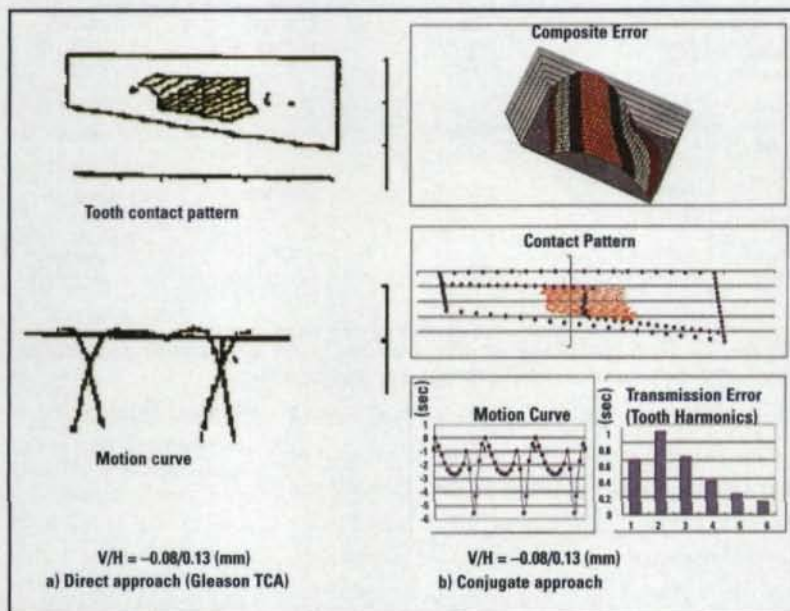


Figure 6—TCA of direct approach and conjugate approach.

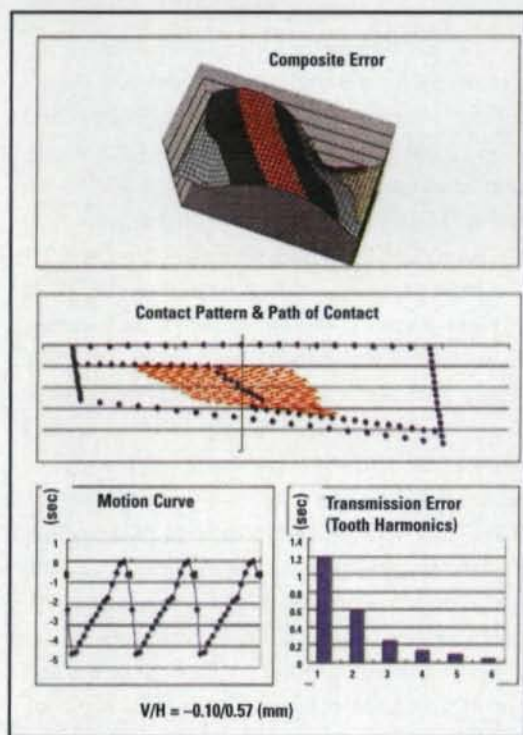


Figure 7—Simulation for pinion cutting change (conjugate approach).

At the first stage of hypoid gear development, cutting machines for pinion teeth are adjusted to get the optimum tooth contact pattern as seen in Figure 6(b). Figure 7 shows the simulated effect of a change in pinion cutting conditions. In this case, a 60° bias-in change is given according to the Gleason cutting sheet (Summary).

By this change, the path of contact (the trace of dots in the figure) is inclined and the contact pattern becomes longer, but the contact position is shifted in the toe direction. In this case, the mounting position should therefore be adjusted to get the contact pattern in the middle of the tooth flank.

#### Tooth Contact Analysis of Actual Gear

Using the conjugate approach, tooth contact analysis of actual hypoid gears based on the measured tooth flank form data becomes easy. An example of analysis of actual gears that had tooth wear problems in the field is discussed in the following sections.

**No load analysis.** The sample gear set used in Figures 8–10 is gear set 2 in Table 1. Figure 8(a) shows the composite error surface for the theoretical tooth flank form calculated from the machine setting parameters for ring gear and pinion. That surface is expressed by the form deviation curve on each contact line, neglecting its actual length.

The contact lines are set in the direction of the progress of gear mesh and the composite error values are plotted on these contact lines. The length of contact line near toe and heel of a tooth is stretched, which differs from the actual state. But by this expression, the contact point of tooth flanks can be more vividly recognized. The effect of toprem® is well recognized from the figure. Toprem® is a protuberance on the hypoid cutting blade that produces an undercut on the gear being cut.

From Figure 8(a), we can directly predict the contact pattern under no-load conditions to some extent. Figures 8(b) and 8(c) show the path-of-contact point and contact pattern expressed on the ring gear tooth flank under no load. Figure 8(d) shows the motion curve (transmission error) for this case.

This gear set has experienced trouble with tooth flank wear. Figure 9 shows the state of the initial tooth flank, i.e. of fresh gears just produced, in terms of measured composite error surface, observed no-load contact pattern obtained on the meshing frame (rolling tester) and the no-load contact pattern calculated by using the measured tooth flank form data (Ref. 4). The transmission error calculated for no-load conditions by

using measured tooth flank forms is also shown. Figure 10 shows those aspects for the worn tooth flank of the troubled gears. Usually the mounting position of hypoid gears, normally expressed by (V, H) value, is different at lapping, at the tooth contact pattern checking on a rolling tester, and in an actual gearbox's assembled condition. This difference leads to the same kind of difficulty in carrying out the simulation using the measured data of tooth flank form deviation of hypoid gears. In the simulation, some adjustment of (V, H) values from those nominal mounting distances ( $V_n$ ,  $H_n$ ) at designing or on the rolling tester is usually necessary to obtain the nearest contact pattern between observed and simulated figures.

**On load analysis.** Figure 11 shows the comparison between the measured and simulated contact pattern under load, and the transmission error for that case. Figure 12 shows the distribution of normal load, flash temperature (a), and Hertzian stress on the tooth flank.

Flash temperature  $T_f$  (in degrees) is obtained as a function of the flash temperature index  $T_f^*$  by the following equation:

$$T_f = T_f^* (1.11\mu \sqrt{\pi \cdot \text{rpm}/30}) / \sqrt{\lambda \cdot \rho \cdot c},$$

where  $\mu$  = functional coefficient of tooth flank; rpm = rotational speed of pinion;  $\lambda$  = normal conductivity;  $\rho$  = specific weight; and  $c$  = specific heat.

The sample gear set used in Figures 11 and 12 is gear set 3 in Table 1. Figure 13 compares the simulated, FEA calculated and measured tooth fillet stress (Ref. 6) for gear set 4 from Table 1. Figure 13(a) shows the changing state of tooth fillet stress with the progress of meshing at three longitudinal fillet positions along the tooth width, i.e. at 1/6 from toe, 1/2 from toe and 5/6 from toe.

Figure 13(b) arranges the same results in the form of fillet stress distribution along tooth width. These figures for tooth fillet stress show a good agreement among the FEA, measured and simulated results. Figure 14 shows a simulated frequency component of vibrational excitation for gear set 2 in Table 1, using measured tooth flank form deviation of the initial gear flanks shown in Figure 8. The vibrational excitation is calculated in terms of the tooth meshing frequency components. The actual frequency component in Hertz (1/s) of the vibrational excitation caused by the tooth meshing of these hypoid gears can be well predicted from these results and the driving speed of this gear set in operation. All of these quantities concerning the performance of gears are simulated-calculated at the same time. This is an extraor-

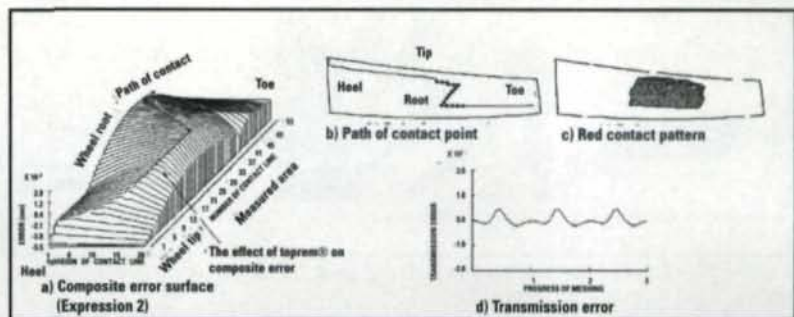


Figure 8—Composite error surface, path of contact point and red contact pattern expressed on the ring gear tooth flank (for gears cut by standard machine setting summary data, and  $V=0.00$  mm,  $H=0.00$  mm).

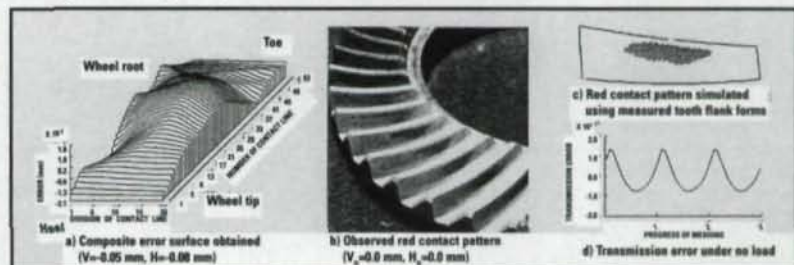


Figure 9—Composite error surface and red contact pattern for initial gear flank.

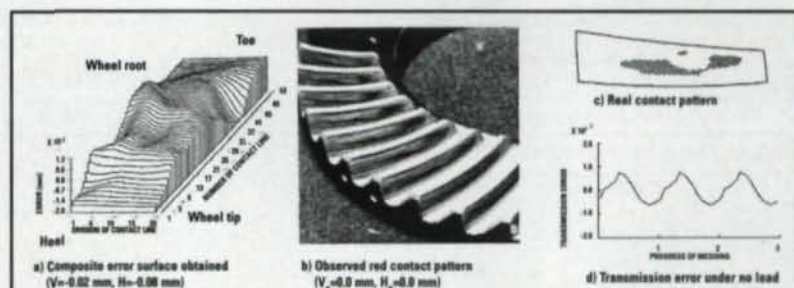


Figure 10—Composite error surface and red contact pattern for worn flank of troubled gears.

dinarily big difference from conventional methods of hypoid gear designing and load carrying capacity calculation.

Using measured tooth flank form deviation and the performance simulation software, the performance of hypoid gears with actual tooth flank forms under certain gear alignment conditions can be well predicted at the designing stage and in the case of troubleshooting.

Taking an example of tooth flank wear trouble mentioned in the "No load analysis" section, the vibrational excitation is calculated with the progress of tooth flank wear as shown in Figure 15. We can predict how the vibrational excitation of hypoid gears changes with long-running usage (Ref. 5).

### Conclusions

The merit of quality control of hypoid gears by measuring the tooth flank form is discussed. The scanning measuring method can realize tooth end-to-end measurement, which is very effective for knowing the detailed condition of the gear tooth. A performance simulation program of hypoid



Figure 11—Loaded contact pattern on ring gear flank and the transmission error (input torque = 30 N-m).

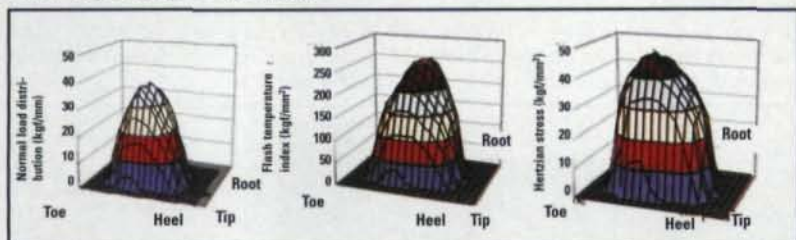


Figure 12—Simulated stress state on tooth flank of hypoid gears.

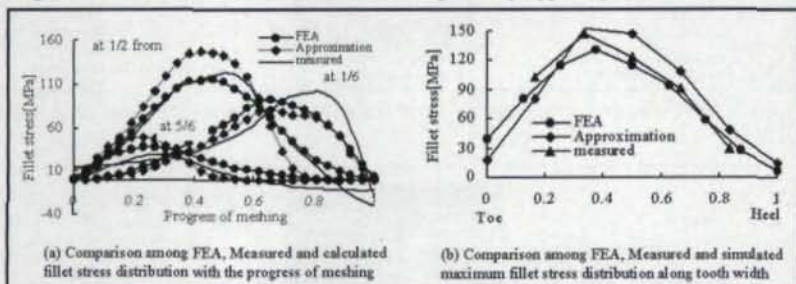


Figure 13—Ring gear tooth fillet stress (torque = 777 N-m).

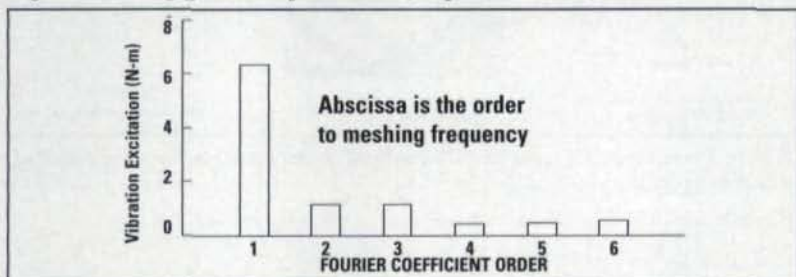


Figure 14—Result of the vibrational excitation.

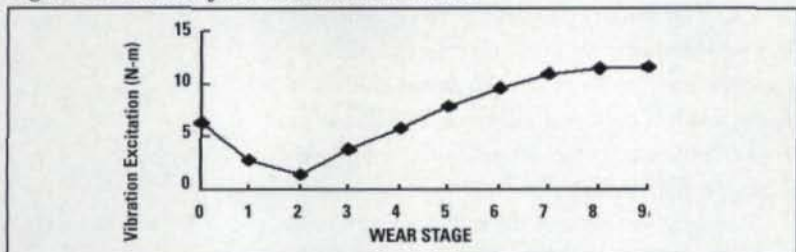


Figure 15—Change of the meshing frequency component of vibrational excitation with progress of tooth flank wear.

Table 1—The Dimensions of Hypoid Gear Sets.				
Gear Set	1	2	3	4
Gear Type	Helixform®	Formate®	Formate®	Formate®
No. of Teeth	9/37	6/40	7/43	6/37
Module	4.11	3.90	4.186	8.243
Face Width	24.0 mm	27.0 mm	31.0 mm	40.0 mm
Pinion Offset	27.0 mm	22.0 mm	22.0 mm	31.75 mm
Figures in Which Gear Set Was Used	Fig. 6, Fig. 7	Fig. 8, Fig. 9, Fig. 10, Fig. 14, Fig. 15	Fig. 11, Fig. 12	Fig. 13

gears that can accept measured form deviation of real tooth flank form is developed. The gear performance—such as transmission error, contact pattern under load and no load, normal load distribution on tooth flank, contact stress, tooth fillet stress, flash temperature and vibrational excitation for actual production gears—can now be analyzed. Some simulated results are compared with actual results in the field and from experience. Also, the effectiveness of the measurement of tooth flank form accuracy of hypoid gears is discussed. ⚙

## References

- Saito, Y. et al. "Check of 3D Form Accuracy of Hypoid & Bevel Gear Teeth for the New Generation of Quality Control," AGMA Fall Technical Meeting, 1998.
- Kubo, A. et al. "A Computer Based Approach for Evaluation of Operating Performance of Bevel and Hypoid Gears," *JSME International Journal* (Series C), Vol. 40, No. 4, 1997, pp. 749-758.
- Kagimoto, H. et al. "Development of Tooth Surface Measuring Machine for Hypoid Gears," Toyota Motor Co. Technical Review, Vol. 43, No. 1, May 1993, pp. 112-117.
- Wang, Zhonghou, A. Kubo, et al. "Simulation of Deterioration of Hypoid Gear Performance due to Tooth Flank Wear (1st Report, Data Processing of Measured Tooth Flank Wear)," *Transactions of the Japan Society of Mechanical Engineers*, Vol. 66, No. 647, 2000, pp. 2354-2362.
- Wang, Zhonghou, A. Kubo et al. "Simulation of Deterioration of Hypoid Gear Performance due to Tooth Flank Wear (2nd Report, Model for Tooth Flank Wear and the Change of Gear Performance Resulted)," *Transactions of the Japan Society of Mechanical Engineers*, Vol. 66, No. 647, 2000, pp. 2363-2370.
- Wang, Zhonghou, A. Kubo et al. "Tooth Root Stress Analysis of Hypoid Gears (2nd Report, Introduction of Predicting Method for the Tooth Root)," *Transactions of the Japan Society of Mechanical Engineers*, Vol. 66, No. 652, December 2000, pp. 221-227.
- Wang, Zhonghou, A. Kubo et al. "Measurement of 3D Tooth Flank Form of Hypoid Gears and its Usage for High Quality Production and Performance Prediction," International Conference on Mechanical Transmissions, Chongqing, China, April 2001.

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