Design of High Contact Ratio Spur Gears Cut With Standard Tools

Evgueni Podzharov and Almantas Mozuras

Abstract

In high precision and heavily loaded spur gears, the effect of gear error is negligible, so the periodic variation of tooth stiffness is the principal cause of noise and vibration. High contact ratio spur gears can be used to exclude or reduce the variation of tooth stiffness.

A simple method of designing high contact ratio spur gears cut with standard tools of 20° profile angle is presented in this paper. It consists of increasing the number of teeth on mating gears and simultaneously introducing negative profile shift in order to provide the same center distance.

Computer programs to calculate static and dynamic transmission error of gears under load have been developed to evaluate dynamic properties of gears. The analysis of gears using these programs showed that gears with high contact ratio of 1.96 have much less static and dynamic transmission error than standard gears.

Introduction

The periodic change of tooth stiffness, gear errors and friction force impulse at the pitch point are the principal causes of vibration and noise in gears. In high precision and heavily loaded gears, the effect of gear errors is insignificant, so the periodic variation of tooth stiffness and friction force impulse are the most significant causes of noise and vibration.

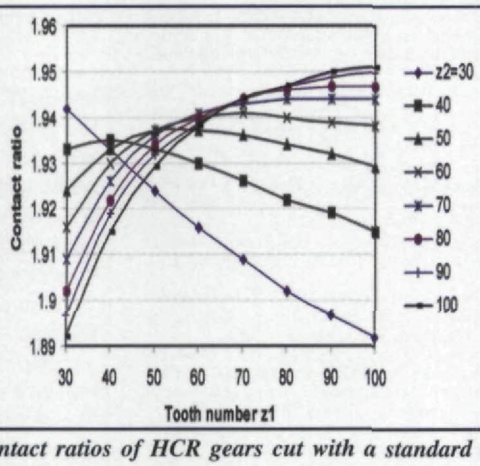

Figure 1—Contact ratios of HCR gears cut with a standard rack tool of 20° pressure angle.

High contact ratio spur gears can be used to exclude or reduce the variation of tooth stiffness. Kasuba (Ref. 1) established experimentally that the dynamic loads decrease with increasing contact ratio in spur gearing. Sato, Umezawa, and Ishikawa (Ref. 2) demonstrated experimentally that the minimum dynamic factor corresponds to gears with a contact ratio slightly less than 2.00 (1.95). The same result was found experimentally by Kahraman and Blankenship (Ref. 3) and theoretically by Lin, Wang, Oswald, and Coy (Ref. 4).

The increase in contact ratio can be implemented by decreasing pressure angle and increasing tooth height. In the previous works (Refs. 2–4), the increase in the tooth contact ratio was implemented by increasing tooth height. Vulgakov (Ref. 5) proposed a method of designing nonstandard gears in generalized parameters and found that spur gears with a contact ratio of more than 2.0 and a pressure angle more than 20° worked considerably quieter. Rouverol and Watanabe (Ref. 6) proposed maximum-conjugacy gearing, which has a low pressure angle at pitch point and increases slowly at the tip and at the root. The measurements also showed a considerable reduction in the noise level compared to standard gears.

Nevertheless, the use of standard pressure angle and standard tool is preferable. In the author's previous work (Ref. 7), a simple method of designing high contact ratio spur gears with a standard basic rack of 20° profile angle was presented. This method allows us to design gears with a contact ratio of nearly 1.95. In this paper, an analysis of static and dynamic transmission error in standard gears with 20° pressure angle and high contact ratio gears is given.

Method of Design of High Contact Ratio Spur Gears Cut with a Standard Tool

An increase of the contact ratio can be carried out by the method (Ref. 7) in the existent gears by incrementing the sum of numbers of teeth in the gears by two and then simultaneously introducing a negative displacement of the gear tooth profile in the following way. The operating transverse pressure angle is equal.

$$\alpha_{tw} = \arccos(a \cdot \cos\alpha_t / a_w) \qquad (1)$$

where α_{tw} is the operating transverse pressure angle;

α_t is the transverse pressure angle;

$a = 0.5m_t + (z_1 + z_2)$ is the standard center distance;

a_w is the operating center distance;

m is the module;

$z_{1(2)}$ is the tooth number of pinion (gear).

Then, the sum of profile shift coefficients is determined by

$$\Sigma x = (z_1 + z_2 + 2)(inv\alpha_{tw} - inv\alpha_t) / 2tg\alpha_n \qquad (2)$$

where

inv() is the involute function of the angle;

α_n is the normal pressure angle.

The profile shift coefficients of the pinion x_1 and gear x_2 can be selected, balancing specific sliding (Ref. 8). The conditions of interference absence must also be checked.

Contact ratio of a spur gear:

$$\varepsilon_\alpha = Z / p_b \qquad (3)$$

where Z is the active length of the line of contact;

$$p_b = \pi\, m \cos\alpha - \text{base pitch} \qquad (4)$$

The calculated values of contact ratio for the range of tooth numbers from 30–100 for both gears are presented in Figure 1. These values vary from 1.89 to 1.95.

The real value of contact ratio is slightly higher due to the consequence of the tooth deformation under load and edge contacts at the beginning and the end of contact. The absence of tooth undercut gives the range of values of tooth numbers (approximately $z > 26$) for these high contact ratio gears cut by a rack-type tool of 20° profile angle.

The pressure angle in HCR gears is lower than in standard gears. The lower pressure angle leads to greater sliding velocity and increases the risk of scuffing. The negative shift coefficient which must be introducted in this type of gear decreases the bending resistance of a pair of gear, but because of better load distribution between two pairs of teeth, this can be partly compensated. So, the HCR gears must be checked for bending and scuffing resistance of teeth.

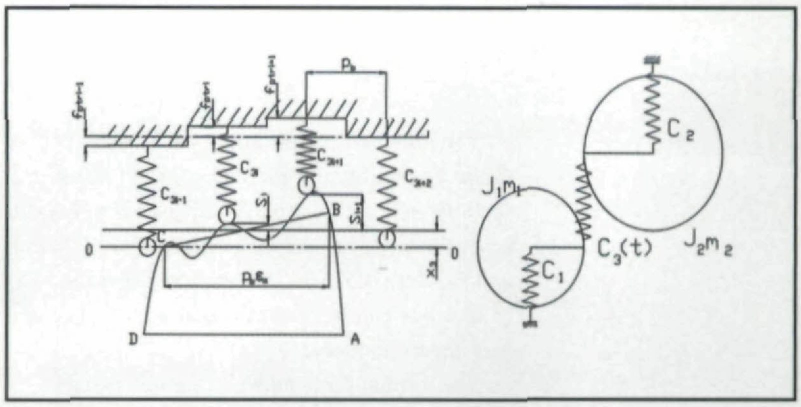

Figure 2—Gear dynamic model: (a) model of tooth engagement, (b) dynamic model.

Determination of Static Transmission Error in Spur Gears

The tooth engagement model presented in Figure 2a shows the influence of load distribution between teeth on effective gear errors. Following Yelle and Burns (Ref. 9) and Remmers (Ref. 10), the tooth profile is represented as a slide and the teeth as springs with rollers. The pitch error is modeled as a step base for spring and profile error, and base pitch error is displayed as an undulated inclined slide surface. From the analysis of this model, it is evident that having finish tooth errors, tooth edge contact designed as slopes AB and CD on the slide, the tooth load is not distributed uniformly between teeth.

Here C_{3i} is the stiffness of the *i-th* pair of teeth;

S_i is the kinematic error of the *i-th* pair, composed by base pitch error and profile error;

f_{ptri} is the circular pitch error difference of the *i-th* pair of teeth;

x_3 is the static transmission error under load;

f_{iri} is the local kinematic error of the *i-th* pair of teeth;

f_{pb} is the base pitch error; and

ε_a is the contact ratio.

In Figure 2a, the positive error is directed outward from the slide and the negative error is directed into the slide. As a consequence, a positive error corresponds to spring compressions. With these definitions, the tooth deflection, which appears as the result of the action of positive tooth error, is also positive. Then, the transmission error x_3, can be expressed by current errors of several pairs of teeth and its deflections x_{3i} in the following way:

$$x_{3i} = x_3 + f'_{iri} \qquad (5)$$

where

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$$f'_{iri} = S_i + f_{ptri} \qquad (6)$$

The kinematic error during tooth edge contact at the beginning and the end of tooth mesh (the sections AB and CD on the slide, see Fig. 2a) can be evaluated using the method exposed by Seireg and Houser (Ref. 11). The error in the section BC of the slide can be determined as a sum of cosine and linear functions.

The normal force between teeth is equal.

$$F_n = \sum_{i=1}^{n} C_{3i} x_{3i} = \sum_{i=1}^{n} C_{3i}(x_3 + f'_{iri}) \qquad (7)$$

Transforming Equation 7, we have

$$x_3 = (F_n - \sum_{i=1}^{n} C_{3i} f'_{iri}) / \sum_{i=1}^{n} C_{3i} \qquad (8)$$

Using formulas in Equations 5–8, one can find the kinematic error of tooth engagement under load at any moment in time, which is static transmission error. At first, the calculation is done with $n = 1$, assuming that $f'_{ir1} = f'_{ir\,max}$ is the maximum tooth error in the tooth engagement. Then, if $x_3 + f'_{ir2} > 0$, we accept $n = 2$ and continue the calculation. The method developed by Weber and Bonaschek (Ref. 12) is used to determine the tooth stiffness at any position. Calculation of static transmission error can be used in designing a gear with the purpose of selecting geometric and precision parameters, which assures a minimum excitation of vibration in gear engagement.

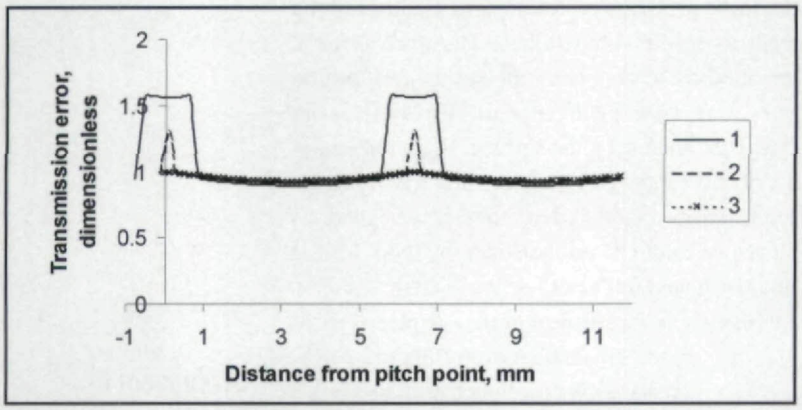

Figure 3—Static transmission error in gears: 1–standard gear, 2–high contact ratio gear with standard tooth height, 3–high contact ratio gear with increased tooth height.

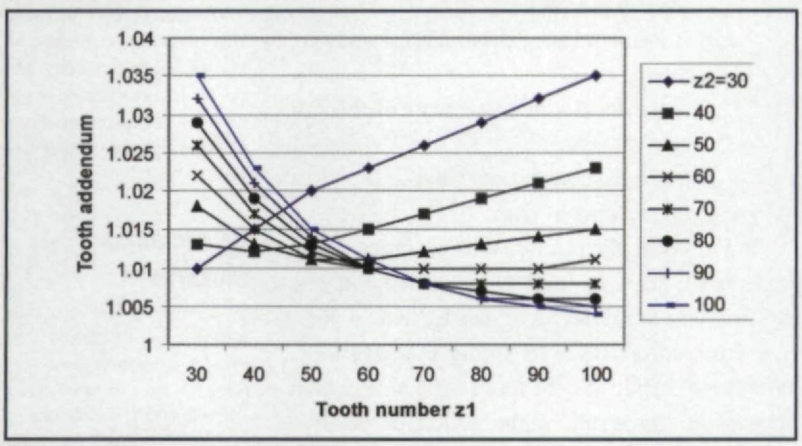

Figure 4—Tooth addendum corresponding to contact ratio of 1.96.

Table 1

No.	m, mm	a_w, mm	z_1	z_2	α_{tw}	x_1	x_2	h_a^*
1	3.0	135	40	52	20°	0.000	0.000	1.000
2	3.0	135	41	53	16.24°	−0.393	−0.520	1.000
3	3.0	135	41	53	16.24°	−0.393	−0.521	1.018

Analysis of Static Transmission Error of Standard and High Contact Ratio Spur Gears

The geometric parameters of gears analyzed here are shown in Table 1.

Here m is the module of gears;

a_w is the center distance;

$z_{1(2)}$ is the tooth number of pinion (gear);

α_{wt} is the operating transverse pressure angle;

$x_{1(2)}$ is the profile shift coefficient of pinion (gear);

$h_a^* = h_a/m$ is the addendum coefficient; and

h_a is the addendum, pinion (gear).

The results of the calculations of static transmission error for the gears with parameters shown in Table 1, without tooth errors, are presented in Figure 3. In the graphics of Figure 3, the values of transmission error are presented in dimensionless form, where $x_3^* = x_3 / x_{30}$ where $x_{30} = F_n/C_{30}$ mean tooth deflection, and C_{30} = mean tooth stiffness of gear mesh.

It can be seen from the figure that the variation of transmission error of the gear with standard pressure angle and standard tooth height (without tooth errors) has a stepped form (curve 1). The stepped form of transmission error is due to a change in tooth stiffness between one-pair tooth contact zone and two-pair tooth contact zone. The static transmission error of this form can excite high-level vibration. The gear pair with high contact ratio and standard tooth height (Equation 2) has an increased tooth contact ratio (ε_a = 1.93). This contact ratio was obtained by increasing the number of teeth of the pinion and the gear each by one, and introducing negative profile tooth shifts (x_1 = −0.393 y x_2 = −0.52), according to the method proposed (Ref. 7).

The static transmission error here is not large, but it has a peak that can cause vibration excitation. To further reduce the transmission error, we must increase tooth addendum by 0.018 of its value (h_a^* = 1.018). The contact ratio of this gear is 1.96. In this case, the static transmission error almost completely disappears (curve 3 in Figure 3).

The increments of tooth addendum which correspond to a contact ratio of 1.96 were calculated by iteration and presented in Figure 4. For the range of tooth numbers from 30–100, the required increments of addendum vary from 1.005–1.035 of its value, and these increments can be made using standard tooth tools.

Dynamic Transmission Error

The dynamic model of a gear pair is presented in Figure 2b. Here, $C_{1(2)}$ is stiffness of support of a pinion (gear), $C_3(t)$ is stiffness of tooth mesh, $m_{1(2)}$ is mass of pinion (gear), $J_{1(2)}$ is moment of inertia of pinion (gear). These parameters have the following values $C_1 = 45.5$ MN/m, $C_2 = 81$ MN/m, $C_{30} = 36.3$ MN/m, $m_1 = 3.38$ kg, $m_2 = 4.5$ kg, $J_1 = 0.297 \cdot 10^{-3}\ kg\ m^2$, $J_2 = 0.835 \cdot 10^{-3}\ kg\ m^2$. The dissipative coefficient was expected to be 0.05. In this dynamic model, the tooth engagement is represented by the structural model shown in Figure 2a. The whole dynamic model is described by three differential equations with periodic functions that were solved by a program based on the use of the Runge-Kutta method (Ref. 13).

The results of the solution of the equations for three types of gears without errors are shown in Figure 5, for one period of stationary vibrations, and for tooth mesh frequencies from 0–3,000 Hz. The dynamic transmission error is also represented here in dimensionless form.

It can be concluded from the figures that:

1. Standard gears have very high amplitudes of vibrations (Figure 5a).
2. Amplitudes of vibrations diminish in high contact ratio gears with standard tooth height (Fig. 5b).
3. Vibrations completely disappear in the case of high contact ratio gears with slightly increased tooth height and contact ratio equal to 1.96 (Fig. 5c).
4. Curves at zero frequency are identical to the static transmission error curves in Figure 3.

Conclusions

Methods and programs have been developed to calculate static and dynamic transmission errors under load in spur gears. A tooth mesh of periodic structure, which takes into account deflection and errors of each pair of teeth in the enagement, is used.

The analysis of static and dynamic transmission errors in high-precision, heavy-loaded standard gears, high contact ratio gears of standard tooth height and high contact ratio gears with slightly increased tooth addendum showed that, in the last type of gears, the static and dynamic transmission errors can be almost completely excluded. ○

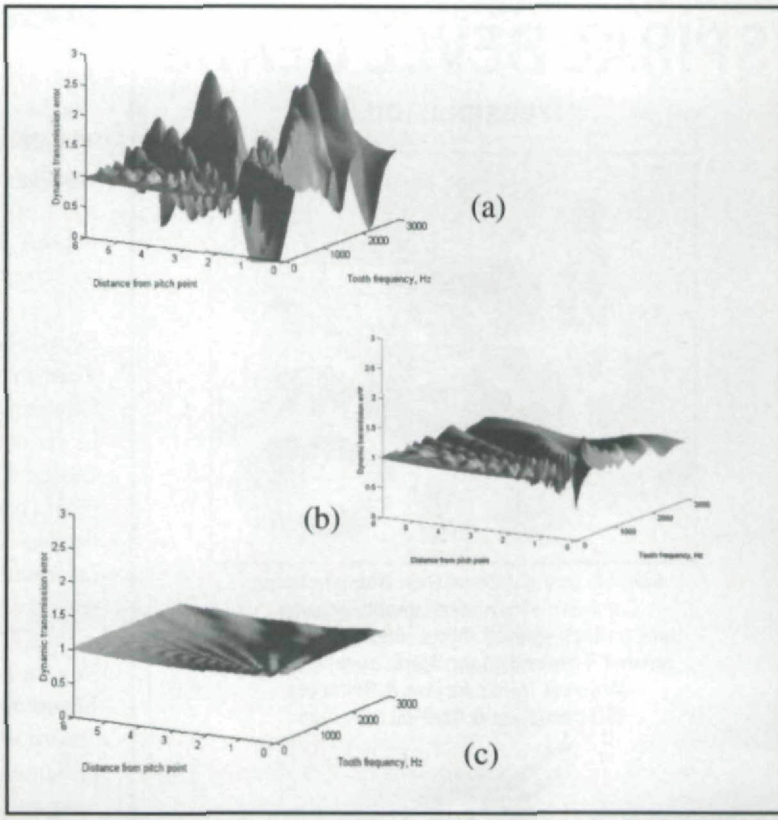

Figure 5—Dynamic transmission error: (a) Standard gear, (b) High contact ratio gear with standard tooth height, (c) High contact ratio gear with contact ratio of 1.96.

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