Systematic Investigations on the Influence of Case Depth on the Pitting and Bending Strength of Case Carburized Gears

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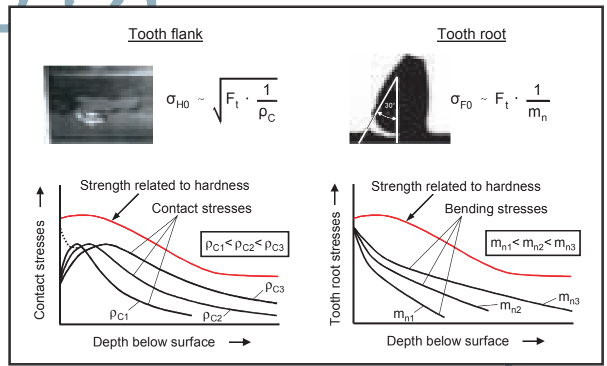

Figure 1—Basic principles of tooth flank and tooth root stresses (schematically).

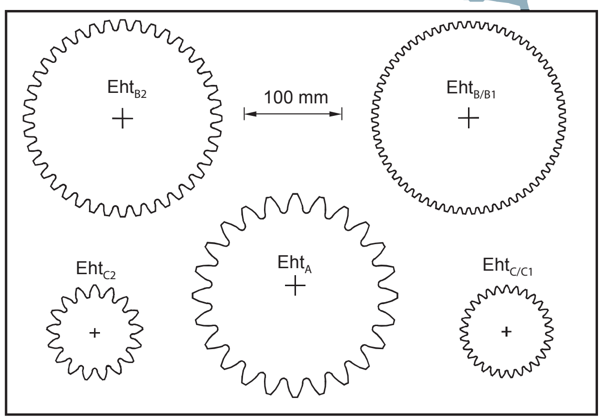

Figure 2—Gear types (pinion) of the test program.

Management Summary

High power transmitting gears are nowadays nearly always case carburized and hardened. The value of case depth is one important parameter that has to be specified by the gear designer for the heat treatment process. On the one hand, the available gear load capacity can be reduced with a case depth that is too small. On the other hand, unfavorable influences on the material properties and possible increased distortion by hardening and increased requirements for grinding may result from a case depth that is too large. In times of modern and increasingly optimized gear manufacturing, there is a fundamental need for the gear designer to know how to determine an appropriate case depth for his actual gear application in order to guarantee the required load capacity and taking into consideration the different basic principles in the nature of contact and bending stresses that are most relevant for gear load capacity.

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Introduction

In modern gear manufacturing, power-transmitting gears are nearly always made of case carburized steels, which are particularly suitable for withstanding high local stresses without sustaining damage. The heat treatment process of case carburizing is an exceedingly demanding process, requiring a high level of technical knowledge and experience.

Gears are case carburized to increase surface hardness, improve wear resistance and achieve high contact and bending strength. The hardness distribution in general is described by the characteristic parameters of surface hardness, case depth (*Eht*) and core hardness, and is usually seen as an approach for the strength distribution in the case hardened layer. While surface and core hardness are restricted to narrow limitations, case depth can be varied in a wide range. Thus the value of case depth decisively influences the hardness (strength) profile in the case carburized layer.

Failure modes of pitting and tooth root breakage are affected by the value of case depth. Whereas the pitting load capacity is a function of Hertzian contact stresses, depending on the square root of applied load and reciprocal of equivalent radius of flank curvature, the tooth root strength is related to bending stresses and directly to the applied load and gear module.

These differences in the nature of contact and bending stresses result in different requirements regarding the strength profile for tooth root and tooth flanks of a gear and have to be taken into consideration when choosing an appropriate case depth (see Fig. 1).

Since the costs of a case carburized gear are influenced significantly by the value of case depth, experimentally verified and easily applicable rating formulas are required to evaluate the influence of case depth in order to guarantee required load capacity regarding pitting resistance and tooth root bending strength of a gear.

For this purpose, the pitting and the bending strength of case carburized gears were investigated (Ref. 14). Gears of different sizes and different gear geometry were included in the test program in order to determine the basic principles for the influence of case depth on the gear load capacity. Residual stress and further characteristics of the case hardened layer that are also influenced by the value of case depth were examined.

Test Programs and Test Gears

The investigations have been carried out on several gear types, different in gear size and gear geometry. Figure 2 shows the test pinions of the gear types.

From each gear type, several test series of gears having the same geometry but different case depth were investigated. Table 1 shows the complete test program.

Test series	1	2	3	4	5
	Case depth in mm (drawing specification)				
Eht_A	0.3	0.6	0.9	1.4	2.0
$Eht_{B/B1}$	0.2^{*}	0.4	0.7^{*}	1.1	1.6^{*}
Eht_{B2}	0.2^{**}	-	0.7^{**}	-	1.6^{**}
$Eht_{C/C1}$	0.2^{*}	0.4	0.7^{*}	1.1	1.6^{*}
Eht_{C2}	0.2^{**}	-	0.7^{**}		1.6^{**}

* only bending fatigue tests ** only pitting fatigue tests

Table 1—Test Program: Influence of Case Depth on Pitting and Bending Strength.

Nomenclature	
Eht	Case depth at Vickers hardness 550HV1
Eht_{Fopt}	Optimum case depth for maximum bending strength
Eht_{Hopt}	Optimum case depth for maximum pitting resistance
F_t	Nominal tangential load
S_F	Safety factor—bending
S_H	Safety factor—pitting
Y_{Eht}	Case depth factor—bending strength
Y...	Influence factor—bending, according to DIN 3990 (Ref. 4)
Z_{Eht}	Case depth factor—pitting resistance
Z...	Influence factor—pitting, according to DIN 3990 (Ref. 4)
a	Center distance
m_n	Normal module
z	Number of teeth
ρ_c	Relative radius of flank curvature at pitch point
σ_F	Bending stress number
σ_{Flim}	Allowable bending stress number
σ_H	Contact stress number
σ_{Hlim}	Allowable contact stress number
*Further symbols according to DIN 3990/ISO 6336 (Refs. 4, 9).	

Tooth root bending strength was investigated on gear types Eht_A, Eht_B and Eht_C. Essential data for the bending gears are listed in Table 2.

Pitting resistance was investigated on test series of all gear types but with special focus on test series with center distance of 200 mm. The design parameters for the pitting gears are given in Table 3.

All test gears were made from one batch of 16MnCr5 steel, comparable to SAE 5115. The chemical composition of the gear material is shown in Table 4.

All gears were hobbed, carburized and hardened with the carburizing process, which was varied in order to obtain the desired different case depth values. After heat treatment, the test gears were mechanically (shot) cleaned. The flanks of the pitting gears were additionally finished by grinding (MAAG–0°) to surface roughnesses of R_a = 0.2–0.4 µm (*a* = 91.5 mm) and R_a = 0.3–0.5 µm (*a* = 200 mm), respectively, and a gearing accuracy of 4–6, according to ISO 1328 (Ref. 8). The peak-to-valley roughness R_z in the unground tooth root of the bending gears is $R_z \approx 5$ µm.

Test gears were manufactured according to industrial practice and fulfill the requirements for case carburized gears of quality MQ according to DIN 3990/ISO 6336 (Refs. 4, 9).

Parameter		Unit	Eht_A	Eht_B	Eht_C
Normal module	m_n	mm	8	3	3
Number of teeth	z	-	24	67	29
Pressure angle	α	°	20	20	20
Helix angle	ß	°	0	0	0
Face width	b	mm	30	30	20
Add. mod. factor	x	-	0.27	-0.60	0.56
Tip diameter	d_a	mm	212.3	201.0	96.3

Table 2—Gear Data of Bending Test Gears.

Parameter		Unit	Eht_A	Eht_{B1}	Eht_{B2}	Eht_{C1}	Eht_{C2}
Center distance	a	mm	200	200	200	91.5	91.5
Normal module	m_n	mm	8	3	5	3	5
Number of teeth	z_1	-	24	67	40	29	17
	z_2	-	25	69	41	30	18
Face width	b	mm	18	18	18	12	14
Pressure angle	α	°	20	20	20	20	20
Helix angle	ß	°	0	0	0	0	0
Contact ratio	ε_α	-	1.50	1.50	1.50	1.51	1.38
Relative radius of flank curvature	ρ_C	mm	19.5	14.3	15.4	9.5	10.0

Table 3—Gear Data of Pitting Test Gears.

Element composition wt%									
C	Si	Mn	P	S	Cr	Al	Ni	Mo	Cu
0.17	0.37	1.20	0.02	0.03	1.17	0.04	0.15	0.04	0.15

Table 4—Chemical Composition of 16MnCr5 Steel.

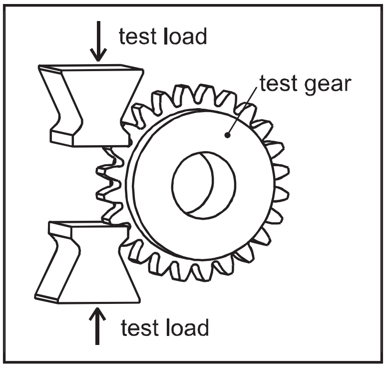

Figure 3—Clamping of test gear.

Test Conditions

Each test series repeated single stage tests in the range of endurance limit and low- and high-cycle fatigue.

Bending fatigue tests were carried out in pulsator test rigs of 100 and 250 kN capacity. The frequency was about 110–120 Hz. The gear teeth were clamped between two contact jaws as shown in Figure 3 and loaded in such a way that the load direction was tangential to the base circle. The endurance limit was assumed to be 6 x10^6 stress cycles without breakage. The endurance strength in bending was calculated according to the method in DIN 3990/ISO 6336 (Ref. 9).

Pitting fatigue tests were performed on FZG gear test rigs (see Fig. 4). The gear center distances were 200 mm and 91.5 mm, respectively. A detailed description of the test rig is given in Reference 5. The gears were spray lubricated with refined mineral oil ISO VG100 (viscosity v = 100 mm^2/s at 40°C) with a 4% sulfur-phosphate additive. Oil injection temperature was 60°C. All tests were performed at rotational speed of 3,000 rpm at the pinion of the driving gear. The gears were loaded to various Hertzian stress limits until failure occurred. An endurance limit was considered to be reached when the test pinion ran for 100 x 10^6 cycles without damage. Test gears were deemed to have failed when 4% of the active working flank area of a single tooth was damaged by pitting. The applied contact pressure and Hertzian stresses were calculated according to the method of DIN 3990/ISO 6336.

Test Results—Bending Strength

Figure 5 shows the hardness distribution of bending gears type Eht_C. Surface hardness and core hardness of the different test series are comparable. The case depth values are clearly different.

Surface hardness of all test gears type Eht_A and type Eht_B is also in the same range, 720 +/– 50 HV1.

Core hardness of test gears type Eht_A is about 350 HV10 and, due to the larger size, is somewhat less than core hardness of gear types Eht_B and Eht_C.

Figure 6 shows test results for the influence of case depth on the bending strength of all test series. Each point represents the tooth root endurance limit of one test series as determined by the S-N curve and is related to the maximum bending strength of each investigated gear type. Results of some former investigations (Ref. 2) are shown.

Maximum bending strength was achieved for a case depth of 0.1...0.2 · m_n. In the range of case depth < 0.1 · m_n, bending strength strongly decreases with reduced case depth. In the range of case depth > 0.2 · m_n, the bending strength decreases with increasing case depth but was more moderate compared to the range of too small a case depth. The actual results are in good agreement with those from former investigations.

Test results clearly demonstrate that the bending strength of case carburized gears is influenced significantly by the ratio of case depth to gear module. This corresponds with the basic principles for tooth root bending stresses, as a module of a gear is a relevant parameter for the dimension of the critical cross-section in the tooth root area. Increasing the module causes a decreasing stress gradient over the material depth. With the same maximum tooth root bending stresses at the surface, a larger gear will therefore have higher stresses at a given distance below the surface than a smaller gear (see Fig. 1).

Compared to DIN 3990/ISO 6336 standards for case carburized gears, all test series with a case depth of 0.1...0.2 · m_n show a bending fatigue strength equal to or even higher than specified by the DIN/ISO field for allowable stress number σ_{Flim} of quality MQ case carburized gears.

Investigations of material properties, on the one hand, gave no indication of a relevant influence of carbon content (C approximately 0.65–0.85%) or residual austenite content (< 5–20%) on the test results for the investigated gears. On the other hand, material investigations showed that with increasing case depth and thus also increasing duration of the carburizing process, intergranular oxidation as well as grain size of the former austenite increased (see Fig. 7).

Residual stress distribution in the case carburized layer was determined by X-ray diffraction.

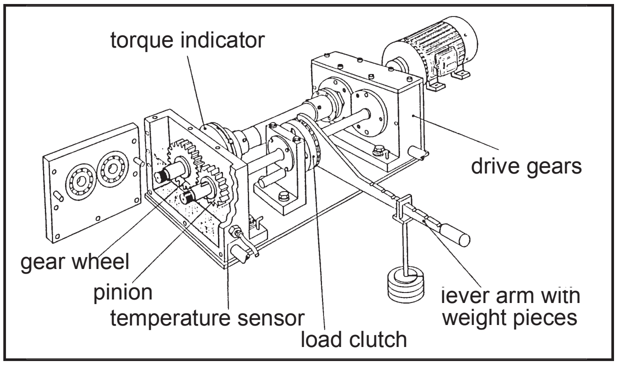

Figure 4—FZG gear test rig for pitting endurance tests.

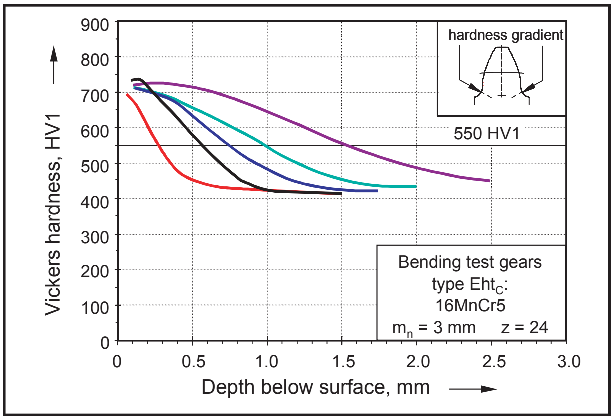

Figure 5—Hardness distribution of bending test series, gear type *Eht_C*.

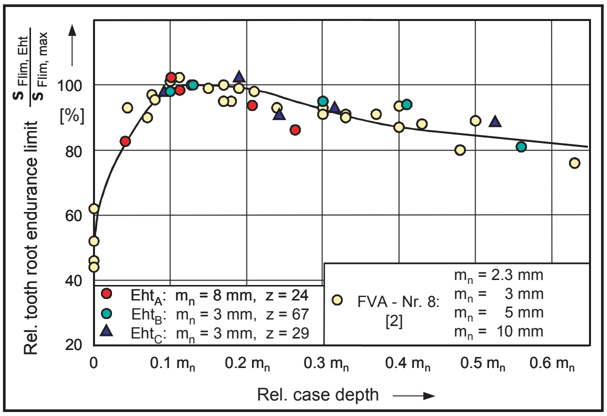

Figure 6—Test results for the influence of case depth on the tooth root bending strength (endurance limit).

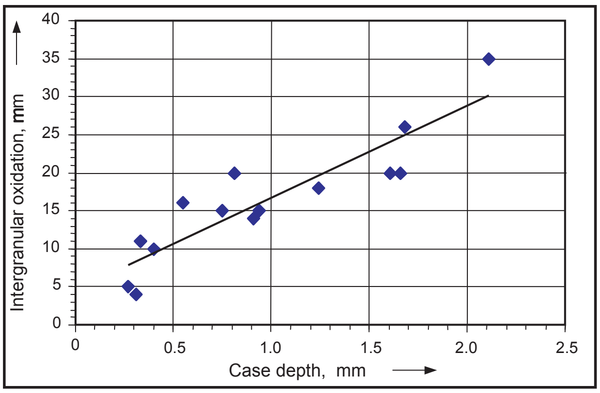

Figure 7—Case depth and intergranular oxidation of bending test gear types *Eht_A*, *Eht_B*, *Eht_C*.

Figure 8 shows the residual stress distribution for different test series of gear type Eht_C. Residual stress distribution has the typical form known for case carburized and shot cleaned gears. Residual compressive stresses at the surface and in the near surface area, especially maximum values, are smaller for test series with higher case depths and longer carburizing times than for gears with smaller case depths.

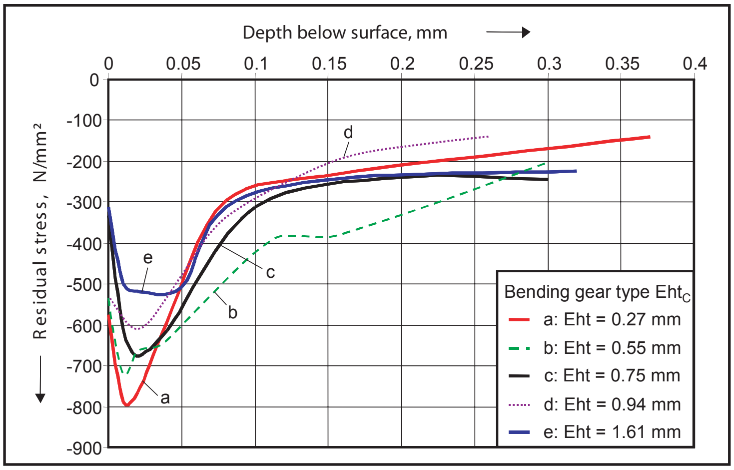

Figure 8—Residual stress distribution for test gears with different case depth (bending gear type *Eht$_C$*).

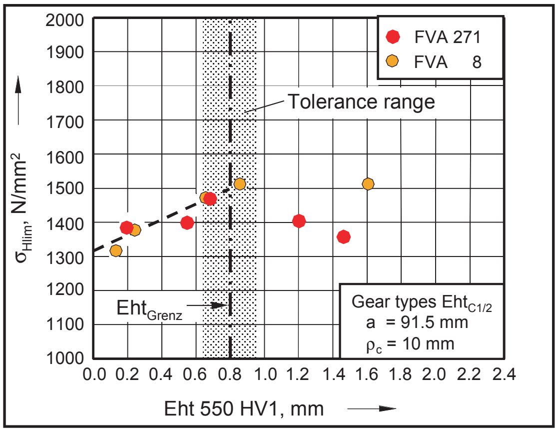

Figure 9—Test results for the influence of case depth on the pitting resistance (gear size ρ_C = 10 mm).

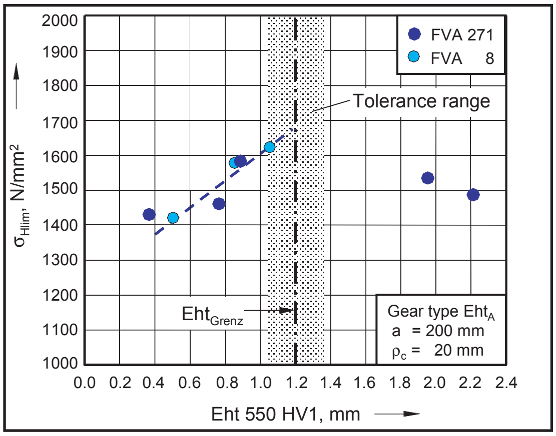

Figure 10—Test results for the influence of case depth on the pitting resistance (gear size ρ_C = 20 mm) .

It is well known that these influences—higher intergranular oxidation, larger grain size, smaller residual compressive stresses in the tooth root area of a case carburized gear—may result in reduced bending strength (Refs. 1, 3 and 6). As all test gears were made of the same batch of steel and manufactured under equivalent mechanical conditions, results are related to case depth carburizing time and not separated into individual influence parameters.

Test Results—Pitting Resistance

In former investigations on the influence of case depth on the pitting resistance of case carburized gears (Refs. 2, 11), an optimum case depth to ensure the maximum allowable contact stress number has been established as:

$$Eht_{Grenz} = \frac{\rho_C + 10}{25} \pm 0.15 \text{ mm} \qquad (1)$$

The test results are mainly based on smaller gears. In Figure 9, results of investigations on the influence of case depth on pitting resistance (Ref. 14) are compared with the results from other investigations (Ref. 2). Results are given as allowable stress numbers, which are derived from the pitting fatigue limit of S-N curves for the investigated test series of gear types Eht_{C1} and Eht_{C2}. The highest fatigue limit was achieved for test series with case depth in the range of optimum case depth Eht_{Grenz}. Test series with smaller or larger case depth than the optimum depth (Eht_{Grenz}) achieved lower fatigue limits. Results in Figure 10 are based on larger gears (gear type Eht_A) from other investigations (Refs. 11, 14). Tendencies for the influence of case depth on the pitting resistance are the same as for smaller gears. However, the highest fatigue limit was achieved for larger optimum case depth. These findings are also confirmed by the results of the investigations on the test series of gear type Eht_{B2}.

Figure 11 summarizes the experimental results on the influence of case depth on the pitting resistance for test series of different gear types. The achieved contact fatigue limit (surface pitting) of each test series is related to maximum fatigue limit of the relevant gear type for $Eht \approx Eht_{Grenz}$.

Figure 11 shows that all gear types achieved maximum pitting resistance if case depth was in the range of optimum case depth Eht_{Grenz} as defined in References 2, 10 and 11. An approximately linear decrease of pitting resistance with the difference of actual and optimum case depth was found.

Several guidelines given in literature (Refs. 12, 13) recommended case depth as a function of module. Comparing test results of gear types Eht_{B2} and Eht_{C2}, both with the same module but different radii of flank curvature, indicates that the gear module may not be sufficient for choosing appropriate case depth regarding contact fatigue life. Especially for gears with small ratios of m_n/ρ_C, often used in high speed gears, discrepancy will arise if choosing case depth as a function of relative radius of flank curvature or if choosing case depth based on module.

Compared to DIN 3990/ISO 6336 standards for case carburized gears, test series with case depth in the range of Eht_{Grenz} achieve allowable contact stress numbers as specified in DIN/ISO standards for quality MQ case carburized gears. Fatigue limits (pitting) of other test series, in particular with smaller case depth, fell mostly below the upper limit of the DIN/ISO allowable field for material quality MQ.

Thus, the results indicate that optimum case depth for maximum pitting resistance is a function of the relative radius of flank curvature as described by Equation 1.

Accompanying investigations on the material properties indicated for most gear types a slight increase of surface carbon content and consequently higher content of residual austenite with increasing case depth. On the other hand, the investigations showed no relevant—and from the value of case depth—independent influence of these specific parameters on the achieved pitting resistance (see Fig. 12). Only two test series of gear type Eht_A with a large case depth showed a relatively high surface carbon content, but also showed case depth to be the dominant influence on the achieved fatigue limits.

Residual stress distribution was measured using X-ray diffraction. Figure 13 shows measurement results for a different test series of gear type Eht_{B2}. For test series with smaller case depth, relatively high compressive residual stresses were measured in the near-surface region. Larger case depth, especially on test gears with higher surface carbon and higher residual austenite content, caused mostly a reduction of compressive residual stress in the case hardened layer. In some cases, test series with larger case depth and high surface carbon content

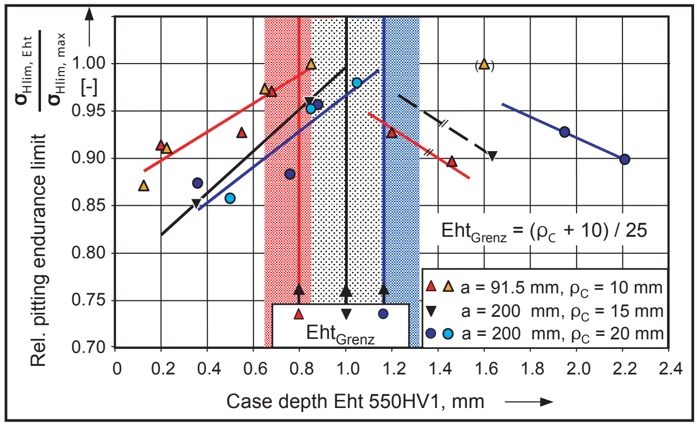

Figure 11—Comparison of test results for the influence of case depth on pitting resistance for different gear sizes.

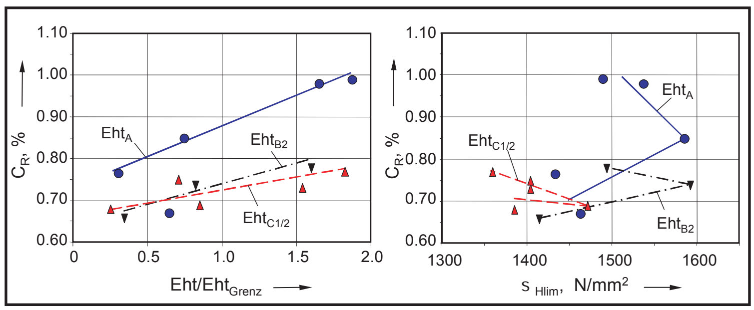

Figure 12—Case depth (*Eht*), surface carbon content (C_R) and achieved pitting fatigue limit (σ_{Hlim}) of investigated test series.

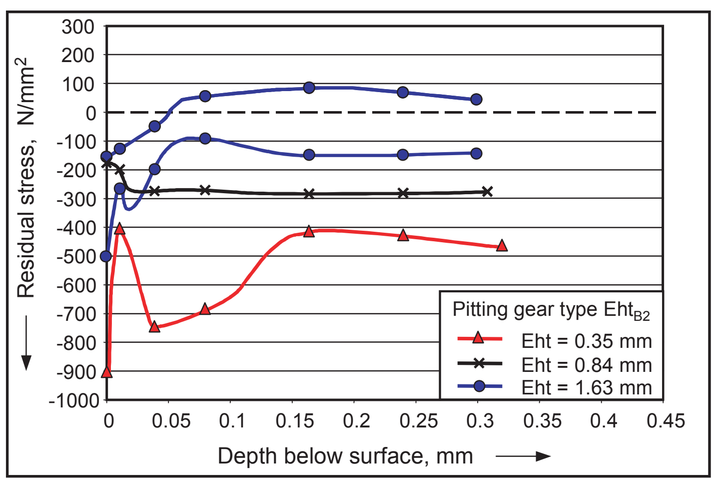

Figure 13—Residual stress distribution for test gears with different case depth (pitting gear type Eht_{B2}).

showed even small tensile residual stresses below the surface.

The results presented in Figures 9–11 are based on typical pitting failures. Analysis of the damaged gear flanks showed that these failures originated at the surface or at least in the near-surface region. The given stress values therefore have to be regarded as surface contact fatigue limits.

Test series of gear type Eht_{B1}, and in some cases also test gears of gear type Eht_{B2}, failed due to a special type of tooth breakage where the fracture occurred above the tooth root, frequently halfway down the tooth tip (see Fig. 14). Analysis of the fractured surfaces showed that the fracture was starting at a small inclusion in the material, generally at the transition between the case hardened layer and the softer core material.

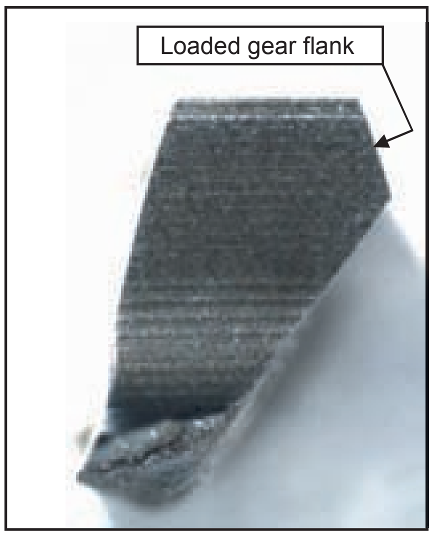

Figure 14—Special tooth breakage on test gear type Eht_{B1}.

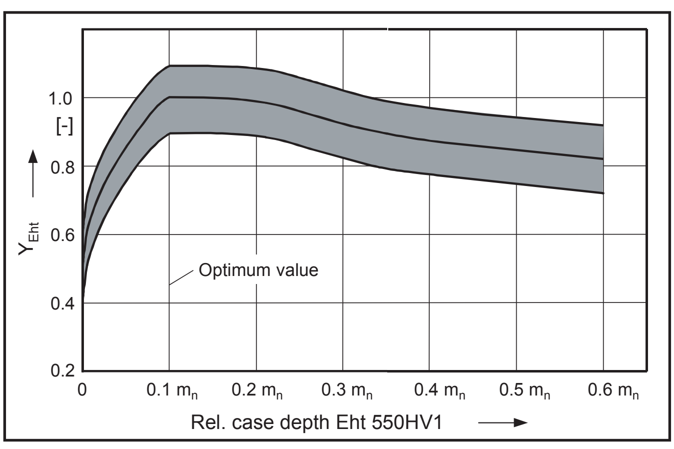

Figure 15—Influence factor Y_{Eht} for the influence of case depth on tooth root bending (endurance) strength.

These tooth breakages appeared suddenly, often after a high number of load cycles and without any indication of previous surface (pitting) damage. Gear type Eht_{B2} and especially gear type Eht_{B1} are characterized by a relatively small module but a high number of teeth (high relative radius of tooth curvature). Tooth breakage appeared on each of the two test series of gear type Eht_{B1} with case depths of 0.5 mm and 1.3 mm, respectively. As the nature and the mechanisms of this special type of tooth fracture are not fully understood, results of gear type Eht_{B1} were not taken into consideration in results on the influence of case depth on the surface contact (pitting) fatigue.

Results of the influence of case depth on the load capacity of the tooth flank agree with the accompanying investigations. These theoretical studies show that the variation of case depth influences the stress as well as the strength distribution over material depth, especially if residual stresses connected with the value of case depth are taken into consideration. Computations demonstrate that adequate case depth, depending on the relative radius of flank curvature and applied load, leads to a peak value of stress/strength ratio at or near the surface so that pitting will be initiated in this area. Smaller values of case depth or unfavorable residual stresses due to large case depth can result in a higher stress/strength ratio, or a lower load capacity. It may also lead, especially for gears with small ratios of m_n/ρ_C to a relocation of the maximum value of stress/strength ratio to a greater distance below the surface. This relocation may lead to gear damage that is initiated below the surface. Results of the theoretical studies have been published in detail (Refs. 7, 15).

Application of the Test Results on the Influence of Case Depth on Gear Load Capacity

Influence factor Y_{Eht} for tooth root bending strength. Test results indicate tooth root bending strength is influenced by the ratio of case depth to gear module. Optimum case depth for maximum tooth root bending strength (Eht_{Fopt}) is evaluated as

$$Eht_{Fopt} = 0.1...0.2 \cdot m_n$$

Gears with case depth in the range of optimum case depth Eht_{Fopt} should certainly achieve the allowable stress number according to standards for material quality (MQ).

For case depth values different from the optimum ($Eht \neq Eht_{Fopt}$), achievable tooth root bending strength is reduced. When evaluating the influence of case depth on tooth root bending strength, the influence factor Y_{Eht}, as defined in Figure 15, depends on the ratio of the case depth to the gear module. All test results fall into the given tolerance field. Y_{Eht} may be integrated in the standardization calculation method for rating gears according to DIN 3990/ISO 6336, shown in Equation 3 (Refs. 4, 9).

$$S_F = \frac{\sigma_{Flim} \cdot Y_{ST} \cdot Y_{\delta relT} \cdot Y_{RrelT} \cdot Y_X \cdot Y_{Eht}}{\sigma_F} \qquad (3)$$

Influence factor Z_{Eht} for surface contact (pitting) fatigue strength. Test results show that pitting resistance is influenced by case depth. Optimum case depth regarding the maximum pitting resistance of the tooth flank (Eht_{Hopt}) is a function of relative radius of flank curvature according to Equation 4.

$$Eht_{Hopt} = Eht_{Grenz} = \frac{\rho_c + 10}{25} \pm 0.15 \text{ mm} \qquad (4)$$

Gears with case depth in the range of Eht_{Hopt} should achieve the allowable stress number for case carburized gears of material quality MQ according to DIN 3990/ISO 6336 (Refs. 4, 9).

Smaller or larger case depth values than the optimum lead to a decrease of pitting resistance. Influence of case depth on allowable contact stress number (pitting) is described by the influence factor Z_{Eht}. Z_{Eht} is established as a function of the optimum case depth regarding maximum pitting resistance Eht_{Grenz}—that depends on the gear geometry, described by ρ_c—and the relevant case depth of the actual gear application. Z_{Eht} may be approximated from Figure 16.

According to DIN/ISO, the influence of case depth on pitting load capacity can be taken into consideration by introducing factor Z_{Eht} into Equation 5.

$$S_H = \frac{\sigma_{Hlim} \cdot Z_w \cdot Z_L \cdot Z_v \cdot Z_R \cdot Z_X \cdot Z_{Eht}}{\sigma_H} \qquad (5)$$

Optimized case depth regarding maximum pitting and bending strength. Equations 2 and 4 and influence factors Y_{Eht} and Z_{Eht} may be used to calculate optimum case depth for maximum load capacity of tooth root or tooth flank as well as to determine adequate case depth for actual gear application if geometry, relevant stresses and minimum required safety factors are known. Consequently, lightly loaded gears will tolerate less case depth. On the other hand, safety factors S_H and S_F for a gear with a given case depth may be calculated by using Y_{Eht} and Z_{Eht}.

Especially for critical gear applications and special gear geometries, an optimized load capacity may be evaluated by using

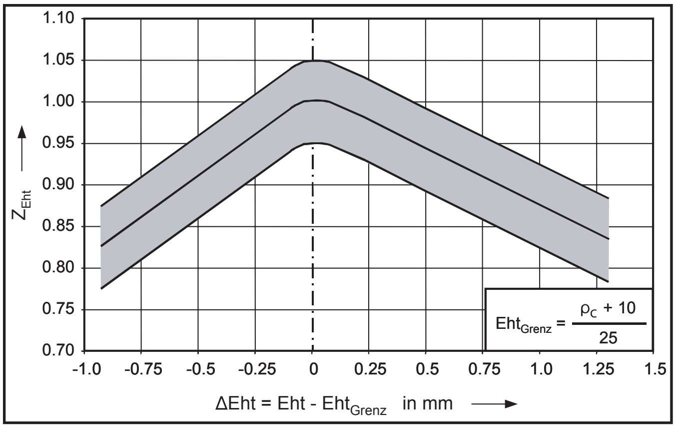

Figure 16—Influence factor Z_{Eht} for the influence of case depth on the pitting resistance (endurance strength).

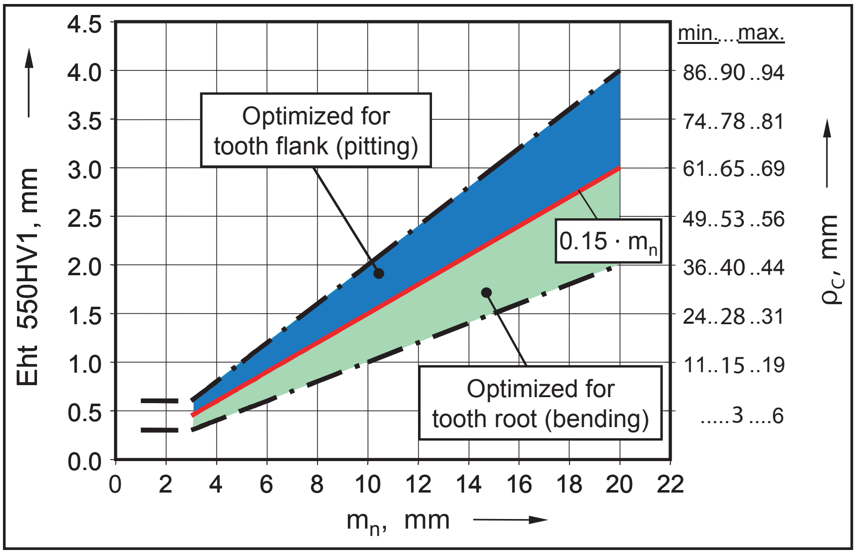

Figure 17—Basic recommendation for simplified determination of "optimized" case depth regarding maximum load capacity for tooth flank (pitting) and tooth root (bending) of case carburized gears with usual ratio of m_n/ρ_C.

defined influence factors.

For practical use, easily applicable guidelines are required. As tooth root and tooth flank of a gear cannot be loaded independently from each other, an adequate case depth for a gear has to consider requirements for surface contact fatigue (pitting) as well as for tooth root bending strength. Often, simple empirical methods that are based on long practical experience—mostly case depth as a function of gear module—are used (Refs. 12, 13). For a wide range of standard gears, these recommendations also agree with the results of the presented investigations. Figure 17 shows a simplified guideline for practical use in order to determine optimized case depth for a gear regarding maximum load capacity for tooth flank and tooth root. Given values are based on test results and with special regard to fatigue limits as stated in the standards (Refs. 4, 9).

Choosing case depth according to Figure 17 requires that module and relative radius of flank curvature of a gear be within the limits of the specified range. For other gear geometries as well as critical gearing, it is recommended to evaluate an optimized case depth with regard to the defined influence factors Y_{Eht} and Z_{Eht}. In case of a given gear geometry and a case depth outside the specification, a decrease of the gear load capacity is expected.

Conclusions

The influence of case depth on the bending strength and pitting resistance of case carburized gears was systematically investigated in a number of test series with different gear sizes and geometries.

Test results show that the case depth influences both bending and surface (contact) load capacity but in different ways. Maximum load capacity is achieved for an optimum value of case depth, but optimum values for maximum tooth root bending strength and pitting resistance of a gear need not necessarily be the same. An unfavorable case depth, smaller or larger than the optimum, leads to a decrease of achievable load capacity.

Based on the results, rating formulas were derived which can be used to calculate optimum case depth for maximum load capacity of tooth root and tooth flank of a gear as well as to determine adequate case depth in order to guarantee required load capacity.

By introducing the defined influence factors into the standardized calculation method, the influence of case depth on bending and surface (contact) load capacity can be taken into consideration if rating a gear according to DIN/ISO.

For practical use, a basic recommendation for choosing optimized case depth regarding maximum gear load capacity is given, applicable for a wide range of standard gears.

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