Load Sharing Analysis of High-Contact-Ratio Spur Gears in Military Tracked Vehicle Applications

M. Rameshkumar, P. Sivakumar, S. Sundaresh and K. Gopinath

Nomenclature

a	addendum of gear tooth
b	face width
CR	contact ratio
F_t	tangential force
G_r	gear ratio
h	height of the parabola at the critical section
J	geometry factor
k_f	stress correction factor
m	module
r	minimum radius of curvature of root fillet
t	tooth thickness at the critical section
Y	tooth form factor
Y_a	addendum factor
Y_L	y coordinates of the vertex of the parabola
Z_1	number of teeth of pinion
α	pressure angle
φ_L	load angle
φ	pressure angle (same as α)
r_1, r_2	operating pitch radius of pinion and gear

Management Summary

Military tracked vehicles demand a very compact transmission to meet mobility requirements. A compact transmission with low operating noise and vibration is desirable in military tracked vehicles to reduce weight and improve power-to-weight ratio. It is also necessary to increase the rating of existing transmissions in military tracked vehicles, like prime movers, to accommodate the additional weight required for ballistic protection. Hence, it was decided to apply a high-contact-ratio (HCR) spur gearing concept that will reduce noise and vibration and enhance load carrying capacity for a 35-ton, tracked vehicle final drive. In HCR gearing, the load is shared by a minimum two pairs of teeth, as in helical gears. It was decided to analyze the load sharing of the normal-contact-ratio (NCR) gearing used in the sun/planet mesh of the existing final drive; and, to analyze the load sharing of the HCR gearing that will be used to replace the NCR gears without any change in the existing final drive assembly except sun, planet and annulus gears. This paper deals with analysis of the load sharing percentage between teeth in mesh for different load conditions throughout the profile for both sun and planet gears of NCR/HCR gearing—using finite element analysis (FEA). Also, the paper reveals the variation of bending stress, contact stress and deflection along the profile of both NCR and HCR gearing.

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Introduction

Contact ratio is defined as the average number of tooth pairs in contact under static conditions, and without errors and tooth profile modifications. A majority of the current gearboxes for tracked vehicle applications have contact ratios ranging from 1.3 to 1.6; the number of tooth engagements is either one or two. The term high-contact-ratio (HCR) applies to gearing that has at least two tooth pairs in contact at all times—i.e., a contact ratio of two or more. As the percentage change in mesh stiffness for HCR meshes is lower than the percentage change in mesh stiffness for normal-contact-ratio (NCR) meshes, one can expect high-quality HCR gear meshes to have lower mesh-induced vibration and noise than NCR gear meshes. A high-powered, compact transmission (Ref. 1) is essential to enhance the mobility of military tracked vehicles. This requirement is partially met by improvements applied to NCR gearing. A literature survey indicated that HCR gearing was designed (Ref. 2) and successfully used in helicopter transmissions (Ref. 3) to improve further the power-to-weight ratio of the transmission. In HCR gears, since a greater number of teeth share the load (Ref. 4), the concept appears to be simple and has wide, potential applicability. It has not to date been applied to military tracked vehicle transmissions.

A detailed study of the existing final drive planetary gear assembly (Fig. 1) of a 35-ton military tracked vehicle was carried out to apply the HCR concept. The final drive is an independent unit that has been isolated for separate testing to apply the concept of HCR. This paper provides an approach to arrive at the HCR gears for the final drive of a tracked vehicle—i.e., a detailed analysis of the load sharing percentage of teeth in mesh throughout the profile for both sun and planet gears of NCR/HCR gearing using finite element analysis (FEA). Bending stress, contact stress and deflection of gear teeth were also calculated.

An attempt has also been made to prove the new concept of HCR gearing using *ANSYS* software (Ref. 5).

Evolution of Design for HCR Final Drive

The mechanical schematic of the entire transmission is shown in Figure 1. NCR gearing was originally used through the transmission and final drive. The HCR gearing concept is applied to the final drive of the 800 hp automatic transmission.

The final drive is located outside the main transmission and is fixed to the vehicle hull. It serves as an additional reduction unit in multiplying the driving and braking torque for the tracks. The final drive (Fig. 2) consists of three gear elements, namely: sun gear (23 teeth), planet gear (22 teeth) and annulus gear (69 teeth) of module 4 mm with a contact ratio of 1.343. The transmission left-hand (LH) and right-hand (RH) outputs are connected via toothed, sliding couplings to the sun gears of the LH and RH final drives, respectively. The annulus gears of the two final drives are fixed, and they provide the reaction torque. The output power from the LH and RH final drives is taken from the planet carriers, which are connected to the LH and RH sprockets driving the tracks.

In order to increase the load carrying capacity of the existing NCR final drive—keeping the same weight and volume envelope—introducing HCR gearing was proposed.

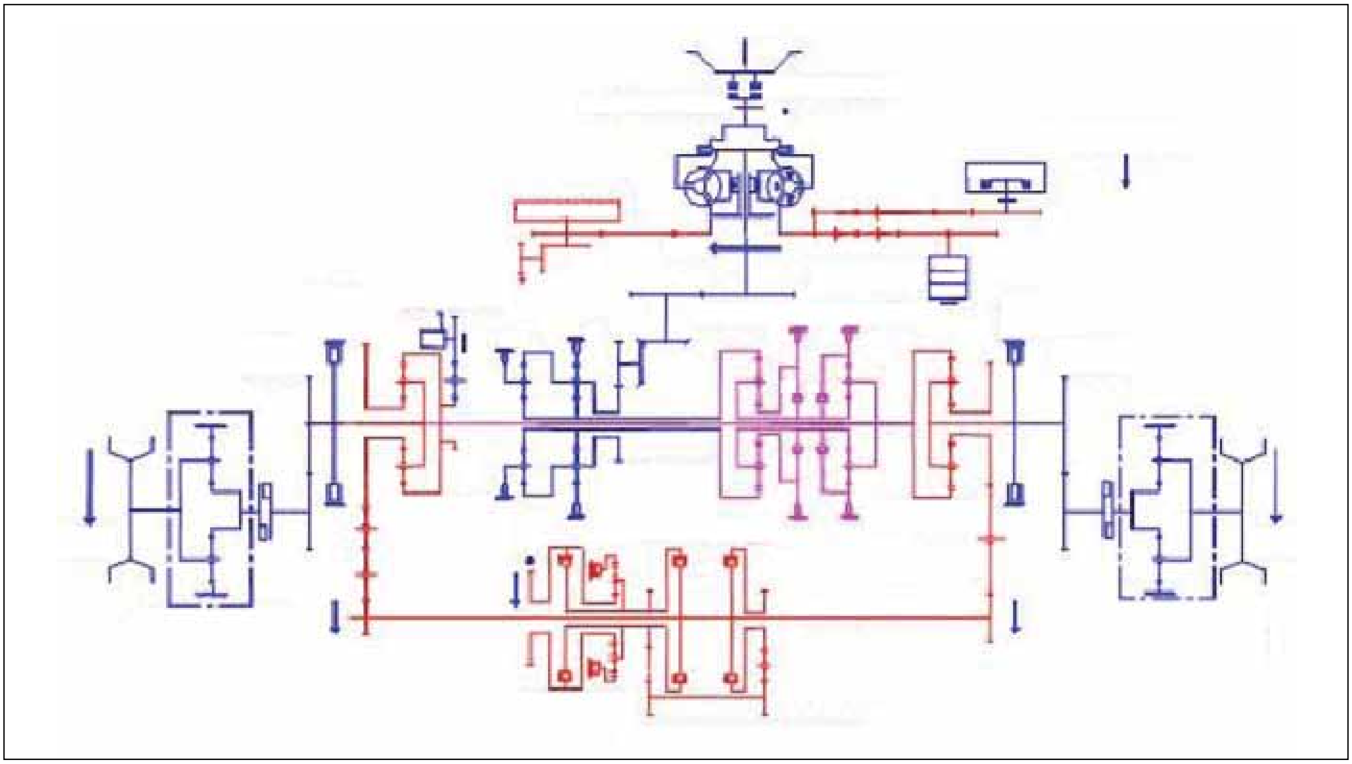


Figure 1—Mechanical schematic of 800 hp automatic transmission.

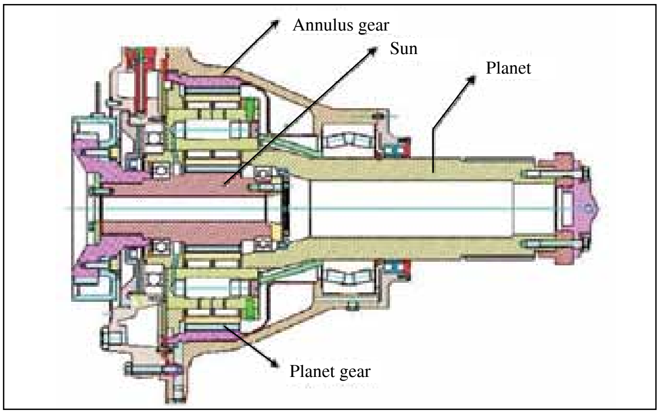


Figure 2—Cross section of final drive.

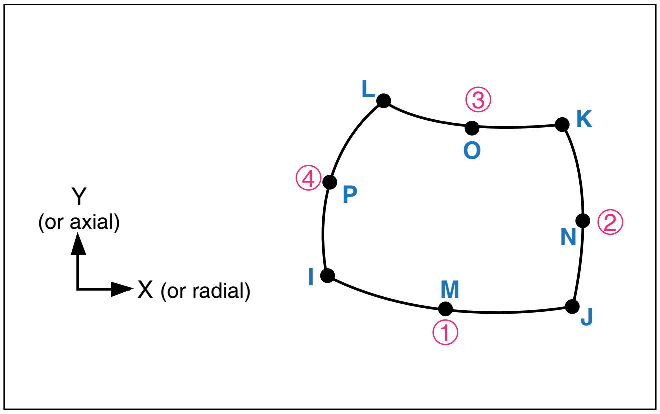
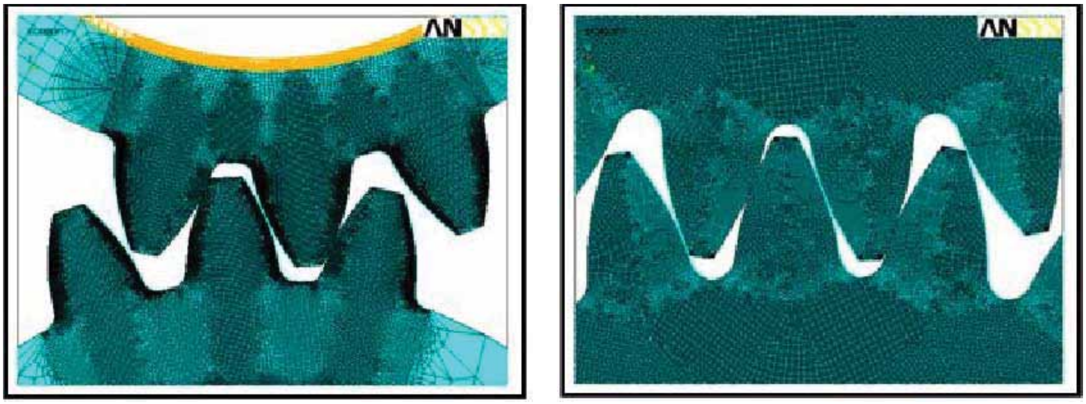


Figure 3—PLANE 82 higher-order plane strain element.



a) NCR gear

b) HCR gear

Figure 4—Finite element-meshed model.

Various combinations of number of teeth, module, profile correction, addendum factor, etc., were analyzed to achieve a design very close to the existing NCR final drive data. The HCR gearing is designed in such a way that only sun, planet and annulus gears of the NCR final drive are changed by keeping the same center distance (93.013 mm), face width (64 mm) and keeping other members the same. A minor variation in the gear reduction ratio was necessary since it is very difficult to achieve the same ratio in view of various other constraints, such as center distance. This implies that to make use of the same gearbox for HCR gearing, with a restriction on contact ratio of 2.0106, the center distance should not be altered. However, considering the various parameters, such as tolerance on tip circle diameter and center distance, top land edge chamber, thermal effects, profile tip and modification, it is advisable to select a contact ratio greater than 2.2 wherever possible. Spur gears are used since their production is simpler and more economical than helical gears; moreover, they are free from axial loads. The concept for various factors that aid in obtaining HCR for spur gears is evolved from the contact ratio (*CR*) given in Equation 1—i.e., where r_1 and r_2 are the operating pitch radius of the pinion and gear, a is the operating pressure angle, m is the module and α is the addendum (based on the operating pitch radius), which is equal to one module for standard gears.

Since for the present investigation the center distance

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Table 1—Gear Parameters

Sl no.	Parameters	NCR	HCR
1.	Profile	Involute	Involute
2.	DIN accuracy class	7	7
3.	Module, ***m***	4.0 mm	2.5 mm
4.	Number of teeth in sun, ***Zs***	23	38
5.	Number of teeth in planet, ***Zp***	22	36
6.	Number of teeth in annulus, ***Az***	69	110
7.	Profile correction in sun, ***Xs***	0.3	0.109
8.	Profile correction in planet, ***Xp***	0.539	0.1
9.	Profile correction in annulus, ***Xa***	-0.3024	-0.3096
10.	Center distance, ***Cd***	93.013 mm	93.013 mm
11.	Reduction, ratio, ***Gr***	4.0	3.894
12.	Addendum factor, ***Ya***	1.0	1.25
13.	Contract ***CR***	1.343	2.0106
14.	Facewidth, ***F***	64 mm	64 mm

should not be altered, a close observation of the equation suggests that the contact ratio of spur gearing can be increased by several ways: (a) by reducing module; (b) by increasing the number of teeth; (c) by lowering the pressure angle; and (d), by increasing the addendum. In this paper the HCR is obtained by increasing the addendum factor and number of teeth by reducing the module. The important gear parameters of both NCR and HCR gear designs are shown in Table 1.

Material properties of the gear are taken to be Young's Modulus = 2.1×10^7 MPa and Poisson's ratio = 0.30.

FEA of NCR/HCR Gear Design

FEA is used to study in detail various parameters such as bending stress, contact stress, deflection, etc., for both NCR and HCR gearing.

Equation 1 see below (1)

Spur gear geometry. The profile of an involute spur gear tooth is composed of two curves. The working portion is the involute, and the fillet portion is the trochoid. Theoretical limit radius (Ref. 6) is an important parameter when gear kinematics is considered. It is the radius at which the involute profile on a gear should start in order to make use of the full length of the involute profile of the mating gear. The trochoidal tooth fillet, as generated by a rack cutter, is modeled exactly using the procedure suggested by Buckingham (Ref. 7). A *C* computer language code was developed for generating exact tooth profile with trochoidal fillet. The trochoidal fillet form is generated from the dedendum circle up to the limiting circle, where it meets the involute profile at the common point of tangency and the involute profile extends up to the addendum circle.

Gear models. The gear tooth under consideration for NCR gearing is a standard one, with a full depth of 2.25 times the module and the addendum of one unit module, and the gear tooth for HCR gearing is a full depth of 2.75 times the module and addendum of 1.25 times the module. Each generating cutter having a tip radius of 0.8 mm and 1.0 mm is used for the generation of HCR and NCR gearing, respectively. Since the gear fillet assumed as a constant radius curve by AGMA in its layout procedure is not the true representation of the spur gear geometry (with generated fillets), it was decided to consider the trochoidal fillet as a fair model in this paper. AGMA (Ref. 8) uses Newton's method of iterations while this paper deals with polynomial equations for direct calculations of the AGMA geometry factor, with minimum process time for computerized gear analysis.

In order to make the perfect gear geometry for NCR/HCR design data mentioned in Table 1, a *C* code is developed for generating the complete gear profile of both NCR and HCR with trochoidal fillet. The sun gear (23 teeth) and planet gear (22 teeth) mesh for NCR gearing, and the sun gear (38 teeth) and planet gear (36 teeth) mesh for HCR gearing, and are generated in a single-window FEA environment of *ANSYS Version 11.0*.

FEA models and meshing. The gears (both NCR and HCR) are kept in contact by positioning at the stipulated center distance (93.013 mm) with respect to the global coordinate system, and only the plane area models are used for the FEA. Quadratic, two-dimensional (2-D), eight-noded higher-order-plane strain elements (PLANE 82 of *ANSYS*) are used, as shown in Figure 3. To promote convergence of the contact solution, the finite element models are meshed with a very fine mesh where the tooth will experience the contact. The total number of elements used in HCR gearing is 38,044 and the total number of nodes is 115,118.The finite element meshed models of NCR and HCR gears are shown in Figure 4.

Loading and boundary conditions. The maximum torque (Tc) on the carrier of each final drive with a factor of safety of 1.17 is 44,975 Nm, and the sun gear is in mesh with four planet gears (N). Thus, the torque applied per unit face width on the sun gear (38 teeth) is 45 Nm (i.e., T_c / NFG_r). The load is applied in the form of torque in a clockwise direction from the input (coupling) side, and the planet gear (36 teeth) is fully constrained. Both the sun and the planet gears are arrested in radial direction with respect to a local

Equation 1:

$$CR = \frac{\sqrt{(r_1 + a)^2 - r_1^2 \cos^2\alpha} + \sqrt{(r_2 + a)^2 - r_2^2 \cos^2\alpha} - (r_1 + r_2)^2\alpha}{\pi\, m \cos\alpha}$$

Table 2—Results Based on FEA for HPSTC/HPDTC.						
	Teeth	Bending Stress, MPa		Deflection - 100% load, mm	Deflection - 52% load, mm	Contact Stress, MPa
		100% load	52% load	Deflection Vector Sum	Deflection Vector Sum	
NCR	22	608.4	-	30.94E-03	-	1504.8
	23	673.2	-	49.15E-03	-	
HCR	36	-	482.1	-	21.22E-03	1074.8
	38	-	390.2	-	21.37E-03	

	Teeth	Bending Stress, MPa		Deflection - 50% load, mm	Deflection - 20% load, mm	Contact Stress, MPa
		50% load	20% load	Deflection Vector Sum	Deflection Vector Sum	
NCR	22	386.9	-	23.9E-03	-	929.6
	23	406.7	-	32.65E-03	-	
HCR	36	-	282.5	-	17.39E-03	914.9
	38	-	255.5	-	17.75E-03	

Table 3—Results Based on FEA for Tip Loading.

cylindrical coordinate system.

Solution and Post Processing

Load sharing. The gear pair is rotated as a rigid body according to the gear ratio. The solution is repeated for both gears rotated with some amount of angular increment according to the gear ratio. Approximately 45 angular increments with a 0.5° step are used for this analysis, and the analysis is carried out with the help of customized *APDL* (*ANSYS Parametric Design Language*) looping program. Transmission error, torsional mesh stiffness, root stress, contact stress and load sharing ratio are obtained for all the positions. The nodal force at each node has been obtained for each individual gear tooth. By this methodology, the percentage of load sharing between the teeth for both sun and planet gears at any position was determined. Accordingly, the maximum percentage of tooth load for NCR and HCR gear designs—at the tooth tip and highest point of single-tooth contact (HPSTC) for NCR and the highest point of double-tooth contact (HPDTC) for HCR gearing—was determined. The individual tooth loads have been determined by comparing equally the total normal load with the sum of the normal loads contributed by each pair in contact.

It is observed from the above FEA that for NCR gearing, a 100% load is applied at the HPSTC condition and a 50% load is applied at the tip load condition. For HCR gearing, a 52% load is applied at the HPDTC condition and only a 20% load applied at the tip load condition.

Bending stress, contact stress and deflection. In this study, the stresses were calculated for corrected gears considering the trochoidal fillet form and using FEA, which includes the size and shape effects as well as stress concentrations. The ratio of the stress determined by FEA to that of the modified Lewis equation is considered a stress correction factor.

The FEA-calculated bending stress in the gear tooth fillet, contact stress and vector sum deflection were determined using *ANSYS Version 11.0* for both NCR/HCR gears and tabulated in Tables 2 and 3. Figure 5 shows the nodal stress plot for HPSTC in the case of NCR with the maximum stress occurring at the trochoidal fillet of the 22-tooth NCR gears. Figure 6 shows the nodal stress plot HPDTC in the case of HCR gears with the maximum stress occurring at the trochoidal fillet of the 36-tooth HCR gears.

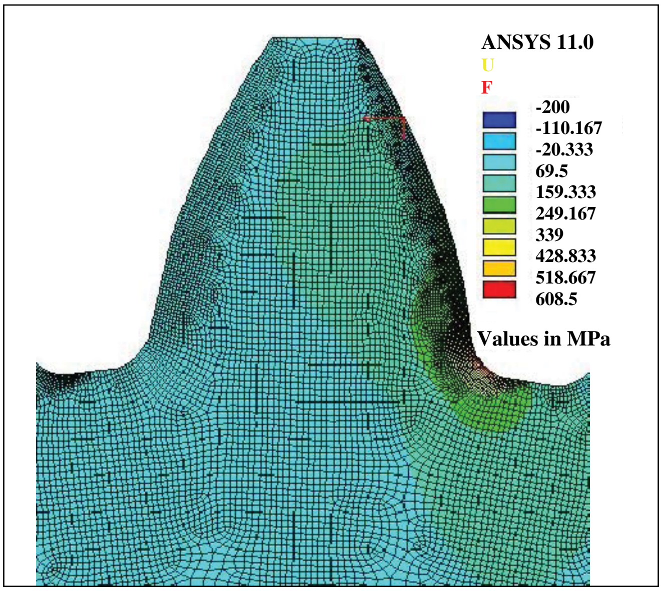

Figure 5—Stress plot for 22-tooth NCR gear with HPSTC loading.

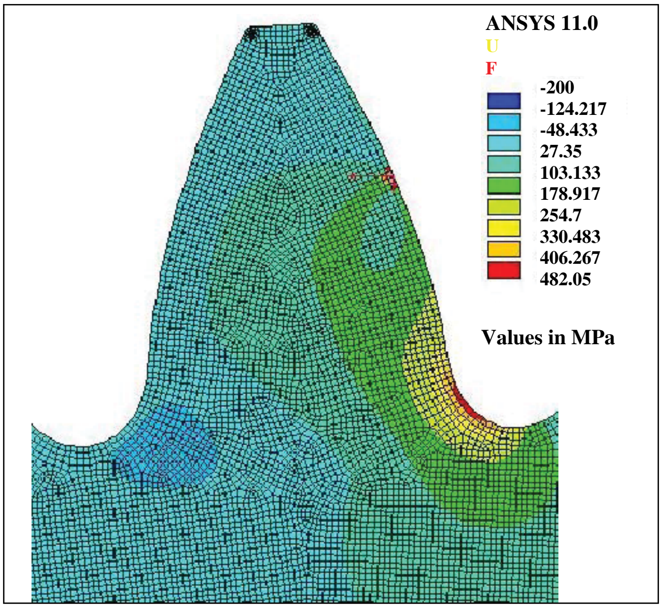

Figure 6—Stress plot for 36-tooth HCR gear with HPDTC loading.

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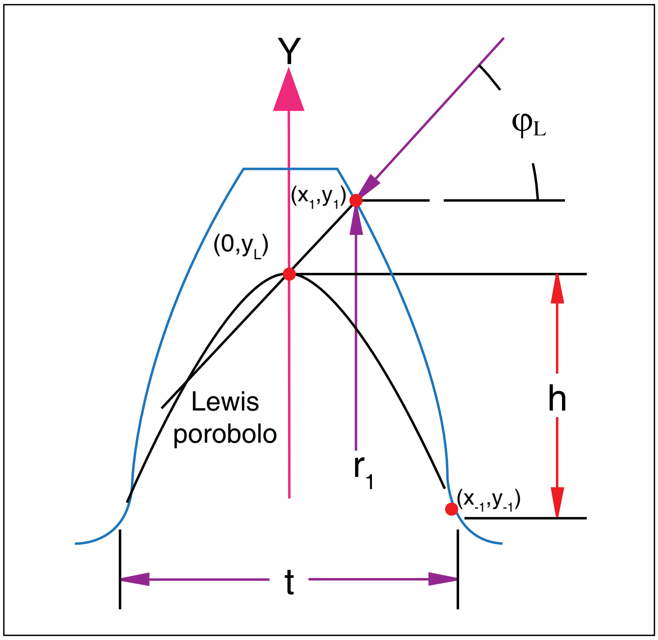

Figure 7—Critical section parameters.

AGMA approach. AGMA's specification provides charts for uncorrected gears and a semi-analytical method for corrected gears with constant radius fillet to calculate the geometry factor. AGMA's mathematical procedure was written in the form of *C* computer language code in this paper so that the geometry factor can be calculated analytically for gears with trochoid fillet.

Critical parameters. A *C* code was developed to calculate the geometry factor using the mathematical procedure specified by AGMA (Ref. 8). The procedure takes into account the effects of tooth shape, worst-load position, stress concentration and both tangential (bending) and radial (compression) components of the tooth load.

According to AGMA, a parabola (Refs. 9–10) is inscribed inside the gear tooth profile with its vertex at the sharp point and in tangent with the root fillet profile (Fig. 7). The plane passing through this point of tangency—and perpendicular to the tooth centerline— defines the critical section. In this standard, a semi-analytical method is used to calculate the geometry factor through the evaluation of r, t, h, φ_L. The height of the parabola (h) and corresponding tooth thickness (t) at the critical cross section can be calculated by finding the coordinates of the tangency point of the parabola and trochoid (Fig. 7).

The Y-coordinate of the vertex of the parabola Y_L can be calculated from the line equation whose slope is tan φ_L and passing through a point on the involute tooth profile (x_i, y_i). The values of x_i and y_i can be calculated from the parametric involute:

$$Y_L = y_i - x_i \tan \varphi_L \tag{2}$$

The vertex of the parabola (0, y_L) is calculated from Equation 2.

The coordinates of trochoid (x_t, y_t) are a straight line with two end points, and so the slope of the line is obtained from Equation 3:

$$\tan \theta = \frac{y_2 - y_1}{x_2 - x_1} \tag{3}$$

where:

x_1,y_1 are the coordinates of the trochoid and x_2,y_2 are the coordinates of vertex of the parabola.

The procedure for calculating the slope is repeated for all the points along the trochoid and slope where one point of the trochoid is compared with the other, and the maximum slope and the coordinates of the trochoid (Ref. 6) are considered. The length of the line joining the vertex of parabola (L) and the maximum coordinates of the trochoid are calculated from the straight line:

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4}$$

Thus, the height and tooth thickness of the parabola are obtained from h = Lcosθ and t = 2Lsinθ.

The critical parameters—diameter, height, thickness, stress correction factor, tooth form factor and *J* factor—are calculated based on *C* computer language code and tabulated in Tables 4 and 5 for tip and HPSTC/HPDTC condition, respectively.

Stress correction factor. The stress correction factor plays a major role when a machine member is subjected to fatigue-type loading. As gear teeth have to withstand repeated and fluctuating types of load, their failure is essentially attributable to fatigue phenomena. Abrupt changes in a cross section of any stressed member—such as occur in the root fillet area of gear teeth—give rise to large irregularities in stress distribution. It depends on the radius of curvature of the fillet at the critical section in the gear. The basic Lewis equation doesn't consider the size and shape effects of the rapid change in the fillet form at the root portion and the stress induced due to the radial component at the normal load. So, the bending stress obtained using this equation is less than the localized stress at the fillet. Studies through photo elasticity have revealed that stress concentration occurs mainly at the point of loading and at the root fillet.

According to AGMA, the stress correction factor (k_f) is obtained from an empirical relation gleaned from the experiments conducted by Dolan and Broghamer (Ref. 11) for a = 20° is given as:

$$k_f = 0.18 + \left[\frac{t}{r}\right]^{0.15} \left[\frac{t}{h}\right]^{0.45} \tag{5}$$

Geometry factor. For spur gears, the definition of the AGMA (Ref. 9) geometry factor *J* reduces to equation 6:

$$J = \frac{Y}{k_f} \tag{6}$$

The following reduced equations can be used for tooth form factor Y and stress correction factor k_f. Tooth dimensions used in calculation of Y are shown in equation 7:

$$Y = \frac{1}{\frac{\cos \varphi_L}{\cos \varphi}\left(\frac{6h}{t^2} - \frac{\tan \varphi_L}{t}\right)} \quad (7)$$

Considering the stress correction effect, the fillet stress is given by:

$$\sigma_{act} = \left(\frac{F_t}{b\, m\, Y}\right) k_f = \frac{F_t}{b\, m\, J} \quad (8)$$

In calculation of the AGMA bending stress number, HPSTC is considered the most critical point—provided there is load sharing between adjacent contacting tooth pairs. In case of not having load sharing, the tip of the tooth should be taken as the most critical point. Critical loading points for tooth stress analysis are HPSTC and the tooth tip.

Results and Discussions

In the continuous action of meshing, the load traverses throughout the involute profile of the gear. In the case of load sharing, the maximum stress occurs above the pitch circle (Ref. 12).

The bending geometry factors are evaluated for profile-corrected gears generated with rack-type cutters for both conditions where the load is applied at the tip of the tooth and at the HPSTC for NCR gears and HPDTC for HCR gears.

The planet gear (22 teeth) of module 4 mm, meshed with the sun gear (23 teeth) of the NCR type, and the planet gear (36 teeth) of module 2.5 mm meshed with the sun gear (38 teeth) of the HCR type, are all considered for analysis to evaluate various parameters for tip and HPSTC/HPDTC loading. All the data such as stress, deflection, stress correction factor, critical diameter and the corresponding tooth form factor, *J*-factor, height and thickness are tabulated in Tables 2–5.

Bending stress. Generally, bending stress will increase at the root with the addendum factor in view of the cantilever effect. However, in the case of HCR (CR > 2) gears, the load applied at the tip/HPDTC is less compared to NCR gears and thus the net effect will reduce the stress. It can be seen from Table 2 that the maximum load applied at the HPDTC—where two teeth are in contact—is only 52% in the case of the HCR planet gear (36 teeth) and so the maximum bending stress is 482.1 N/mm^2 as against 608.4 N/mm^2 in case of the NCR planet gear (22 teeth), where the maximum load is 100% in view of one tooth contact and results in a 26.2% lower stress value.

Similarly, the bending stress in the case of the HCR planet gear (36 teeth) is 37% less compared to the NCR planet gear (22 teeth) with the maximum load applied at the tip due to the load sharing of three teeth in contact. This load is only 20% compared to 50% for the NCR planet gear (22 teeth) in view of two teeth in contact as shown in Table 3.

Contact stress. This paper calculates the contact stress of contact teeth through calculating the tooth load distributed on the unit contact area of the tooth surface. Tooth contact stresses were analyzed using the Hertzian formula, which proved to be less than precise (Ref. 13). It can be seen from Table 2 that the maximum load applied at the HPDTC—where two teeth are in contact—is only 52% and therefore the maximum tooth contact stress is less—by 40%—in HCR compared to NCR. The reduction in contact stress is due to an increase of the addendum factor and the number of contact teeth, resulting in less load.

Deflection. As explained above, the load is less in the case of HCR gears compared to NCR gears in both conditions—i.e., tip loading and HPDTC. Also, in view of the higher number of teeth in HCR gearing, the tooth height is less and the tooth is more rigid. It can be seen from Tables 2

continued

Table 4—Results Based on *C*-Code for Tip Loading.

	No. of Teeth	Critical Diameter, mm	Thickness, *t*, mm	Height, *h*, mm	Stress Correction factor, K_f	Tooth Form Factor, *Y*	J-factor
NCR	22	41.33	8.83	7.67	1.369	0.5375	0.3928
	23	42.45	8.45	7.67	1.338	0.4827	0.3928
HCR	36	41.92	5.41	6.03	1.236	0.3764	0.3046
	38	41.92	5.47	6.03	1.243	0.3823	0.3076

Table 5—Results Based on *C*-Code for HPSTC/HPDTC Loading.

	No. of Teeth	Critical Diameter, mm	Thickness, *t*, mm	Height, *h*, mm	Stress Correction factor, K_f	Tooth Form Factor, *Y*	J-factor
NCR	22	41.12	9.09	5.18	1.629	0.8230	0.5053
	23	42.29	8.69	5.31	1.566	0.7032	0.4490
HCR	36	41.61	5.94	2.89	1.777	0.9090	0.5114
	38	44.11	5.96	2.88	1.784	0.9205	0.5161

and 3 that the deflection is 31.4% less in the case of HPDTC and 27.2% less in the case of the tip loading condition for the HCR planet gear (36 teeth) versus the NCR planet gear (22 teeth).

Stress correction factor. It can be seen from the stress correction factor equation that k_f is a function of *h*, *t* and *r*. HCR gears are designed by increasing the number of teeth and decreasing module. Therefore, the values of thickness and height are less in the case of HCR gears compared to NCR gears. Also, the cutter edge radius of an HCR gear is 0.8 mm for a 2.5-module gear, whereas the cutter edge radius of an NCR gear is 1.0 mm for 4-module gears.

The length and radius of the curvature at the critical point of the trochoidal fillet portion will decrease with an increase in the number of teeth, since the difference between form diameter and base circle diameter is reduced. Also, the increase in the number of teeth will increase the width at the critical cross section, and the slope of the line joining the critical point and load point will gradually increase. Table 4 reveals that the stress correction factor for tip loading is 9.7% less for the HCR planet gear (36 teeth) compared to the NCR planet gear (22 teeth). However, the stress correction factor for HPDTC is 9.1% greater in the case of the HCR planet gear (36 teeth) compared to the NCR planet gear (22 teeth), as shown in Table 5. Even though the stress correction factor is more, the bending stress will be much less (26.2%) in the case of HCR gears for the same tip-base pitch loading, given the 52% loading versus the 100% in NCR gears.

Conclusions

The FEA of a final drive gear assembly for both NCR and HCR gearing reveals that both the bending and the contact stresses are more than 25% less in HCR gears compared to NCR gears. The load carrying capacity of HCR gearing could be increased by at least 25% for the same weight and volume. Therefore, the concept of HCR gearing can be adapted to all the gear assemblies of the automatic transmission for increasing the load carrying capacity of the transmission within the same envelope, as well as increasing the mesh efficiency and decreasing dynamic loads and noise level.

In this paper, the actual shape of the trochoid is considered, whereas the fillet radius is assumed as a constant radius curve for calculating the geometry factor presented by AGMA (Ref. 8). With the aid of recent computing advances, finite element numerical methods were applied in the research of high-contact-ratio gears that accounted for the actual profiles, thus providing more realistic results.

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