The Basics of Gear Theory, Part 2

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Bevel Gears: By the Book

Introduction (Chapter 1, Part 2)

The first part of this publication series covered the general basics of involute gearing and applied the generating principle of cylindrical gears analogous to angular gear axis arrangements the kinematic coupling conditions between the two mating members have been postulated in three rules. Entering the world of bevel gears also required to dwell somewhat on the definition of conjugacy. The second part is devoted to the different generating gears and the chain of kinematic relationships between the gear - gear generator - pinion generator and pinion. In addition to the kinematic coupling conditions, the five geometric flank form rules are discussed in order to lay the grounds for the understanding of the different kinds of generating gears. After this, the generating gears of the most common bevel and hypoid gear design and manufacturing methods are discussed. The lineup covers face milling and face hobbing as well as generated and non-generated pinion-gear systems. This section will provide the reader with the deeper understanding of the strength of the different systems and their limitations. It is also explained, how some of the limitations can be overcome if certain measures are applied. **—Hermann J. Stadtfeld**

Geometric Flank Form Rules

1. Flank lines 2 times steady differentiable
   - No steps
   - No edges (elbows)
   - No steps in curvature changes
2. Steady monotonic rising or falling flank lines
   - No inflection points
   - No maxima and no minima
   - Special case straight bevel gears
3. Spiral angle rises along flank line from inside to outside
   - Special case straight bevel gears
4. Pressure angle not allowed below limit pressure angle (meshable profile)
5. Spiral angle limit not exceeded

Violation of the rules will reduce the mesh performance and can even result in the complete loss of the ability to transmit motion and torque. An example of this is Zerol gears *(see Chap. 4, General Explanations, Fig. 26)*. Zerol gears have an arc-shaped flank line with zero degree spiral angle in the middle of the face width; this violates rules 2 and 3. The consequence is an instant contact zone moving during meshing from the outside to the inside, and then back to the outside. This gives a consolidated contact area (tooth bearing) with a tendency to split into parts. The results are mesh disturbances that increase with higher flank line curvatures.

Generating Gears of Bevel Gears with Parallel Depth Tooth

The kinematic requirements conditions are applied in this next section to the four methods in Figures 12–15. It has been assumed that conformance to the geometric flank form rules is given in all cases. In order to achieve congruent generating gears, certain geometric and kinematic features in the bevel gear generators are required. The greatest influence is the tooth depth characteristic along the face width (tapered or parallel).

Method "A" (Fig. 12) was already globally discussed (Figs. 9–10). The generating gear axis of the pinion is labeled in the top part of the figure as "Pinion Generating Gear Axis." The pinion cutter rotates around the "pinion cutter axis," which is parallel to the generating gear axis. The generating gear plane is equal to the pitch plane. It contains the axis Z_4 and stands perpendicular to the drawing plane. The same rules explained for the pinion generation apply also for generating the ring gear. In addition to the generating gear orientation and axis location, the blade profiles are also congruent (see pinion cut-

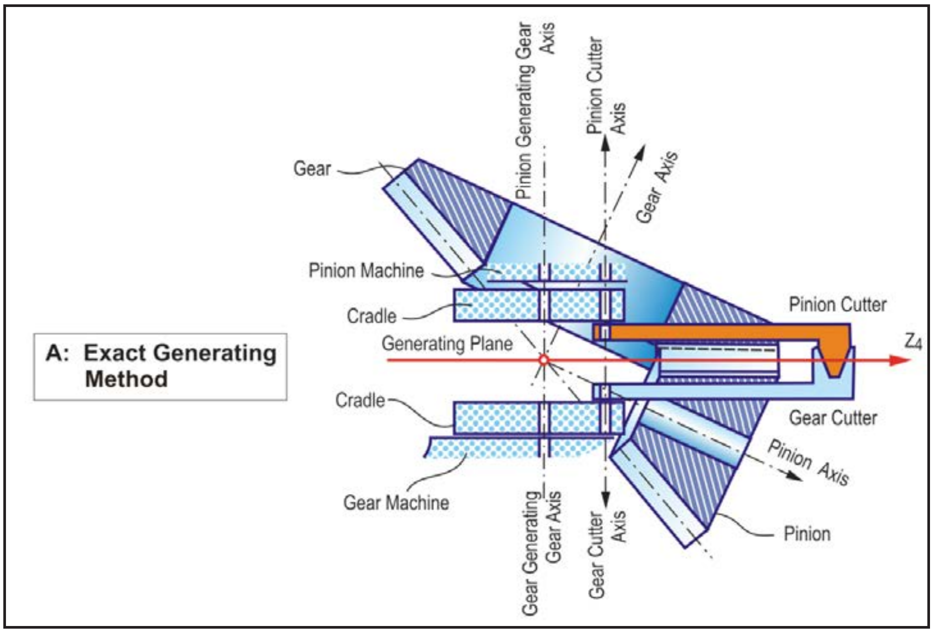

Figure 12 Generating model for bevel gears with parallel tooth depth Method A, generated pinion and ring gear.

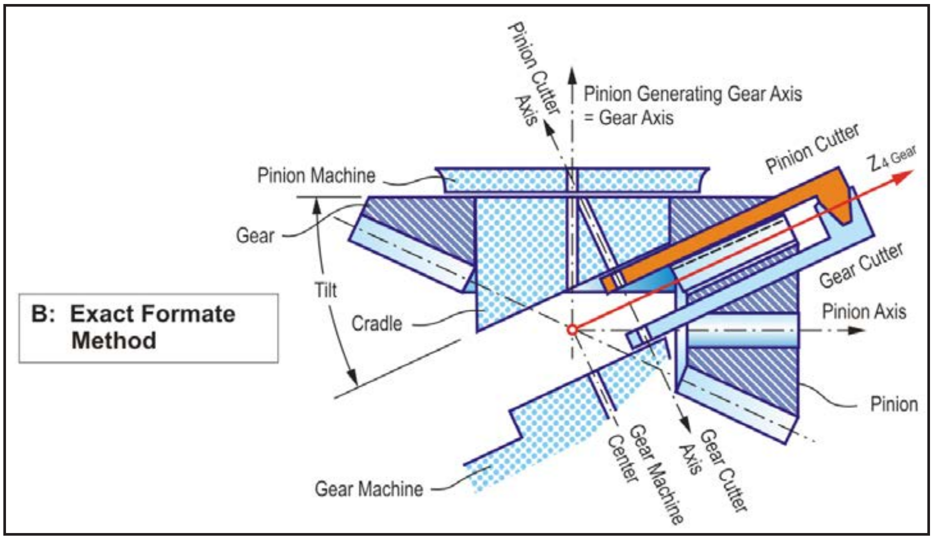

Figure 13 Generating model for bevel gears with parallel tooth depth Method B, formate.

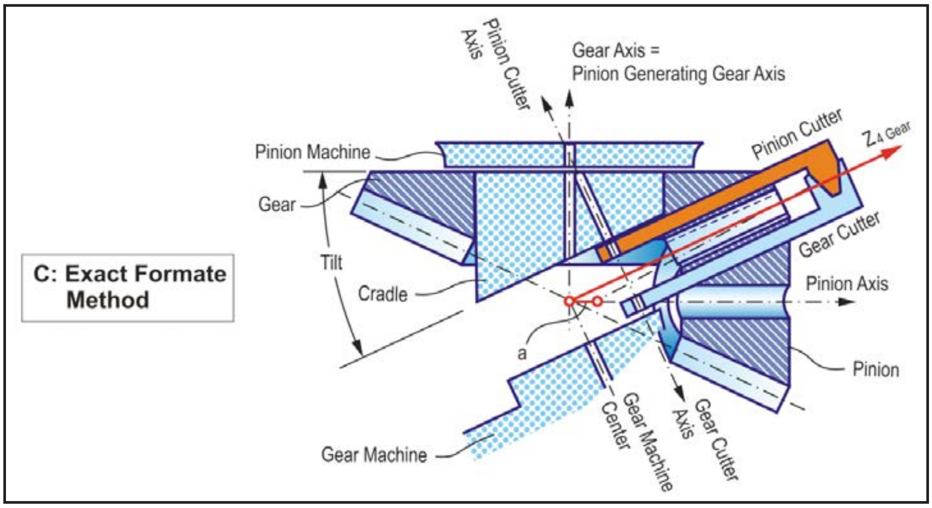

Figure 14 Generating model with parallel tooth depth and offset Method C, formate.

ter and gear cutter, to the right in Figure 12). The kinematic coupling requirements for method "A" are entirely fulfilled.

The orientation of pinion and ring gear (Fig. 12) for this method is identical to the orientation of pinion and ring gear in their final gearbox assembly. Pinion and ring gear have in most cases a different number of teeth but their common generating gear does not. The number of generating gear teeth has to be calculated in order to determine its precise rotation dependent on the rotation of pinion or ring gear. The pitch surfaces of the two meshing gears roll onto each other without sliding (like a slip-free traction drive). At a certain radius *R*, the relationship between the circumference of the generating gear to the circumference of the work gear (on the pitch cone element) must be determined. The work gear pitch angle has the value *γ*; the generating gear pitch angle is 90°.

$$z_E/z_W = [R \ast \sin(90°) \ast 2 \ast \pi]/[R \ast \sin(\gamma) \ast 2 \ast \pi] \tag{10}$$

or:

$$z_E/z_W = 1/\sin(\gamma) \tag{11}$$

which delivers:

$$z_E = z_W/\sin(\gamma) \tag{12}$$

$$UDIF = z_w/z_E \tag{13}$$

where:

z_E number of teeth generating gear
z_W number of teeth work gear
R observed radius
90° pitch angle generating gear
γ pitch angle work gear

UDIF ratio of roll (generating gear rotation/work gear rotation)

The angular velocities of two meshing gears have the opposite relationship than their number of teeth; the work gear rotation is therefore calculated by dividing the generating gear rotation with *UDIF*.

Figure 13 shows a different realization, which will also satisfy the kinematic coupling requirements (method "B"). The gear cutter on the gear machine (without generating provision) cuts the ring gear (top) without generating motion. The Formate ring gear (see chapter 0) produced this way does not include any involute curvature in the flank profiles. In order to generate the pinion, a virtual version of exactly this ring gear is used as generating gear. In order to do this, the pinion cutter has to be adjusted with its axis perpendicular to the pitch angle of the ring gear and has to rotate during the pinion generation around the pinion generating gear axis (which is identical to the gear axis). The blades simulate one tooth of the generating gear, while the cutter rotates around its axis. The blade profiles (Fig. 13, right) are congruent. The generating rotation around the pinion generating gear axis (= gear axis) simulates the rotation of the generating gear. With this arrangement a conical generating gear was created that cuts and rolls conjugate pinion tooth slots if the pinion blank is placed correct relative to the generating gear. The orientation of pinion and ring gear in Figure 13 for this method is identical to the orientation of pinion and ring gear in their final gearbox assembly. Method B is called "Formate" (non-generated ring gear - generated pinion).

The ring gear requires no generating roll during its creation, while the pinion ratio of roll is calculated as:

$$UDIF = z_{Pinion}/z_{Gear} \tag{14}$$

where:

z_{Gear} number of teeth generating gear
z_{Pinion} number of teeth work gear

Both methods "A" and "B" are valid for continuous as well as for single indexing processes. The differences between those cutting processes are explained in detail in Chapters 4, 5 and 9.

The case of a hypoid offset between pinion and ring gear (*see also chap. 4.6 "Hypoid Gears"*) is investigated with the two generating models (Figs. 14 and 15; Methods C and D). If the model of the ring gear, like in the case of method "B," should be used as a generating gear (Formate), then the generating gear model is easy to understand. A pinion blank is positioned in front of the generating gear while its axis is not intersecting

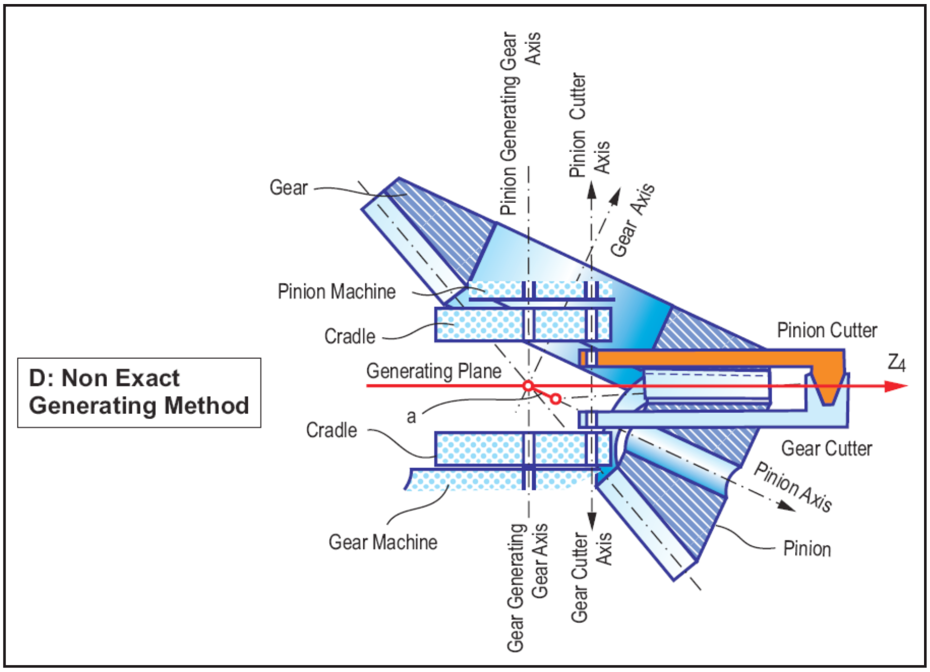

Figure 15 Generating model with parallel tooth depth and offset Method D, generated pinion and ring gear.

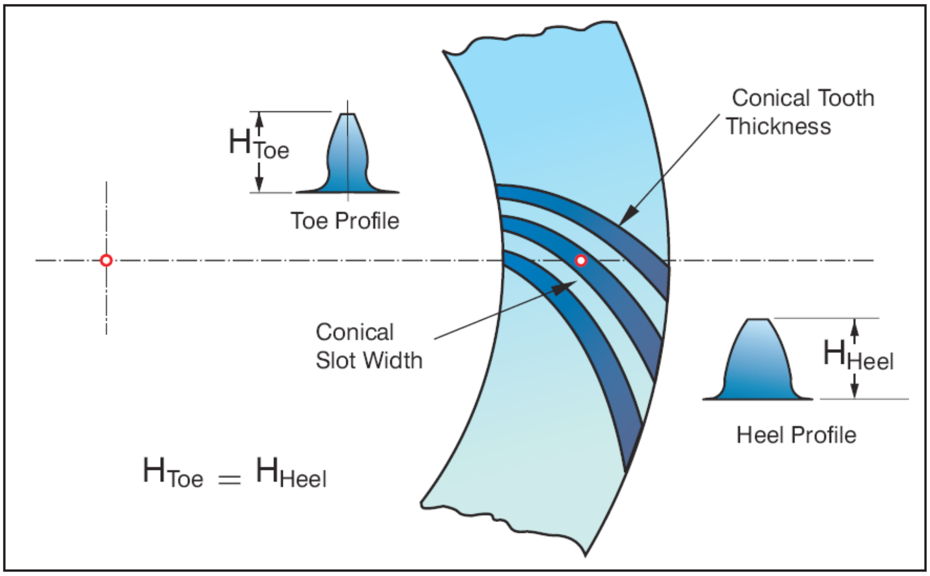

Figure 16 Conical tooth thickness change and conical change of the slot width in case of parallel-depth teeth and a continuous indexing process.

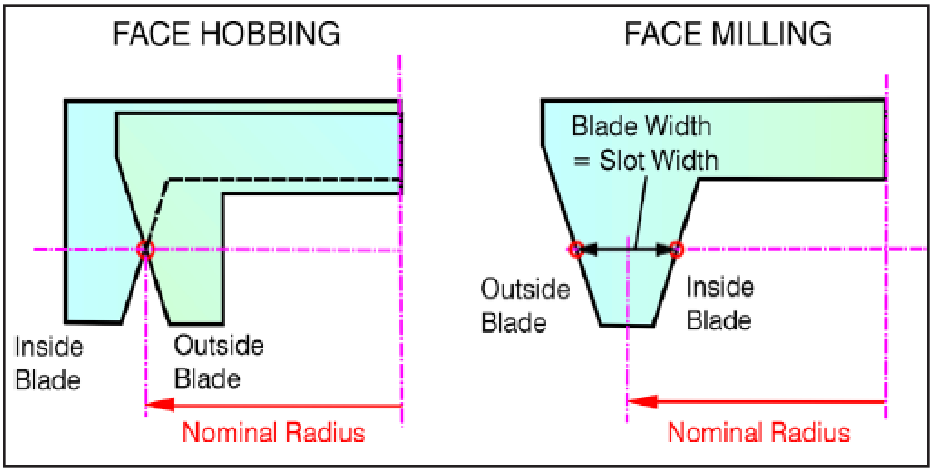

Figure 17 Blade orientation — left for face hobbed, and right for face milled bevel gears.

with the generating gear axis, but crosses it under a certain distance (offset). The difference from Figure 13 is given with the distance "*a*" between the pitch cone apexes in Figure 14. The generating gears for pinion and ring gear, as well as their axes, are identical, since with this method the generating gear is equal to the ring gear. It can be concluded that all kinematic coupling requirements for method "C" are fulfilled.

In the case where the pinion and ring gear are manufactured by rolling them both on a plane generating gear like in method "A," but with an offset between their axes, deviations from a conjugate pair will occur.

Method "D" (Fig. 15) applies the same generating gear for the generation of both, pinion and ring gear, which satisfies the first two kinematic coupling requirements. The surfaces of engagement between generating gear and ring gear and between generating gear and pinion are not congruent because they lie about the axes offset apart (in offset direction). It is possible to rotate them "into" each other, but they are still not exactly congruent. Although the blade profiles in Figure 15 are congruent, the generating gear flank surfaces will still deviate from each other due to the axes offset. The non-conformance with one of the kinematic coupling requirements causes, in this case — surface deviations — which can be compensated to a large extent by first order corrections.

The pitch line (flank line through the pitch point in Figure 5) in case of parallel depth teeth is parallel to the root line. Identical generating gear axes and congruent generating gear flank surfaces can therefore be achieved and the kinematic coupling conditions 1 and 2 can be satisfied.

In order to achieve a proportional and balanced relationship between tooth thickness and slot width along the tooth face it has been shown that bevel gears manufactured with a continuous indexing process (face hobbing) require a parallel tooth depth to fulfill those requirements and deliver at the same time conjugate flank pairs. Bevel gears manufactured by face hobbing have in general a flank line with an epicyclic form. Tooth thicknesses and slot widths are the result of an even split of the gears circumference due to the process' kinematics. Also between outside diameter (heel) and inside diameter (toe) a proportional adjustment of tooth thickness and slot width depending on the radial position occurs (Fig. 16).

Face hobbed bevel gears can be lapped after heat treatment in a short time with good results. The precise bevel gear grinding of epicycloids in a completing process to the contrary is not possible. A precisely defined flank form of the hard finished face hobbed bevel gears can be achieved by (hard) skiving (*see also Chapters 9 "Cutting Methods"; and 11 "Hard Finishing, Grinding and Skiving"*).

Generating Gears of Bevel Gears with Tapered Depth Teeth

Bevel gear sets manufactured in the single indexing process (face milling) have circular flank lines. A proportional tooth thickness and slot width split like in face hobbing is not acceptable. If the objective is a tooth thickness and slot width change along the pitch line similar to that of face hobbed gears, it is necessary to use convex and concave flanks cutter heads with different radii and also different machine settings (*see also Chapter 5, Practical Characteristics*). Cutting of convex and concave flanks has to be done in this case using two separate cutting cycles. If both sides of a slot are machined with only one cutter head having outside and inside blades (Fig.17, right side), then a parallel slot width and a conical (tapered) tooth thicknesses will result (Fig. 18).

Since this applies initially also for the mating gear, a pinion and a gear manufactured this way would not fit together. A tapered depth tooth, by lifting up the root towards the smaller diameter, will still maintain a parallel root width but also achieve a proportionally reducing (conical) slot width from outside to inside (Fig. 19).

Lifting the root up is possible via the dedendum angle (Fig. 20); this is so only with generating gear configurations different from those as previously shown (Figs. 12–15). As a result, the introduction of a dedendum angle requires also the introduction of a corresponding addendum angle. This is necessary in order to avoid interferences of the toplands with the root fillets of the mating gear (which also requires a tapered depth

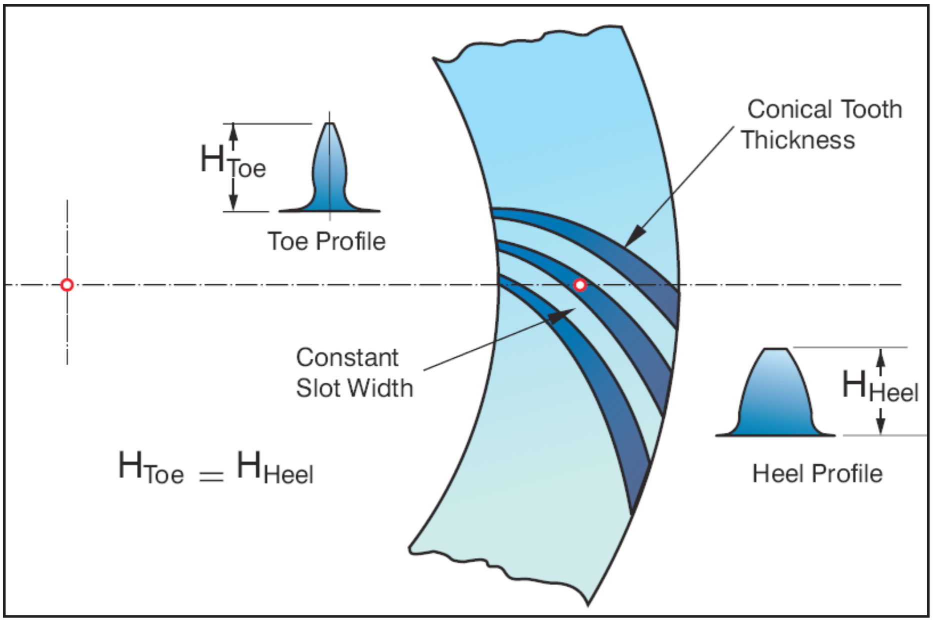

Figure 18 Tapered tooth thickness and constant slot width (face milling).

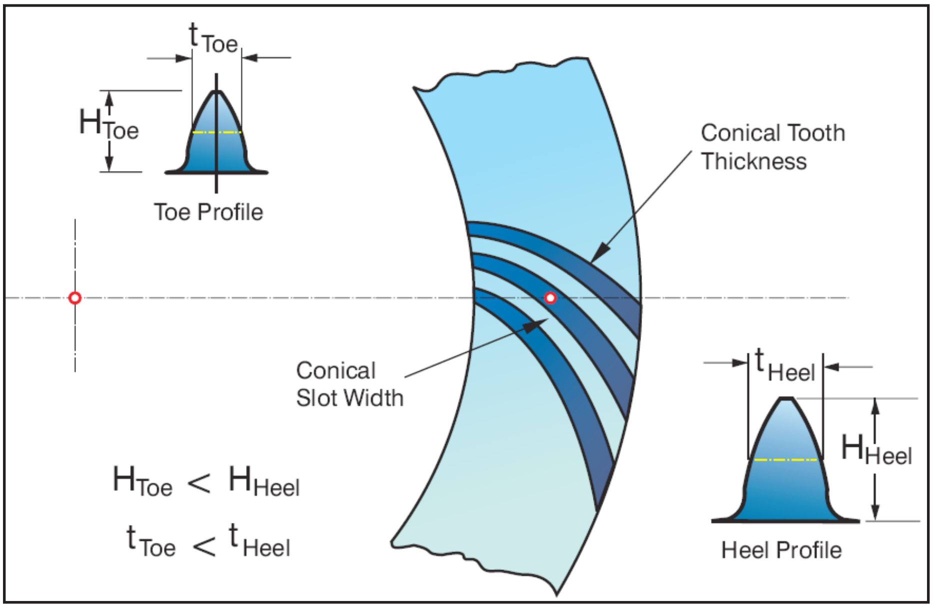

Figure 19 Tapered tooth depth change causes conical slot width along the pitch line (face milling).

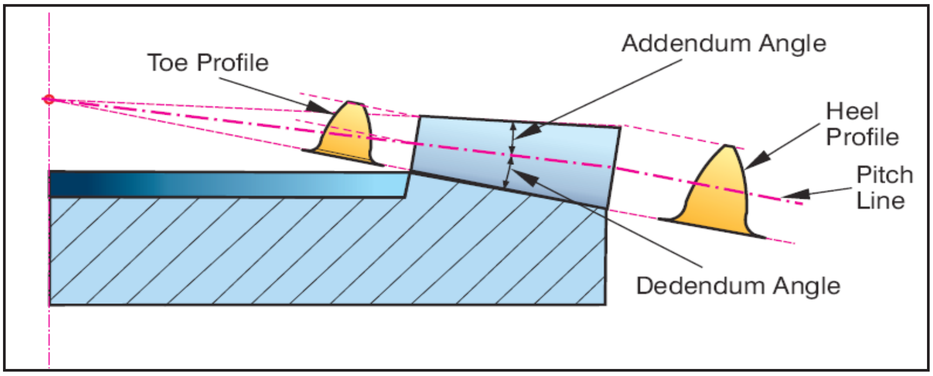

Figure 20 Tapered tooth depth change by addendum and dedendum angle.

tooth (Fig. 20). The tapered depth tooth has a number of advantages based on the original idea of the spherical involute. The tooth depths and the tooth profiles have proportions connected to the distance from the gear axes. The phenomenon known as undercut (left tooth profile, Fig. 16) is virtually eliminated or reduced.

However, the generation of bevel gears with tapered depth teeth causes conflict between the desired generating gear axis and the practical possible generating gear axis orientation. The methods E, F, G and H present different solutions for this conflict which are compared based on their kinematic coupling conditions.

Graphic "E" (Fig. 21) would require a horizontally oriented generating gear plane, which is perpendicular to the presentation plane and includes the pitch line. The employed machine design allows the tilting of the cutter head about $_{K}$ into the root line direction only in connection with a generating gear orientation — which is also parallel to the root line. The results are two non-matching generating gear axes for pinion and gear.

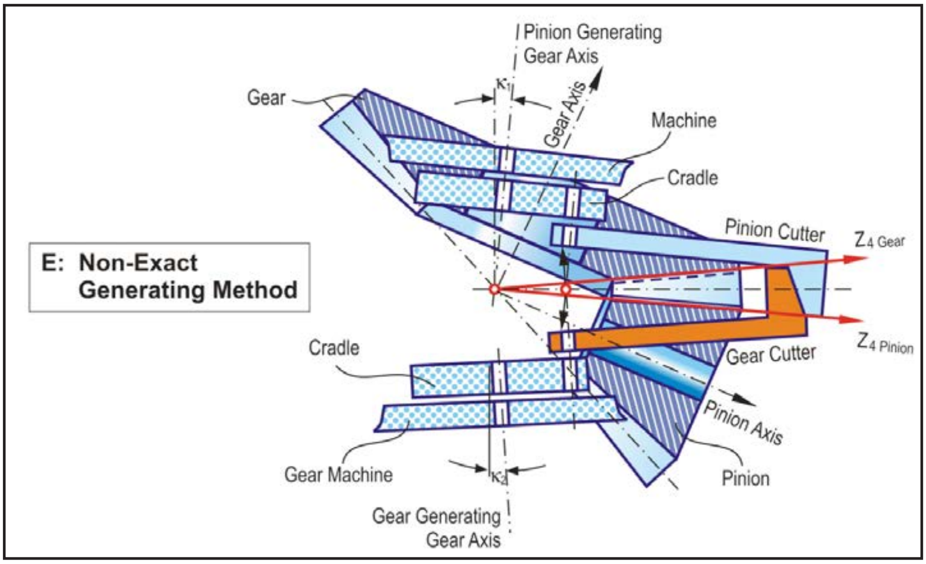

Figure 21 Generating model for bevel gears with tapered depth teeth — Method E, octoid of the second order.

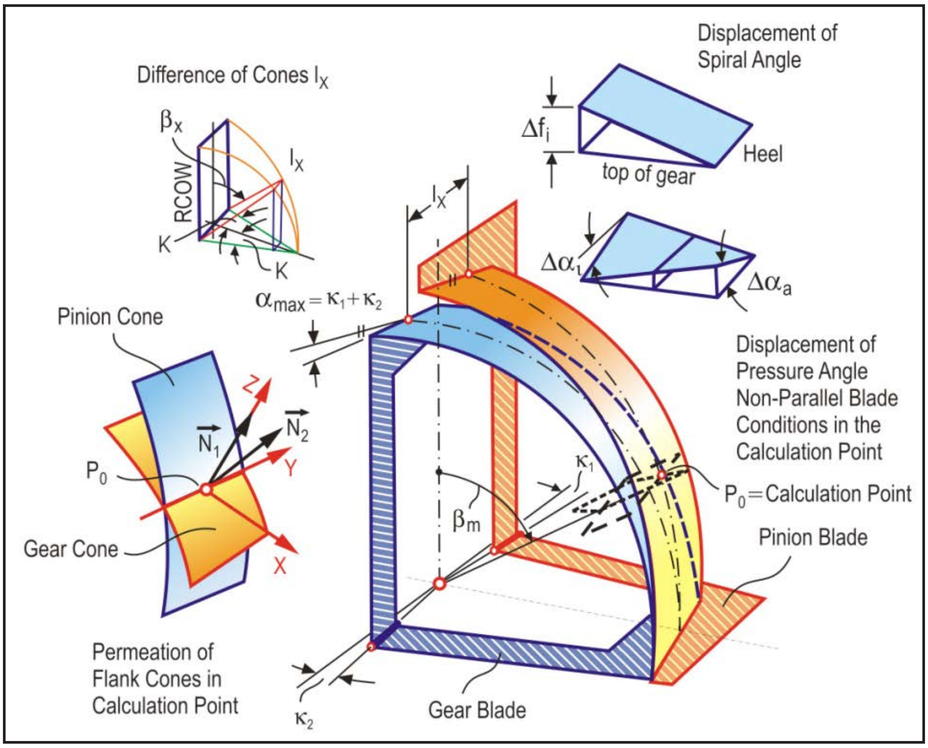

Figure 22 Blade cone element deviation in case of different axes of rotation.

Although both cutting edges match at the calculation point, the cone elements generated by the pinion and gear cutter deviate from each other due to a cutter axis orientation difference of $_{K1} + _{K2}$ (Fig. 22; Ref. 5). The kinematic coupling requirements 1 and 2 are not satisfied, whereas coupling requirement 3 is only slightly violated. Method "E" exists as a production process with and without a hypoid offset. The profiles of the resulting non-conjugate flank forms are octoids of the second order. The flank form deviations of method "E" are a maximum compared to the other methods discussed in this chapter. With the configuration of method "F" (Fig. 23) the attempt is made to keep the systematic errors as small as possible (Refs. 6–7). In spite of the collinear generating gear axes, both cutter heads are tilted about the angles $_{K1} + _{K2}$ in order for the blade tips to follow the root lines of the work gears. Coupling requirement 2 is fulfilled, the generating gear axes are identical, and the cutter cone elements match perfectly in the area of the calculation point. However, the cutter head tilt creates two slightly internal conical generating gears, which is why the conical generating tooth surfaces increasingly deviate with increasing distance from the calculation point. Coupling conditions 1 and 3 are not precisely fulfilled. The generated profile form is consistent with an octoid of the first order. Method "F" creates small flank form deviations that consist mostly of profile crowning.

Arrangement "G" (Fig. 24) shows the form cutting of a ring gear and the generating of a pinion with a tilted cutter head. The tilt angle $_{K1}$ is equal to the root angle $_{K1}$ of the pinion (in case of a gear box shaft angle of 90°). Although the two cutting edges match in the calculation point, the generating gear flank cone elements are deviating from each other with distance from the calculation point. Coupling requirement 2 is not satisfied, while the coupling requirements 1 and 3 are fulfilled.

By applying the artifice in Figure 25, a nearly exact bevel gear pair is created in spite of the tapered depth teeth and the plain generating gears (Method "H"). The crossing angle of the generating gear axes is like in case of method "E," or the sum of the dedendum angles. The particular artifice bases on the choice of curved blades whose radii originate in the intersecting point of the two correctly oriented cutter head axes. The result is a spherical generating gear flank surface which is perfectly congruent in the calculation point. The two surfaces of engagement intersect in this roll position along the center contact line. The coupling requirements 1 and 3 are fulfilled at the calculation point. Moving from the roll position that includes the calculation point will however, show differences in the surfaces of engagement and misalignment of the spherical generating flanks because the intersecting point of the cutter head axis is shifting during a generating cradle rotation. Eventually, none of the kinematic coupling requirements is fulfilled any longer.

The roll quality of uncorrected gearsets manufactured with method "H" (Gleason UNITOOL) is similar to the roll quality of gearsets manufactured with Method "F," but Method "H" can be performed on a less complex machine tool.

Bevel gears with tapered depth teeth present a number of advantages that are based on the balanced tooth cross-sections between heel and toe. Their manufacturing is limited until today, to face milled bevel gear sets. The reason for this is that changes in tooth thickness (i.e., slot width along the face width) cannot be compensated with a face hobbing process.

Already in the 1920s, Gleason developed mathematics for first- and second-order flank modifications via geometrical and kinematical corrections in cutting machines. These corrections made it possible to compensate flank form errors and additionally allowed the application of crowning to the flank surfaces. Crowning is necessary to avoid edge contact between the pinion and gear flanks in case of load-inflicted deformations and manufacturing tolerances.

Today's Phoenix free-form bevel gear cutting machines use a combination of cutter head tilt and helical motion (axial shifting of the generating gear during roll rotation) in order to manufacture bevel gears with tapered depth teeth and conical slot width while using a face milling completing process (Fig.20). With this technology the rolling quality of bevel gears with tapered depth teeth (cut in a single indexing process) is comparable

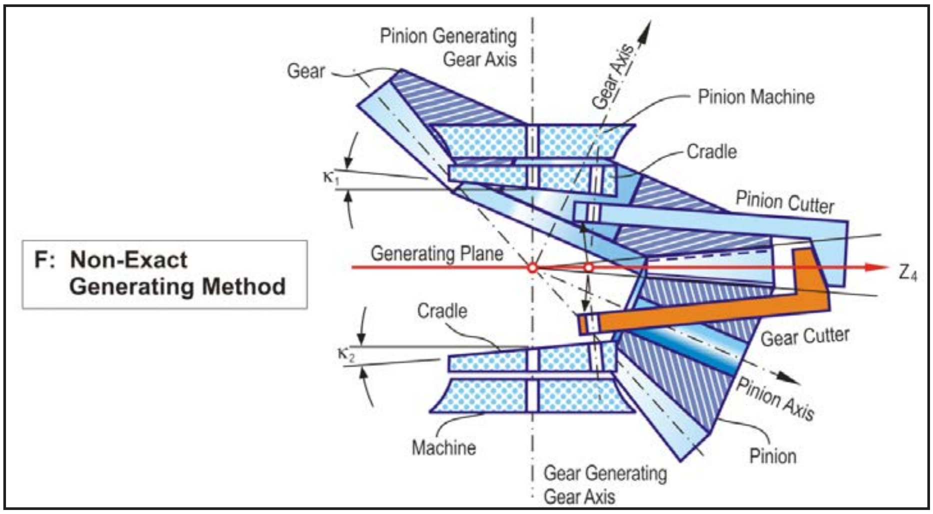

Figure 23 Generating model for bevel gears with tapered depth teeth — Method F, octoid of the first order.

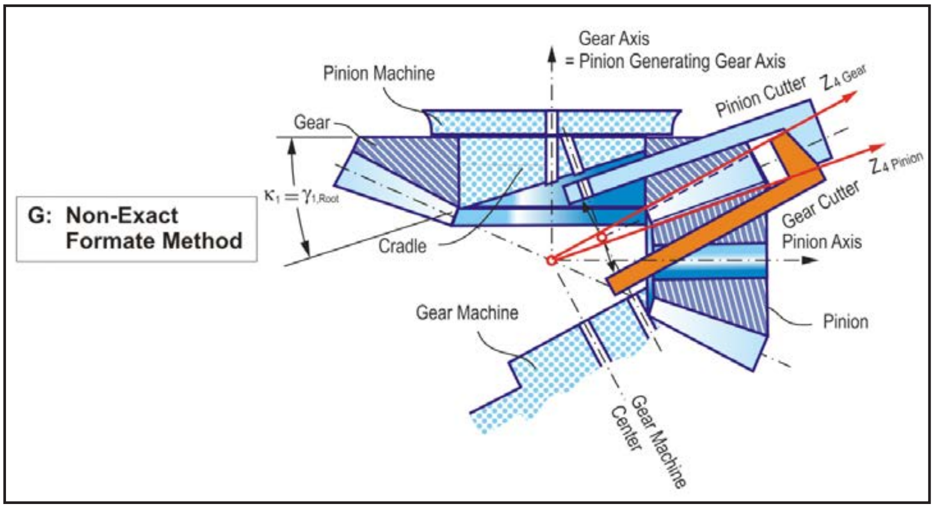

Figure 24 Generating model for bevel gears with tapered depth teeth — Method G, formate.

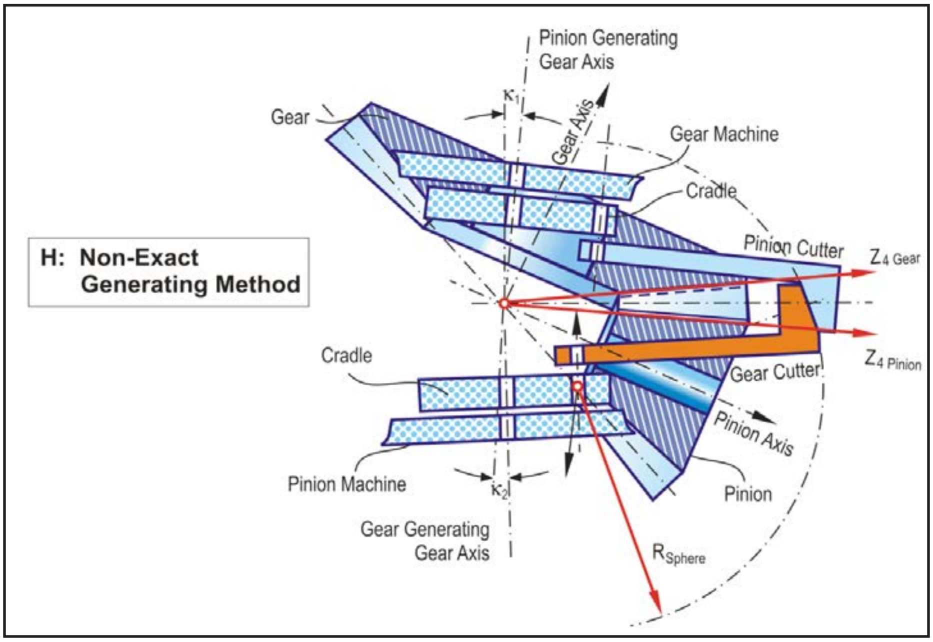

Figure 25 Generating model for bevel gears with tapered depth teeth — Method H, spherical flanks.

with the rolling quality of bevel gears with parallel depth teeth (cut in a continuous cutting process). Also, the cutting times of the two methods with modern machines and tools are basically identical.

A further advantage of the single indexing (face milling) method lies in the possibilities for hard finishing after soft cutting and heat treatment. The flank lines of face milled bevel gears are circular arcs, which make it possible to use grinding (not only lapping) as a hard finishing process. A suitable grinding wheel duplicates the silhouette of the cutting edges in a cutter head (stock allowance taken into account). The grinding wheel profile is basically dressed like the profile at the right side in Figure 17. The crossed profiles required in the continuous cutting process (face hobbing; left, Fig. 17) make it clear that it is physically impossible to dress those profiles onto a suitable grinding wheel.

Summary

- At the beginning of this chapter some thoughts about plausible explanations of the gearing law were discussed.
- Involute gearing was then presented as the consequential result of the engineering demand for a robustly functioning, easy-to-manufacture tooth form.
- A simplified explanation of the analogy between the cylindrical gear and bevel gear generating principle helps clarify things in making the bevel gear generating methods easier to understand. Based on this general understanding garnered at this point, a closer relationship of how the different bevel and hypoid gear generating methods are conducted is developed.
- The chapter continues to a deeper comprehension of the theory and understanding the pros and cons of the different methods.
- There is an acknowledgement that face hobbed bevel gears always feature parallel depth teeth and are not suitable for grinding due to their flank form and tooth thickness taper.
- Hard finishing of face hobbed bevel gears is generally done by lapping. In cases of smaller batches, a skiving with coated carbide blades is also possible.
- The goal with regards to face milled bevel gears was to convey the knowledge that they have, with only some unimportant exceptions, a tapered tooth depth form. It is possible to grind face milled gears very precisely and efficiently based on their tooth depth taper and circular flank lines. Lapping as well as skiving of face milled bevel gears are today's only exceptions — which are not often applied.

References

1. Buckingham, E. *Spur Gears*, McGraw-Hill Book Company, Inc., New York and London 1928.
2. Niemann, G. and H. Winter. *Maschinenelemente I, II & III*, Springer-Verlag Berlin, Heidelberg, New York, Tokyo, 1983.
3. Dudley, D. *Dudley's Gear Handbook*, McGraw-Hill, Inc. New York 1991.
4. Schriefer, H. "Verzahnungsgeometrie und Laufverhalten bogenverzahnter Kegelradgetriebe," Dissertation, RWTH Aachen, 1983.
5. Stadtfeld, H.J. "Anforderungsgerechte Auslegung bogenverzahnter Kegel-Radgetriebe," Dissertation, RWTH Aachen, 1987.
6. Brandner, G. "Kreisbogenverzahnte Kegelräder," *Maschinenbautechnik, 3.* Jg. Issue 5 May 1954.
7. Richter, E.H. "Geometrische Grundlagen der Kreisbogenverzahnung und ihre Herstellung," *Konstruktion*, Issue 3, 1958.

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