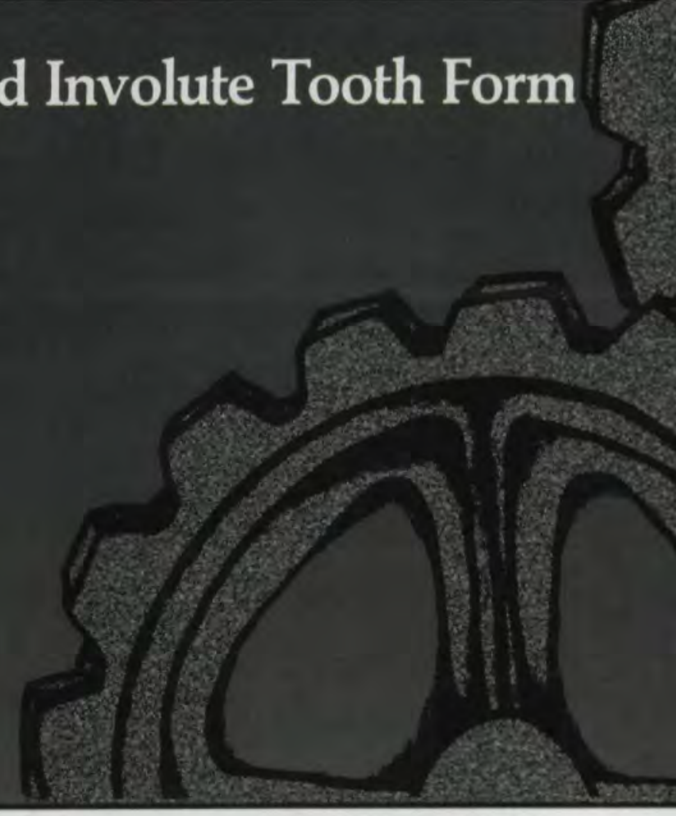
Dynamic Analysis of Straight and Involute Tooth Form



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Abstract

The effect of load speed on straight and involute tooth forms is studied using several finite-element models. It is found that for rapidly rotating gears and sprockets, the load speed along the tooth surface can significantly affect the tooth vibration. Indeed, it is found that for sufficiently high load speeds and for sufficiently slender tooth forms, the tooth deflection can, at times, be directed opposite to the load direction.

Comparisons are made of various dynamic models of gear and sprocket teeth. It is shown that for stubby tooth forms there is considerable difference between results obtained with finite element models and results obtained with Timoshenko beam models.

Finally, it is shown that gear or sproket vibrations can be induced by the shape of the tooth form itself. This effect becomes increasingly significant at highėr speeds.

Introduction

Recently there has been increased interest in the dynamic characteristics of gear and sprocket teeth. Performance, noise, wear, and life considerations have all stimulated interest in tooth dynamics. In this paper, we consider the dynamical effects of a moving load along the surfaces of straight and involute tooth forms.

Several researchers have considered various aspects of this problem. For example, Nagaya and Uematsu[1] recently (1981) studied effects of moving load speeds on the deflection of a tapered, cantilever, Timoshenko beam. Their analysis is based upon approximations developed earlier by Nagaya.[2] Much earlier research by Attia[3] (1959) and Utagawa and Harada[4] (1961) led to an analysis of tooth response as a function of load postion but not load speed. Later Wallace and Weireg[5] (1973) used the finite element method together with Hertzian type contact to study dynamic loading. Still later, Cornell and Westervelt[6] (1978), using a method developed by Richardson[7] and Howland[8] studied dynamic tooth stresses by modeling meshing gear teeth by variable springs with inertias of rigid bodies. They showed that tooth profile modification, inertia, damping, and resonance can all affect the tooth stresses.

TABLE I. Interaction of Dynamic Parameters of Gears and Sprocke

	Tooth Errors and Modifications	Gear Speed	Transmitted Load	Contact Ratio	Conta Positic
Dynamic Load	L	N	I	I	U
Efficiency	U	L	I	N	U
Dynamic Stress	N	U	L	I	U
Tooth Deflection	U	U	L	I	U
Gear or Sprocket Vibration	U	N	U	U	U

NOTATION: L – Linearly Proportional Relation
N – Nonlinear Relation

A summary of the interation of the various parameters affecting tooth dynamics, as reported in the literature, is given in Table I.

In the analysis of this paper, we use the finite element method to study the effect of load speed and slenderness ration on the dynamic responses of tapered beams. We then examine the dynamic response of involute tooth forms to moving loads.

Dynamic Analysis of Tapered, Truncated Beams

Using Timoshenko beam theory, Nagaya and Uematsu[1] considered the problem of a force P moving with constant speed V along the centerline of a tapered, truncated, cantilever beams as depicted in Fig. 1. The beam cross section is rectangular. To simulate a gear tooth the half angle β is taken as 20°. The truncated length $(L-a)$ is 2.25M where M is the

Mesh Stiffness	Rim Thickness	Gear or Sprocket Inertia	Damping	References
U	U	L	I	3, 4, 6, 10, 12, 13, 15
U	U	U	U	14, 22, 23
U	I	U	U	6, 9, 16, 17
I	I	U	U	20
U	U	U	U	18, 25

I – Inversely Proportional Relation
U – Unknown or Random Relation

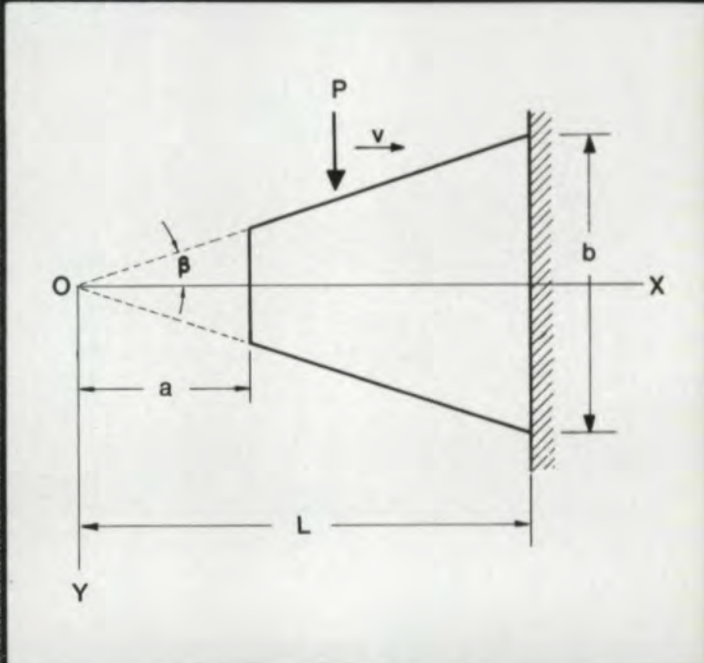

Fig. 1 – Tapered, Truncated, Cantilever Beam.

module of the gear. Then the base b of the beam, simulating the root of the tooth, is 2.48M. Nagaya and Uematsu[1] have shown that as V is increased, the centerline deflection differs substantially from the static deflection. Indeed, when V is 1% of the longitudinal wave propagation speed ($\sqrt{E/p}$), the centerline deflection is in the negative Y direction for a major portion of the span at various load positions, and even beneath the load itself.

To further examine this phenomenon, we constructed a three-dimensional, finite-element model of the beam as shown in Fig. 2. The mesh contains 120 "8-node brick" elements and 288 nodes. The model was given a unit thickness in the Z-direction. The elastic constant E was 30×10^6 psi (s.07x10″ N/m^2), the weight density γ was 0.283 lb/in^3 (7.83×10^3 kg/m^3), giving a wave propagation speed V* of 2.024×10^5 in/sec (5.14×10^3 m/sec). The beam was loaded in the positive Y-direction with a unit load moving at 0.001V*, 0.005V*, and 0.01V*. The centerline displacement was then determined. The results are shown in Fig. 3 for four load positions.

By comparing these results with those of Nagaya and Uematsu[1] it is seen that the magnitude of the displacements differ substantially. Moreover, the displacements are always positive with the finite-element model. The primary reason for the difference is the effect of the thickness of the beam. Indeed, for this beam the slenderness ration, defined as $\sqrt{AL^2/I}$, is only 6.0, where A is the average beam cross section area, L is the beam length and I is the average second moment of area of the beam cross section about a line parallel to the Z-direction and intersecting the centerline.

To examine the effects of the slenderness ratio, three other finite-element models were developed. First, a stepped beam model with a slenderness ratio of 49.0 was constructed. The model consists of 15 beam elements with different thicknesses as depicted in Fig. 4. The model was given the same loading as the tapered beam of Fig. 1. The displacements of the centerline and the tip were computed. Figs. 5 and 6 show the

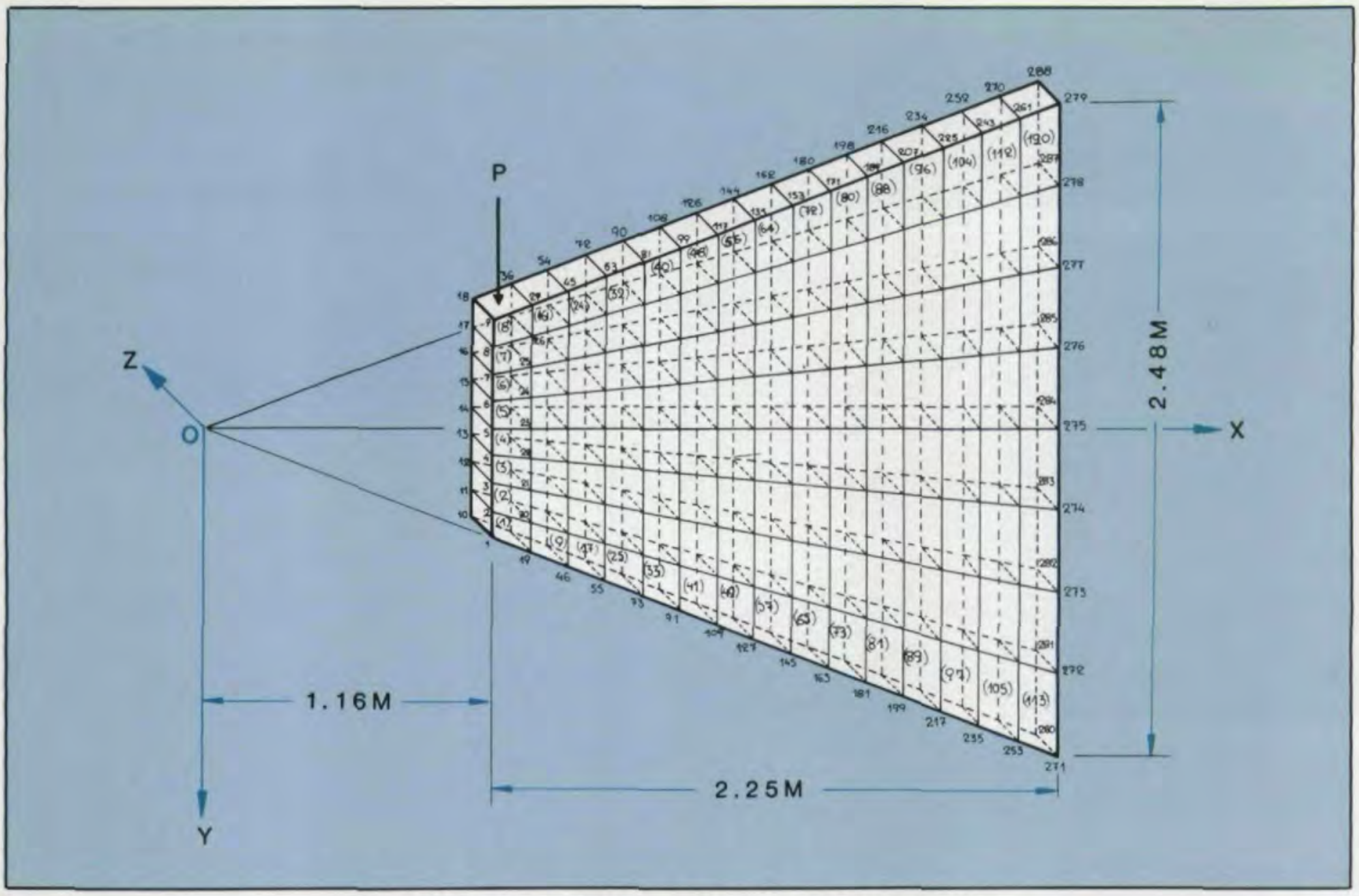

Fig. 2—A Finite Element Mesh for the Beam of Fig. 1 (M is Gear Module).

results. Finally, straight, thin beam models were constructed with slenderness ratios of 26 and 78. With the same loading as before, the centerline and tip deflections were computed. Figs. 7, 8, and 9 show the results.

A comparison of the results of Figs. 3, 5, 6, 7, 8, and 9, together with those of Nagaya and Uematsu[1], show that, as expected, the slenderness ratio has a significant effect upon the beam response to the moving load. However, the finite-element models show that a relatively high slenderness ratio is required for significant negative V-deflection of the centerline.

Dynamic Analysis of Involute Tooth Forms

Many gears and sprockets have involute tooth forms. That is, the tooth profile is in the shape of an involute to a circle. If a gear or sprocket has an infinite radius (a "rack"), the involute form is a straight line. Thus, a tapered, truncated cantilever beam is a good model of a rack gear or sprocket tooth. However, for circular gears and sprockets, a preferable model has the curved shape of an involute. To develop such a model, we constructed a finite-element grid as shown in Fig. 10. The model consists of 104 8-node brick elements with 242 nodes. The model represents a 20° involute 20-tooth gear or sprocket with a diametral pitch of 1.0.

A unit load was applied to the surface to simulate the meshing loading. The load was applied near the root of the tooth, at the base circle, and then moved with a speed V along the surface toward the tip. The local speed was not constant, but instead was governed by the expression:

$$V = R_B \omega^2 t \qquad (1)$$

where R_B is the base circle radius and ω is the gear or sprocket angular velocity. When the time t is zero, the load is at the base circle. [Equation (1) is developed in the Appendix.]

Figs. 11 and 12 show the radial and transverse displacements of various surface nodes for an angular speed of 100 rpm.

Discussion and Conclusions

The foregoing results show that slender beams may produce erroneous conclusions for many tooth forms — especially for short stubby teeth. Indeed, if the root thickness is approximately equal to the tooth depth, the slenderness ratio is too small for conventional beam theory to be valid — even with thickness modifications.

Next, it is seen that the load speed along the tooth can have a significant effect on the deformation and it can induce vibrations. For sufficiently thin tooth forms, or for sufficiently high load speeds, the centerline deflection can even be directed *opposite* to the load. This is especially important for high speed

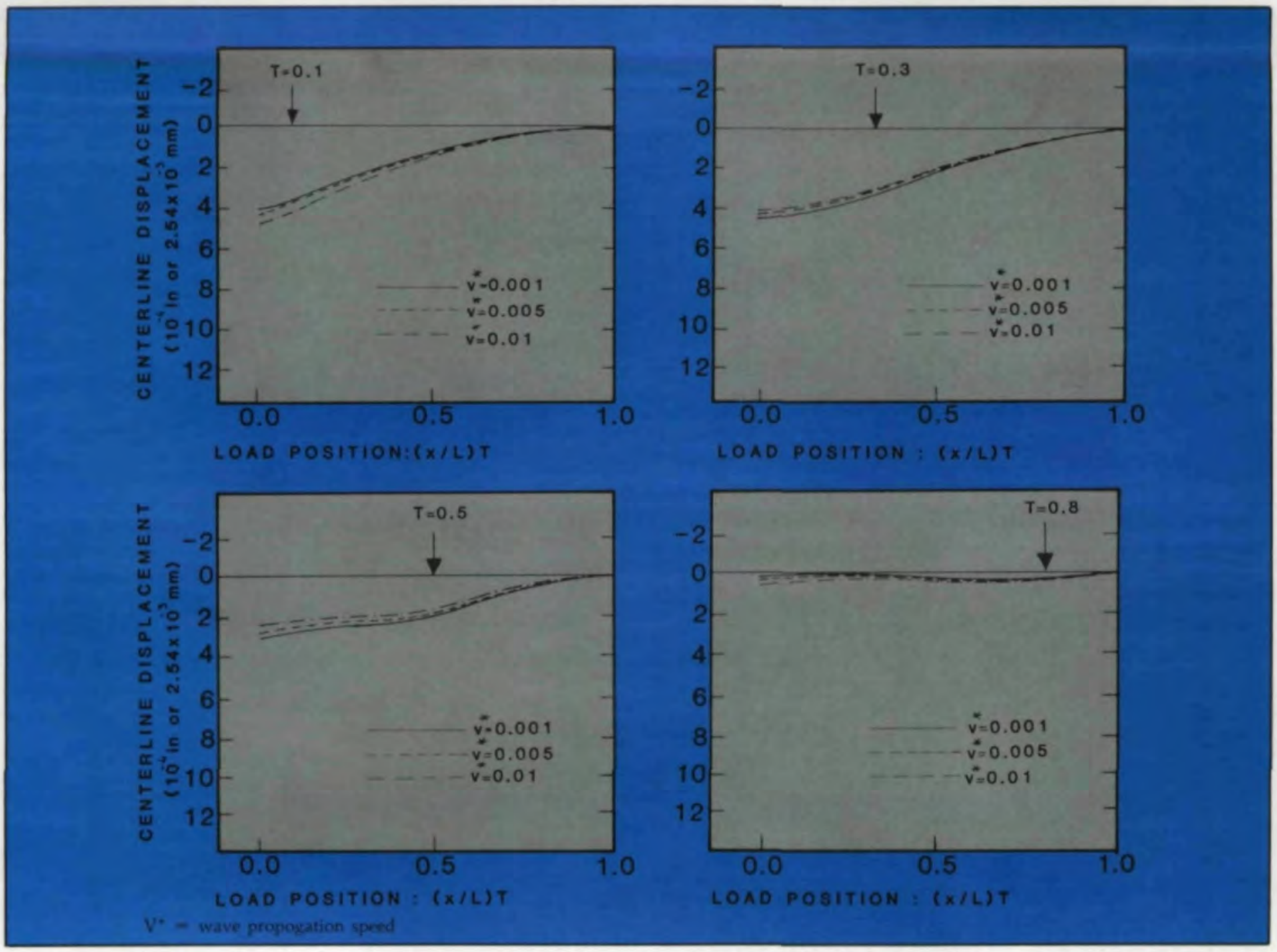

Fig. 3 – Centerline Displacements for the Beam of Fig. 1 for Various Load Speeds and Positions.

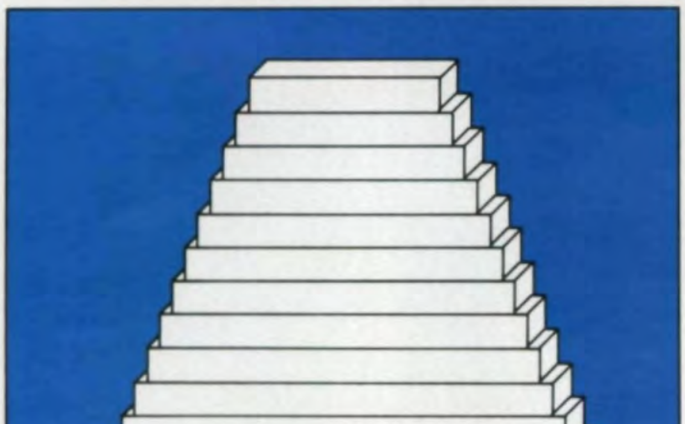

Fig. 4 – Stepped Beam Model.

gears and sprockets since tip relief and tooth design modifications for dynamic effects are generally based upon the static load deflection curve. This curve closely resembles the low speed curves of Figs. 3, 5, 7, and 8. Thus, for higher load speeds (for example, greater than 0.005V*), the design modifications may lead to *deleterious* as opposed to beneficial effects.

Third, Figs. 11 and 12 show that disengagement of the load may also induce significant vibration. Indeed, for rapidly rotating gears and sprockets, this vibration could lead to interference and impact during subsequent engagement.

Finally, the shape of the tooth itself can affect the load speed and thus induce vibration. For involute tooth forms, the load speed is proportional to the time of engagement and to the square of the rotation speed. This means that the induced vibration and other dynamic effects become much more pronounced as the gear or sprocket rotational speed is increased.

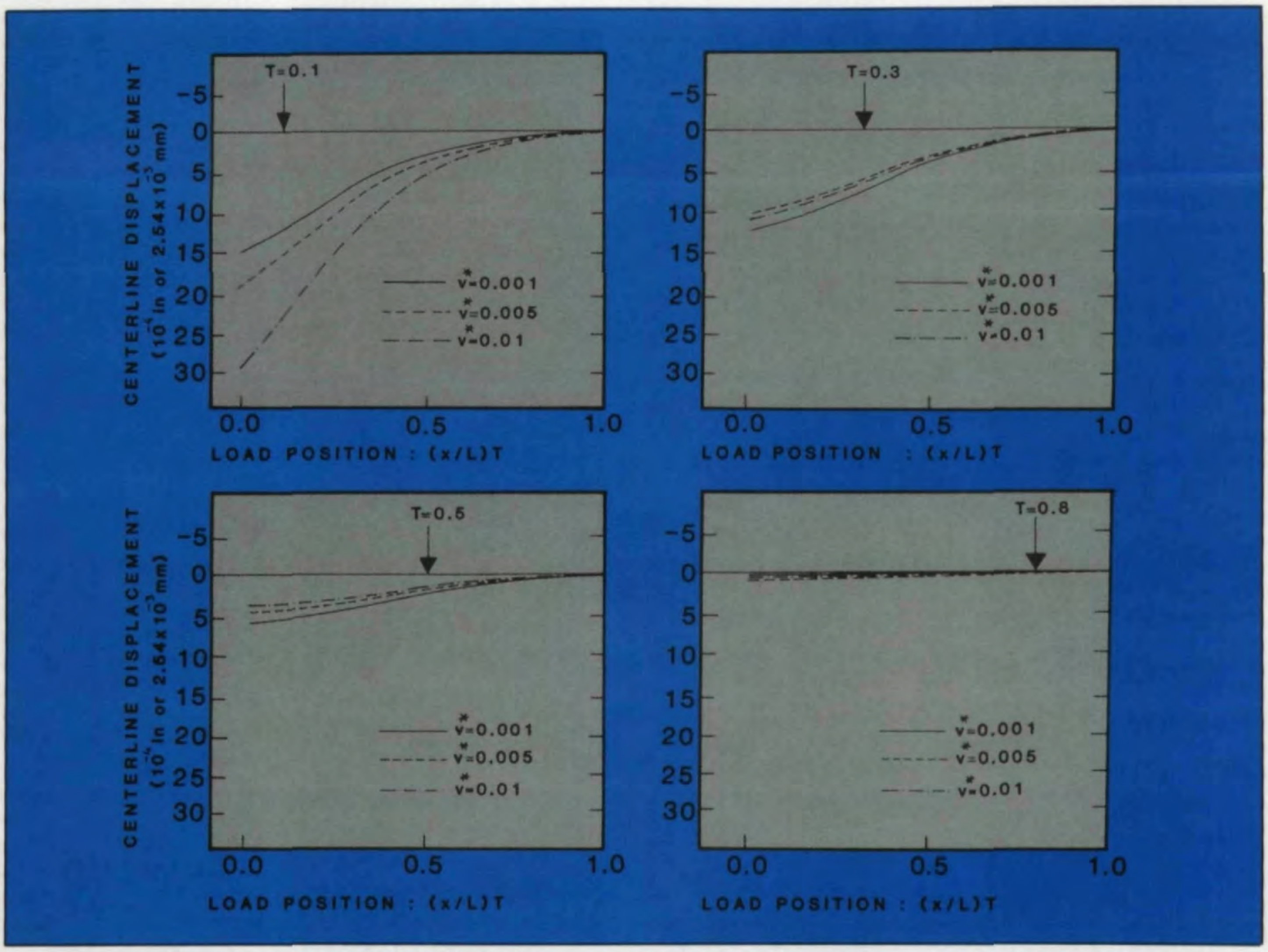

Fig. 5 – Centerline Displacements for the Stepped Beam of Fig. 4 for Various Load Speeds and Positions.

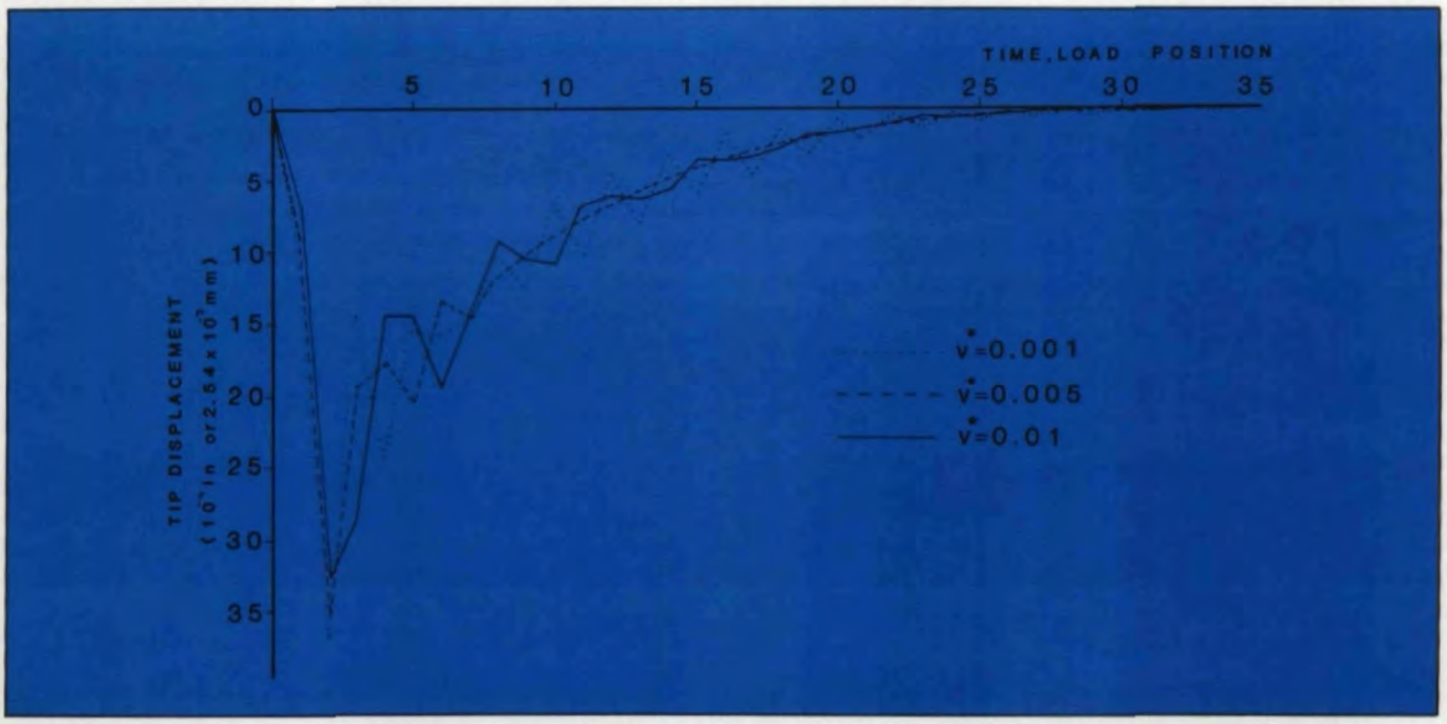

Fig. 6 – Tip Displacements for the Stepped Beam of Fig. 4 for Various Load Speeds.

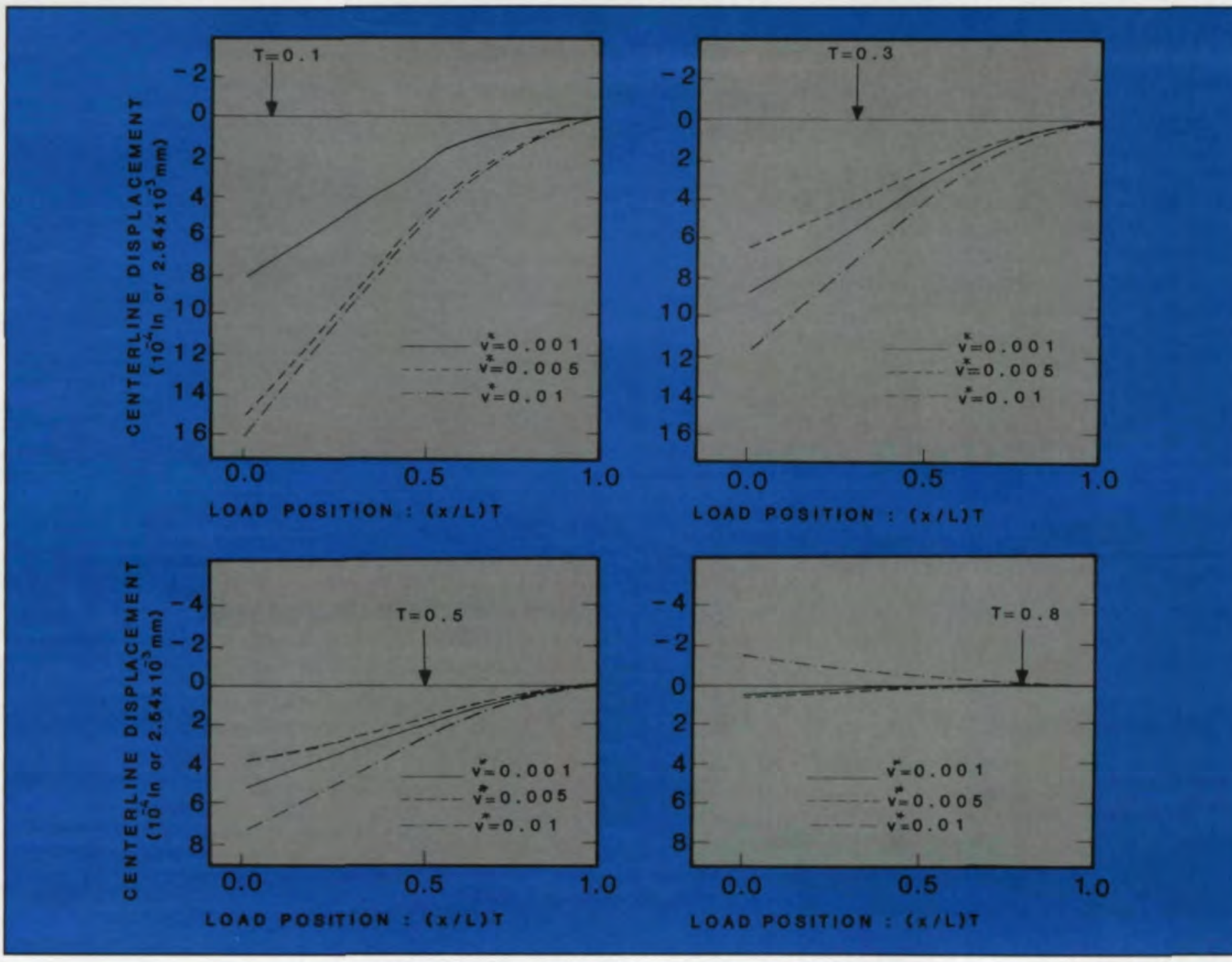

Fig. 7 – Centerline Displacement for a Straight Beam with Slenderness Ratio: 26.

APPENDIX

Moving Load Along an Involute Tooth Form

Recall that an involute to a circle may be viewed as the locus of the end point of a cord being unwrapped around the circle. Consider the clockwise involute I to the circle C shown in Fig. 13. Let $\underset{\sim}{p}$ locate a typical point P of I relative to O, the center of C. Then, if $\underset{\sim}{r}$ and $\underset{\sim}{t}$ are the radial and tangential vectors as shown, $\underset{\sim}{p}$ may be written as:

$$\underset{\sim}{p} = \underset{\sim}{r} + \underset{\sim}{t} \qquad (A1)$$

Let X and Y be Cartesian coordinate axes with origin at O and with accompanying unit vectors $\underset{\sim}{n}_x$ and $\underset{\sim}{n}_y$. Let the Y-axis pass through the origin O of I. Finally, let ϕ be the angle between $\underset{\sim}{r}$ and the X-axis. Then, $\underset{\sim}{p}$ may be expressed in terms of $\underset{\sim}{n}_x$, $\underset{\sim}{n}_y$, and ϕ as:

$$\underset{\sim}{p} = (r\sin\phi - r\phi\cos\phi)\underset{\sim}{n}_x + (r\cos\phi + r\phi\sin\phi)\underset{\sim}{n}_y \qquad (A2)$$

where r is the radius of C. Hence, the X-Y coordinates of P are:

$$x = r(\sin\phi - \phi\cos\phi) \text{ and } y = r(\cos\phi + \phi\sin\phi) \qquad (A3)$$

The arc length along I from its origin O to a typical point P is then:

$$s = \int_O^\phi [(dx/d\phi)^2 + (dy/d\phi)^2]^{1/2} d\phi = r\phi^2/2 \qquad (A4)$$

From the properties of involute geometry [26], since the normal to I at P is tangent to C (See Fig. 13), P is the contact point of the involute tooth form with its mating tooth form. Therefore, the speed V of the contact point, and hence the load, along the tooth is:

$$V = ds/dt = r\phi(d\phi dt) = r\omega^2 t \qquad (A5)$$

where $\omega(= d\phi dt)$ is the angular speed of the gear or sprocket.

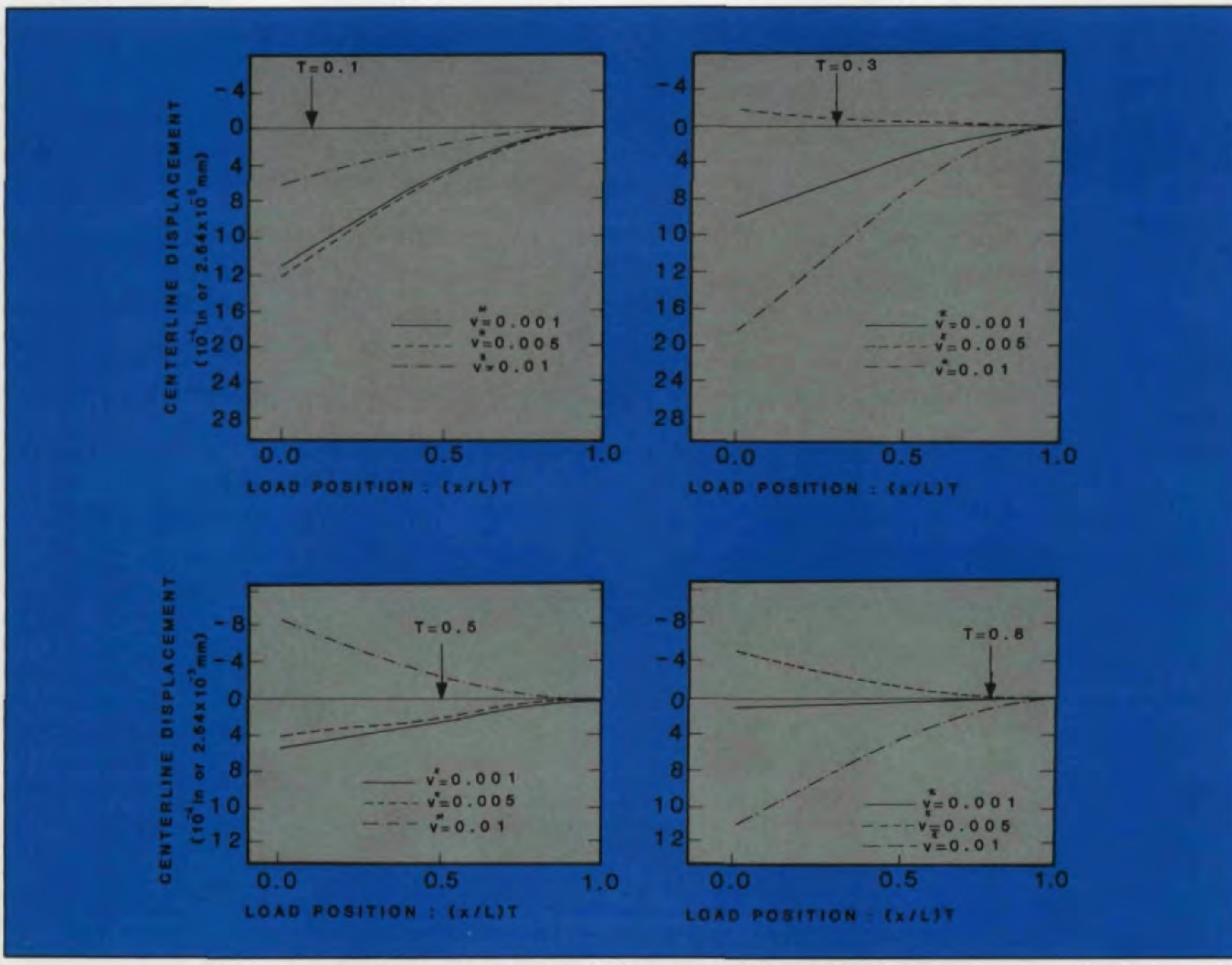

Fig. 8—Centerline Displacement for a Straight Beam with Slenderness Ratio: 78.

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DYNAMIC ANALYSIS OF STRAIGHT . . .
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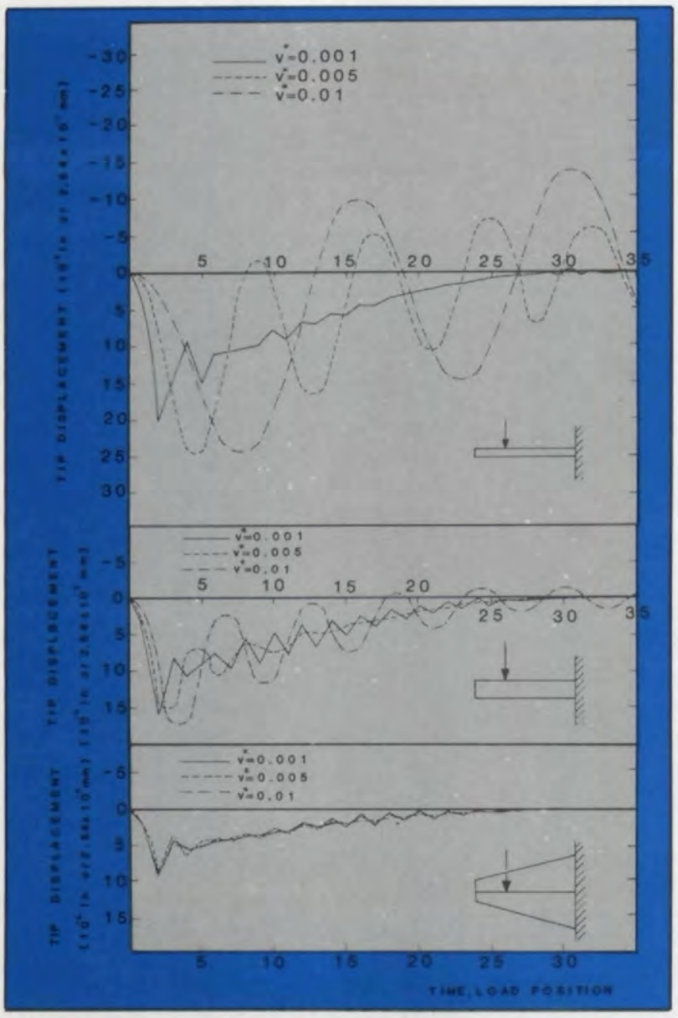

Fig. 9 – Comparison of Tip Displacements for Thin and Tapered Beam Models.

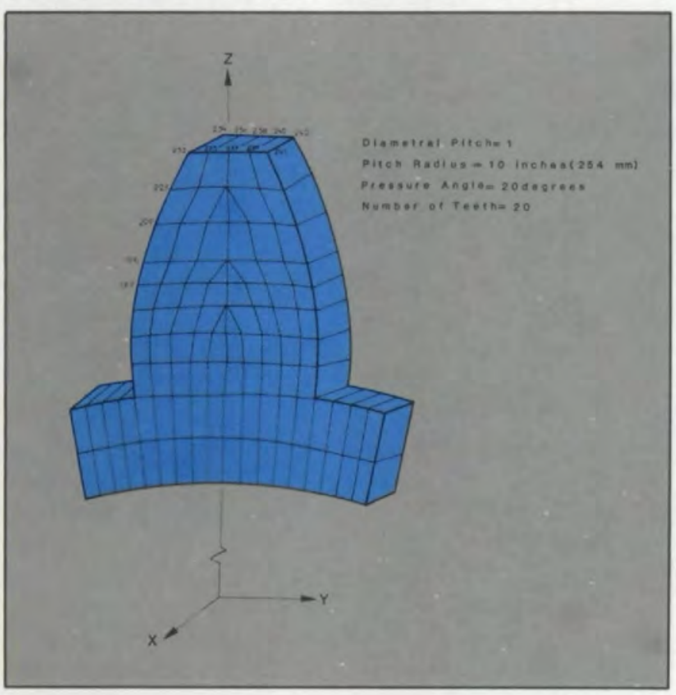

Fig. 10 – Finite Element Model of an Involute Tooth Form.

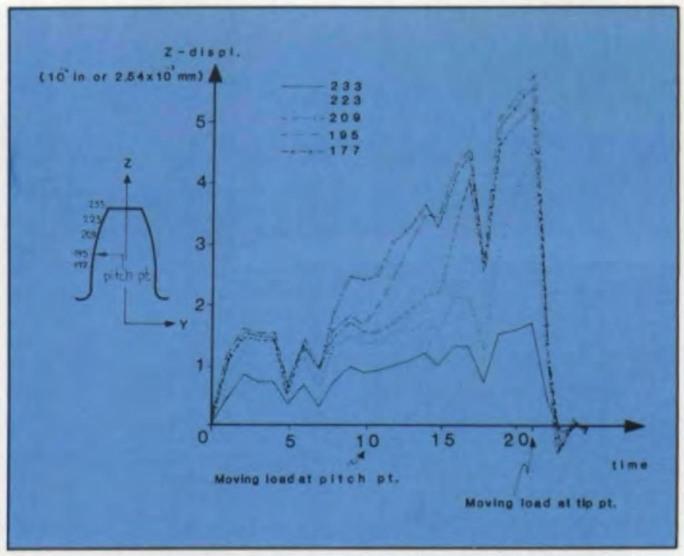

Fig. 11 – Radial Displacement of Involute Tooth.

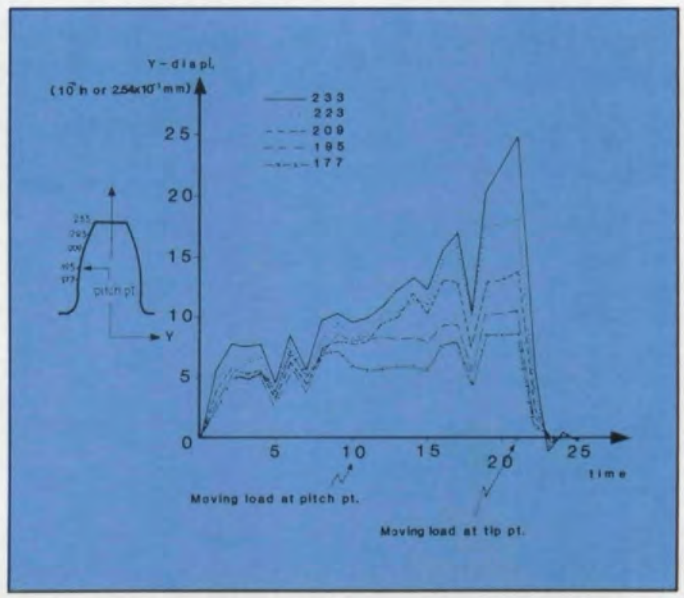

Fig. 12 – Transverse Displacement of Involute Tooth.

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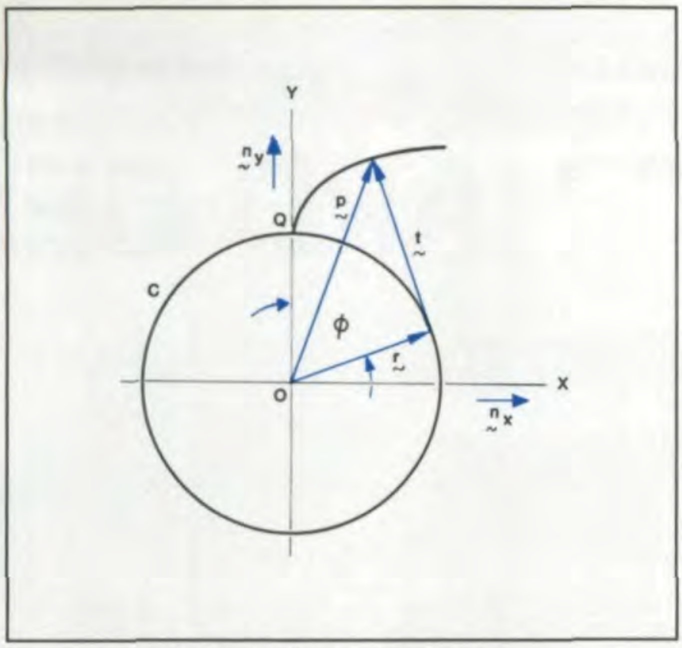

Fig. 13 – Involute I and Base Circle C.

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