Longitudinal Load Distribution Factor of Helical Gears

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This paper deals with the longitudinal load distribution and the bending moment distribution of a pair of helical gears with a known total alignment error. The load distribution along the contact lines is calculated by the finite element method based on the plate theory including transverse shear deformation. Empirical formulas for both longitudinal load distribution factor and bending moment distribution factor are proposed for practical use. The load distribution factor in AGMA 218.01 is examined, and it is concluded that the load distribution factor is close to the calculated results if the value of unity is taken as the transverse load distribution factor.

Introduction

The contact lines of a pair of helical gears move diagonally on the engaged tooth faces and their lengths consequently vary with the rotation of the gears. The load distribution along the contact lines is one of the most important factors for gear design, and some investigators have analyzed this problem.

Hayashi[1] and Niemann and Schmidt[2] solved numerically integral equations to obtain the load distribution. Niemann and Richter[3] proposed an experimental formula of the load distribution which was obtained by the photoelastic method. Conry and Seireg[4] developed a mathematical programming technique to estimate the load distribution and to obtain optimum profile modification. Kubo and Umezawa[5] obtained tooth bearings by means of the finite difference method. The authors developed a finite element technique based on the plate theory including the transverse shear deformation to calculate the deflection of gear teeth,[6] then estimated the longitudinal load distribution factor $K_{H\beta}$ and determined the optimum amount of arc shaped crowning for both spur gears[7] and helical gears

In a previous article[8], the longitudinal load distribution factor was defined as the ratio of the maximum load intensity to the average load on the contact lines at the worst position. Although the definition is logical, it is difficult to foresee the worst position. The average load is, therefore, generally unknown and the load distribution factor in the previous article is inconvenient for the practical use in gear design. In this article, this weak point is improved by introducing the average load on the contact lines of the minimum length. Formulas for both load distribution factor and bending moment distribution factor are proposed. A comment is also given on the transverse load distribution factor in AGMA 218.01[9].

Assumptions for the Calculation of the Load Distributions

The load distributions discussed in this article are for the involute helical gears which are generated by the basic rack (pressure angle = 20 deg, whole depth = $2.25m_n$ and the radius of tip corner = $0.375m_n$) recommended in ISO 53-1974 as well as JIS B 1701-1973.

Although the tooth of helical gears is essentially twisted, the effect of twist on the flexibility of tooth and the bending moment is assumed to be negligible. The thrust component of transmitted load is also assumed to be neglected. According to the assumptions, the cantilever plate with the flexural rigidity of the tooth is adopted as an adequate model. The plate is approximately represented by assembling 12 (in the direction of tooth height) x 21 max (in the direction of face

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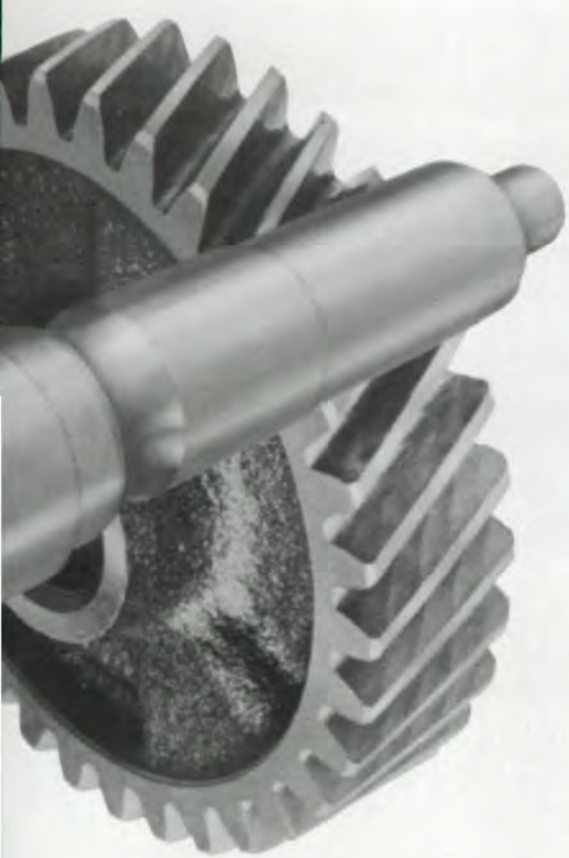

width) rectangular elements whose thicknesses vary linearly in the direction of tooth height. The deflection of the plate was calculated by FEM including both the transverse shear deformation and the deformation at the elastic built-in edge of the plate[6]. Since the helical gear tooth does not have full thickness near the end of tooth trace, the thickness at the centroid of element is adopted to estimate the flexibility at the part of tooth. The characteristic of a helical gear tooth is mainly involved in the inclination of the contact lines. In the middle plane of a tooth, the angle β_{tm} between the contact line and the tooth trace is presented by the following expression,

$$\tan \beta_{tm} = \sin \beta_b \tan \alpha_t \cos \alpha_n \qquad (1)$$

where α_t is the transverse pressure angle. The fundamental equations[7, 8] are summarized in Appendix 2.

Variations of the Load Intensity and the Bending Moment With the Rotation of Gears

An example of the contact lines in the plane of action is illustrated in Fig. 1. The transverse base pitch p_{bt} is divided into six equal parts. The lines with the same number are a set of contact line and the mesh advances in numerical order. The position of each line is indicated by the distance ζ along the side of the plane of action.

The load distributions of the pair of gears: $m_n = 5$, $z_1 = z_2 = 20$, $\beta = 20$ deg, $b_1 = b_2 = 68.89$ ($\epsilon_\beta = 1.5$), were calculated at every position of mesh shown in Fig. 1. The variations of the maximum load p_{max} and the maximum bending moment m_{max} of a tooth are shown in Fig. 2. The transmitted load is $P_{nt}/b = 600$ N/mm. The direction of total alignment error F_β and the rotation of gear 1 are illustrated in the figure. The abscissa indicates the position of the contact line, that is, $\zeta/(\epsilon_\alpha + \epsilon_\beta)\, p_{bt} = 0$ and 1 mean the initiation and the end of meshing, respectively. Since a set of contact lines whose interval is p_{bt} are in mesh simultaneously, the maximum load on the contact lines and the maximum bending moment of gear 1 vary as shown in Fig. 3. The total length of contact lines L, the mean load p_m and the load sharing factor ψ are also shown in the figure. In the case of $F_\beta = 0$, p_{max} and M_{max} reach maximum at the position where L is minimum. When the gears have total alignment error, the worst meshing positions for the load distribution are fairly close to the position of $L = L_{min}$. The

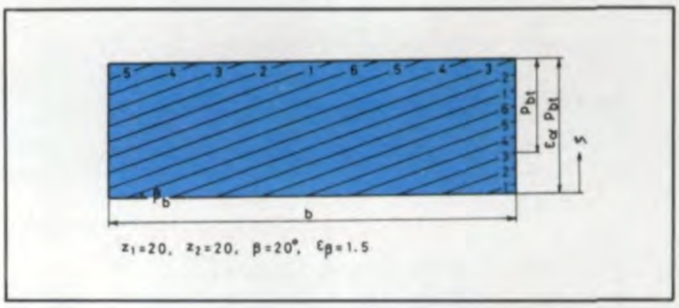

Fig. 1 – Contact lines on the plane of action ($m_n = 5$, $b = 68.89$, $p_{bt} = 15.59$, $\epsilon_\alpha = 1.44$)

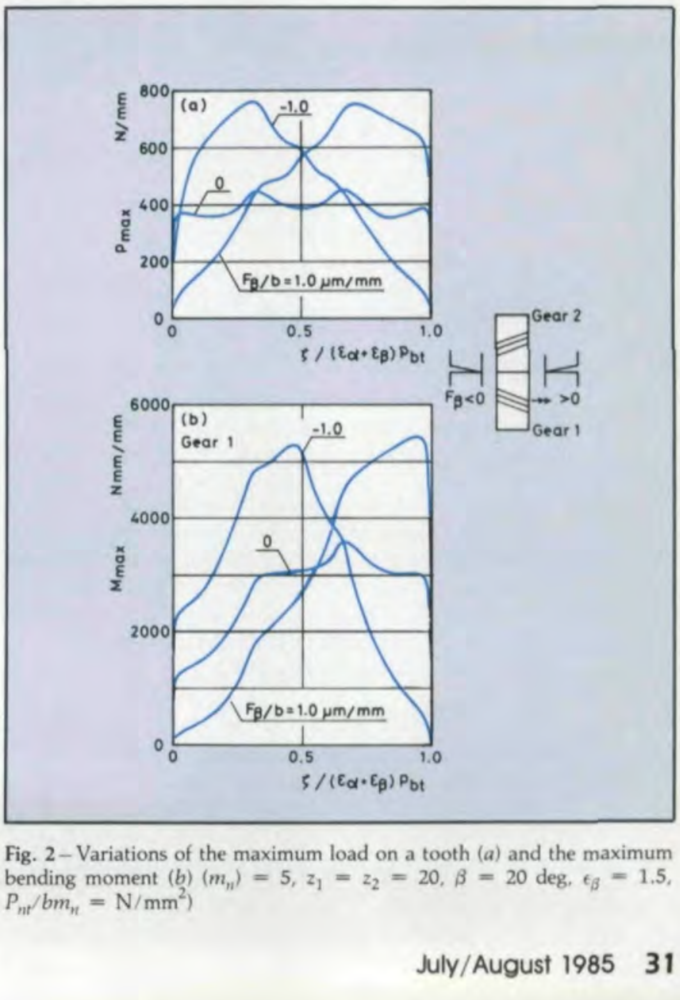

Fig. 2 – Variations of the maximum load on a tooth (*a*) and the maximum bending moment (*b*) ($(m_n) = 5$, $z_1 = z_2 = 20$, $\beta = 20$ deg, $\epsilon_\beta = 1.5$, $P_{nt}/bm_n = $ N/mm^2)

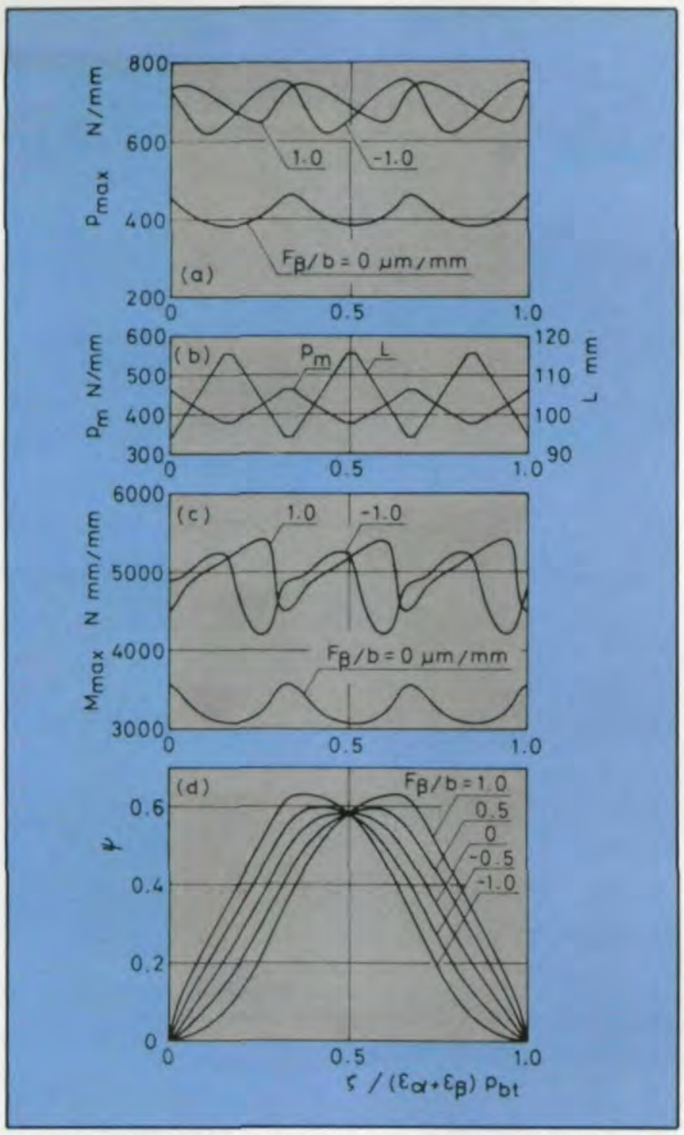

Fig. 3 – Variations of the maximum load intensity p_{max} (*a*), mean load p_m and the total length of contact lines L (*b*), the maximum bending moment M_{max} (*c*) and the load sharing factor ψ (*d*) of the pair of gears shown in Fig. 2.

worst positions for the bending moment, on the contrary, are shifted and they do not coincide with the worst positions for the load distribution. The worst positions ζ^* of both load distribution and bending moment are shown in Fig. 4. In the case of $F_\beta/b = 1.0$ μm/mm, ζ^* increases linearly with the increase of the face width. On the contrary, ζ^* in case of $F_\beta/b = -1.0$ μm/mm is approximately constant. The increase, like a step shown in the figure, means the boundary where the worst position shifts from the region of the single-tooth meshing to double-teeth meshing.

Longitudinal Load Distribution Factor

In the previous paper,[8] the longitudinal load distribution factor was defined as the ratio of the maximum load intensity to the average load which was uniformly distributed on the contact lines at the worst position. Although the definition is logical, the worst position may not be foreseen and the average load is generally unknown.

In order to improve this weak point, the following definition of the longitudinal load distribution factor is adopted in this paper:

$$K_{H\beta} = p_{max}/p_{ref} \tag{2}$$

where p_{max} is the maximum load intensity and p_{ref} is the reference load intensity which is represented as follows:

$$p_{ref} = p_n/L_{min} = (P_{nt}/\cos\beta_b)/L_{min} \tag{3}$$

The load distribution factor $K_{H\beta}$ of the pair of gears $z_1 = z_2 = 20$, $\beta = 20$ deg is shown in Fig. 5. The direction of total alignment error had little effect on $K_{H\beta}$. In most cases of $F_\beta = 0$, $K_{H\beta}$ is not equal to unity. However, $K_{H\beta}$ for $F_\beta = 0$ is assumed to be unity in this paper, since the error is not very significant. From the calculated results in the figure, the following expression can be obtained:

$$K_{H\beta} = 1.00 + \alpha_H\,(|F_\beta|/b)^{1.2} \tag{4}$$

Nomenclature

A = dimensionless value in relation to the ratio of L_{min} to face width, see equation[5] and Appendix 1
b = face width, (mm)
C_m = load distribution factor in AGMA 218.01
C_{mf} = face load distribution factor in AGMA 218.01
C_{mt} = transverse load distribution factor in AGMA 218.01
F_β = total alignment error, (μm)
$K_{H\beta}$ = longitudinal load distribution factor
$K_{m\beta}$ = bending moment distribution factor
L_{min} = minimum total length of lines of contact, (mm)
m_n = normal module, (mm)
M = bending moment at the root per unit length (N mm/mm)
p = load intensity or tooth normal load per unit length of the contact line (N/mm)
p_{bt} = transverse base pitch (mm)
p_n = tooth normal load in the normal plane, (N)
P_{nt} = tooth normal load in the transverse plane, (N)
z = number of teeth
α_n = normal pressure angle, (deg)
β = helix angle, (deg)
β_b = base helix angle, (deg)
ϵ_α = transverse contact ratio
ϵ_β = overlap ratio
ζ = distance from the initiation of meshing to the position of contact line, (mm)
ψ = load sharing factor

Subscripts 1 and 2 represent pinion and gear, respectively.

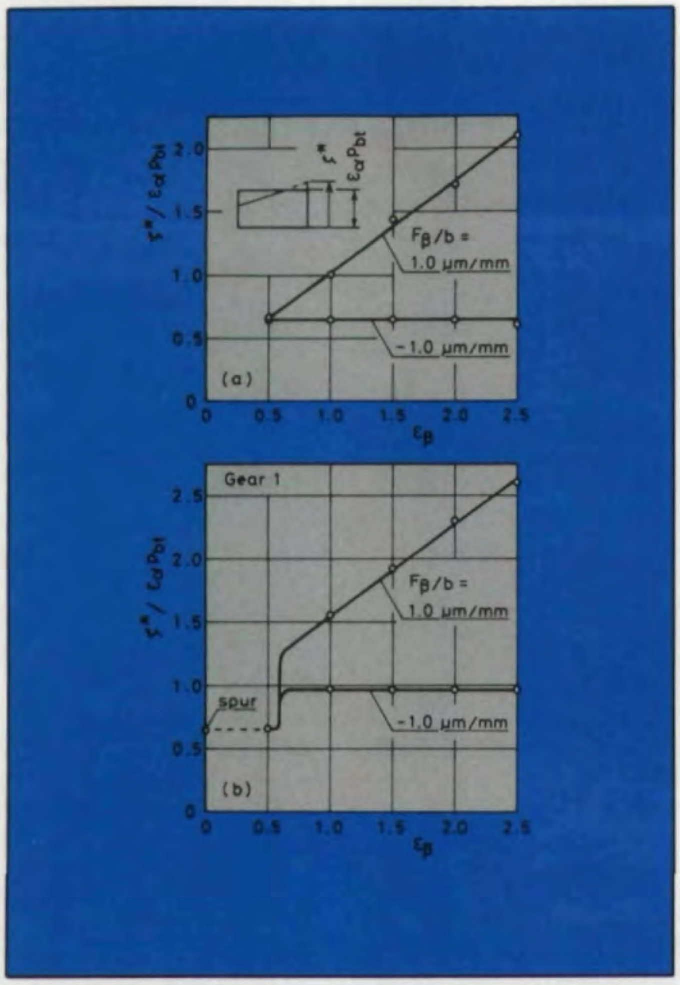

Fig. 4 – Worst position ζ^* of helical gears in relation to load intensity (*a*) and bending moment (*b*)

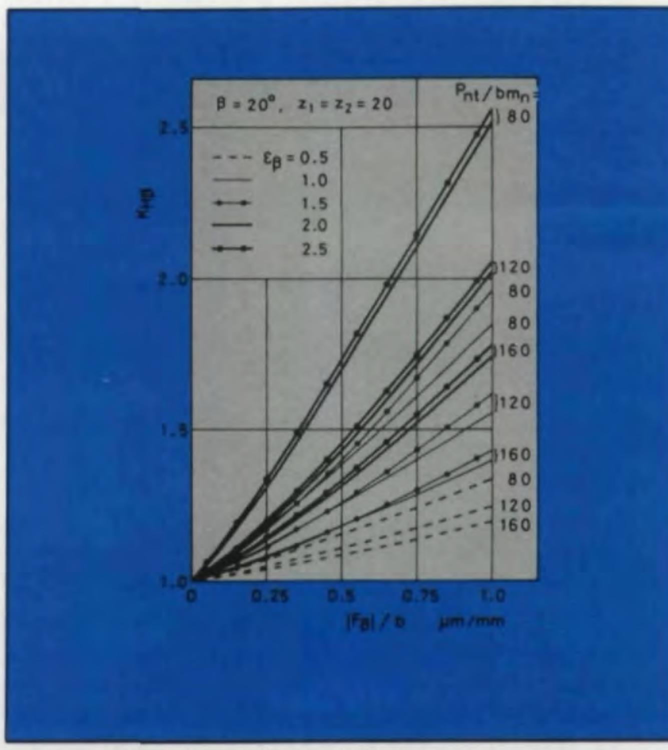

Fig. 5 – Longitudinal load distribution factor $K_{H\beta}$

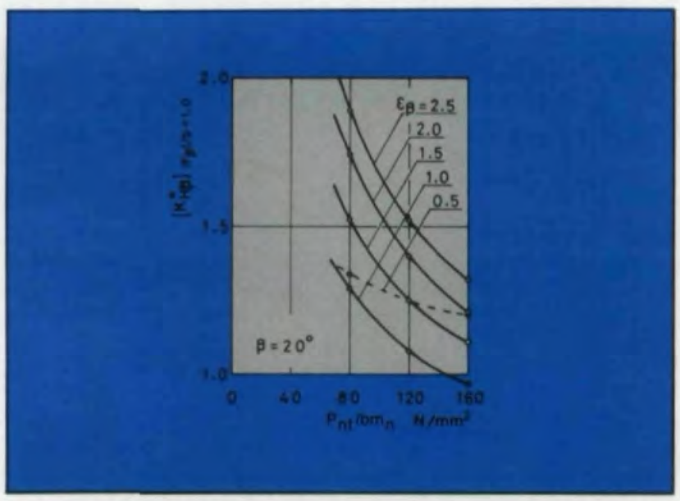

Fig. 6 – Modified load distribution factor $K_{H\beta}^*$ of gears $z_1 = z_2 = 20$ with the effective alignment error $|F_\beta|/b = 1.0$ µm/mm

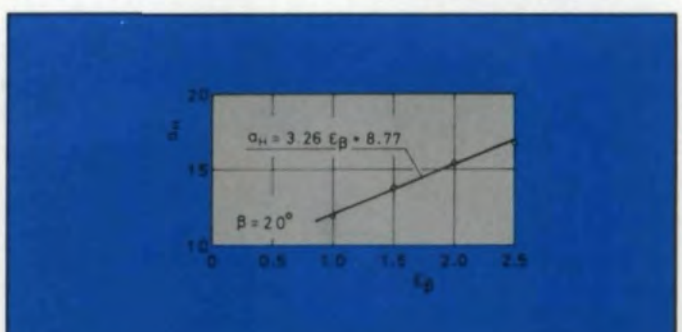

Fig. 7 – Value of α_H ($\alpha_H = [K_{H\beta}^*]_{|F_\beta|/b\,=\,1.0} \sqrt{P_{nt}/bm_n}$)

where α_H is estimated from the value of $K_{H\beta}$ for $|F_\beta|/b$ = 1.0 µm/mm. Introducing the dimensionless value

$$A = (L_{\min} \cos\beta_b)/b \quad (5)$$

equation (2) is transformed as follows:

$$K_{H\beta}^* = K_{H\beta}/A = p_{\max}/(P_{nt}/b) \quad (6)$$

$K_{H\beta}^*$ for $|F_\beta|/b$ = 1.0 µm/mm is shown in Fig. 6. The relation between $\alpha_H = [K_{H\beta}^*]_{|F_\beta|/b\,=\,1.0}\sqrt{P_{nt}/bm_n}$ and ϵ_β is shown in Fig. 7 and the following expression can be derived for $\epsilon_\beta \geqq 1.0$:

$$[K_{H\beta}^*]_{|F_\beta|/b\,=\,1.0} = (3.26\epsilon_\beta + 8.77)/\sqrt{P_{nt}/bm_n} \quad (7)$$

From equations (4) to (7), the approximate expression of $K_{H\beta}$ for the pair of gears of β = 20 deg is obtained. In the same way, similar expressions for gears of β = 10 deg and 30 deg are obtained. These are arranged and the empirical formula is finally determined as follows:

$$K_{H\beta} = 1.00 + \left(\frac{\phi_H \epsilon_\beta + 8.77}{\sqrt{P_{nt}/bm_n}} A - 1.00\right)(|F_\beta|/b)^{1.2} \quad (8)$$

$$\phi_H = 160\,\beta^{-1.3}$$

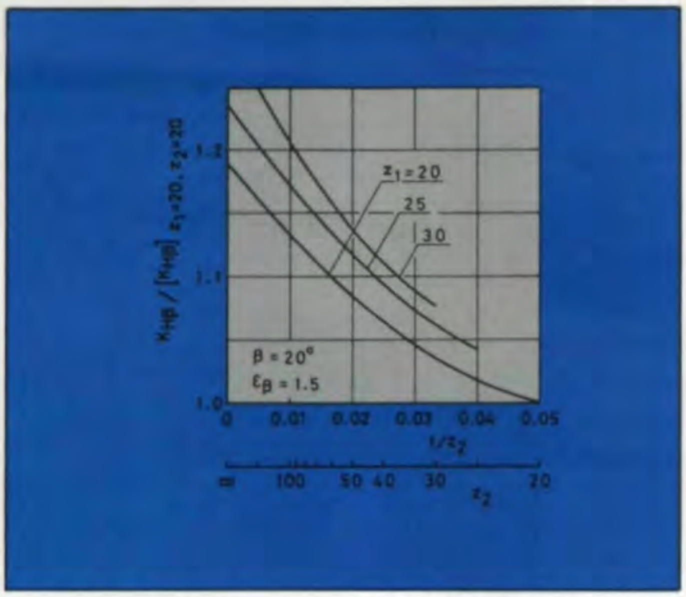

Fig. 8—Effect of gear ratio on the load distribution factor

The formula is valid for gears of $z_1 = z_2 = 20$, 10 deg $\leqq \beta \leqq$ 30 deg, $1.0 \leqq \epsilon_\beta \leqq 2.5$; $80 \leqq P_{nt}/bm_n \leqq 160$ N/mm² with the restriction that the value in the first parentheses of expression[8] is positive. The maximum error is about 5 percent except for the gears of narrow face width.

An example of the effect of gear ratio on $K_{H\beta}$ is shown in Fig. 8. It is obtained for the gears with the total alignment error of $|F_\beta|/b = 0.5$ and 1.0 μ/mm. The transmitted load is P_{nt}/bm_n = 80 to 160 N/mm.² The effect shown in the figure is rather significant. It is the reason that the reference load of gears with larger number of teeth is light since the L_{min} is proportional to the transverse contact ratio ϵ_α. The maximum load intensity p_{max}, however, is not strongly influenced by gear ratios. For example, p_{max} of gears $z_1 = 20$ and $z_2 = 100$ is only about 5 percent greater than that of gears $z_1 = z_2 = 20$.

The effect of shaft stiffness for straddle- and overhung-mounted gears on the load distribution factor has already been reported.[8] The load distribution can be estimated from the resultant error which is the sum of the initial alignment error and the additional alignment error due to shaft deflection. The formula,[8] therefore, is valid for straddle- and overhung-mounted gears by substituting the resultant error into F_β.

The comparison between $K_{H\beta}$ of the present method and the load distribution factor C_m in AGMA 218.01 is shown in Table 1. The value of AGMA 218.01 (the stiffness G = 1.4 x 10⁴ MPa is used) are close to the calculated results, especially in the case of β = 20 deg.

Comments on the Transverse Load Distribution Factor in AGMA 218.01

In AGMA 218.01, the load distribution factor C_m is defined by the product of the transverse load distribution factor C_{mt} and the face load distribution factor C_{mf}.

$$C_m = C_{mt}C_{mf} \tag{9}$$

Table 1 Comparison of the load distribution factor

(1) Empirical formula (8) (z_1 = 20, z_2 = 20)

β	P_{nt}/bm_n \ $\vert F_\beta \vert/b$	ϵ_β = 1.0: 0.5	1.0: 1.0	1.5: 0.5	1.5: 1.0	2.0: 0.5	2.0: 1.0	2.5: 0.5	2.5: 1.0
β = 10°	80	1.81	2.87	1.95	3.18	2.41	4.24	2.57	4.61
	120	1.58	2.34	1.70	2.60	2.07	3.45	2.20	3.76
	160	1.45	2.03	1.54	2.25	1.87	2.99	1.98	3.26
15°	80	1.54	2.25	1.59	2.35	1.89	3.04	1.96	3.21
	120	1.36	1.84	1.40	1.92	1.64	2.48	1.70	2.62
	160	1.26	1.59	1.29	1.66	1.50	2.15	1.55	2.27
20°	80	1.41	1.93	1.42	1.97	1.63	2.46	1.68	2.55
	120	1.25	1.58	1.27	1.61	1.44	2.01	1.47	2.09
	160	1.16	1.37	1.17	1.40	1.32	1.74	1.35	1.81
25°	80	1.31	1.72	1.32	1.74	1.48	2.10	1.50	2.16
	120	1.18	1.41	1.18	1.42	1.31	1.71	1.33	1.76
	160	1.09	1.22	1.10	1.23	1.21	1.48	1.23	1.53
30°	80	1.24	1.55	1.24	1.56	1.36	1.83	1.38	1.88
	120	1.12	1.27	1.12	1.27	1.21	1.49	1.23	1.53
	160	1.04	1.10	1.04	1.10	1.13	1.29	1.14	1.33

(2) AGMA 218.01 (G = 1.4 x 10⁴ MPa)

β	P_{nt}/bm_n \ $\vert F_\beta \vert/b$	ϵ_β = 1.0: 0.5	1.0: 1.0	1.5: 0.5	1.5: 1.0	2.0: 0.5	2.0: 1.0	2.5: 0.5	2.5: 1.0
β = 10°	80	1.64	2.29	1.97	2.78	2.27	3.21	2.54	3.59
	120	1.43	1.86	1.64	2.27	1.86	2.62	2.07	2.93
	160	1.32	1.64	1.48	1.97	1.64	2.27	1.81	2.54
15°	80	1.42	1.85	1.64	2.25	1.85	2.60	2.06	2.91
	120	1.28	1.56	1.42	1.85	1.56	2.12	1.71	2.38
	160	1.21	1.42	1.32	1.64	1.42	1.85	1.53	2.06
20°	80	1.31	1.62	1.47	1.93	1.62	2.23	1.78	2.49
	120	1.21	1.41	1.31	1.62	1.41	1.83	1.52	2.03
	160	1.16	1.31	1.23	1.47	1.31	1.62	1.39	1.78
25°	80	1.24	1.48	1.36	1.72	1.48	1.96	1.60	2.20
	120	1.16	1.32	1.24	1.48	1.32	1.64	1.40	1.80
	160	1.12	1.24	1.18	1.36	1.24	1.48	1.30	1.60
30°	80	1.19	1.39	1.29	1.58	1.39	1.77	1.48	1.97
	120	1.13	1.26	1.19	1.39	1.26	1.52	1.32	1.64
	160	1.10	1.19	1.15	1.29	1.19	1.39	1.24	1.48

C_{mf} is defined as the ratio of the peak load intensity to the average load. C_{mt} is related to the load sharing, but the definition is not given. The value of unity is used because standardized procedures to evaluate the influence of C_{mt} have not been established.

The contact stress number s_c can be represented as follows:

$$s_c \propto \sqrt{\frac{W_t}{F}\,\frac{C_m}{C_v}\,\frac{m_N}{C_\psi^2}} \tag{10}$$

In the case of m_F (= ϵ_β) > 1.0, C_ψ = 1.0 and $m_N = F/L_{min}$. Substituting these values into equation,[10] the following expression is obtained:

$$s_c \propto \sqrt{\frac{W_t}{L_{min}}\,\frac{C_m}{C_v}} \tag{11}$$

In order to estimate s_c on the basis of the maximum tangential load w_{max}, C_m should equal to w_{max} / (W_t/L_{min}) and it coincides with the definition of $K_{H\beta}$ in this paper.

If C_{mf} is assumed here to be defined as the ratio of the peak load to the mean load on the contact line where the peak load exists

$$C_{mf} = w_{max}/(\psi W_t/l) = p_{max}/(\psi P_n/l), \tag{12}$$

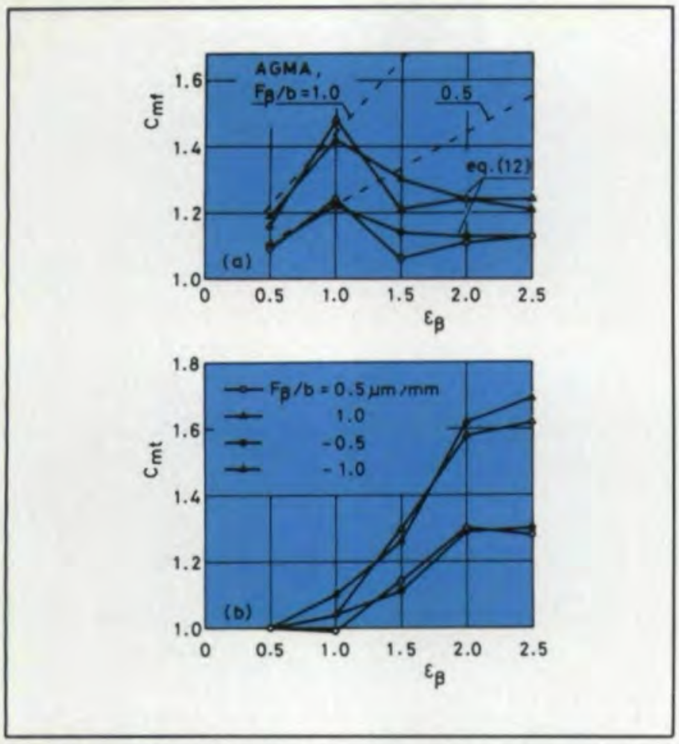

Fig. 9 – Estimated face load distribution factor C_{mf} (a) and transverse load distribution factor C_{mt} (b) of gears β = 20 deg, P_{nt}/bm_n = 120 N/mm[2]

C_{mt} is represented as follows:

$$C_{mt} = C_m/C_{mf} = (w_{max}/(W_t/L_{min})/C_{mf} = \psi/(l/L_{min}) \quad (13)$$

where ψ and l denote the load sharing factor and the length of contact line on the tooth where w_{max} exists. This idea, apart from the propriety of equation,[12] would be consistent with the definition of C_{mt} which is related to the load sharing. Following these definitions, C_{mf} and C_{mt} are estimated from the results of calculation and they are shown in Fig. 9. In the case of $\epsilon_\beta \leqq 1.0$, estimated C_{mf} is approximately equal to the values of AGMA 218.01 and estimated C_{mt} is close to unity. C_{mt} in equation[13] is, however, exactly equal to unity only when the load distribution is uniform or the gears are in single-tooth meshing. Consequently, in the case of larger ϵ_β, estimated C_{mt} is greater than unity as shown in the figure and C_{mf} in equation[12] is too small in comparison with C_{mf} in AGMA 218.01 because of larger C_{mt}. The foregoing discussion, therefore, leads to the following conclusion: the supposed transverse load distribution factor is not unity owing to the definition, and C_{mt} should be taken as unity if the formula of C_m in AGMA 218.01 is used to estimate the maximum load intensity.

Bending Moment Distribution Factor

In AGMA's formula, the load distribution factor for bending stress K_m is equal to the load distribution factor for surface durability C_m. In ISO's formula,[10] on the contrary, the load distribution factor for bending stress $K_{F\beta}$ is reduced by the expression $K_{F\beta} = K_{H\beta}{}^N$. The authors have reported the bending moment distribution factor $K_{M\beta}$ for spur gears[7] and it was less than ISO's $K_{F\beta}$. In the case of helical gears, the meshing position where the maximum bending moment arises is generally different from the worst position of load intensity as illustrated in Figs. 3 and 4. It shows that the relation between $K_{H\beta}$ and $K_{M\beta}$ has less physical meanings as compared with the case of spur gears.

The following definition of $K_{M\beta}$ for the bending moment distribution is adopted in this paper:

$$K_{M\beta} = M_{max}/M_{ref} \quad (14)$$

M_{ref} is the reference bending moment due to the uniform load P_n/L_{min} which is imaginarily distributed along the tip

$$M_{ref} = (P_n/L_{min})l_p \quad (15)$$

where l_P is the length of moment arm and it is presented using tooth height h, chordal thickness at the tip s_{tip}, and the normal load angle at the tip μ_n,

$$l_P = h\cos\mu_n - (s_{tip}/2)\sin\mu_n \quad (16)$$

calculated $K_{M\beta}$ of the pair of gears: $z_1 = z_2 = 20$, $\beta = 20$ deg is shown in Fig. 10. The following expression can be obtained from the result

$$K_{M\beta} = [K_{M\beta}]_{|F\beta|/b=0} + ([K_{M\beta}]_{|F\beta|/b=1.0} - [K_{M\beta}]_{|F\beta/b=0}) (|F_\beta|/b) \quad (17)$$

Using A in equation,[5] $K_{M\beta}$ is transformed as follows:

$$K_{M\beta}{}^* = K_{M\beta}/A = M_{max}/((P_{nt}/b)\ l_p) \quad (18)$$

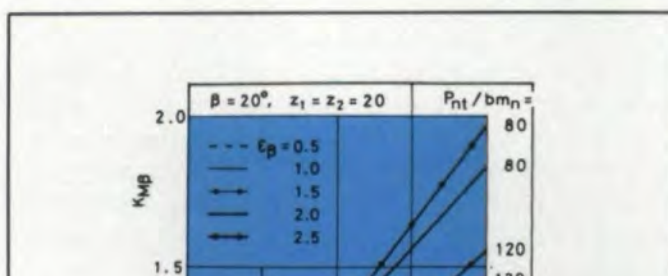

Fig. 10 – Bending moment distribution factor $K_{M\beta}$

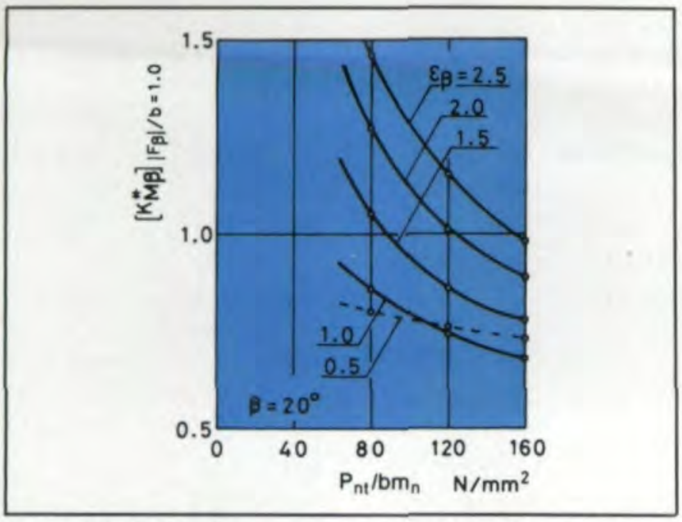

Fig. 11 – Modified bending moment distribution factor $K_{M\beta}$ of gears $z_1 = z_2 = 20$ with the effective alignment error $|F_\beta|/b = 1.0$ μm/mm

$K_{M\beta}{}^*$ for $|F_\beta|/b = 0$ μm/mm in approximately equal to 0.5 and the value for $|F_\beta|/b = 1.0$ μm/mm is illustrated in Fig. 11. The relation between $a_M = [K_{M\beta}{}^*]_{|F_\beta|/b=1.0}\sqrt{P_{nt}/bm_n}$ and ϵ_β is shown in Fig. 12 and the following expression can be derived for $\epsilon_\beta \geqq 1.0$:

$$[K_{M\beta}{}^*]_{|F_\beta|/b=1.0} = (3.09\epsilon_\beta + 5.05)/\sqrt{P_{nt}/bm_n} \qquad (19)$$

From equations[17] to,[19] the approximate formula of $K_{M\beta}$ for the pair of gears of $\beta = 20$ deg is obtained. In the same way, similar formulas for the gears of $\beta = 10$ deg and 30 deg are obtained and the following formula is finally determined:

$$K_{M\beta} = A\left\{0.5 + \left(\frac{\phi_M\epsilon_\beta + 5.05}{\sqrt{P_{nt}/bm_n}} - 0.5\right)(|F_\beta|/b)\right\} \qquad (20)$$

$$\phi_M = 83.2\,\beta^{-1.1}$$

The formula is valid for the gears of $z_1 = z_2 = 20$, 10 deg $\leqq \beta \leqq 30$ deg, $1.0 \leqq \epsilon_\beta \leqq 2.5$; $80 \leqq P_{nt}/bm_n \leqq 160$ N/mm² with the restriction of $(\phi_M\epsilon_\beta + 5.05)/\sqrt{P_{nt}/bm_n} - 0.5 > 0$. The maximum error is about 6 percent except for a part of light load where the error exceeds 10 percent. Since the bending moment distribution factor is less than the load distribution factor, the effect of gear ratio shown in Fig. 8 can also be adopted in this case as the value of the safe side.

It should be noted that the factor $K_{M\beta}$ is obtained at the worst position of gears with the alignment error. As the position does not generally coincide with the worst position in the case of $F_\beta = 0$, the helical factor C_h in AGMA strength rating formula is still valid for the gears without the alignment error. The helical factor calculated by the present method has already been shown in the previous paper.[8]

Conclusions

The longitudinal load distribution on the contact lines and the bending moment distribution along the root of helical gears are calculated by FEM which is based on the plate

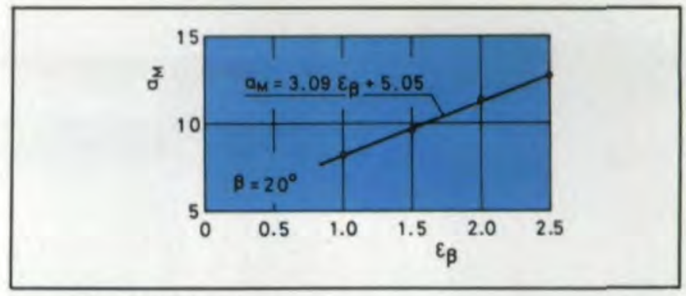

Fig. 12 – Value of a_M ($a_M = [K_{M\beta}{}^*]_{|F_\beta|/b=1.0}\sqrt{P_{nt}/bm_n}$)

theory including the transverse shear deformation.

The longitudinal load distribution factor $K_{H\beta}$ caused by the effective alignment error is obtained and an empirical formula of $K_{H\beta}$ is proposed. The load distribution factor C_m in AGMA 218.01 is close to the values calculated by the present method. A formula is also proposed for the estimation of the maximum bending moment of gears with the alignment error.

A supposed definition of the transverse load distribution factor is examined and it leads to the conclusion that the transverse load distribution factor in AGMA 218.01 should be taken as unity if the formula of load distribution factor C_m is used to estimate the maximum load intensity.

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Appendix 1

The minimum of total length of contact lines L_{min} is calculated by the following equation:[11]

(*a*) if $\mathrm{frc}(\epsilon_\alpha) + \mathrm{frc}(\epsilon_\beta) < 1$,

$$N_b = 1 - \frac{\mathrm{frc}(\epsilon_\alpha)\mathrm{frc}(\epsilon_\beta)}{\epsilon_\alpha\epsilon_\beta} \qquad (\epsilon_\beta \geqq 1)$$

$$N_b = 1 - \mathrm{frc}(\epsilon_\alpha)/\epsilon_\alpha \qquad (\epsilon_\beta < 1) \qquad \text{(A.2)}$$

(*b*) if $\mathrm{frc}(\epsilon_\alpha) + \mathrm{frc}(\epsilon_\beta) \geqq 1$,

$$N_b = 1 - \frac{[1 - \mathrm{frc}(\epsilon_\alpha)][1 - \mathrm{frc}(\epsilon_\beta)]}{\epsilon_\alpha\epsilon_\beta}$$

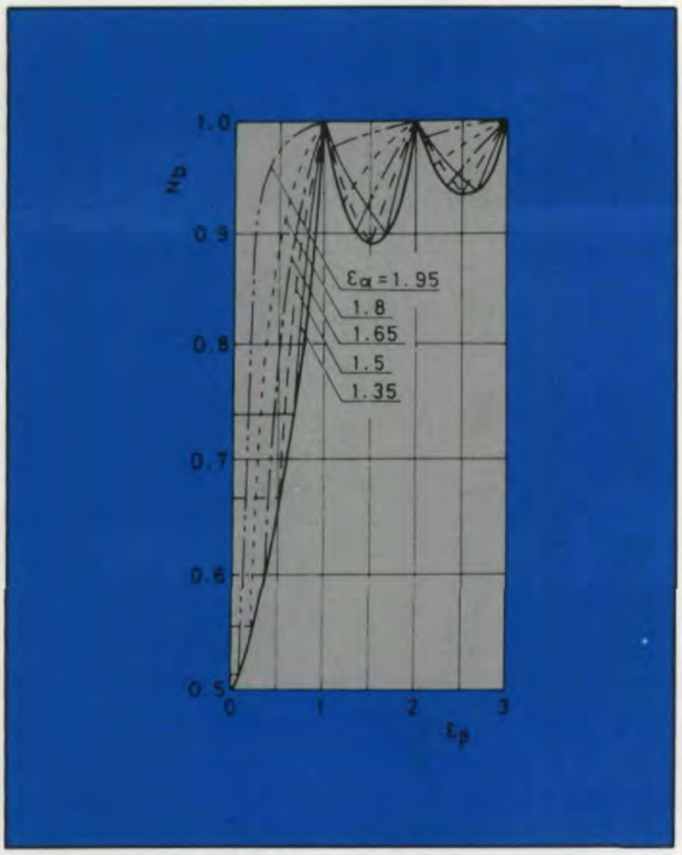

Fig. A.1 – Value of N_b

where frc (X) denotes the fraction of X. The function N_b is shown in Fig. A. 1. The dimensionless value A in equation(5) can be calculated by using N_b

$$A = \frac{L_{min} \cos\beta_h}{b} = N_b \epsilon_\alpha \tag{A.3}$$

In the case that the face contact ratio m_F ($=\epsilon_\beta$) in AGMA 218.01 is greater than unity, the load sharing ratio m_N is defined by $m_N = F / L_{min}$. Therefore, A is expressed as follows:

$$A = \frac{\cos\beta_h}{m_N} \tag{A.4}$$

Appendix 2

The matrix $[H_k]$ for gear $k(k = 1, 2)$ is defined by $w_{k,ij}$, which is the deflection at node j on the contact line due to a unit normal load applied to node i.

$$[H_k] = [\{W_{k,1}\}, \{W_{k,2}\}, \ldots, \{W_{k,i}\}, \ldots]$$

$$\{W_{k,i}\} = (w_{k,i1}, w_{k,i2}, \ldots, w_{k,ij}, \ldots)^T \tag{A.5}$$

$(\quad)^T$ = transposed matrix

The deflection $w_{k,ij}$ is calculated by FEM. When a pair of teeth are in mesh, the distributed load $\{P\}$ along the contact line is related to the sum of the deflection of the teeth and the relative approach due to elastic contact.

$$[H]\{P\} = \{w\} \tag{A.6}$$

The elements of matrices $[H]$ and $\{w\}$ are

$$H_{ij} = H_{1,ij} + H_{2,ij} + \delta_{ij} \frac{w_{P,i}}{P_i}$$
$$w_i = w_{1,i} + w_{2,i} + w_{P,i} \tag{A.7}$$

where $w_{P,i}$ is the relative approach at node i and σ_{ij} is Kronecker's delta. When some pairs of teeth I, II, . . . are in mesh matrices $[H_I]$, $[H_{II}]$, . . . are separately obtained. If the load on a pair of teeth is assumed to have little effect on the deflection of other pair of teeth, the matrix $[H]$ in equation (A.6) is diagonally constructed as follows:

$$[H] = \begin{pmatrix} [H_I] & 0 & \\ 0 & [H_{II}] & \\ & & \ldots \end{pmatrix} \tag{A.8}$$

The equation (A.6) is solved under the following conditions:

$$\Sigma P_i = P_n$$
$$w_i + \frac{s_i}{1000} = (r_{b1}\theta_1 + r_{b2}\theta_2)\cos\beta_b \qquad \text{(node in contact)} \tag{A.9}$$
$$P_i = 0 \qquad \text{(node not in contact)}$$

where s_i $[\mu m]$ is the spacing at node i caused by the effective alignment error, r_b is the radius of base cylinder and θ (rad) is the rotating angle of gear.

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MATERIAL SELECTION . . .

(continued from page 46)

and accuracy, and improved lubrication – rather than changes in material – are required to solve this problem.

Scoring

In some heavily loaded or high-speed gearing, scoring may occur under boundary film conditions. This is believed to be caused by frictional heat which reduces the lubricant protection sufficiently to allow welding and tearing of the profile.

Materials selection alone will not prevent scoring; proper lubricants and design geometry are required. This difficulty is seldom encountered in the conventional industrial gear drive. *AGMA 217.01, Oct. 1967, "AGMA Information Sheet – Gear Scoring Design Guide for Aerospace Spur and Helical Power Gears"* provides helpful recommendations for avoiding scoring.

(This article will be continued in the September/October 1985 issue of GEAR TECHNOLOGY.)

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