Influence of Relative Displacements Between Pinion and Gear on Tooth Root Stresses of Spiral Bevel Gears

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Introduction

The manufacturing quality of spiral bevel gears has achieved a very high standard. Nevertheless, the understanding of the real stress conditions and the influences of certain parameters is not satisfactory. This refers to the influence of geometric data (like flank curvature, crowning, spiral angle, etc.) or mounting conditions on stress distribution and load capacity. The design of bevel gears in general is based on experience. The investigation of tooth root stresses at spiral bevel gears, described in this paper, was initiated under these aspects. This research program started in 1981 at the FZG, Munich (Forschungsstelle für Zahnräder und Getriebebau – Gear Research Laboratory). It is financed by the DFG (Deutsche Forschungs-Gemeinschaft – German Research Association). The research method follows a line of similar basic studies on tooth root stresses for spur [1] and helical[2] gears and is also mainly based on experimental investigations.

The main objective was to gain reliable information on the amount, direction, and distribution of root stresses in spiral bevel gears by direct measurements. Especially the influence of size and location of the contact pattern on the working conditions of a bevel gear set should be investigated. In this connection, the influence of the amount of crowning of the pinion teeth should be examined. The results of this investigation were intended to provide data for verification of similar theoretical calculations[3] and, therefore, to present a contribution to a more accurate and realistic calculation of the load capacity of bevel gears.

The measuring method is explained and some general results on the distribution of tooth root stresses are discussed. In particular, the influence of various contact patterns achieved by defined relative displacements between pinion and gear are described.

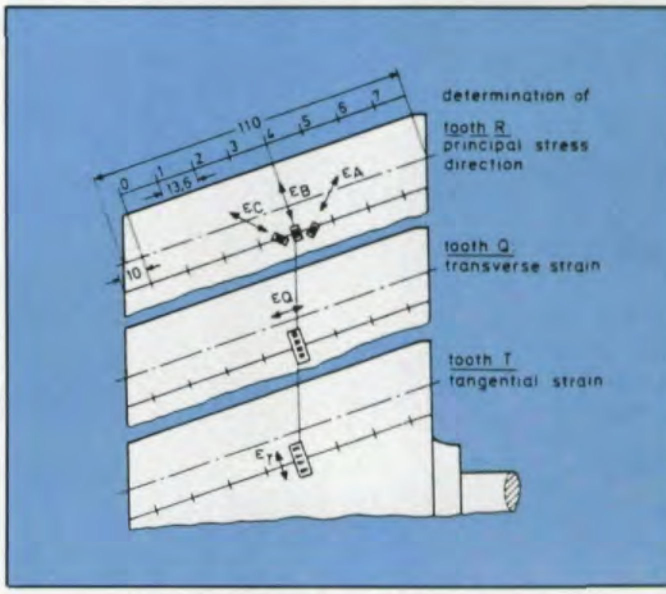

Fig. 1 – Type and position of the strain gages.

Measuring Method

In the fillet surface of a loaded tooth, there is a two-dimensional stress status. Therefore, the principal (maximum) stresses can be evaluated by following the rules of Mohr's circle:

$$\epsilon_{1,2} = \frac{\epsilon_T - \epsilon_Q}{2} \pm \frac{\epsilon_T - \epsilon_Q}{2\cos(2\psi)} \quad (1)$$

$$\sigma_{12} = E(\epsilon_{12} + \nu\epsilon_{21})/(1 - \nu^2) \quad (2)$$

For an experimental determination of the principal stresses $\delta_1$and δ_2 at any point in the root fillet, three measurements (ψ, ϵ_T, ϵ_Q) must be determined. ψ specifies the principal stress direction. ϵ_T and ϵ_Q are strains oriented in perpendicular directions.

Corresponding to these relations different types of strain gages had to be used to determine the strain elements. The arrangement of the gages is shown in Fig. 1. On tooth *R*, strain gage rosettes are applied to evaluate the principal stress direction (angle ψ). These rosettes are approximately positioned at the height of the 30 deg-tangent-point (see Fig. 4). The teeth *Q* and *T* are fitted by 10-element strip gages having a grid length of 0.51 mm (0.020 in.). On tooth *Q* , the

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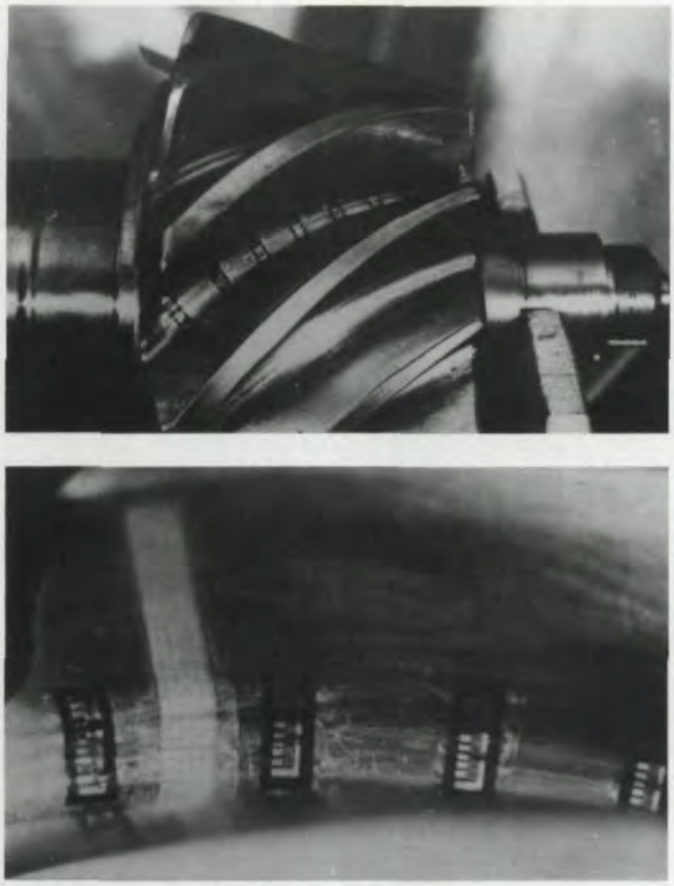

Fig. 2 – Pinion tooth with 10-element strip gages (before they were wired).

measuring direction of the grids is oriented lengthwise, and on tooth T in the direction of the tooth height. These strip gages cover nearly the whole root curvature, so that the stress distribution across the root fillet can be determined.

To determine the stress distribution across the facewidth, eight of these gages are fitted on each measuring tooth. Fig. 2 shows the strain strips on one pinion tooth before they were wired. Wiring such a large number of strain gages was a severe problem since the normal meshing of the mating teeth could not be disturbed. Only the small clearance space provided at the bottom was available. Considering this fact, a test gear set having a large module (normal module in midfacewidth = 12 = 2.12 DP) had to be chosen. For essential geometric data, see Fig. 3. This test gear set represents a typical gear design. The gears were manufactured by Klingelnberg und Söhne, Germany.

Fig. 4 gives a summary of the relationship between the different measuring data in detail. Determination of the principal stress direction ψ by using three rosette strains is explained in Fig. 4(b). Fig. 4(a) shows the relationship between ψ and the strip gage strains ϵ_T and ϵ_Q.

It has to be considered that the principal directions estimated by the rosettes are theoretically valid for only one level of the root depth. The transverse and tangential strains were measured across the whole area of the root fillet; so the combination of data taken from different tooth height levels might not be correct. To estimate the eventual error,

nomenclature	pinion		gear
number of teeth	11		36
normal module in midfacewidth		12	
outer transverse module		17.50	
outer pitch diameter /mm/	192.50		630.00
mean spiral angle		34.597	
cutter radius /mm/		*210.00*	
excentricity (pinion) /mm/		2.5	
total contact ratio (theoretical)		2.8	
addendum modification	0.34		-0.34
thickness modification	0.025		-0.025
material		17 CrNiMo 6 (through hardened)	

Fig. 3 – Main data of the test gear set.

on one tooth the rosettes were bonded on different levels, and it was found that the difference of the principal directions in the area covered by one strip gage is largely negligible. Therefore, this approximation seems to be tolerable.

The combination of measuring data received from three different teeth requires that loading conditions are equal on these teeth. Fig. 5 shows how the measuring teeth of the gear were selected considering the adjacent pitch error. These teeth which were selected, showed a minimum pitch fluctuation with their neighboring teeth. So for these teeth, one can assume that the conditions referred to the load distribution on several pairs, in contact at the same time, might be largely the same.

Loading Device

The loading of the large test gears required a rigid test rig. This is shown in Fig. 6. To apply torque, the gear is loaded against the blocked pinion by using a hydraulic system.

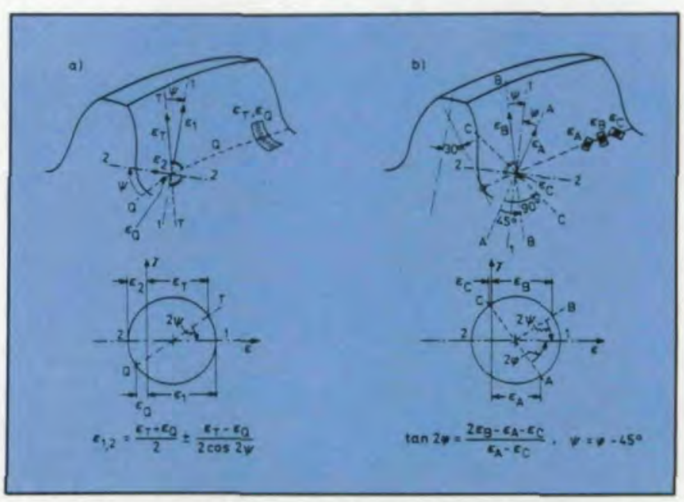

Fig. 4 – Determination of the prinicpal strains from the measuring data.

Nomenclature

f_a = pinion offset deviation, mm
$f_{v1,2}$ = axial displacement of pinion/wheel, mm
f_Σ = shaft angle deviation, degree
E = modulus of elasticity, N/mm² (E = 210 000 N/mm²)
T_1 = pinion torque, Nm
$\epsilon_{1,2}$ = principal strains, μm/m
$\epsilon_{A,B,C}$ = rosette strains, μm/m
ϵ_Q = transverse strains, μm/m
ϵ_T = tangential strains, μm/m
ν = Poisson's ratio (ν = 0.3)
$\sigma_{1,2}$ = principal stresses, N/mm²
σ_T = stress in depth wise direction, N/mm²
φ = angle between ϵ_A and ϵ_1, deg
ψ = angle between ϵ_B and ϵ_1 deg

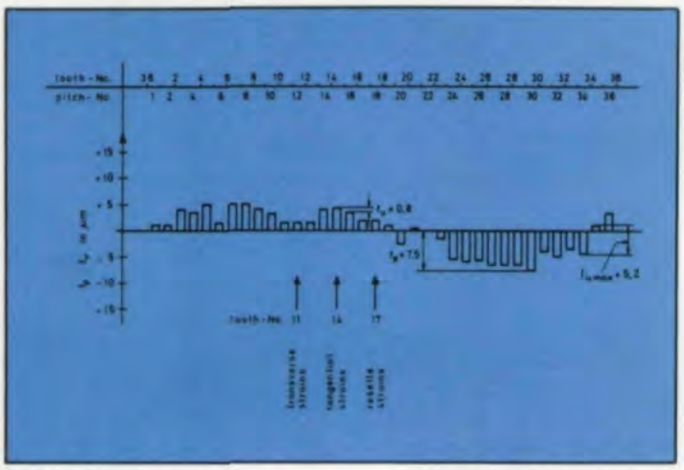

Fig. 5 – Determination of the measurement gear teeth suitable for considering the adjacent pitch errors.

Measurements along the whole path of contact were made stepwise while holding the pinion in different rotary positions. That meant that the load was applied statically on discrete lines of contact to which the corresponding stress distribution is related. Therefore, all results discussed in this paper refer to the static loaded condition.

The test rig provides for accurate adjustments of defined positions and displacements of the pinion and gear. By an additional mechanism, the deflections under load can be set back.

General Results on the Distribution of Tooth Root Stresses

Before discussing the influence of displacements on tooth root stresses, some general results should be mentioned. Fig. 7 shows the distribution of the tensile strains across the root fillets of pinion and gear, plotted in the normal section. The stress curves are nearly the same as those of cylindrical gears. Looking at the gear, it becomes obvious that the maximum tension appears almost exactly at the 30-deg. tangent point.

Fig. 6 – Loading device for determination of tooth root stresses at bevel gears.

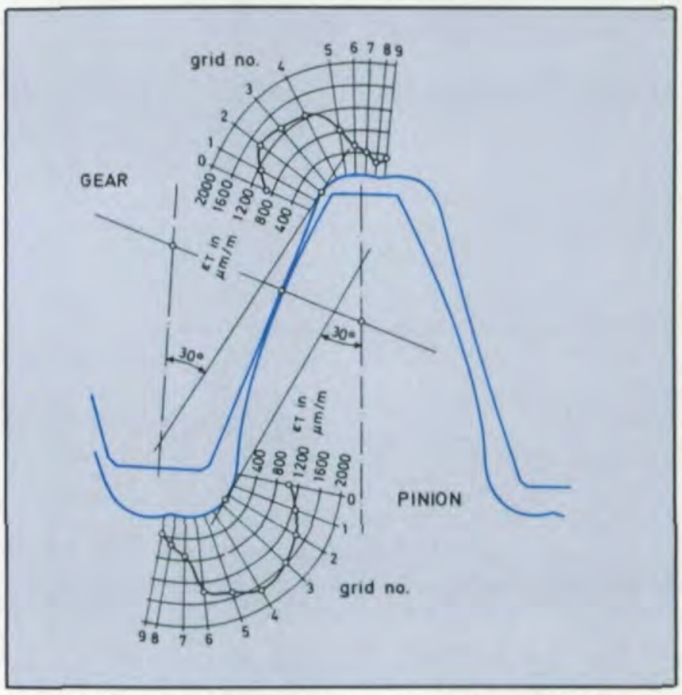

Fig. 7 – Distribution of the tangential strains over the root fillet: (measuring point 4; mesh position 4, compare Fig. 9; torque T_1 = 8000 Nm).

On the pinion, the point of maximum stress is somewhat closer to the root circle. These results were well confirmed by finite element analysis applied to the actual tooth profiles. Moreover, it was found that the point of maximum strain is nearly independent of the bending arm and largely constant across the face width.

Therefore, in the following evaluation, the strain and stress distributions are plotted for the level in the root fillet where the maximum stresses occur.

In Fig. 8, the distributions of the tangential and transverse strains are plotted for different mesh positions, i.e. for different lines of contact. It can be seen that the maximum tangential strains occur approximately at the center of the line of contact. At that point, the transverse strains have their (negative) minimum. That means that the depthwise tension is accompanied by a lengthwise contraction.

It has to be pointed out that the scales used in Fig. 8 for the transverse and tangential strains differ in about one magnitude. The transverse strains are relatively small and are equal to about 5 percent of the maximum tangential strains. This is an effect of the relatively short line of contact compared to the total face width; so the unloaded ends of the teeth largely obstruct the transverse strains.

The principal stresses received by a combined evaluation of the tangential, transverse, and rosette strains are plotted for the pinion in Fig. 9. The arrows qualitatively show the

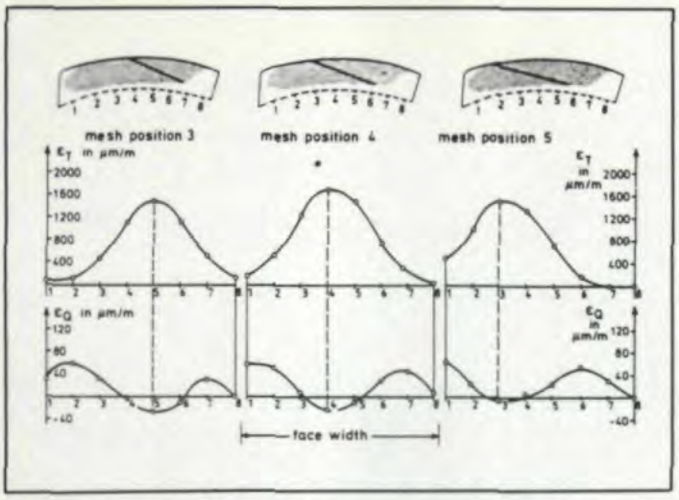

Fig. 8 – Distribution of the tangential and transverse strains over the facewidth at the pinion (torque $T_1 = 8000$ Nm).

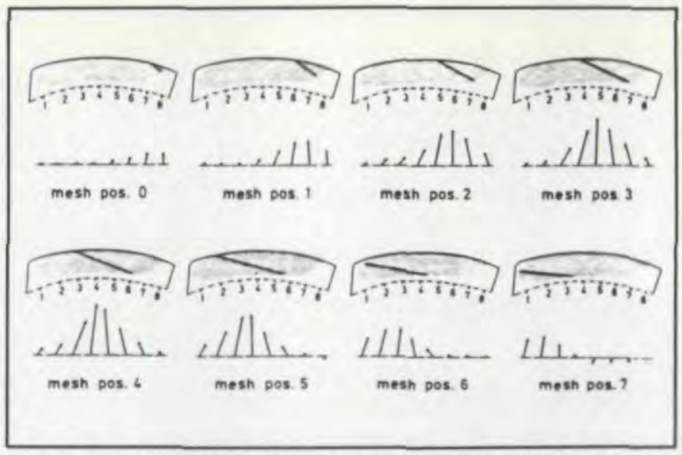

Fig. 9 – Value (qualitatively) and orientation of the absolute greater principal stresses.

magnitude and orientation of the (absolute) greater principal stresses. It is obvious that in each mesh position the stresses are directed toward one common point. This point is located approximately at the center of the corresponding line of contact. That means that the maximum stress in each position is oriented approximately in the depthwise direction. Therefore, the real tension status can be closely approximated by straight bending, if only the maximum stresses are of interest. The calculation methods, which are based on such a straight bending, seem to represent a good approximation. The same results were found for the gear.

In Fig. 10, the stress distribution over the facewidth and for the whole contact cycle is plotted for the pinion and the mating gear. These measurements were made with the original rating position of the contact pattern. The pinion torque was 8000 Nm, which corresponds approximately to the fatigue limit of an equivalent case hardened gear set.

Looking first at the pinion it can be seen that the contact begins at the toe with light load (mesh position 0; corresponding lines of contact see Fig. 9). After that, the stresses gradually increase while the stress concentration shifts toward the heel. The maximum stress appears in mesh position 4, nearly in the middle of the face width (see the hatched area). At this position, a nearly symmetric stress distribution over the facewidth is observed. Subsequently, the stresses decrease until the heel goes out of contact.

The corresponding plot for the gear looks quite different. We observe an extensive area of compressive stresses at the beginning of the contact (see screened field). These stresses are caused by the loading of the neighboring tooth. They amount to about 35 percent of the maximum tensile stress, i.e. in this region there is an alternating loading in the root of gear. On the pinion, only a small area of compressive stresses is found amounting to about 15 percent of the maximum tensile stress.

Corresponding to the position of the line of contact, the tensile stresses at the gear first appear at the toe and then extend gradually over the facewidth. The maximum stresses appear in mesh positions 4 and 5. Both have about the same amount and they are located at midfacewidth, respectively, shifted somewhat toward the heel.

In general, the plotted stress curve for the gear is flatter than for the pinion. In the test gear set, the maximum stress at the gear was about 25 percent lower.

Load Distribution With Multiple Teeth Contact

The total contact ratio of spiral bevel gears normally exceeds a value of 2. That means that there are always two, sometimes three, teeth in mesh simultaneously. The tangential force is distributed on two or three teeth.

The influence of load distribution on the tooth root stresses was determined by measurements with single and multiple tooth contact.

Fig. 11 shows the stress curve for single (broken line) and multiple tooth contact (full line) plotted across the face width. In mesh positions 2 and 6, there are two pairs of teeth in contact and the load is distributed almost equally. If one tooth is removed (by milling) the stresses at the root of the remaining tooth increases to about twice the value with double contact. With mesh position 4, where three pairs of teeth are simultaneously in contact, the root stresses in the case of single contact are about the same as with multiple contact.

The teeth before and behind, having short lines of contact only, carried a very small portion of the total load.* As the maximum stress occurs in this position, it can be stated that a high theoretical contact ratio (in case of the test gears ≈ 2.8) does not essentially reduce the maximum tooth root stresses.

In Fig. 12, the percentages of load carried by one tooth in the course of the contact is plotted. The load is smoothly taken over from the tooth before (*V*) and given further to the tooth behind, and there is a short period of time when tooth (*M*) has to carry nearly the total load (for this example about 98 percent).

*The complete distribution of root stresses in the loaded area is much more complicated than can be described by Fig. 11. So tensile and compressive tooth root stresses overlap each other when the teeth before and behind carry a portion of the total load. This has to be taken into account with regard to the load capacity. Moreover, it has to be considered that the length of the line of contact is increased if the total load is applied to one pair of teeth only. The results in Fig. 12 include this more detailed consideration.

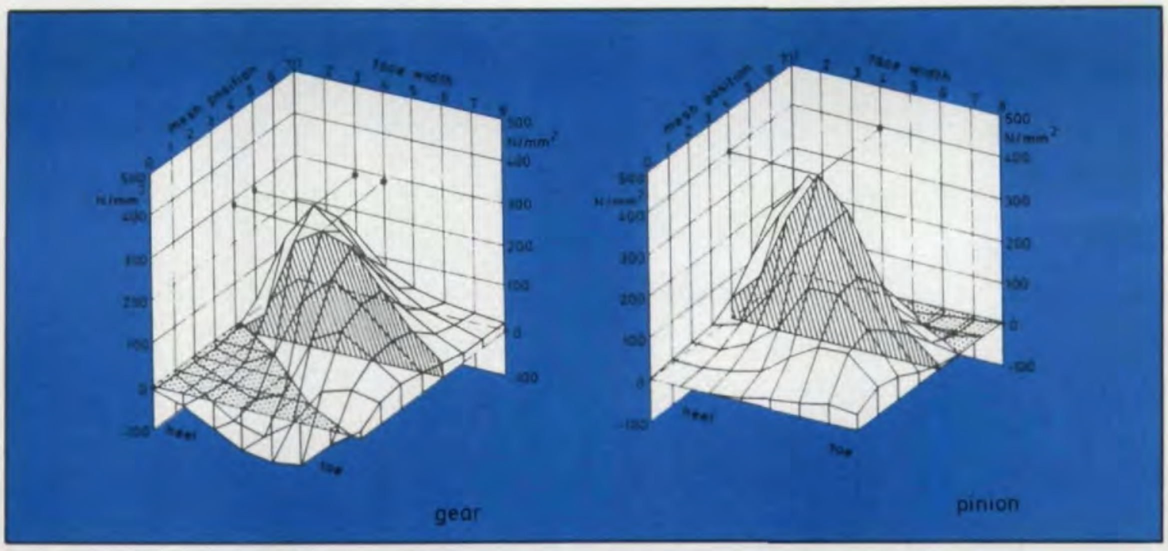

Fig. 10 – Stress distribution over facewidth and time of contact (torque $T_1 = 8000$ Nm).

Influence of Pinion and Gear Displacements on the Tooth Root Stress

Fig. 13 shows the definitions for the various relative displacements between pinion and wheel. All these displacements can be exactly adjusted in the test rig. As an example, the influence of an axial displacement of the pinion (f_{vl}) on the tooth root stresses shall be discussed. The results were achieved with a gear set having a usual length and position of the contact pattern.

Fig. 14 shows the contact patterns occurring when the mounting distance of the pinion was varied. It is easy to recognize that the light load contact pattern shifts appreciably axially and also in the depthwise direction.

If the discrete lines of contact arising from loading the teeth in the different mesh positions are combined, the contact pattern for the loaded flank is obtained. This pattern tends to move in the same way as with a light load, but the amount of the displacement is smaller. Even if the extreme positions ($f_{vl} = \pm$ 0, 8mm) are considered, the contact patterns under load nearly cover the total facewidth. On the other hand, quite a different load pattern in the depthwise direction is obtained.

Fig. 14 indicates that the stress at the end of the teeth is strongly influenced by these displacements. Fig. 15 demonstrates this for the pinion in mesh position 6. At that moment, the corresponding line of contact is positioned toward the heel. The curves indicate that there is a large increase in stress for the position of $f_{vl} = -$ 0, 8, i.e. when the contact pattern is shifted toward the heel. Compared to the stress curve for the nominal contact pattern, (full line in Fig. 15) the increase is in the order of 60 percent.

When evaluating this effect, one has to consider that the maximum stress over the whole time of contact does not appear at the moment shown in Fig. 15 (see Fig. 10).

To give a more general view in Fig. 16, the stresses were compared for those mesh positions where the maximum stresses occur. These maximum stress positions always are observed in mesh positions 4 or 5; i.e. when the lines of contact are positioned approximately in midfacewidth. As shown in Fig. 11, the tooth has to carry nearly the total load at that moment. Hence, it follows that in spite of the displacements, and although the centers of the contact patterns are spread over a wide range, the point of stress concentration is hardly moved from midfacewidth. On the pinion, this area of

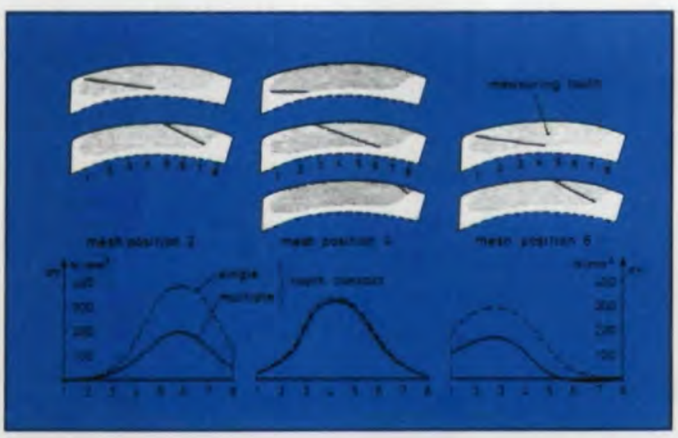

Fig. 11 – Stress distribution across the facewidth at the pinion root in case of single and multiple tooth contact (torque $T_1 = 4000$ Nm).

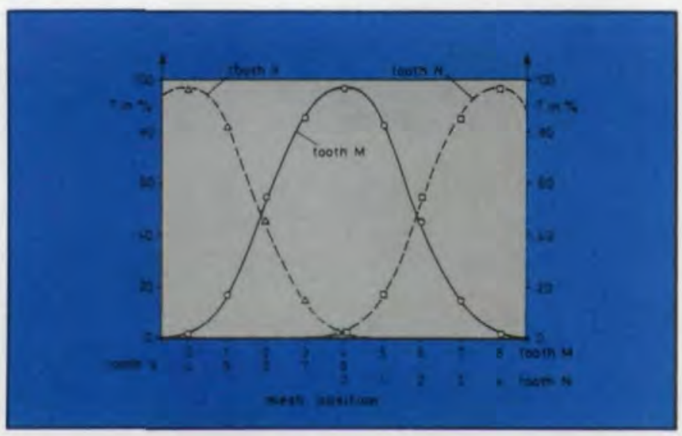

Fig. 12 – Load distribution onto meshing teeth in multiple teeth contact (torque $T_1 = 4000$ Nm).

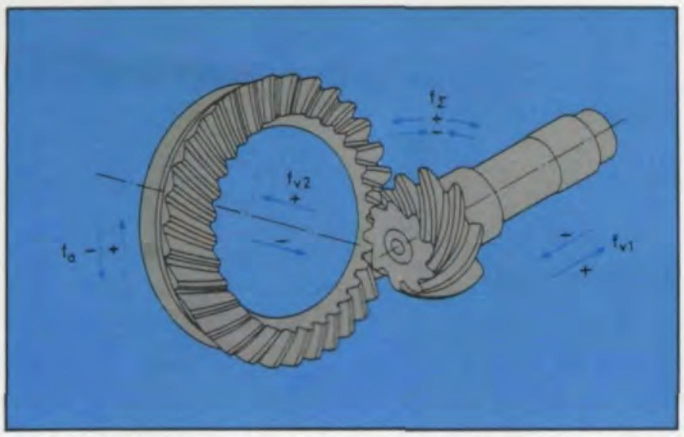

Fig. 13 – Definitions of relative displacements between pinion and wheel

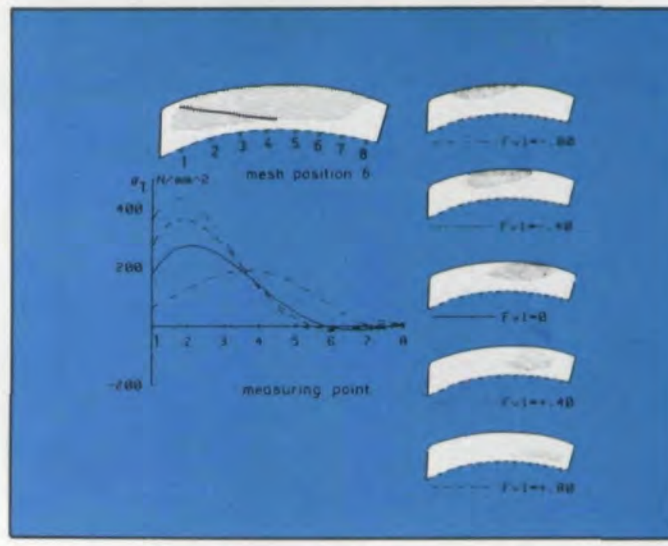

Fig. 15 – Stress distribution due to axial displacement of the pinion.

maximum stress is quite narrow. This observation corresponds with experience from a large number of damage cases at bevel or hypoid gears in service. Nearly always the cracks started at the pinion teeth in the middle of the face width.

Due to the axial displacement of the pinion (f_{vl}), the maximum stresses vary in a range of about 16 percent for the gear and 20 percent for the pinion.

Fig. 17 covers a summary of the influences of the other displacements, defined in Fig. 13. For this comparison, the stress distribution curves were evaluated in the same way as shown in Fig. 15. From such curves, the maximum stresses were taken. It becomes obvious that axial displacements of the pinion (f_{vl}) and misalignments (offset) of the axes (f_a) have the strongest influence. Deviations of the shaft angle (f_{Σ}) and of the mounting distance of the wheel (f_{v2}) have a minor influence.

Further, one can see that the pinion stresses are always higher than the gear stresses. Of course, this effect is partly due to the design of the present test gear set, but obviously the main reason is the special load distribution over the line of contact. Previous measurements have shown that the load is more or less concentrated at the outer end of the line of contact for *all* displacements. For a left-hand spiral on the pinion, the outer end of the line of contact is located at the pinion tip and near the root of the gear (see lines of contact in Fig. 14). Therefore, very different bending arms are acting at the pinion and the gear, and the root stress differs correspondingly. It is intended to continue investigations on this interrelationship between load distribution and different relative deviations and deflections of the gears.

Moreover, it was found that a shifting of the contact pattern toward the toe (positive values of f_{vl} and f_a) results in a decrease of pinion root stress. In case of the variation of f_{vl}, this is accompanied by an increase of the gear root stresses. For the test gear set, optimum conditions were achieved for a (light load) contact pattern shifted approximately 25 percent of the facewidth toward the toe.

Nearly the same conditions, according to contact pattern and root stresses, were obtained by an offset deviation of $f_a \approx +0.5$ mm.

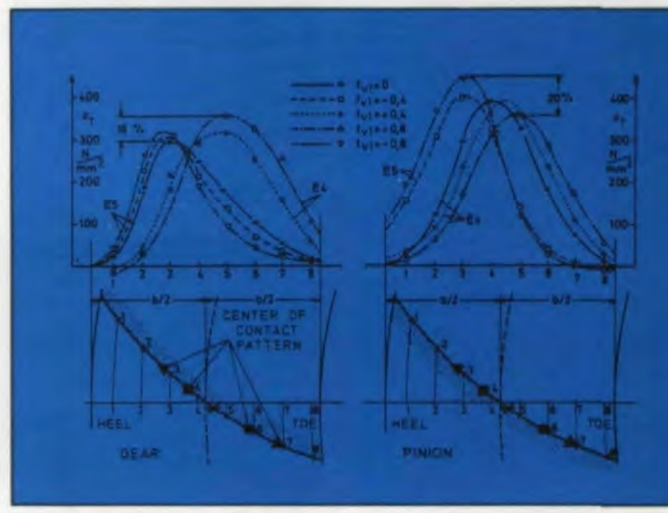

Fig. 16 – Maximum tooth root stress for various pinion mounting distances.

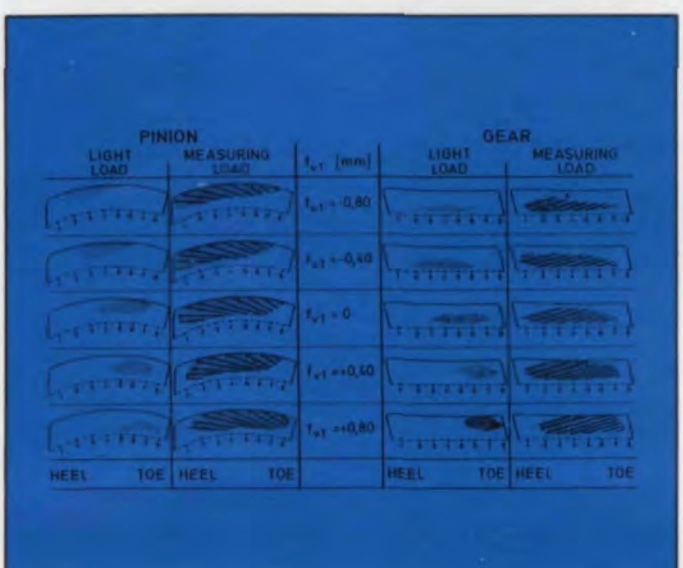

Fig. 14 – Contact patterns for different axial displacement of the pinion.

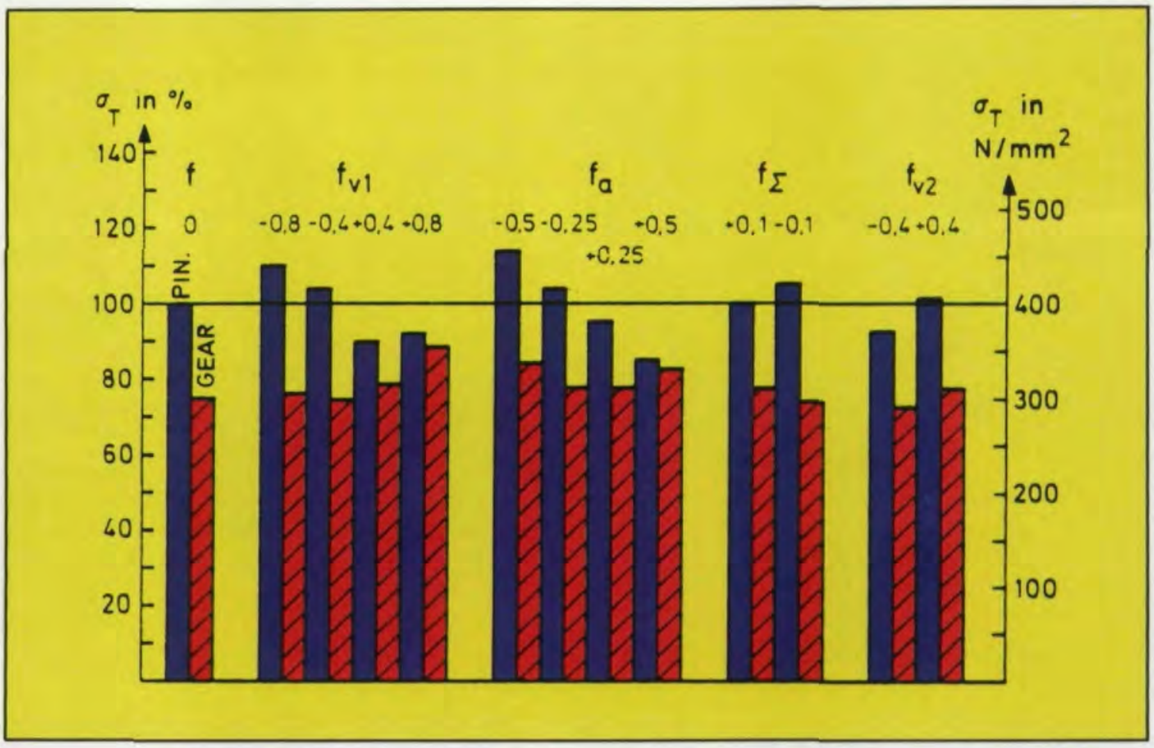

Fig. 17 – Influence of relative displacement between pinion and wheel on tooth root stresses.

Of course, the quantitative results referring to the relationship between the amount of displacement and the amount of stress increasing or decreasing are influenced by the special rigidity and mounting conditions of the test stand. This test rig is very rigid; therefore, the deflections under load are quite small. Since larger displacements, which for instance might correspond to weak aluminum housings, can be adjusted, also results valid for weak mounting conditions are achieved. For this the range of displacements adjusted for the measurements, as shown in Fig. 17, was chosen so wide that it exceeds the amount of deflection which usually might appear even in very weak configurations.

These results give a survey on the behavior of the test gear set under different working conditions. And as these results are intended to be a basis for the verification of theoretical calculations by finite element method, (e.g. the research work "Kegelradkette" of the Technical University Aachen) they can make a contribution to an improvement of the calculation of load carrying capacity of bevel gears in general.

Summary

A method for the experimental examination of tooth root stresses on spiral bevel gears has been described. The principal stresses were derived from strip strain gages and strain gage rosettes measurements.

Some basic results, referring to the stress distribution over the root fillet and along the facewidth and to the load distribution between the meshing teeth, have been discussed. It was found that the maximum stresses appear approximately at the middle of the facewidth, if the total load is carried by one pair of teeth. This result was hardly influenced by different positions of the contact patterns due to various displacements between pinion and wheel. The maximum stresses were always observed in (approximately) midfacewidth.

The maximum differences in root stresses could be achieved by varying the mounting distance of the pinion and the offset between the axes of pinion and gear. Differences up to about 20 percent in maximum stress were observed.

Common experience was confirmed on the test gear set: under light load the contact pattern should be displaced somewhat towards the toe. In this case, balanced stresses with relative low values on pinion and wheel were observed.

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This article was contributed by the Power Transmission & Gearing Committee and was presented at the Design Engineering Technical Conference of the American Society of Mechanical Engineers. Paper No. 84-Det-98.

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