Longitudinal Load Distribution Factor for Straddle- and Overhang-Mounted Spur Gears

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Abstract

Longitudinal load distribution and bending moment distributions at the root of a pair of spur gears with a known effective lead error are calculated by the finite element method based on plate theory. A convenient empirical formula for the longitudinal load distribution factor is proposed and compared with the formulas recommended by ISO and AGMA. The relation between the maximum bending moment at the root and the longitudinal load distribution factor is also presented. The effect of crowning on the longitudinal load distribution is investigated and the amount of arc-shaped crowning needed, which is frequently determined by the experience of gear designers, is determined by minimizing the longitudinal load distribution factor.

Introduction

A pair of spur gears generally has an effective lead error which is caused, not only by manufacturing and assembling errors, but also by the deformations of shafts, bearings and housings due to the transmitted load. The longitudinal load distribution on a contact line of the teeth of the gears is not uniform because of the effective lead error. Longitudinal load distribution factors $K_{H\beta}$ and $K_{F\beta}$ are used in the ISO strength rating formula[1] to account for the effects of the non-uniform distribution of the load on the contact stress and the bending stress at the root.

Hayashi[2] solved integral equations to calculate the load distribution of helical gears. Conry and Seireg[3] developed a mathematical programming technique to estimate the load distribution and optimal amount of profile modification of spur and helical gears. In these studies, the deflection of gear teeth was estimated from that of thin cantilever plates of uniform thickness. Niemann and Reister[4] proposed an experimental formula for the factor of spur gears. The authors[5,6] solved some problems of the load distribution using the finite element method (FEM). Although the papers involve many interesting results, they are not sufficient for wide application to designing of gears for strength.

This article summarizes, from a design point of view, the analysis and the results and proposes a formula for the longitudinal load distribution factor for straddle- and overhang-mounted steel spur gears. The deflection of gear teeth is calculated by FEM based on plate theory including the effect of transverse shear deformation. Gears dealt with in this article are generated by the basic rack (pressure angle 20°; top clearance 0.25m; and radius of tip corner 0.375m) recommended in JIS B 1701-1973 as well as in ISO 53-1974. Finally the determination of the optimal amount of the arc-shaped crowning and the effect of the crowning on the reduction of $K_{H\beta}$ of spur gears with effective lead error are presented.

Fundamental Equations

A pair of straddle- or overhang-mounted spur gears (Fig. 1), which are in mesh at the highest point of single tooth contact of pinion, is taken as the typical example of calculation. Each tooth is regarded as a cantilever plate of varying thickness, and it is divided into 10 (in the direction of tooth height) by 20 (in the direction of face width) rectangular elements to analyze by FEM.[7] If $b_1 \neq b_2$, tooth 2 is divided into 10 x 24 elements so that teeth 1 and 2 can come in contact at the 21 nodes.

When a unit normal load is applied to node i of gear k ($k = 1, 2$, corresponding to pinion and gear, respectively), the deflection at node j, $w_{i,j}^{(k)}$, in the direction of the line of action can be determined as the sum of the deflection of the

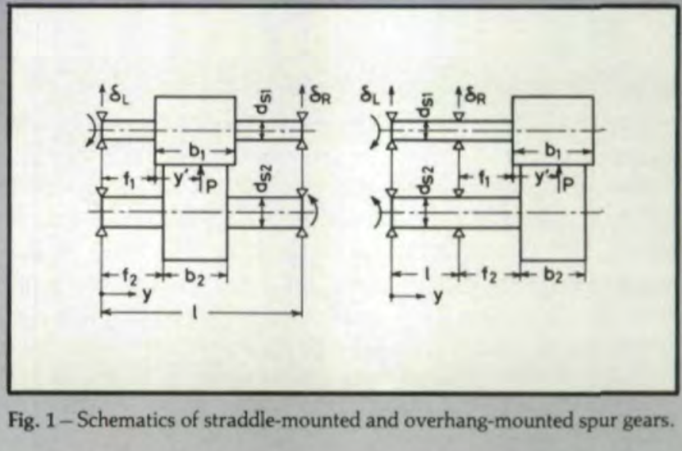

Fig. 1 – Schematics of straddle-mounted and overhang-mounted spur gears.

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tooth and the additional displacement due to the bending and torsional deflection of gear bodies and shafts obtained by Equations (A.1) to (A.4) in the Appendix. Introducing matrix $[H_{(k)}]$, which is defined by $w_{i,j}^{(k)}$, in the following form

$$[H_{(k)}] = [W_1^{(k)}, W_2^{(k)}, \ldots, W_{21}^{(k)}] \quad (1)$$

where

$$\{W_1^{(k)}\} = (w_{i,1}^{(k)}, w_{i,2}^{(k)}, \ldots, w_{i,21}^{(k)})^T$$

and $(\ldots)^T$ is a transposed matrix. Any distributed load $\{p^{(k)}\}$ along the contact line is related to the deflection $\{w^{(k)}\}$ at each node according to the equation

$$[H^{(k)}]\{p^{(k)}\} = \{w^{(k)}\}. \quad (2)$$

When these teeth are engaged, a certain distributed load $\{p\}$ arises along the contact line and necessarily $\{p^{(1)}\} = \{p^{(2)}\}$. Then the relation between the load distribution and accompanying deflection including relative approach can be represented by the following equation

$$[H]\{p\} = \{w\} \quad (3)$$

where the elements of matrices [H] and {w} are given by

$$H_{i,j} = H_{i,j}^{(1)} + H_{i,j}^{(2)} + \delta_{ij}(w_i^P/p_i)$$

$$w_i = w_i^{(1)} + w_i^{(2)} + w_i^P \quad (4)$$

where δij is Kronecker's delta, p_i is the element of {p} and w_i^P is the relative approach of teeth due to elastic contact.

The equilibrium equation and the condition of contact are given by

$$\sum_{i=1}^{21} p_i = p_n \quad (5)$$

$$w_i + (s_i/1000) = r_{g1}\theta_1 + r_{g2}\theta_2 \ \text{(contact)}$$
$$p_i = 0 \qquad \text{(non-contact)} \quad (6)$$

where θ is the rotating angle of gears and s_i is the spacing at the node i caused by the effective lead error and any crowning. The load distribution {p} can be determined from Equation (3) under the conditions in (5) and (6).

The relative approach is estimated in this article by applying Lundberg's formula[8] to the virtual cylinders with the same length as the face width.

Comparison of FEM Solutions With the Experimental Formula by Niemann and Reister

The load distribution and the maximum load intensity p_{max} of the gears used in the experiment are shown in Fig. 2. The deformation of shafts, bearings and housing is neglected in the FEM calculation because the data are not given in their article. The results obtained by FEM are very close to the results obtained by their experimental formula over the load range of 20.6 to 345.2 N/mm. p_{max} obtained by FEM is close to the value calculated by the AGMA strength rating formula (9) (where stiffness is assumed to be $G = 1.2 \times 10^6$ lb/in^2 [10]) under the heavy transmitted load. On the other

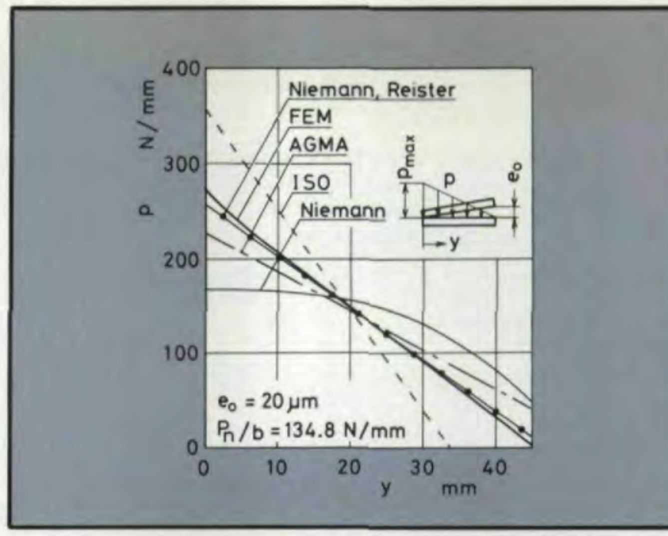


Fig. 2a – Comparison of the load distribution.

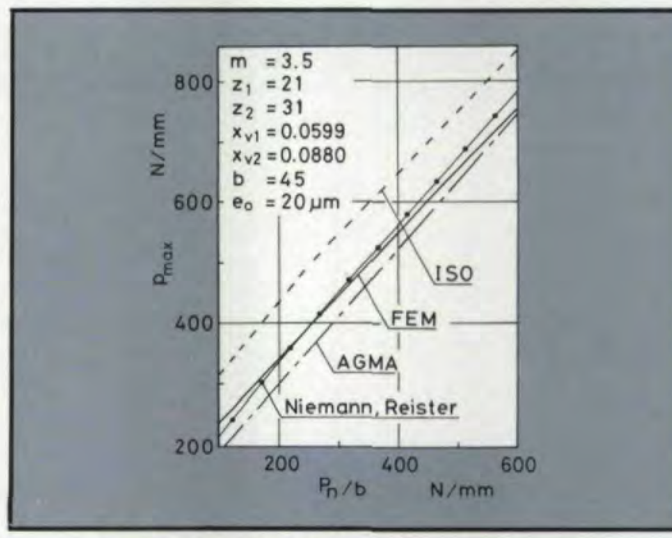


Fig. 2b – Comparison of the maximum load intensity.

hand, the ISO strength rating formula (1) overestimates p_{max} about 49 to 8.4 per cent in comparison with the experimental formula.

Longitudinal Load Distribution Factor

Empirical Formula for Longitudinal Load Distribution Factor. In this section, the longitudinal load distribution factor

$$K_{H\beta} = \frac{p_{max}}{p_{mean}} \text{ where } p_{mean} = \frac{P_n}{b} \quad (7)$$

is calculated and a formula for the factor is proposed.

$K_{H\beta}$ neglecting the effect of shaft stiffness. The longitudinal load distribution factor is affected by the total stiffness. To simplify the effect of the total stiffness on the load distribution, it is assumed in this article that the formula for $K_{H\beta}$ may be represented as the product of two terms: one is the longitudinal load distribution factor of the pair of standard gears $z_1:z_2 = 18:18$, and the other is the modification factor of the gear ratio and the addendum modification.

The longitudinal load distribution factor $K_{H\beta}$ for the standard gears $z_1:z_2 = 18:18$ is shown in Fig. 3. The calculation was performed for the various combinations of the tooth dimension: m = 2.5 to 10 mm and b = 30 to 120 mm, and the transmitted load P_n/b = 100 to 600 N/mm. From these results, the following empirical formula was derived.

$$[K_{H\beta}]z_1:z_2=18:18 = 1.00 + \varkappa(e_o/b)^{0.95} \tag{8}$$

where $\varkappa$ is evaluated by the following equation.

$$\varkappa = \{3.26(b/m) + 8.00\}\ (P_n/bm)^{-0.87} \tag{9}$$

The influence of the gear ratio and the addendum modification on $K_{H\beta}$ is then examined and the following formula is obtained.

$$K_{H\beta} = \{1.00 + \varkappa(e_o/b)^{0.95}\}\ \{1.00 + \phi(e_o/b)^{0.5}\} \tag{10}$$

The second factor of Equation (10) is the modification factor. This accounts for the effects of the gear ratio and the addendum modification on $K_{H\beta}$, where ϕ is found from Fig. 4 as a function of b/m and γ. An example of the stiffness ratio γ is shown in Fig. 5 where every pair of gears is engaged at the pitch point. In this figure, total stiffness k is calculated by the empirical formula of the tooth deflection obtained by two-dimensional FEM[11] and by Lundberg's formula when m = 5, b = 60 and P_n/b = 400 N/mm.

Although the value of total stiffness k depends on m, b and P_n/b, the stiffness ratio γ does not vary so much, and Fig. 5 may be valid for common gears. The error of Equation (10) is about three per cent.

An example of the effect of the difference of the face width $\Delta b = b_1 - b_2$ on $K_{H\beta}$ is shown in Fig. 6, where the gears are $z_1:z_2 = 18:40$ and $b_2 = 60$.

$K_{H\beta}$ including the effect of shaft stiffness. Examples of longitudinal load distribution factor of both straddle- and overhang-mounted spur gears are shown in Fig. 7, where the elastic deformation of bearings and housing is neglected. In

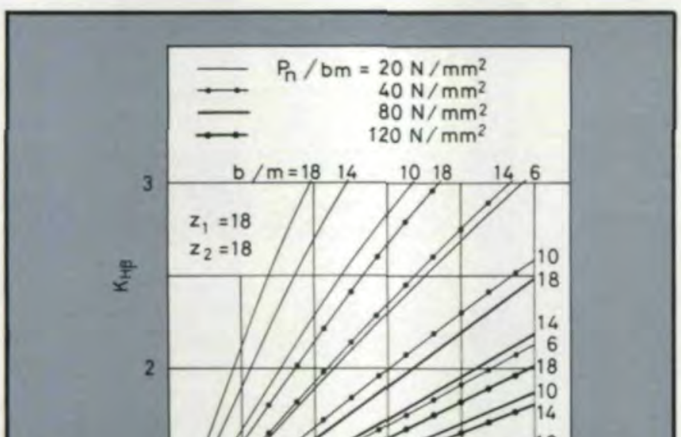


Fig. 3 – Longitudinal load distribution factor of standard gears $z_1:z_2 = 18:18$, $b = b_1 = b_2$ (Deformation of gear bodies and shafts are neglected.)

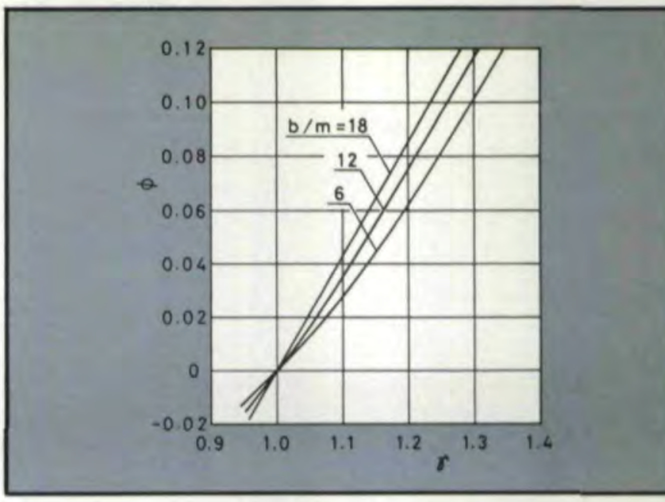


Fig. 4 – Coefficient ϕ.

Nomenclature

- b = face width of gear tooth (mm)
- c = distance from the side edge of tooth to the center of arc-shaped crowning (mm)
- d_s = diameter of shaft (mm)
- e = amount of crowning (μm)
- e_o = effective lead error under no – load (μm)
- e_{eq} = equivalent effective lead error (μm)
- k = total stiffness of a pair of gears, consisting of the deflection of teeth in mesh and the relative approach [N/(mm μm)]
- $K_{H\beta}$ = longitudinal load distribution factor for contact stress
- $K_{M\beta}$ = bending moment distribution factor
- m = module (mm)
- p = load intensity or distributed load per unit length along the contact line (N/mm)
- P_n = transmitted normal load (N)
- r_g = base radius (mm)
- s = spacing between tooth surfaces (μm)
- w = deflection of tooth or deflection of shaft (mm)
- $\overline{w}_b$ = mean displacement of gear caused by the bending deflection of shaft due to uniformly distributed unit load (μm/N)
- Δw = displacement difference between the side edges of gear caused by the deflection of shaft due to uniformly distributed unit load (μmm/N)
- x_p = distance from the root to the meshing position along the tooth height (mm)
- x_v = addendum modification coefficient
- z = number of gear teeth
- γ = ratio of total stiffness of a pair of gears to that of the pair of standard gears $z_1:z_2 = 18:18$
- η^* = position of the point where crowned teeth come into contact (mm)
- ξ = bending moment reduction coefficient

Suffixes 1 and 2 represent pinion and gear, respectively.

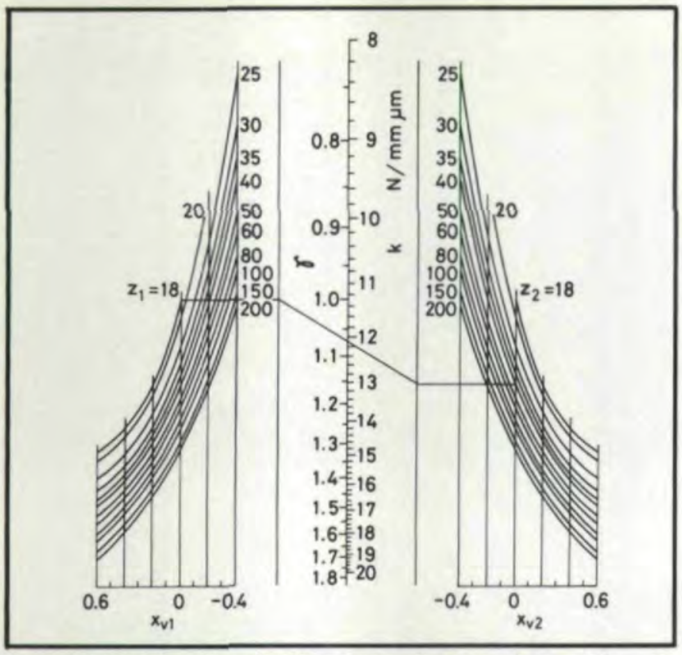


Fig. 5 – Total stiffness k and stiffness ratio γ for gears of $m = 5$, $b = 60$ and $P_n/b = 400$ N/mm (Example: $z_1 = 18$, $x_{v1} = 0$, $z_2 = 40$, $x_{v2} = 0$; $k = 12.08$ N/mm µm, $\gamma = 1.07$).

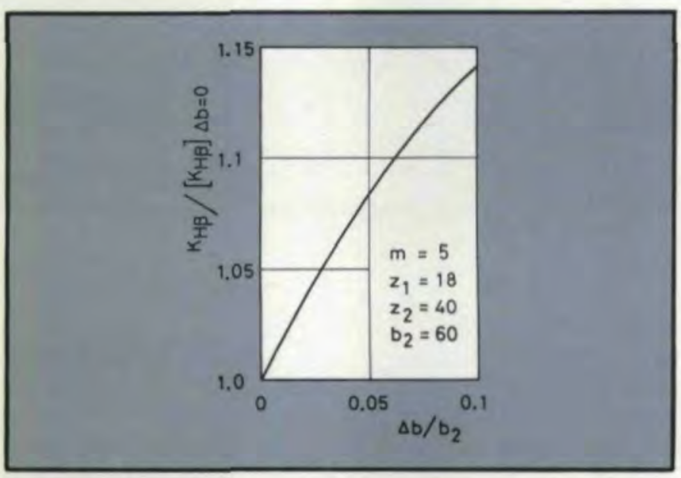
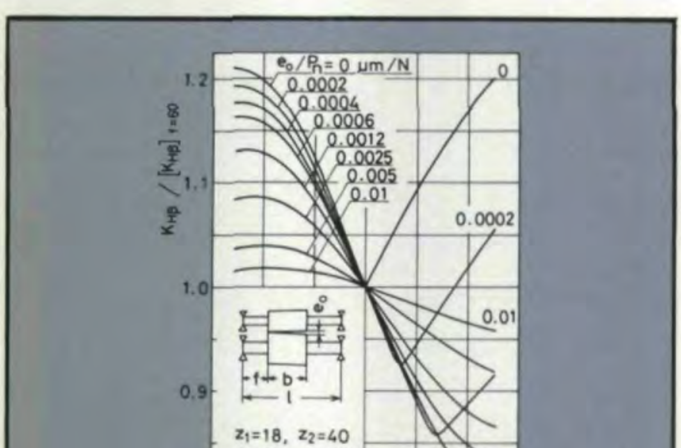


Fig. 6 – Example of the influence of the difference of face width Δb on $K_{H\beta}$.



these cases, the deflection of the shafts exerts a great influence on longitudinal load distribution factor. When the spacing between tooth surfaces increases because of the deflection of the shafts the factor $K_{H\beta}$ increases, and vice versa. The turnings of the curves in these figures are caused by both the compensation of the initial lead error and the inversion of the direction of the lead error by the deflection of the shafts.

It is essential, therefore, to find the equivalent effective lead error e_{eq} under loading. Therefore, the standard gear $z_1{:}z_2 = 18{:}18$ was again adopted, and the longitudinal load distribution factor $K_{H\beta}$ for both straddle- and overhang-mounted gear with the shafts of various length and diameter was calculated. The value of $K_{H\beta}$ obtained, was substituted in Equation (8), and the value of lead error e_o in the equation, namely, the equivalent effective lead error e_{eq}, was estimated. In most strength rating formulas, the error e_o', which is the sum of the effective lead error e_o under no-load, and the displacement difference Δw between the side edges of the gears due to the deflection of the shafts,

$$e_o' = e_o + (\Delta w_1 + \Delta w_2)\, P_n \qquad (11)$$

is used as the equivalent lead error to calculate the longitudinal load distribution factor. The error e_o' is, however, larger than the equivalent lead error e_{eq} estimated above, except for the very rigid mounting. The relation between e_{eq} and e_o' is expressed by the following equation.

$$e_{eq} = \left| e_o' \right|^{\chi} \qquad (12)$$

Coefficient χ is shown in Fig. 8. Consequently, when the equivalent effective lead error e_{eq} is estimated by Equations (11) and (12), the longitudinal load distribution factor $K_{H\beta}$ is evaluated by the following formula:

$$K_{H\beta} = \{1.00 + x(e_{eq}/b)^{0.95}\}\,\{1.00 + \phi(e_{eq}/b)^{0.5}\} \qquad (13)$$

Equation (13) was tested for other pairs of gears. Its error is about six per cent, unless mountings of small rigidity or extreme asymmetry are used.

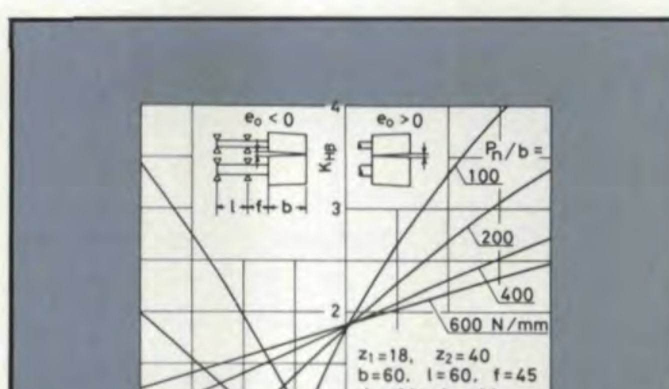


Fig. 7a & 7b – Examples of longitudinal load distribution factor for straddle-mounted (left) and overhang-mounted gears (right).

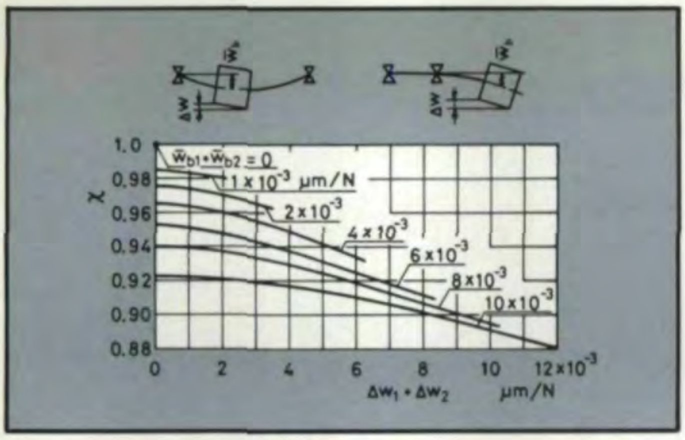


Fig. 8—Coefficient χ.

Table 1 Dimensions of gears (straddle-mounted)

	gear 1	gear 2
m	5	
z	18	40
x_v	0	0
b	60	60
l	180	
f	40	40
d_s	50	75

Table 2 Comparison of longitudinal load distribution factor $K_{H\beta}$

	FEM			Formula (13)			ISO			AGMA (K_m)		
P_n/b (N/mm)	200	400	600	200	400	600	200	400	600	200	400	600
e_o = 0 μm	1.19	1.20	1.20	1.15	1.15	1.16	1.25	1.25	1.25	1.18	1.18	1.18
15 μm	1.39	1.15	1.07	1.36	1.13	1.05	1.69	1.22	1.06	1.33	1.16	1.08
30 μm	1.89	1.41	1.24	1.78	1.37	1.22	2.55	1.69	1.37	1.61	1.33	1.22

*Comparison of $K_{H\beta}$ with the Values in ISO and AGMA**

Taking the example started in Table 1, $K_{H\beta}$ values are compared in Table 2. The factor $K_{H\beta}$ using the Formula (13) is very close to the calculated values. ISO formula give 10 to 30 per cent larger values except for the case of small lead error. The AGMA formula gives fairly close values of K_m as a whole. The lead error e_o' was used in the calculation of $K_{H\beta}$ in ISO and K_m in AGMA. The fundamental formulas for the factor in ISO and AGMA are same, but the values of the mesh stiffness are different. The stiffness $c_\gamma = c'(0.75\epsilon_\alpha + 0.25)$, recommended in ISO(1), is about 2.8 times the stiffness G in AGMA(10) and is about 2 times the stiffness k obtained by our finite element analysis.(11) This is the reason that the ISO formula gives large $K_{H\beta}$ as compared with the AGMA formula and the proposed Formula (13).

Bending Moment Distribution Factor

The plate theory gives more gentle distribution of bending moment at the root than the beam theory, and the effect is represented by the coefficient ξ defined as follows:

$$\xi = \frac{M_{x,\,max}}{M_{x,\,o}} = \frac{M_{x,\,max}}{p_{max,\,lp}} \qquad (14)$$

where $M_{x,\,max}$ and $M_{x,\,o}$ are the maximum bending moment at the root calculated by the plate theory and by the beam theory, respectively, and lp is the length of moment arm. The

*Details shown in Appendix B.

bending moment distribution factor $K_{M\beta}$ is defined as follows:

$$K_{M\beta} = \frac{M_{x,\ max}}{M_{x,\ mean}} = \frac{M_{x,\ max}}{P_n/b)\ lp} \qquad (15)$$

If the loading position and the root location adopted in this article and in ISO are same, $K_{M\beta}$ in our definition may be equal to $K_{F\beta}$ in the formula of ISO.** From Equations (14) and (15), $K_{M\beta}$ is derived as follows:

$$K_{M\beta} = \xi K_{H\beta} \qquad (15')$$

For standard gears $z_1{:}z_2 = 18{:}18$ engaged at the highest point of single tooth contact of gear 1, the calculated coefficient ξ of gear 1, $\xi_o = [\xi]z_1{:}z_2 = 18{:}18$, is shown in Fig. 9. Using this result, the coefficient ξ of any gears can be approximately estimated by the following equation:

$$\xi = 1.00 - (1.00 - \xi_o)\,\frac{X_p/m}{1.59} \qquad (16)$$

The value 1.59 indicates Xp/m of the gear 1 mentioned above. The value of ξ obtained by Equation (16) is valid to estimate the bending moment distribution factor $K_{M\beta}$ of both straddle- and overhang-mounted gears without any crowning.

Total Stiffness of Gears with Effective Lead Error

When a pair of gears with effective lead error is in mesh, the maximum deflection at the meshing position is larger than the deflection of the same gears without lead error.

The total stiffness, which is defined in this article as the ratio of the transmitted load and the maximum deflection, decreases with the increase of the effective lead error or $K_{H\beta}$, as shown in Fig. 10. In these cases, the relative approach estimated from Lundberg's formula is included and the deflection of shafts is neglected. In the case of $K_{H\beta} = 1$, the stiffness is about six to twelve per cent larger than the stiffness which is estimated by two-dimensional FEM.[11] The empirical formula is obtained from Fig. 10 as follows:

$$k = (P_n/b)/[w_1 + w_2 + w^P]_{max}$$

$$k = K_{H\beta}{}^{-0.96}[k]_{K_{H\beta}=1} \qquad (17)$$

Optimal Amount of Crowning

To minimize the longitudinal load distribution factor of a pair of gears with effective lead error e_{eq}, this section determines the optimal amount of arc-shaped crowning. The center of the curvature of the crowning is assumed to lie between the side edges of the tooth.

Referring to Fig. 11, the total spacing between tooth sur-

**$K_{F\beta}$ is the longitudinal load distribution factor for bending stress in ISO[1] and accounts for the effect of load distribution across the face width on the bending stress at the tooth root. It is given by the following equation:

$$K_F = K_H{}^N$$

$$N = \frac{(b/h)^2}{1 + b/h + (b/h)^2}$$

b/h = ratio of face width to tooth height, the minimum of b_1/h_1 or b_2/h_2

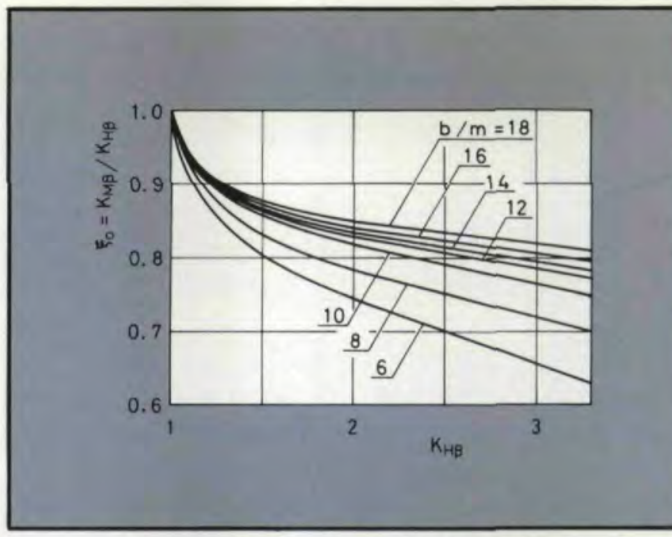


Fig. 9 – Coefficient ξ_o for the calculation of $K_{M\beta}$.

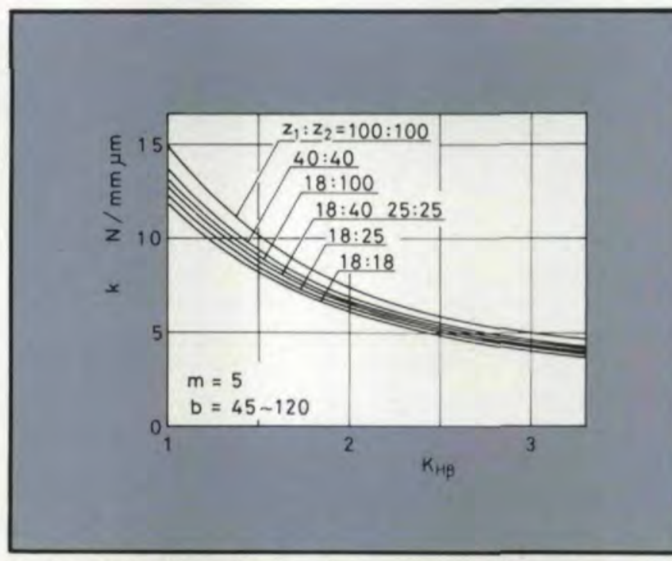


Fig. 10 – Total stiffness of a pair of spur gears with effective lead error.

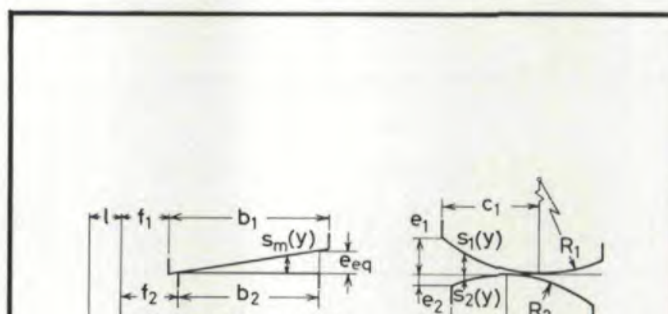


Fig. 11 – Spacing between tooth surfaces caused by the effective lead error (left) and the crowning (right).

faces due to the lead error and the crowning is expressed in the following form;

$$s_o(\eta) = s_m(\eta) + s_1(\eta) + s_2(\eta)$$

$$= e_{eq}\frac{\eta - f_2}{b_2} + e_1(\frac{f_1 - c_1 - \eta}{c_1})^2 + e_2(\frac{f_2 - c_2 - \eta}{c_2})^2 \qquad (18)$$

where $\eta = y$ for straddle-mounted gears
$\eta = y - 1$ for overhang-mounted gears.

When both tooth surfaces just come into contact at $\eta = \eta^*$, *the position* η^* is obtained from

$$\left.\frac{ds_o(\eta)}{d\eta}\right|_{\eta=\eta^*} = 0$$

as follows:

$$\eta^* = \ldots. \qquad (19)$$

The spacing $s(\eta)$ is therefore obtained by subtracting the minimum of s_o from $s_o(\eta)$ and represented in the following expression.

$$s(\eta] = s_o(\eta) - s_o \min$$

$$= s_o(\eta) - s_o(\eta^*)$$

$$= \frac{1}{c_1^2c_2^2(e_1c_2^2+e_2c_1^2)}[(e_1c_2^2+e_2c_2^1)\eta - \{e_1(f_1+c_1)c_2^2 + e_2(f_2+c_2)c_1^2 - \frac{c_1^2c_2^2}{2b_2}e_{eq}\}]^2 \qquad (20)$$

To locate the maximum load intensity at the required position $\eta = \eta^*$, the shapes of crowning of both pinion and gear, (e_1, c_1) and (e_2, c_2), should satisfy the following equation, which is derived from Equation (19).

$$e_1c_2^2(f_1+c_1-\eta^*) + e_2c_1^2(f_2+c_2-\eta^*) - \frac{c_1^2c_2^2}{2b_2}e_{eq} = 0 \qquad (21)$$

And to minimize the longitudinal load distribution factor for the given value of η^*, the coefficient of η^2 in the Equation (20), namely,

$$e_1/c_1^2 + e_2/c_2^2 \qquad (22)$$

has to be minimized.

For example, the longitudinal load distribution of the pair of gears studied in Table 1 is shown in Fig. 12. In this calculation, only pinion is given the arc-shaped crowning listed in Table 3. The solid curves 1,2,3 and 4 in Fig. 12 show the load distribution of the gears with the optimal amount of crowning. The maximum load p_{max} is reduced about 40 per cent as compared with p_{max} of the gears without crowning. On the other hand, the load distribution of the gears with the larger value of e_1/c_1^2 is not so reduced. Note the broken curves in Fig. 12.

Although the method above determines the optimal amount of crowning, it requires η^*, and it is not easy to determine η^* to minimize the longitudinal load distribution factor. Another simple method is needed to estimate the optimal

Table 3 Shapes of crowning (e_1,c_1) and $K_{H\beta}$ of gears shown in Table 1
(P_η/b = 200N/mm, e_o = 20μm, e_{eq} = 26.18μm)

Case	$\eta^* - f_1$	c_1/b	e_1/e_{eq}	$(e_1/c_1^2)/(e_{eq}/b^2)$	$K_{H\beta}$
1	12	1.0	0.625	0.625	1.14
2	18	1.0	0.714	0.714	1.12
3	24	1.0	0.833	0.833	1.12
4	30	1.0	1.0	1.0	1.14
1'	12	0.45	0.405	2.0	1.51
2'	18	0.55	0.605	2.0	1.39
3'	24	0.65	0.805	2.0	1.31
4'	30	0.75	1.125	2.0	1.29

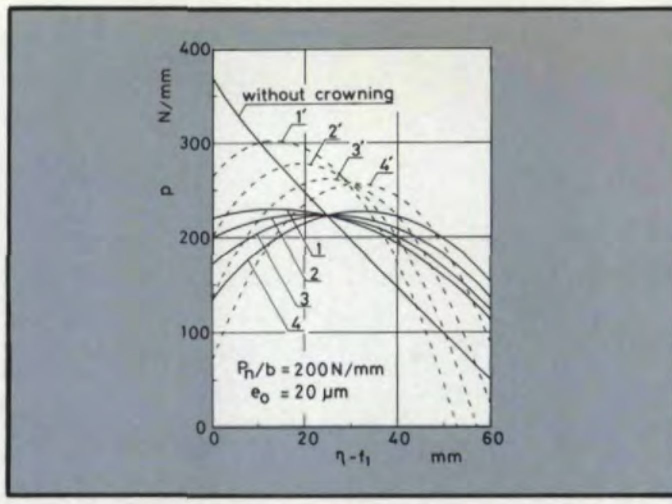


Fig. 12 – Longitudinal load distribution of the gears with the arc-shaped crowning in Table 3.

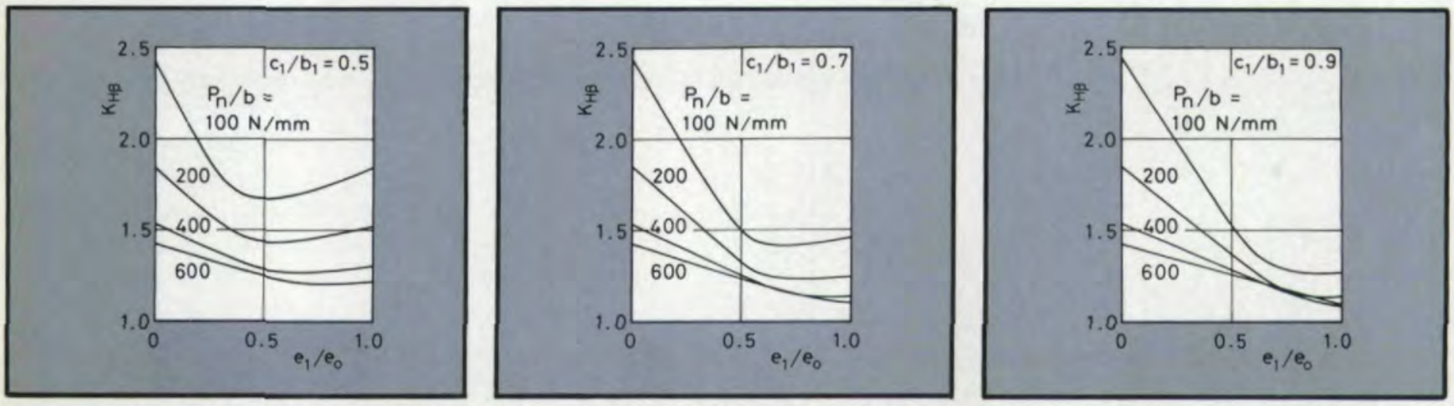


Fig. 13 – Longitudinal load distribution factor of straddle-mounted gears with the arc-shaped crowning (m = 5, z_1:z_2 = 18:40, b = b_1 = b_2 = 60, 1 = 180, f_1 = f_2 = 40, d_{s1} = 50, d_{s2} = 75, e_o = 20μm).

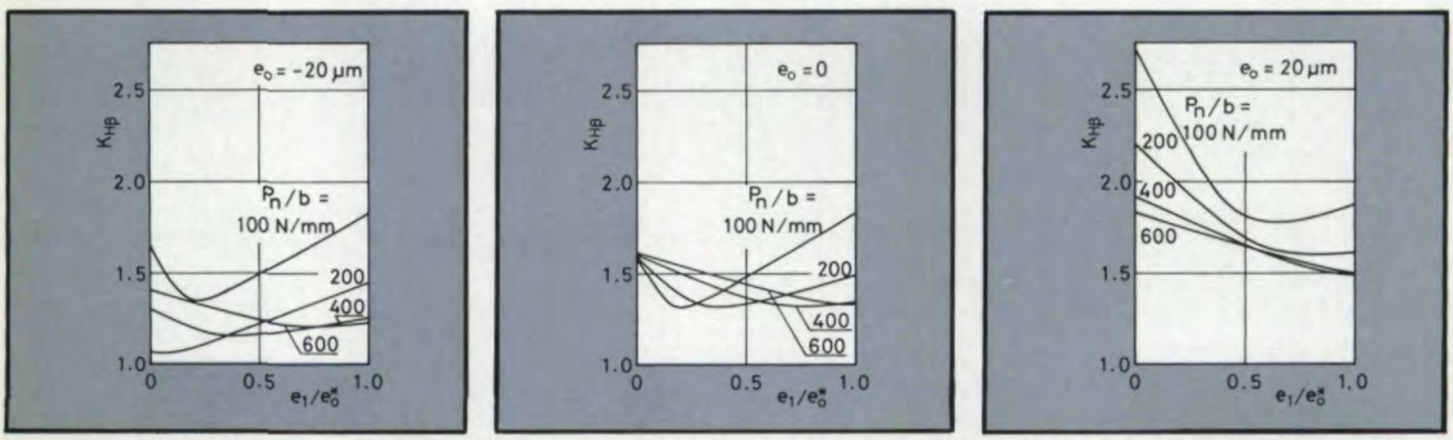


Fig. 14 – Longitudinal load distribution factor of overhang-mounted gears with the arc-shaped crowning (m = 5, z_1:z_2 = 18:40, b = b_1 = b_2 = 60, 1 = 60, f_1 = f_2 = 30, d_{s1} = 60, d_{s2} = 90, c_{1/b_1} = 0.5, e_o·20μm).

amount of crowning without η^*. Fig. 13 and Fig. 14 show some results of $K_{H\beta}$, where only pinion is crowned with various amount of crowning e_1. From these results, the relation between the optimal amount of crowning and the equivalent lead error e_{eq} is obtained as follows:

$$e_{1\ opt}/e_{eq} = 0.92(c_1/b_1) \qquad (23)$$

If the position of the center of crowning is given, the optimal amount of crowning $e_{1\ opt}$ can be found from Equation (23).

When the equivalent effective lead error e_{eq} is adopted instead of the error e_o', ISO recommendation for the crowning, or $c/b = 0.5$ and $e/e_{eq} = 0.5$, is reasonable to minimize $K_{H\beta}$ approximately $c/b = 0.5$.

Some results of $K_{H\beta}$ and ξ of both straddle- and overhang-

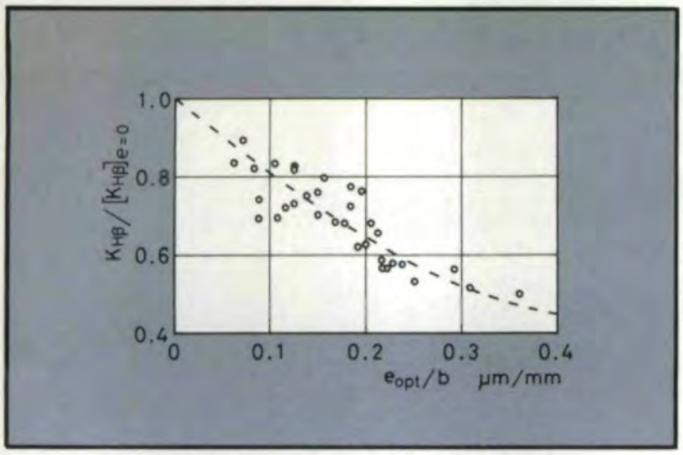


Fig. 15a – $K_{H\beta}$ of the gears with the optimal crowning.

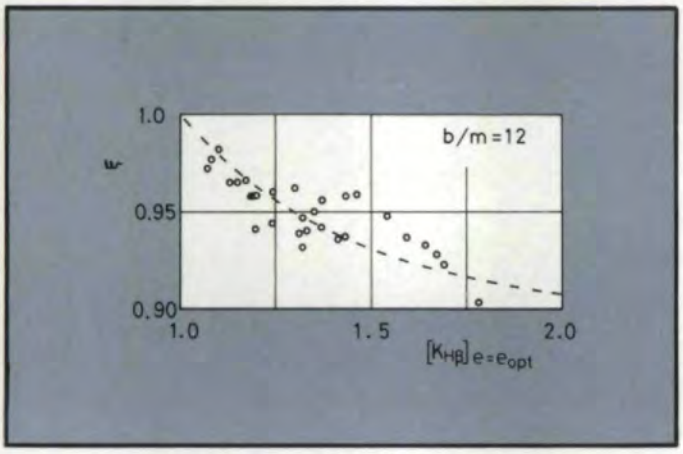


Fig. 15b – ε of the gears with the optimal crowning.

mounted gears with the optimal amount of crowning are plotted in Fig. 15. The pair of gears used in these calculations is $m = 5$, $z_1{:}z_2 = 18{:}40$. The bending moment distribution at the root of gears with the optimal crowning is nearly uniform. (See Fig. 15)

Conclusion

Longitudinal load distribution and bending moment distribution at the root are calculated for the straddle- and the overhang-mounted spur gears.

The longitudinal load distribution factor $K_{H\beta}$, the bending moment distribution factor $K_{M\beta}$ and the total stiffness K are given in the illustrations. A formula for the estimation of $K_{H\beta}$ is proposed. The formula is very useful to estimate $K_{H\beta}$ of spur gears whose effective lead error can be evaluated. When $K_{H\beta}$ is compared with the values calculated by ISO and AGMA formulas, the load distribution factor K_m obtained by AGMA formula is fairly close to $K_{H\beta}$ in our calculation.

A method is proposed to determine the optimal amount of arc-shaped crowning of spur gears with the effective lead error. The ISO recommendation for the determining of optimal crowning is reasonable, and it approximately minimizes the longitudinal load distribution factor.

Appendix A:

[I] Straddle-mounted

(1) Bending deflection

$$w_b = -\frac{K}{6}\frac{1-(f+y')}{1}p_n y^3 + a_1 y + a_2$$

$$(f \leqslant y \leqslant f+y')$$

$$w_b = \frac{K}{6}\frac{f+y'}{1}p_n y^3 - \frac{K}{2}(f+y')p_n y^2 + a_3 y + a_4$$

$$(f+y' \leqslant y \leqslant f+b)$$

$$a_1 = -\frac{K}{2}(f+y')^2 p_n + a_3$$

$$a_2 = \frac{K_S-K}{3}\frac{1-(f+y')}{1}f^3 p_n + \delta_L$$

$$a_3 = \frac{K_S-K}{2}(f+b)\,(f+y')\,(\frac{f+b}{1}-2)p_n + a_5$$

$$a_4 = -\frac{K}{6}(f+y')^3 p_n + a_2$$

$$a_5 = \frac{K_S}{3}(f+y')1p_n - \frac{a_6}{1} + \frac{\delta_R}{1}$$

$$a_6 = \frac{K_S-K}{6}(f+b)^2(f+y')\,(2\frac{f+b}{1}-3)p_n + a_4$$

(A.1)

(2) Torsional deflection

$$w_t = [J_S f + J(y-f)]p_n r_g^2 \qquad (f \leqslant y \leqslant f+y')$$

$$w_t = (J_S f + Jy')p_n r_g^2 \qquad (f+y' \leqslant y \leqslant f+b)$$

(A.2)

[II] Overhang-mounted

(1) Bending deflection

$$w_b = -\frac{K}{6}p_n y^3 + \frac{K}{2}(1-f+y')p_n y^2 + a_1 y + a_2$$

$$(1+f \leqslant y \leqslant 1+f+y')$$

$$w_b = a_3 y + a_4 \qquad (1+f+y' \leqslant y \leqslant 1+f+b)$$

$$a_1 = -\frac{K_S-K}{2}(1+f)^2 p_n + (K_S-K)\,(1+f+y')\,(1+f)p_n + a_5$$

$$a_2 = \frac{K_S-K}{3}(1+f)^3 p_n - \frac{K_S-K}{2}(1+f+y')\,(1+f)^2 p_n + a_6$$

$$a_3 = \frac{K}{2}(1+f+y')^2 p_n + a_1$$

(Continued on page 45)

Longitudinal Load Distribution . . .
(Continued from page 19)

$$a_4 = -\frac{K}{6}(1+f+y')^3 p_n + a_2$$

$$a_5 = -\frac{K_S}{2}(1+f+y')1P_n - \frac{K_S}{6}(f+y')1p_n - \frac{\delta_L-\delta_R}{1}$$

$$a_6 = \frac{K_S}{6}(1+f+y')1^2 p_n + \delta_L \qquad \text{(A.3)}$$

(2) Torsional deflection

$$w_t = [J_S(1+f) + J(y-1-f)]p_n r_g^2$$
$$(1+f \leqslant y \leqslant 1+f+y')$$

$$w_t = [J_S(1+f) + Jy']p_n r_g^2$$
$$(1+f+y' \leqslant y \leqslant 1+f+b)$$

where, $K = \frac{64}{\pi E d_O^4}$, $K_S = \frac{64}{\pi E d_S^4}$, $J = \frac{32}{\pi G d_O^4}$, $J_S = \frac{32}{\pi G d_S^4}$, (A.4)

d_O : pitch diameter [mm]
E : modulus of elasticity = 2.06×10^5 N/mm^2
G : modulus of rigidity = 7.92×10^4 N/mm^2
δ : displacement of bearing [mm]

Appendix B:
Effect of mesh stiffness on the load distribution factor

In the text the load distribution factors calculated by FEM were compared with the values in ISO 6336/I and AGMA 225.01. The latter standard was revised in 1982, and the gear design is now based on the new standard AGMA 218.01. The main effect of this revision in estimating the load distribution factor is in evaluating the mesh stiffness, surely the essential point of the problem. A brief discussion follows to clarify the effect of mesh stiffness on the load distribution factor.

The mesh stiffness G = 0.5 − 2 x 10^6 lb/in^2 was recommended for spur gears in AGMA 225.01, and the authors adopted as a mean value G = 1.2 x 10^6 to demonstrate that the load distribution factor differs from the factor calculated by FEM. The values are compared in Fig. 2 and Table 2. On the other hand, AGMA 218.01 recommends the stiffness G = 1.5 − 2 x 10^6. These values compare well with the stiff-

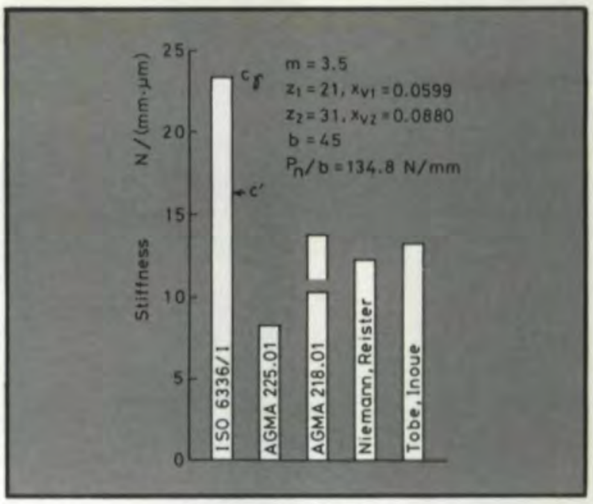


Fig. B.1 – Mesh stiffness

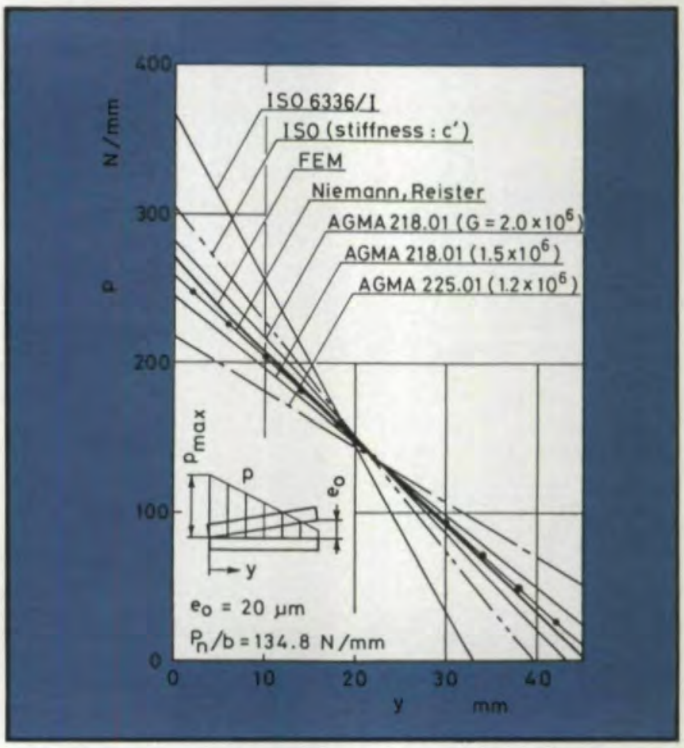


Fig. B.2 – Comparison of Longitudinal Load Distribution

Table B.1 Comparison of longitudinal load distribution factor

	FEM			Formula (13)			ISO 6336/I			AGMA 225.01			AGMA 218.01		
P_n/b (N/mm)	200	400	600	200	400	600	200	400	600	200	400	600	200	400	600
e_0 = 0 μm	1.19	1.20	1.20	1.15	1.15	1.16	1.25	1.25	1.25	1.18	1.18	1.18	1.15	1.15	1.15
15 μm	1.39	1.15	1.07	1.36	1.13	1.05	1.69	1.22	1.06	1.33	1.16	1.08	1.40	1.13	1.04
30 μm	1.89	1.41	1.24	1.78	1.37	1.22	2.55	1.69	1.37	1.61	1.33	1.22	1.95	1.40	1.22

NOTE: AGMA 225.01: G = 1.2 x 10^6 lG/in^2 AGMA 218.01: G = 2 x 10^6 lG/in^2

ness obtained by Niemann and Reister's experiment as shown in Fig. B.1. They are also close to the stiffness calculated by FEM.

In ISO 6336/I, the mesh stiffness c_γ, the mean value of the total stiffness, is used to estimate the load distribution factor. Because of its definition, c_γ may be used to estimate a dynamic load, but it is not logical for the estimation of the load distribution. The single stiffness c′, which is approximately equal to the stiffness of a tooth pair in the phase of single pair contact, should be used instead, because the load distribution factor $K_{H\beta}$ is used to evaluate a contact stress at the operating pitch point. Furthermore, the root stress is calculated for the worst loading condition (loading at the highest point of single tooth contact), and the formula uses the factor $K_{F\beta}$, which is related to $K_{H\beta}$. The stiffness c′ is quite close to the stiffness calcuated by FEM as shown in Fig. A.1.

The load distributions obtained from these stiffnesses are illustrated in Fig. A.2, which corresponds to Fig. 2a in the text. The load distribution obtained by AGMA 218.01 is fairly close to the result calculated by FEM. If the latter is regarded as accurate, the stiffness $G = 1.85 \times 10^6$ is recommended in this case. Using c′ makes the load distribution of ISO very close to the result by FEM. The comparison of the load distribution factors is summarized in Table B.1, which corresponds to Table 2.

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