The Uses and Limitations Of Transmission Error

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Abstract:

Transmission error (TE) is described and its relevance for accuracy, noise and vibration investigations and production control is discussed. Difficulties with attempts to predict TE by computer are explained and guidelines suggested for the conditions when testing at zero load can be used.

Limitations of speed, accuracy and vibration frequency are quoted for two of the commonly used test rig encoders currently available.

Introduction

The concept of "transmission error" is relatively new and stems from research work in the late 1950s by Gregory, Harris and Munro,[1] together with the need to check the accuracy of gear cutting machines. The corresponding commercial "single flank" testing equipment became available in the 1960s, but it was not until about ten years ago that it became generally used, and only recently has it been possible to test reliably at full load and full speed.

The ideas involved are new and the equipment is electronics-dominated. Furthermore, the results do not tie up directly with the traditional pitch, involute and helix measurements; therefore, there is much confusion and, understandably, suspicion about the test, especially as there are not as yet useful industry guidelines on permissible values for test results.

Despite the initial resistance, some industries have found that transmission error is so important a measurement that they must inspect all gears, yet some branches of industry do not need to use it. The difference lies in the varied relative importance of noise, strength and drive position accuracy in the branches of industry. As a rough guide, if drive accuracy is important or if noise and vibration are important, it is absolutely essential to control TE for high quality, but if strength is the only factor and speeds are low enough to avoid large dynamic stresses, then TE is not worth measuring. Sometimes it is only necessary to use TE at the development stage as a research tool, but it may be necessary for full production checking.

Definition of Transmission Error

Transmission Error is defined as the difference between the theoretical position of the output gear if the drive were both perfectly accurate and rigid and the actual output gear position.[1]

In a simple case, such as a 3:1 reduction box, we would expect a 3° movement at input to give exactly 1° at output, so if the measured output were 1°2′, the TE change would be +2′ of arc. The equipment typically checks the correctness of output position every minute of arc, making it convenient to give the output as a continuous trace record rather than a series of numbers. However, when subsequent digital analysis is required (though such analysis is often deceptive), sampling of the numbers is used to give perhaps 1,024 samples per revolution of the output gear.

A typical output from a spur drive is given in Fig. 1 with the short wavelength repetition at once per tooth superposed on a long wavelength at once per revolution. Full scale would be 32′ of arc for a small gear or 10″ arc for a large gear.

It is not very convenient to work in angular measure for gearing purposes so, although equipment measures angular error, we normally multiply by a pitch (reference) circle radius to get error in linear terms, usually μm (0.04 thousandths of an inch). This approach is slightly debatable academically, but has the major advantage that we now get the same "error" figure regardless of which end is input and output of the drive. It has the further advantage that the practical levels of error that can be achieved for "good", industrial or "poor" gears are independent of size, in contradiction to the figures in the various gear accuracy definitions. It may seem ridiculous that the same drive accuracies (in μm) are achieved on a gear 10 mm, 100 mm or 4 m (13 ft) diameter, but this happens.

Relevance of Transmission Error

TE simply and solely measures accuracy and, hence, smoothness of drive; therefore, if accuracy or smoothness are important, TE is relevant. There is, however, no connection fundamentally between strength and smoothness in a drive, so TE cannot give information about strength, and it is very foolish to attempt to use it as the sole production control for low speed gears which are highly stressed.

The only reliable check on strength is a full-torque bedding check to verify that contact is occurring over most of the

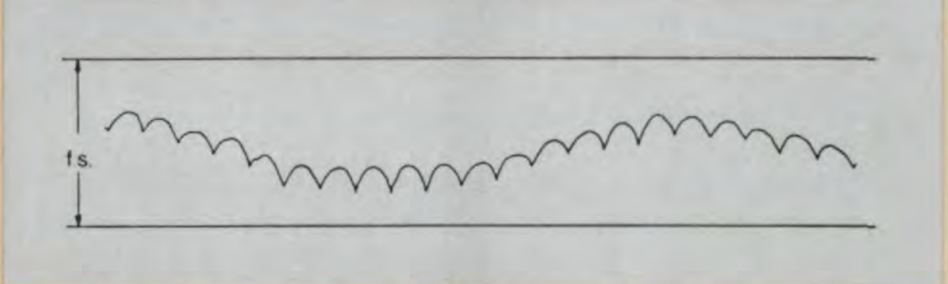

Fig. 1 – Typical transmission error trace; full scale (fs) is 50 μ (2 mil) of error.

tooth face, but this is a time consuming and expensive exercise. Ideally, in critical applications, both TE and bedding must be used.

Using TE for accuracy is an obvious application in industries such as printing, where register between printing cylinders must be accurately maintained or the blue dots may miss the yellow dots and give reject pictures. Accuracy is typically 50 μm (2 thou) maintained through several gear meshes, so that each mesh may have to be accurate to 15 μm. Here the important measurement on the TE check is the peak-to-peak value of the error, since the printing machine is only concerned with the worst relative position. Usually the once-per-revolution components of error are much larger than the once-per-tooth and so dominate the peak-to-peak.

As far as strength is concerned, there is no direct connection with TE so it is possible to design gears which have a very low TE at the expense of gear strength and vice versa. Despite this fundamental lack of correlation with strength, it is preferable to use TE to identify pitch errors, since it is much quicker and cheaper to measure pitch by using TE equipment than to use conventional pitch checkers. A low value of TE (under the correct operating conditions) at the frequency corresponding to once-per-tooth on a helical gear pair infers that either the profile is correct or that the helix matching is good, but does not prove that both are satisfactory. Conversely, a high value of TE infers a poor profile match on the gears *and* a poor helix match.

Vibration and noise are caused primarily by TE, so it is very difficult to investigate or control them without measuring TE. Although relatively few papers have been published on the correlation between noise and TE,[2,3] many checks within industry have confirmed a direct link between them. Gears generate noise for other reasons,[4] but for involute gears of normal accuracy ranges, the other mechanisms may be ignored.

Noise and Vibration Investigation

When a gear drive is too noisy or gives excessive vibration, the initial development work is usually directed towards checking whether there are local resonances which can be eliminated. The presence of such resonances can be checked relatively easily by variation of the operating speed[4] and measurement of natural frequency and mode shape is standard. If there are no "easy" resonances to tackle, or if the excitation, often at once-per-tooth frequency and harmonics, is not near a resonance, then more detailed investigation is necessary.

One approach is to take several complete systems, such as cars or ships, and to instrument each of them thoroughly with as many as 50 accelerometers. Operation under a range of conditions of speed and load with each system will produce a frightening amount of information which can be frequency analysed, correlated, checked for coherence, decomposed into modes, etc.

An alternative approach, which is very popular for work for the state, is to set up a computer model. A very large computer with an extremely large and complex program will compute the output noise and vibration levels from the basic gear shapes. Large numbers of variants on the gear profiles, helices and pitch can be analyzed and the effects on the final vibration levels predicted.

These two approaches both involve massive (and expensive) computation and produce very large and impressive reports which are very good for reassuring customers that the highest possible levels of technological expertise have been applied to the problem. Neither approach, however, provides detailed solutions to the problem, especially when the predictions are that noise levels will be extremely low, but the gear drives do not agree and are not consistent. As this disagreement exists, it is worthwhile looking at the system in more detail to see why the predictive computer approach encounters difficulties.

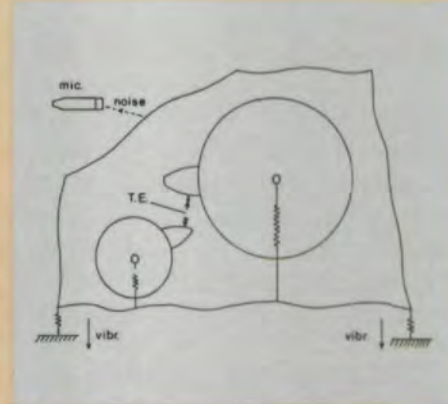

Fig. 2—Sketch of the mechanism of transmission error vibrating the supporting structure and generating noise via internal and external dynamic responses.

The Mechanism of Vibration Generation

Fig. 2 is a sketch of the route by which TE produces noise; this assumes that only TE is relevant (not true for Wildhaber-Novikov gears). The relative

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displacement between the gears (the TE) excites the gears themselves, vibrating mainly on the elasticities of shafts and bearings. Forces are transmitted through the bearings to the gearcase at the bearing housings, the levels of force depending on the size of the original TE and the dynamic response of the gears and their supports. The forces at the bearing housings excite the gearcase, and the vibration from gearcase to the supporting structure may be attenuated by vibration isolators at the gearcase support points.

The two strategic points in this progression from gear shapes to gearcase support vibration are the TE at the gear mesh and the forces which act through the bearings to excite the gearcase. If we put down all the factors which control the stages, the result is as shown in Fig. 3.

A large number of factors contribute to generate the TE, and a relatively small number of factors control the internal dynamic responses of the gears on their shafts. The final response of the gearcase to the bearing forces acting at the bearing depends on a fairly complicated gearcase shape, but relatively few factors, since only stiffness, mass and damping are involved. It is convenient to break down the problem into three stages:

- Gear shapes and design to TE,
- TE to forces through bearings onto gearcase,
- Bearing forces via gearcase to vibration of support.

Of these three stages, the second is relatively simple to predict with reasonable accuracy. The estimates of mass will be very accurate and the estimates of stiffnesses reasonably accurate, since only hydrodynamic bearings will present problems. Estimates of damping will be subject to large errors, but some reasonable guesses can be made based on experimental results from previous designs or, alternatively, variation of speed on a test rig will give useful information about resonances and damping in the internals.

The third stage can be fairly complex if there is a large supporting raft with many resonances, and concepts of statistical energy transfer can be useful. This stage is subject to the same problem that limits the second one in that prediction of the all-important damping is unreliable. The difference between the second and third stages is that it is relatively easy to check the third stage transfer functions experimentally either by running the gearbox at constant torque, but variable speed, or by using electromagnetic vibrators to excite the system to check the force versus displacement transfer function directly.

It is the first stage which presents the really difficult problems due to the accuracy limitations if computer prediction is attempted. Even for the most expensive areas of gearing, such as marine drives, it is difficult to measure to sufficient accuracy, however much care is taken. A rough average rule of thumb for gears of all sizes is that the level of uncertainty in a measurement is of the order of ± 1 μm (0.04 mil). Under typical "precision" conditions it is rather higher at ± 2.5 μm (a tenth of a mil), despite the claims of the manufacturers of the various measuring machines; it is known that some of the measuring machines, though agreeing with others from the same firm, will not agree with machines from rival, equally renowned, firms.

This "uncertainty" of, say, ± 1 μm applies to every one of the many factors contributing to the final TE, so the uncertainty in prediction of TE under operating conditions is likely to be greater than ± 5 μm, even when the averaging effects of the elastic deformations of the stem have been taken into account. This level of uncertainty would not matter too much in a normal industrial gearbox, where the typical level of TE at once-per-tooth frequency might be ± 5 μm. It would matter very much in a very accurate or quiet gearbox, where the requirement could be for an error of not more than ± 1 μm.

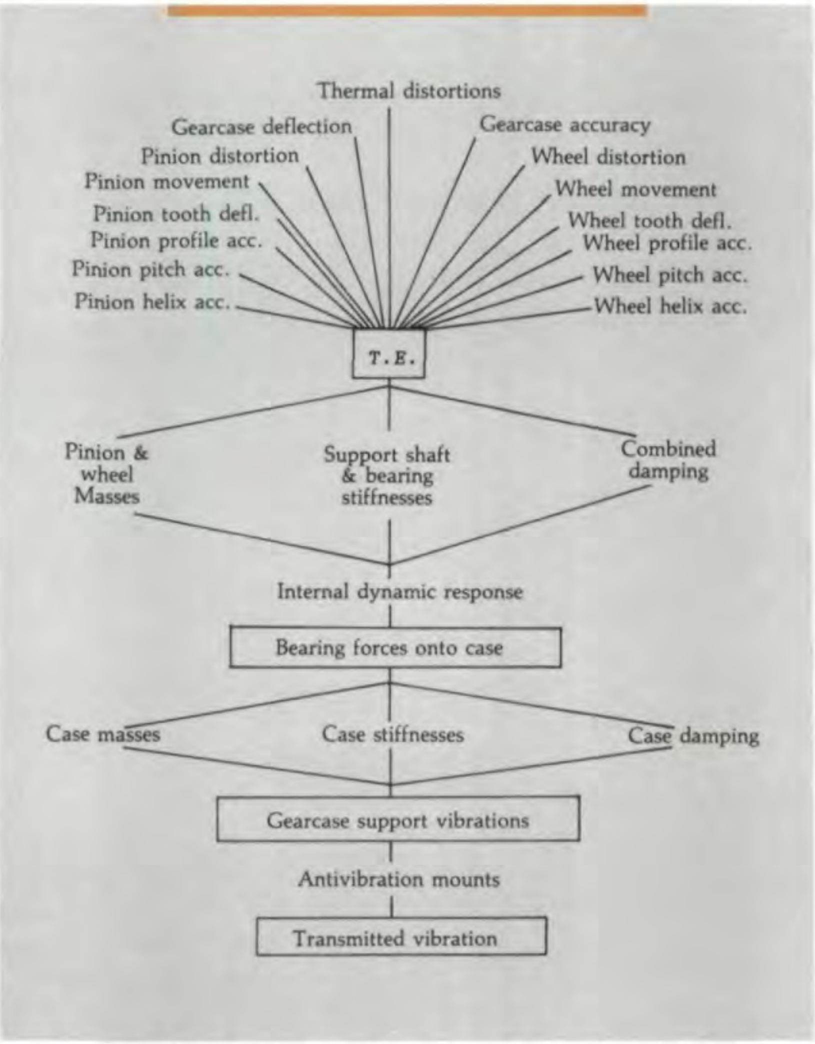

Fig. 3—Factors affecting the vibration generation and transmission.

The practical deduction is that the second and third stages may be predicted with some uncertainty or may be measured rather more reliably. Prediction of the first stages is subject to such high levels of uncertainty that it is difficult to avoid the practical measurement of TE for high quality work on noise or accuracy.

Measurement of Transmission Error

Theoretically, very many possible ways of measuring TE exist, but in practice there are very few which will give a reliable result for accurate gears. The main methods attempted have been:

(a) Magnetic signal methods which "write" a series of pulses onto a magnetic track and decode the resulting "read" signal. These do not appear to work under industrial conditions despite development.
(b) Strain gauges on drive shafts. This approach works for inaccurate drives with known torsional characteristics, but sensitivity is not good enough for precision work.
(c) Torsional vibration transducers of the seismic type. Satisfactory for low frequency work, but not sufficiently sensitive for high frequencies.
(d) Tachometer type devices. Useful for inaccurate work, but not capable of the resolution to one part in 100,000 needed for precision gears.
(e) Tangential accelerometer devices. Cannot be used at once-per-revolution frequencies, but might be suitable for high frequency work.
(f) Grating (rotary encoder) systems. So far this approach has dominated TE testing. A disadvantage is that, like seismic (c) testing, it needs a free shaft end for a coaxial drive to keep torsional frequencies high enough for the system to be adaptable.

To date, the majority of TE testing has been carried out as a production control tool in the inspection shop, either in addition to, or as a replacement for conventional measurement. This type of testing is carried out at very low speeds (10 rpm) and effectively zero torque. The equipment for this is expensive, like all gear measurement equipment, but the testing is so rapid that 100% inspection is possible, and test times of less than a minute can be achieved, so that the cost per gear is low. The gears are mounted on and supported by the spindles of the encoders and driven by an accurate servo-controlled drive which gives a vibration-free input speed. Output can be displayed on a chart recorder or analyzed digitally.

A single TE test on a gear pair is fast and gives a rough check on the combination of helix, profile and pitch errors. In this sense it is similar to the double flank rolling test, though much better in that it gives more reliable information and detects faults which escape the rolling check. A good production compromise is to check 100% with TE checking, since this is so fast and cheap, and to carry out occasional helix and profile checks on a routine basis, or when the TE results show that something is wrong.

In contrast, for development purposes it is sometimes necessary to check at full torque, and an exactly constant speed drive is not realistic. Furthermore, it may be necessary to check the performance of the drive when the gears are mounted in their gearcase, since misalignment errors may be important. For testing under torque it is advisable to run at speeds of above 100 rpm to ensure that an oil film is maintained and to prevent scuffing damage to the teeth surfaces. Provided that free shaft ends are available to connect the rotary encoders, they can be attached to the test rig and give the essential information. Care must be taken that the drive and load are at constant torque or the TE measured will be that of the test rig rather than the gear drive. Costs are relatively low, since the basic encoders are not expensive, and the electronics is much simpler than that used for production testing.

Limitations of Transmission Error Testing

The dominant limitation of TE testing is that it is a vibration measurement and does not give information on strength. Critical applications can overcome this by checking bedding by copper plating and running under load, but, as this requires skill and is time consuming, it is unrealistic for mass production.

Inspection testing is effectively at zero load, whereas, the information is required for operating torques, but at sufficiently low speeds that the system dynamics or resonances do not intrude. There is no general rule and so each case must be assessed separately, but some examples can be quoted.

A vehicle final drive has to be strong enough to withstand full engine torque in bottom gear, yet it is normally only required to be really quiet at 30% torque in top gear. As the torque at cruise is only about 5% of the permissible torque, the cruise deflections are negligible and zero torque testing in inspection is a very good predictor of in-car noise performance. Any other gears, whose noise performance is only relevant at, say, less than 10% of maximum torque, will have small deflections at this torque and so may be tested at zero torque; printing machine gears usually fall into this class because accuracy alone is relevant.

In contrast, a helicopter gearbox is always operating at full torque and will contain gears which have been heavily corrected for wind-up and deflection. Testing at zero torque gives virtually no useful information since the characteristics will be so different at full load.

Many gear drives operate over a wide range of torque, and there is no substitute for testing at low, medium and high load since TE and, hence, gearbox noise may increase, decrease or be independent of load or have a minimum at part loads.

A major limitation of TE testing is seen when shaft free ends are inaccessable, thus ruling out the use of direct coupled rotary encoders. It is possible to use a plain belt drive to a separate bearing block with a rotary encoder at input and /or output. This approach gives rise to two problems: the drive is not exactly synchronous, so much more complicated and temperamental electronics must be used; and, because of the elasticities and masses of the plain belt drive, the maximum reliable frequency for information is limited to about 200 Hz. As an alternative, the tangential accelerometer method can be used, but this needs slip rings or telemetry to transmit the information and does not work for once-per-revolution problems.

Speed and Accuracy of Encoders

Rotary encoders are remarkably cheap considering the accuracy to which they perform. The market is dominated by the firm of Heidenhain, who make two sizes of reasonably accurate encoders which are suitable for gear test rigs.

The smaller ROD 220/260 size is about 100 mm diameter and 50 mm deep and can operate mechanically up to 6,000 rpm without difficulty. Electrically, assuming 9,000 lines, the operating speed is limited to about 1,500 rpm for the 220 and about 6,000 rpm for the faster 260. It is possible to run faster, but the width of the output TTL pulse is then below 0.5 microseconds.

Accuracy is $\pm 5''$ of arc which is satisfactory up to diameters of 80 mm if $\pm 1\ \mu m$ accuracy p-p is required. Noise investigations are not interested in peak-to-peak accuracy, but usually are concerned with once-per-tooth frequencies. The relevant encoder accuracy is, say, that at 20 cycles per revolution and at this frequency the encoders will be accurate to better than $0.2''$ of arc. These relatively small encoders can have their peak-to-peak accuracy greatly improved by using a computer correction on the results, although this requires precalibration and a cumbersome and slow correction of the results.

Although the mechanical and electronic limitation is of the order of 6,000 rpm, the main restriction on the use of the encoder approach is the torsional resonance of the rotary inertia of the disc on the twist stiffness of the 10 mm drive shaft. This imposes a limit of about 1,500 Hz with a well designed coupling and so a practical operating limit for tooth frequency of 1,200 Hz or 400 Hz if third harmonic information is necessary. A 24-tooth pinion at 1,200 Hz allows 3,000 rpm for basic tooth frequency information, or 1,000 rpm for third harmonic, so that the majority of normal industrial drives can be tested.

The larger 800 size of encoder is about 150 mm (6 inches) diameter and 50 mm deep with a lower mechanical limit of the order of 600 rpm for a moderate life. Electronically it is similar to the smaller version, but tends to have higher line numbers of 18,000 or 36,000. It is unlikely to be limited electronically by pulse rate, since it is not used at high speeds.

Torsional frequency limitations apply rather low down the frequency range, since stiffnesses in twist are only about a factor of 4 higher than the 220/260 size, but the inertia is up by a factor of 16, so the torsional resonance will limit the information frequency to about 600 Hz. This would allow only 1,500 rpm on a 24-tooth pinion for tooth frequency; this, of course, exceeds the mechanical limit, but a 200 size encoder could be used on input and the 800 size on the 300 rpm wheel (assuming a 5 to 1 reduction). Checking a hobbing machine drive or a worm and wheel will usually be most satisfactory with a small encoder at input where accuracy is not important.

The accuracy of an 800 size encoder is said to be $\pm 1''$ of arc, and this may be easily verified by cross-checking a pair of encoders. A peak-to-peak measurement of $\pm 1''$ of arc is certainly adequate for most work up to 0.4 m (16 inches) diameter. If greater accuracy is required, $0.1''$ of arc can be achieved without much difficulty either by selecting a higher accuracy encoder or by correcting a standard encoder.

Discussion of accuracy should also involve the accuracy of the critical coupling which connects the encoder to the shaft which is vibrating torsionally. The coupling must accomodate small alignments and be extremely accurate, yet must be very rigid torsionally. This combination can be achieved, but very careful design and manufacture is needed.

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