# Into-Mesh Lubrication of Spur Gears Part 2

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**Abstract:**

An analysis was conducted for into-mesh oil jet lubrication with an arbitrary offset and inclination angle from the pitch point, for the case where the oil jet velocity is equal to or greater than pitch line velocity. Equations were developed for the minimum and the maximum oil jet impingement depth. The analysis also included the minimum oil jet velocity required to impinge on the gear or pinion and the optimum oil jet velocity required to obtain the best lubrication condition of maximum impingement depth and gear cooling. It was shown that the optimum oil jet velocity for best lubrication and cooling occurs when the oil jet velocity equals the gear pitch line velocity. When the oil jet velocity is slightly greater than the pitch line velocity, the loaded side of the driven gear and the unloaded side of the pinion receive the best lubrication and cooling with slightly less impingement depth. As the jet velocity becomes much greater than the pitch line velocity, the impingement depth is considerably reduced and may completely miss the pinion.

## Introduction

In the lubrication and cooling of gear teeth a variety of oil jet lubrication schemes is sometimes used. A method commonly used is a low pressure, low velocity oil jet directed at the ingoing mesh of the gears, as was analyzed in Reference 1. Sometimes an oil jet is directed at the outgoing mesh at low pressures. It was shown in Reference 2 that the out-of-mesh lubrication method provides a minimal impingement depth and low cooling of the gears because of the short fling-off time and fling-off angle.[3] In References 4 and 5 it was shown that a radially directed oil jet near the out-of-mesh position with the right oil pressure was the method that provided the best impingement depth. Reference 6 showed this to give the best cooling. However, there are still many cases where into-mesh lubrication is used with low oil jet pressure, which does not provide the optimum oil jet penetration and cooling. It should also be noted that excessive into-mesh lubrication can cause high losses in efficiency from gear churning and trapping in the gear teeth.[7] In Reference 8 the case for into-mesh lubrication with oil jet velocity equal to or less than pitch line velocity was analyzed, and equations were developed for impingement depth for several jet velocities.

The objective of the work reported here was to develop the analytical methods for gear lubrication with the oil jet directed into mesh and with the oil jet velocity equal to or greater than the pitch line velocity. When the oil jet velocity is greater than the pitch line velocity for into-mesh lubrication, the impingement depth is determined by the trailing end of the jet after it has been cut off or chopped by the following tooth. The analysis is therefore somewhat different from Part I of this article[8] for the case where the oil jet velocity is less than the pitch line velocity. The oil jet location should be offset from the pitch point with an inclined angle to obtain optimum cooling of both gear and pinion for other than one-to-one gear ratios.

The analysis presented here assumes an arbitrary offset and inclination angle to obtain an optimum oil jet velocity for various gear ratios. Further analysis is needed to determine the optimum offset and inclination angle for various gear ratios.

### Analysis

The high-speed cooling jet conditions discussed in this analysis are used only when a range of duty cycle conditions dictate a wide operating speed range with a constant oil jet velocity that must be suitable over the whole range of speeds. Starting with Fig. 1 the sequence of events for the pinion in the case where $V_j > \omega_p r \sec \beta_p$ is shown in Figs. 1 through 4. Here, instead of tracking the head of the jet stream as in Part I of this article,[8] the trailing end or "tail" of the stream will be tracked after it is chopped by the gear tooth.[1] This is shown at "A" in Fig. 3 to the final impingement at a depth "$d_p$" on the pinion tooth 2 as shown in Fig. 4. Initial impingement on the pinion starts as the pinion top land leading edge crosses the jet stream line with inclination angle set at $\beta_p$ and offset $S_p$ as shown in Fig. 1.

The position of the pinion at this time is $\theta_{p3}$, defined (from Fig. 1) as:

$$\theta_{p3} = \cos^{-1}(r_s/r_o) - \text{inv}\,\varphi_{op} + \text{inv}\,\varphi \qquad (1)$$

where:

$$\text{inv}\,\varphi_{op} = \tan\varphi_{op} - \varphi_{op} \text{ and}$$
$$\varphi_{op} = \cos^{-1}(r_b/r_o)$$

$$\text{inv}\,\varphi = \tan\varphi - \varphi$$

Generally the arbitrarily set offset "S" for the gear establishes the value of $\beta_p$ from:

$$\beta_p = \tan^{-1}[S/(R_o^2 - R_s^2)^{1/2}] \qquad (2)$$

Given $\beta_p$, then $s_p$ can be calculated from

$$S_p = [(r_o^2 - r^2\cos^2\beta_p)^{1/2} + r\sin\beta_p]\sin\beta_p \qquad (3)$$

so that:

$$r_s = r - S_p$$

The tail of this jet stream is finally chopped at "A" in Fig. 3 by the gear top land leading edge. The position of the gear at this time is $\theta_{g1}$ calculated:

$$\theta_{g1} = \cos^{-1}(R_s/R_o) - \text{inv}\,\varphi_{og} + \text{inv}\,\varphi \qquad (4)$$

where:

$$\text{inv}\,\varphi_{og} = \tan\varphi_{og} - \varphi_{og} \text{ and}$$

$$\varphi_{og} = \cos^{-1}(R_b/R_o)$$

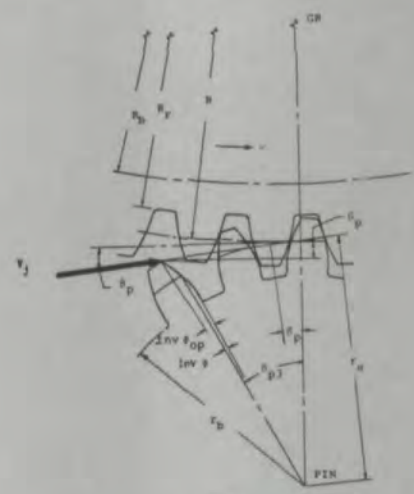

Figure 1.

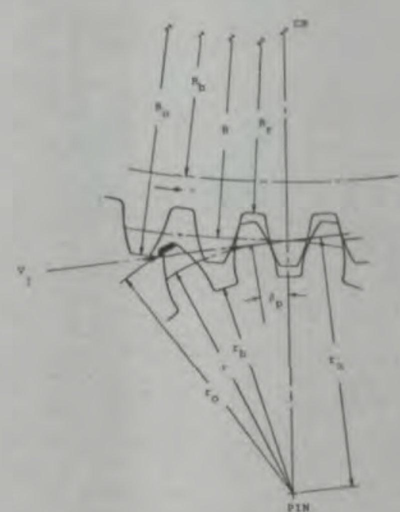

Figure 2.

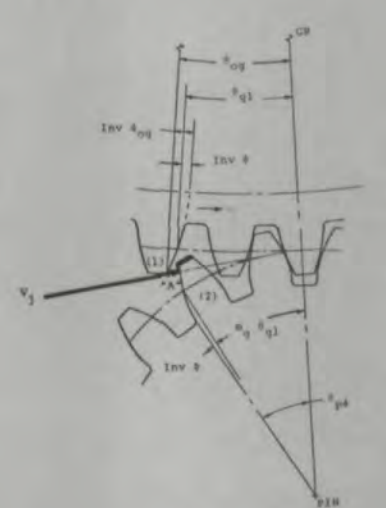

Figure 3.

## Nomenclature

| | | | |
|---|---|---|---|
| $a$ | $1/P_d$ or $(1 \pm \Delta N/2)/P_d$ = addendum | $t$ | time |
| $b_p$, $b_g$ | pinion and gear backlash, respectively | $t_f$, $t_\omega$ | time of flight, rotation |
| $B_p$, $B_g$ | total, pinion, gear backlash at $P_d = 1$ | $V_p = V_g$ | linear velocity of pinion and gear at pitch line |
| $d_p$, $d_g$ | radial impingement depth | $V_j$, $V_{jp}$ | oil jet velocity, general, pinion controlled |
| $L_p$, $L_g$ | pinion, gear final impingement distance | $x$ | distance from offset perpendicular to jet line intersection |
| $L_{ig}$ | intermediate impingement distance | | |
| $m_g$ | $N_g/N_p = R/r = \omega_p/\omega$ = gear ratio | $V_j(\max)_p$ | maximum velocity at which $d_p = 0$ |
| $N_p$, $N_g$ | number of teeth in pinion, gear | $V_j(\min)_p$ | minimum velocity at which $d_p = 0$ |
| $\Delta N$ | differential number of teeth | $\beta$ | arbitrary oil jet inclination angle |
| $P_d$ | diametral pitch | $\beta_p$ | constrained inclination angle |
| $r$, $R$ | pinion and gear pitch radii | $\beta_{pp}$ | inclination angle for pitch point intersection |
| $r_\alpha$, $R_\alpha$ | perpendicular distance from pinion, gear center to jet line | $\varphi$ | pressure angle at pitch circles |
| | | $\varphi_{pi}$, $\varphi_{gi}$ | pinion and gear pressure angle at points specified at i |
| $r_s$, $R_s$ | distance along line of centers to jet line origin | | |
| $r_x$, $R_x$ | distance along line of centers to jet line intersection at x | $\omega_p$, $\omega_g$ | pinion gear and angular velocities |
| | | inv $\varphi$ | $\tan\varphi - \varphi$ = involute function at pitch point or operating pressure angle |
| $r_o$, $R_o$ | pinion and gear outside circle diameter | | |
| $r_b$, $R_b$ | pinion and gear base radii | $V_j(\text{Opt}, U)_p$ | upper limit jet velocity to impingement at pitch line (at upper end of plateau) |
| $S$, $S_o$, $S_p$ | arbitrary jet nozzle offset to intersect O.D.'s offset for pinion only | | |

Then the position of the pinion at time equal to zero ($t = 0$) is calculated from:

$$\theta_{p4} = m_g\theta_{g1} + \text{inv}\,\varphi \tag{5}$$



which locates the pinion at the time of the flight of the *tail* of the jet stream when it is initiated. (See Fig. 3.) The jet tail continues to approach the trailing side of the pinion tooth profile until it reaches the position shown in Fig. 4, when it terminates at time $t = t_f$. The position of the pinion at this time is calculated from:

$$\theta_{p5} = \tan^{-1}\left(\frac{L_p\cos\beta_p}{r - L_p\sin\beta_p}\right) + \text{inv}\,\varphi_{p5} \tag{6}$$

where:

$$\text{inv}\,\varphi_{p5} = \tan\varphi_{p5} - \varphi_{p5}$$

$$\varphi_{p5} = \cos^{-1}\left(\frac{r_b}{[(r - L_p\sin\beta_p)^2 + (L_p\cos\beta_p)^2]^{1/2}}\right) \tag{7}$$

(See Fig. 4.)

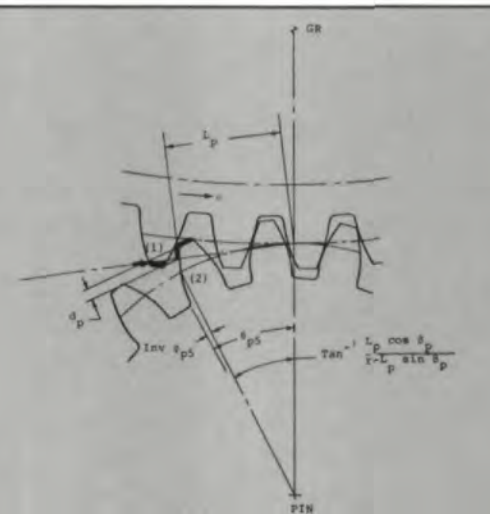


Figure 4.

TABLE 1. – EQUATIONS FOR OIL JET VELOCITY AND PINION IMPINGEMENT DEPTH FOR PITCH LINE AND HIGHER OIL JET VELOCITY

| Relative velocity scale | Oil jet velocity | Pinion Impingement Depth |
|---|---|---|
| Critical high velocity to miss the pinion | $V_j(\max)_p = \frac{*[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - (r_o^2 - r_s^2)^{1/2} \sec \beta_p]\,\omega_p}{\theta_{p6} - \theta_{op}}$<br>$V_j(\max)_p \to \infty$, when $s = s_o$ and $\beta_p = \beta_{pp}$ | $d_p(\min, U) = 0^*$ $(m_g = 1)$<br>*when $m_g \geq m_g(\text{crit})$ and $0 \leq s < s_o$ and<br>$0 \leq \beta_p < \beta_{pp}$ |
| Higher than pitch line velocity up to where the jet starts to miss the pinion | $V_j = \frac{\omega_p[(R_o - R_s)^{1/2} \sec \beta_p - L_p]}{\theta_{p4} - \theta_{p5}}$<br>$V$ = given (when $0 \leq d_p \leq a$ only) | $d_p$ = given (usual design solution and<br>$L_p = [(r_o - d_p)^2 - r^2 \cos^2 \beta_p]^{1/2} + r \sin \beta_p$<br>Iterate $L_p$ from:<br>$[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p]\,\omega_p = (\theta_{p2} - \theta_{p1})V_j$<br>$d_p = r_o - [(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2}$ |
| Slightly higher and pitch line-upper end of velocity plateau | $V_j(\text{opt}, U)_p = \frac{\omega_g (R_o^2 - R_s^2)^{1/2} \sec \beta_p}{\theta_{g1}}$<br>$V_j = \omega_p r \sec \beta_p = \omega_g R \sec \beta_p$ | $d_p(\text{tail}) = a$, (trailing profile only)<br>$d_p = a = (1 \pm \Delta N_p/2)/P_d$, (both profiles) |

The *design* solution to the problem of pinion cooling when "$d_p$" is specified is to solve explicitly for the jet $V_j$ based on the fact that $t_f = t_\omega$ as shown in Part I of this article. Thus, the required jet velocity is calculated from:

$$V_j = \frac{[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p]\omega_p}{\theta_{p4} - \theta_{p5}} \tag{8}$$

where:

$$L_p = [(r_o - d_p)^2 - r^2 \cos^2 \beta_p]^{1/2} + r \sin \beta_p \tag{9}$$

The *analysis* solution to the problem when $V_j$ is specified $[V_j(\text{Opt, U})_p < V_j < \infty]$ so that the resulting impingement depth $d_p$, can be calculated implicitly by solving iteratively for "$L_p$" from:

$$\omega_p[(R_o^2 - R_s^2)^{1/2} \sec \beta_p - L_p] = (\theta_{p4} - \theta_{p5})V_j \quad \text{then} \tag{10}$$

$$d_p = r_o - [(r - L_p \sin \beta_p)^2 + (L_p \cos \beta_p)^2]^{1/2} \tag{11}$$

(See Fig. 1.)

Moving up along the velocity scale of Table 1 from $\omega_p r \sec \beta_p = V_j$, it can be shown that the upper limit $V_j(\text{Opt, U})$ for the "constant impingement depth range" where $d_p = a$, can be calculated for the pinion from:

$$V_j(\text{Opt, U})_p = \omega_g (R_o^2 - R_s^2)^{1/2} \sec \beta_p / \theta_{g1} \tag{12}$$

Thus, if the jet velocity $V_j$ is between the lower limit $V_j(\text{Opt, L})_p < V_j < V_j(\text{Opt, U})_p$, then the impingement depth will be $d_p = a = 1/P_d$ on at least one side of the tooth profile. If $V_j = \omega_p r \sec \beta_p$ exactly, then *$d_p = a$ on both sides of the tooth profile.* Increasing the jet velocity above $V_j(\text{Opt, U})_p$ reduces the impingement depth $d_p$ until at $V_j(\max)_p$ the tail of the jet chopped by the gear tooth is moving so fast as to be just missed by the pinion top land leading edge "A" in Fig. 6 when $m_g > m_g(\text{crit})$. The upper limit critical gear ratio, as a function of $N_p$ and assuming $V_j(\max)_p = \infty$, may be calculated from:

$$m_g(\text{crit}) = \frac{\cos^{-1}\frac{N_p}{N_p + 2} - \text{inv}\left(\cos^{-1}\frac{N_p \cos\varphi}{N_p + 2}\right) + \text{inv}\varphi + \pi/N_p - 2B_p/N_p}{\cos^{-1}\left(\frac{m_g(\text{crit})N_p + 2P_dS}{m_g(\text{crit}) + 2}\right) - \text{inv}\cos^{-1}\left(\frac{N_p \cos\varphi}{N_p + 2/m_g(\text{crit})}\right) + \text{inv}\varphi} \tag{13}$$

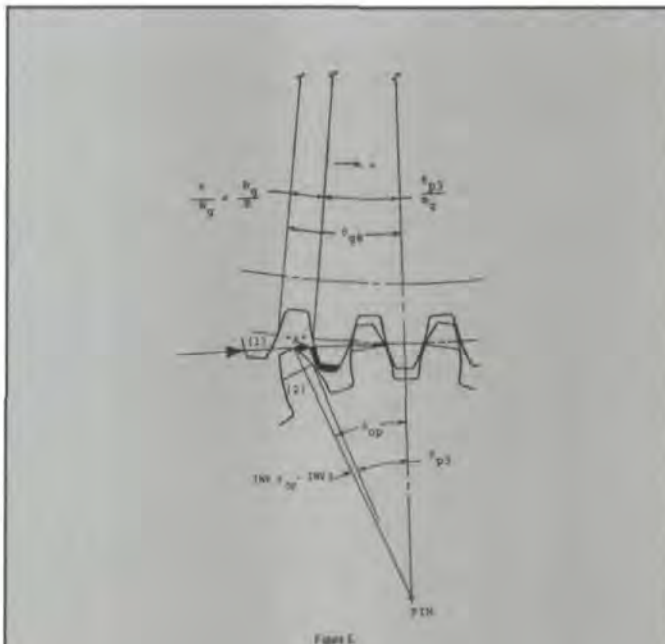

Figure 6

where:

$$N_p' = [N_p \cos^2 \beta_p - \{(N_p + 2)^2 - N_p^2 \cos^2 \beta_p\}^{1/2} \sin \beta_p] \tag{14}$$

and

$$\beta_p = \tan^{-1}\{S\,P_d/[m_g(\text{crit})N_p(1 - P_dS)^2 - (P_dS)^2]^{1/2}\} \tag{15}$$

and

$$P_dS = \frac{(S/S_o)[m_g(\text{crit}) - 1]}{[m_g(\text{crit}) + 1]} \tag{16}$$

when $S = S_o$ and $\beta = \beta_{pp}$, $V_j(\max)_p < \infty$, the $m_g(\text{crit})$ ceases to exist.

When the maximum jet stream velocity ($V_j = V_j(\max)_p$) is reached, the initial position of the pinion as the gear tooth chops the tail of the jet stream may be calculated from:

$$\theta_{p6} = m_g\theta_{g1} - \pi/N_p - \text{inv}\,\varphi + \text{inv}\,\varphi_{op} + 2\,B_p/N_p \tag{17}$$

(See Fig. 5.)

The final position at the point "A" in Fig. 6 is calculated from:

$$\theta_{op} = \cos^{-1}\left(\frac{r_s}{r_o}\right)$$

The maximum jet velocity may then be calculated from:

$$V_j(\max)_p = \frac{\omega_p[(R_o^2 - R_s^2)^{1/2}\sec\beta_p - (r_o^2 - r_s^2)^{1/2}\sec\beta_p]}{\theta_{p6} - \theta_{op}} \tag{18}$$

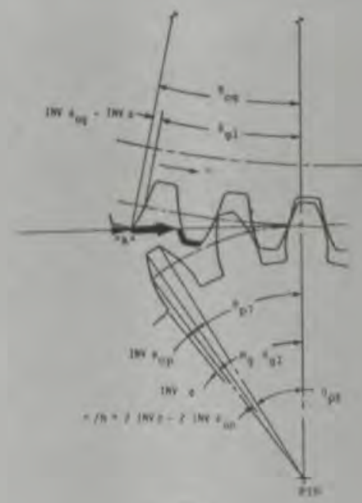

Figure 5.

Therefore, when $V_j \geq V_j(\max)_p$, $d_p = 0$, if $m_g > m_g(\text{crit})$. Also if $S = S_o$ and $\beta = \beta_p = \beta_{pp}$, then $V_j(\max)_p \to \infty$. $V_j(\max)_p = V_j(\text{Opt, U})$ then $V_j(\max)_p$ is set equal to $V_j(\text{Opt, U})$. Stated differently, when $V_j(\text{Opt, U})$ is greater than or equal to the calculated $V_j(\max)_p$, then $V_j(\max)_p$ no longer physically represents the solution, and $V_j(\text{Opt, U})$ is the maximum value that can be allowed for $V_j$.

Again, it should be noted that the selection or specification for $V_j$ must be kept within the bounds of $\omega_p r \sec\beta_p$ and Equation 18 if impingement on the trailing side of the tooth profile is desired.

Equations 8 through 18 have been summarized in Table 1 on a velocity scale to add graphic visibility to their usability range.

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The Gear – When the Jet Velocity is Greater than Pitch Line Velocity ($V_j > \omega_g R \sec \beta_p$).

The sequence of events for the gear in the case where $V_j > \omega_g R \sec \beta_p$ is shown in Figs. 5, 7 and 8. Again, instead of tracking the head, the trailing end or "tail" of the jet stream will be tracked after it is chopped at time ($t = 0$) as shown at "A" in Fig. 7, to the final impingement point at a depth "$d_g$" at time $t = t_\omega$, as shown in Fig. 8. The position of the pinion at time ($t = 0$) may be calculated from $\theta_{p3}$, defined above. The associated gear position can be calculated from:

$$\theta_{g6} = (\theta_{p3}/m_g) + \text{inv}\,\varphi \tag{19}$$

which locates the gear at the time ($t = 0$) when the flight of the *tail* of the jet stream is initiated. (See Fig. 7.) The position of the gear when the jet stream *tail* is terminated on the gear may be calculated from:

$$\theta_{g7} = \tan^{-1}\left(\frac{L_g \cos \beta_p}{R + L_g \sin \beta_p}\right) + \text{inv}\,\varphi_{g7} \tag{20}$$

*where*

$$\text{inv}\,\varphi_{g7} = \tan \varphi_{g7} - \varphi_{g7}$$

$$\varphi_{g7} = \cos^{-1}\left(\frac{R_b}{[(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2}}\right) \tag{21}$$

at time ($t = t_\omega$), as shown in Fig. 8.

Once again, when $0 \leq \beta_p < \beta_{pp}$ and $0 \leq S < S_o$, the *analysis* solution to the problem of cooling the gear is constrained by the jet velocity limits for the pinion to maintain impingement on same. And, as explained above, a given "gear mesh" must have a common jet velocity. Accordingly, a given impingement depth is selected for the pinion. Then, the associated jet velocity $V_{jp}$ is solved for this velocity, which can then be used to find the associated gear impingement depth "$d_g$". Thus, after finding $V_{jp}$, solve for $L_g$ iteratively from:

$$[(r_o^2 - r_s^2)^{1/2} \sec \beta_p - L_g]\omega_g = (\theta_{g6} - \theta_{g7})V_{jp} \tag{22}$$

and

$$d_g = R_o - [(R + L_g \sin \beta_p)^2 + (L_g \cos \beta_p)^2]^{1/2} \tag{23}$$

*(continued on page 45)*

Figure 7.

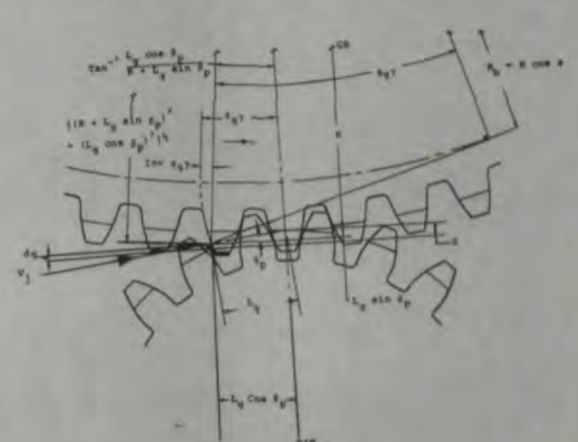

Figure 8.

*(continued from page 33)*

TABLE 2. – EQUATIONS FOR OIL JET VELOCITY AND GEAR IMPINGEMENT DEPTH FOR PITCH LINE AND HIGHER OIL JET VELOCITIES

| Relative velocity scale | Oil jet velocity | Pinion Impingement Depth |
|---|---|---|
| Critical high velocity to miss the gear | $V_j(\max)_g^{*} = \dfrac{\omega_g[(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_o^2)^{1/2}]\sec\beta_p}{\theta_{g1} - \theta_{g8}}$ <br> $= V_j(\max)_p \to \infty$, when $S = S_o$ and $\beta_p = \beta_{pp}$ | $d_g(\min, U) = R_o - \{[R + L_g(\max)\sin\beta_p]^2 + [L_g(\max)\cos\beta_p]^2\}^{1/2}$ <br> *when $m_g \geq m_g(\text{crit})$ <u>only</u> <br> and $d_g(\min, u) = 0$ when $S = S_o$ and $\beta_p = \beta_{pp}$ |
| Greater than pitch line velocity up to where the oil jet starts to miss the gear | $V_j = \dfrac{\omega_g\{(r_o^2 - r_s^2)^{1/2}\sec\beta_p - [(R_o - d_g)^2 - (R\cos\beta_p)^2]^{1/2} - R\sin\beta_p\}}{\theta_{g6} - \theta_{g7}}$ <br> where $V_j(\min)_p \leq V_j \leq V_j(\max)_p$ <br> $V_j$ given | $d_g$ given (usual design solution) <br> $L = [(R_o - d_g)^2 - R^2\cos^2\beta_p]^{1/2} - R\sin\beta_p$ <br> Iterate $L_g$ from: <br> $[(r_o^2 - r_o^2)\sec\beta_p - L_g]\omega_g = (\theta_{g1} - \theta_{g2})V_j$ then <br> $d_g = R_o - [(R + L_g\sin\beta_p)^2 + (L_g\cos\beta_p)^2]^{1/2}$ |
| Slightly higher and pitch line-upper end of velocity plateau | $V_j(\text{opt}, U)_g = \dfrac{\omega_p(r_o - r_s)^{1/2}\sec\beta_p}{\theta_{p3}}$ <br> $V_j = \omega_g R\sec\beta_p$ (@ pitch point) | $d_g = a = \dfrac{1}{P_d} \pm \dfrac{\Delta N}{2P_d}$ (trailing profile) <br> $d_g = a = \dfrac{(1 \pm \Delta N/2)}{P_d}$ (both profiles) |

for $V_j(\text{Opt}, U)_p < V_j < V_j(\max)_p$.

The *design* solution to the problem when $V_j(\min)_g < V_j < V_j(\max)_g$ may be calculated from:

$$V_j = \frac{[(r_o^2 - r_s^2)^{1/2}\sec\beta_p - \{(R_o - d_g)^2 - R^2\cos^2\beta_p\}^{1/2} + R\sin\beta_p]\omega_g}{\theta_{g6} - \theta_{g7}} \quad (24)$$

with the additional restriction that $V_j(\text{Opt}, U)_g < V_j < V_j(\max)_g = V_j(\max)_p$. As for the others, Equation 24 is shown placed on the velocity scale of Table 2.

Also if $V_j$ is specified within the range allowed for $V_{jp}$ for Equation 8 and 24, then $d_g$ can be calculated implicitly by solving iteratively for "$L_g$" from:

$$[(r_o^2 - r_s^2)^{1/2}\sec\beta_p - L_g]\omega_g = (\theta_{g6} - \theta_{g7})V_{jp} \quad \text{and} \quad (25)$$

$$d_g = R_o - [(R + L_g\sin\beta_p)^2 + (L_g\cos\beta_p)^2]^{1/2} \quad (26)$$

<u>When the Jet Velocity is Equal to Pitch Line Velocity ($V_j = \omega_g R\sec\beta_p$)</u>

Continuing up the velocity scale of Table 2 from $V_j = \omega_g R \sec\beta_p$, it can be shown that the upper limit for the constant impingement depth range, where $d_g = a$, can be calculated for the gear from:

$$V_j(\text{Opt}, U)_g = \omega_p[(r_o^2 - r_s^2)^{1/2}\sec\beta_p \div \theta_{p3} \quad (27)$$

Thus, if the jet velocity $V_j$ is between $V_j(\text{Opt}, L)_g < V_j < V_j(\text{Opt}, U)_g$, the impingement depth will be $d_g = a = 1/P_d$ on at least one side of the gear tooth profile. If $V_j = \omega_g R\sec\beta_p$ exactly, the $d_p$ = "a" on both sides of the tooth profile.

Increasing the jet velocity above $V_j(\text{Opt}, U)_g$ reduces the impingement depth $d_g$ until $V_j = V_j(\max)_g$. When $S < S_o$, the tail of the jet chopped by the gear tooth is moving so fast as to be just missed by the pinion top land so that $d_p = 0$ and $m_g = m_g(\text{lim.})$. When $S = S_o$ and $V_j(\max)_g \to \infty$, then $d_p \to 0$.

The initial position of the gear when it chops the tail of the leading jet stream is $\theta_{g1}$ as defined above when $m_g < m_g(\text{crit})$

# Classified



and as shown in Fig. 5 (for $S = 0$). The limit position "A" in Fig. 6 as the jet tail just misses the pinion top land is calculated from:

$$\theta_{g8} = (\theta_{p3}/m_g) + \pi/N_g + 2\,B_g/N_g \qquad (28)$$

$V_j(\max)_g$ in gear parameters may be calculated from:

$$V_j(\max)_g = \frac{\omega_g[(R_o^2 - R_s^2)^{1/2} - (r_o^2 - r_s^2)^{1/2}]\sec\beta_p}{\theta_{g1} - \theta_{g8}} \ (m_g \neq 1) \qquad (29)$$

Note that as $V_g(\max)_g \to \infty$ for $m_g \leq m_g(\text{crit})$ and when $m_g > m_g(\text{crit})$, then $V_j(\max)_g$ is finite at $d_p = 0$.

The impingement distance $L_g(\max)$ when $V_j = V_j(\max)_g$ may be iterated from:

$$\omega_g[(r_o^2 - r_s^2)^{1/2}\sec\beta_p - L_g(\max)] = (\theta_{g6} - \theta_{g7})V_j(\max)_g \qquad (30)$$

Then, when $S < S_o$

$$d_g = d_g(\min, U) = R_o - \{[R + L_g(\max)\sin\beta]^2 + [L_g(\max)\cos\beta]^2\}^{1/2} \qquad (31)$$

If $V_j(\max)_g \leq V_j(\text{Opt}, U)$ then set $V_j(\max)_g$ equal to $V_j(\text{Opt}, U)$. Also, it should be observed that since only one $V_{jp}$ can be used, we must set Equations 18 and 29 equal: $V_j(\max)_p = V_j(\max)_g$ for the given $m_g$, making the design $m_g = m_g(\lim)$ when $m_g > m_g(\text{crit})$.

### Summary

An analysis was conducted for into-mesh oil jet lubrication with an arbitrary offset and inclination angle from the pitch point for the case where the oil jet velocity is equal to or greater than pitch line velocity. Equations were developed for minimum and maximum oil jet impingement depths. The equations were also developed for the maximum oil jet velocity allowed, so as to impinge on the pinion and the optimum oil jet velocity required to obtain the best lubrication condition of maximum impingement depth and gear tooth cooling. The following results were obtained:

1. The optimum operating condition for best lubrication and cooling is provided when the jet velocity is equal to pitch line velocity $V_j = V_g \sec\beta_p = \omega_p r \sec\beta_p = \omega_g R \sec\beta_p$, whereby

both sides of the pinion and gear will be wetted, and the maximum impingement depth to the pitch line will be obtained.

2. When the jet velocity is slightly greater than the pitch line velocity, $\omega_p r \sec \beta_p < V_j < V_j(\text{Opt, U})$, the loaded side of the driven gear is favored and receives the best cooling with slightly less oil impingement than when $V_j = \omega_p r \sec \omega_p$.
3. As the jet velocity becomes much greater than the pitch line velocity, $V_j(\text{Opt, U}) < V_j < V_j(\max)_p$, the impingement depth is considerably reduced. As a result, the pinion may be completely missed by the lubricant so that no direct cooling of the pinion is provided when $V_j(\max)_p \leq V_j$.

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