A New Method of Designing Worm Gears

by Michel Octrue C.E.T.I.M. Senlis, France

AUTHOR:

DR. MICHEL OCTRUE *received his degree in engineering at the E.C.A.M. of Lyon (France), and his degree Thesis at the University of Besançon (France). He is with the Gear Department of the Centre Technique des Industries Mécaniques (CETIM), where he is a specialist in worm gear practice and design. Dr. Octrue is a member of the International Standards Organization (ISO), Technical Committee 60 (Gears), Working Groups WG-4 (Nomenclature) and WG-7 (Worm Gears). He has done extensive work on experimental development of gear programs for computers and is a member of the national standard committees of Association Française de Normalisation (AFNOR) and Union de Normalisation de la Mécanique (UNM) for mechanical transmissions and gears.*

Abstract:

The first part of this article describes the analytical design method developed by the author to evaluate the load capacity of worm gears.

The second part gives a short description of the experimental program and testing resources being used at CETIM to check the basic assumptions of the analytical method; and to determine on gears and test wheels the surface pressure endurance limits of materials that can be used for worm gears.

The end of the article compares the results yielded by direct application of the method and test results.

Introduction

The main form of deterioration observed in worm gears is generally surface damage to the flanks of the gear wheel teeth, comparable to the pitting and flaking found in treated cylindrical gears. It is, therefore, essential to determine the load capacity of these gears by evaluating the torque that can be transmitted, which depends on the surface pressure the materials used can withstand. For this purpose, we developed a design model that can be used to determine the torque that can be transmitted with allowance for the distribution of the pressures along the lines of contact. This model was then experimentally verified using the admissible pressures of the materials as determined by a disk-and-roller simulator. We then compared the results obtained to those yielded by endurance tests carried out on gears.

Presentation of the Design Method

The design method we present here is an analytical method;[3] in other words, the gear is designed with allowance for the meshing conditions at all points of contact and at all times. These meshing conditions depend on the relative positions of the teeth of the gear and the threads of the worm, on the geometry of the contact (curvatures) and on the stiffness of the meshing teeth.

By contrast with some other methods,[6-7] the distribution of contact pressure is not assumed to be uniform at each instant of meshing, but is determined after the transmitted load has been distributed along the instantaneous line of contact. The distribution of the load is determined on the basis of the stiffness of the teeth and the stiffness of contact at each point of contact.

This method was developed in two stages:

First stage: development of a design model making it possible to establish the instantaneous distribution of the transmitted pressure from the geometry of the teeth, the geometry of contact and the mechanical properties of the materials of which the gear and worm are made.

Second stage: determination of the map of pressures along the lines of meshing contact. Since the maximum pressure between the teeth in contact is limited by the admissible pressure of the material, this makes it possible to determine the maximum transmissible load.

First step of the calculation. We first calculate the instantaneous load distribution between the flanks of the worm threads and the flanks of the gear teeth, with allowance for the operating geometry and the materials used.

This geometry results from the contact between the worm threads, generated by grinding or milling (A, I, N, K and other profiles) and the gear teeth, produced by cutting. It is theoretically determined from the basic geometrical characteristics of the worm and gear by calculating, in order:

- the profile of the worm threads according to the cutting method used;
- the transverse path of contact for each rack line of the worm;
- the field of contact or skewed surface on which the lines of contact evolve;
- the lines of contact at each instant of meshing;

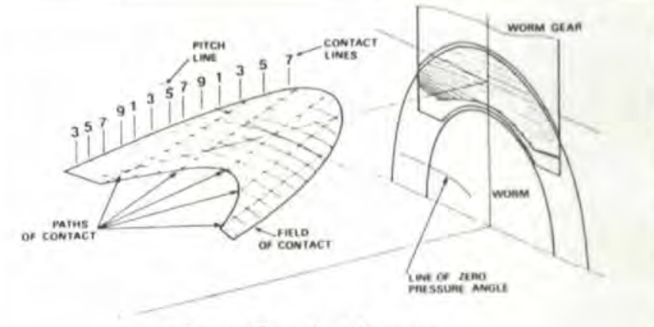


Fig. 1 – Operating Geometry.

- the equivalent radii of curvature and sliding velocities at each point of contact.

All of these calculations are based on the application of envelope theory and analytical geometry. They are made by breaking down the worm gear couple into a succession of elementary rack-and-pinion gears, having variable profiles determined in planes parallel to the midplane of the gear.

Fig. 1 shows the operating geometry of a worm gear having a 40:1 ratio. It shows:

- the transverse paths of contact in seven different rack lines;
- the field of contact with the lines of contact for five relative meshing positions (1, 3, 5, 7, 9);
- the line of zero pressure angle.

These various curves are represented in space and in projection in a plane perpendicular to the axis of the worm. Each point of the zone of contact is identified by two indices, i and j.

- i is the number of the line of contact;
- j is the number of the rack plane.

We define the stiffness of the gear tooth as $(R_R)ij$ and the stiffness of the worm thread as $(R_v ij)$ at each point of contact. To do this, we treat the toothing in each rack plane as a fixed-end beam of variable inertia (cf. Fig. 2).

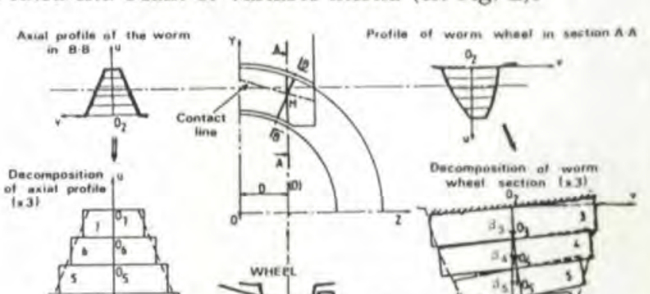


Fig. 2 – Calculation of stiffness.

The bending stiffness of these beams is determined by using Bresse's equations.[2] To determine the equivalent stiffness at the points of contact, we then apply the following two assumptions:

Assumption 1. During meshing, all the points of contact move uniformly, parallel to the operating reference plane of the worm because of the deformation of the teeth (Fig. 3).

Assumption 2. The initial pre-loading contact geometry is maintained during loading. This means that the displacements are sufficiently small and do not modify the initial field of contact.

In addition to the equivalent stiffness (RDeq)ij, the model takes the local contact stiffness (RC)ij into account.

In the zone of application of the load, the contact strains are broken down into deformation resulting from crushing of the teeth and deformation resulting from local compression of the part of the tooth under the contact.

The method used is the one developed and checked experimentally by Weber[8] for cylindrical gears. These deformations are not a linear function of the applied load, so an iterative method must be used to calculate the equivalent stiffness (Fig. 4).

At each point of contact we then have

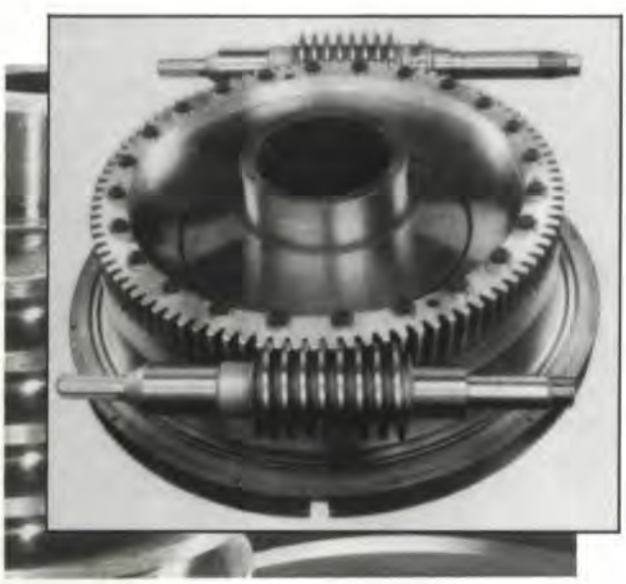
(Req)ij = equivalent radius of curvature

(RDeq)ij = equivalent stiffness of teeth in contact

(RDeqm)ij = mean equivalent stiffness of the teeth in contact calculated along an elementary segment of the line of contact bounded by two successive rack planes.

(qz)ij = transmitted load density along an elementary segment of the line of contact

(RDeq)ij = takes the form



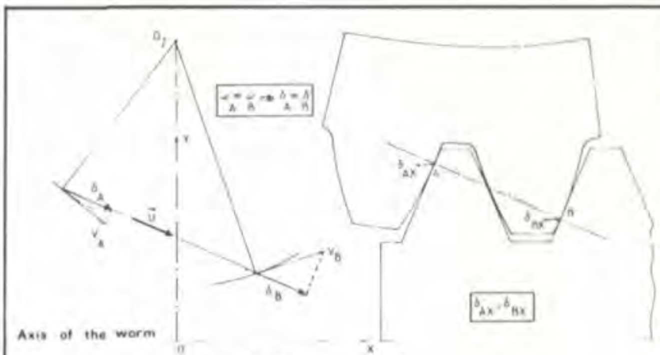


Fig. 3 – Assumption 1.

$$(RDeq)ij = \frac{1}{(R_R)ij} + \frac{1}{(R_v)ij} + \frac{1}{(RC)ij}$$ parallel to the axis of the screw

The total stiffness (RDG) of the gear parallel to the axis of the worm is given by:

$$RDG = \sum_{i_1\ j} (RDeq)ij$$

The total deformation at each point of contact, which we call δij, is the quotient of the mean load density divided by the mean equivalent stiffness. On the basis of Assumptions 1 and 2, we may state:

$$\delta ij = \frac{(qx)ij}{(RDeqm)ij} = \text{constant} = \delta$$

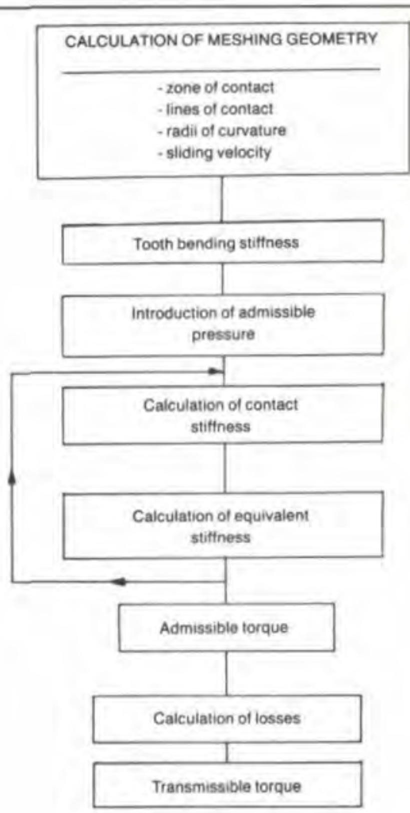


Fig. 4

DETERMINATION OF THE PRESSURES THE MATERIALS CAN WITHSTAND

From the axial force transmitted to the worm and the total stiffness of the worm RDG calculated above, we obtain:

$$\delta = \frac{QX}{RDG} = \delta ij$$

From the mean equivalent stiffness, we determine the distribution of the transmitted load (qx)ij along the lines of contact:

$$(qx)ij = \delta.(RDeqm)ij$$

Second step of the calculation. From the load distribution, which is now known, it is possible to determine the contact pressure at each point of the meshing zone. The maximum pressure found must be less than the admissible pressure of the material σ_{Hlim}. This value is determined experimentally for each material.

Therefore, if we know the materials and the geometry used, it will be possible to reverse the calculation to determine the admissible pressure σ_{Hlim}, to determine the load distribution along the lines of contact and the admissible load on the teeth, and so finally to determine the admissible torque. For this, we use the following procedure. At each point, the contact pressure obtained by applying Hertz's theory (cylindrical contact) is given by

$$(P_M)ij = ZE\sqrt{\frac{QX}{RDG} \cdot \frac{(RDeqm)ij}{(Req)ij\ (X_N)ij}} = ZE\sqrt{\frac{QX}{RDG}\frac{1}{Kij}}$$

where:

$(X_N)ij$ = direction cosine of the normal to the plane of contact

Kij = curvature-rigidity factor $Kij = \left(\frac{Req.\ X_N}{RDeqm}\right)_{ij}$

ZE = elasticity factor such that

$$ZE = \frac{1}{\sqrt{\pi\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)}}$$

with ν_1, ν_2 = Poisson's ratio, E_1, E_2 = Young's moduli } of materials involved

We must verify that

$$(P_M)ij \leqslant \sigma_{Hlim}$$

which allows us to write:

$$QX \leqslant \left(\frac{\sigma_{Hlim}}{ZE}\right)^2 \cdot RDG \cdot Kij$$

At the limit, the maximum load capacity is obtained for the minimum value of $Kij = (Kij)_{mini}$

then

$$QX_{max} = \left(\frac{\sigma_{Hlim}}{ZE}\right)^2 \underbrace{[RDG_P \cdot (Kij)_{minip}]}_{ZR}$$

ZR = pressure distribution factor of the gear.

NOTE: Index p refers to the relative position of the gear and worm. p is chosen so that product $RDG_P \cdot (Kij)_{miniP}$ is minimized, defining ZR. The admissible torque on the gear is then:

$$C = 5.10^{-4} \cdot d_{w2}\left(\frac{\nu_{Hlim}}{ZE}\right)^2 ZR \text{ in m.N}$$

with d_{w2} = pitch diameter of the gear wheel.

Calculation of losses. From the sliding velocity, taking into account the coefficient of friction variation, which is a function of sliding value, it is possible to calculate the power loss at each point of contact and so calculate the instantaneous efficiency of the gearing η.

The true torque that can be transmitted to the gear is then:

$$C_r = \eta.C$$

The calculation method described is, therefore, based on calculating the distribution of stiffness and the distribution of contact pressure during meshing (ZR factor). If the maximum pressure is limited to the admissible pressure of the material δ_{Hlim}, the admissible torque can be deduced and from it, by evaluating the losses in the gearing, the torque that can be transmitted.

The ZR factor is determined using the CADOR-ROUVIS software.

The diagram below shows the various steps of the calculations required for the application of the method described above.

Generally speaking, the load capacity of worm gears is limited by the performance of the material of the gear wheel.

The admissible pressure of this material is determined by evaluating its contact pressure endurance curve. This curve is determined experimentally on a disk roller simulator designed and built at CETIM. *(continued on page 40)*

(continued from page 23)

In worm gears, the sliding induced by the rotation of the worm is greater than the sliding orthogonal to the lines of contact induced by the rotation of the gear wheel. It is essential that the simulator used to evaluate the pressure endurance curve be able to reproduce these components. Conventional roller machines could not be used.

The machine we designed consists of a disk that represents the worm, on which turns and slides a roller that represents the gear.

The axes of the disk and roller are perpendicular (Fig. 5).

It is possible, by adjusting the position of the roller with respect to the disk and the speeds of rotation of the disk and roller, to reproduce any sliding condition that can occur in worm gears.

The test piece is the roller made of the gear material (UE12P bronze, for example). It has the following characteristics:

diameter:	100 mm
transverse crowning radius:	250 mm
width:	25 mm

The disk used is 200 mm in diameter. It is made of ground case-hardened steel.

The speed of rotation used and the corresponding sliding velocity are

wheel:	420 rpm
disk:	855 rpm

giving a mean sliding velocity of 8 m/s.

Obtaining a representative pressure endurance curve for a given pair of materials requires between 7,000 and 10,000 hours of testing, equivalent to the destruction of 30 to 35 rollers. But this method is advantageous because it is faster than bench testing gears. Moreover, the cost of the test pieces and the running costs of the simulator are very low.

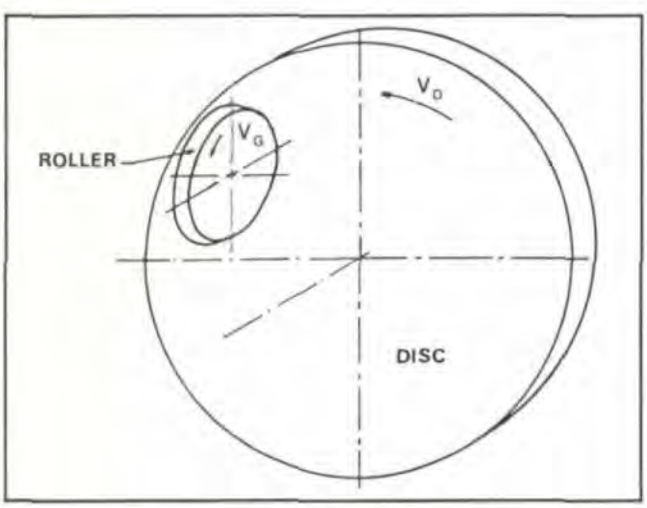


Fig. 5—Principle of disk-roller simulator.

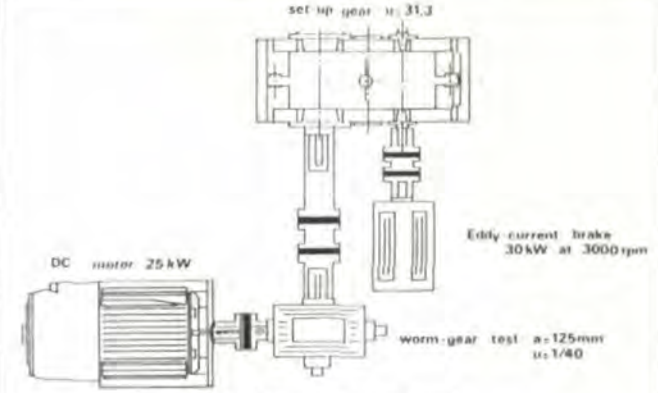


Fig. 6—Worm gear test bench.

Experimental Verification of the Design Method

For a complete verification of the design method, we carried out full-scale tests on worm gears. The test bench we built consists of a variable-speed DC motor that drives the worm of the reducer to be tested directly. The worm wheel at the reducer output is connected to a multiplier that drives an eddy-current brake, used to apply the load (Fig. 6).

The operation of the bench is monitored at all times and the following parameters are measured continually:

- the speed of rotation of the worm,
- the torque delivered by the motor,
- the torque applied by the brake,
- the oil temperature in the reducer housing,
- the temperature of the more heavily loaded worm bearing,
- the instantaneous wear of the test gear.

This last measurement is made by using a special device built by CETIM, based on the use of optical encoders (sensitivity 0.05 mm).

The test bench also has a lubrication system with oil circulation and cooling. The principle of this test bench is shown in Fig. 6.

Characteristics of the test gearing:

• number of threads:	1
• number of teeth:	40
• axial module:	4.95
• center distance:	125 mm
• helix angle:	5.5°
• face width of wheel:	45 mm
• axial pressure angle:	22°
• type of thread profile:	A

- synthetic oil
- worm made of case-hardened steel
- gear made of UE12P bronze

Application of the Design Method

The calculation model described below is contained in a program developed on the VAX 780. If we apply this method to the design of the test gear, we find:

- elasticity factor: $ZE = 498$
- *pressure distribution factor:* $ZR = 1693$

- integrated efficiency (calculated): $\eta = 0.62$
  (mineral oil)
- sliding velocity: 4.4 m/s
- speed of worm: 1600 rpm
- pitch diameter of gear: 198 mm

The measured efficiency was 0.8. The difference we found between the measured and calculated values comes from the fact that the efficiency was calculated on the basis of the "friction versus sliding velocity" curve taken from standard BS 721.

This curve is for a mineral oil, and the result it gives is low when the oil actually used is a synthetic oil with low-friction additives.

Fig. 7 shows the projection along the lines of contact on the flanks of the gear wheel of:
- the distribution of the equivalent radii of curvature;
- the distribution of the mean equivalent stiffness
- the distribution of the contact pressures.

It will be noted that the pressure distribution is to a first approximation the direct combination of the equivalent radii of curvature and the equivalent stiffness.

Comparison of the Results Obtained

The first experimental results obtained on these test benches now cover 12,000 hours for the disk and roller machine and 10,000 hours on the worm gear machine. The grades of materials used on these test benches are strictly identical.

Thirty rollers have been tested on the disk-roller simulator, yielding the pressure versus number of cycles curve given in Fig. 8. The pressure indicated on the y-axis corresponds to the Hertz maximum pressure obtained on the path, which is clearly visible after about four hours running.

The test gearing was subjected to a torque of 115 daN.m on the gear. The first pitting appeared on the flanks of the teeth after 3200 hours of tests, corresponding to $7.68\ 10^6$ loading cycles.

It should be noted that the surface damage found was located primarily at the roots of the gear teeth and on the meshing exit of the worm threads. This finding correlates perfectly with the pressure distribution maps given in Fig. 7.

Looking at the endurance curve obtained on the simulator, we find that $7.68\ 10^6$ cycles corresponds to an admissible pressure of 460 MPa (Fig. 8).

The calculation method, applied to this admissible pressure value, gives us the following transmissible torque:

$$C_r = 5.10^{-4} \cdot d_{w2} \cdot \eta \cdot \left(\frac{\sigma_{Hlin}}{ZE}\right)^2 .ZR =$$

$$C_r = 5.10^{-4} \times 198 \times \left(\frac{460}{498}\right)^2 \times 1693 \times 0.8$$

$$C_r = 114.4 \text{ daN.m}$$

This is very close to the value applied to the gear on the test bench.

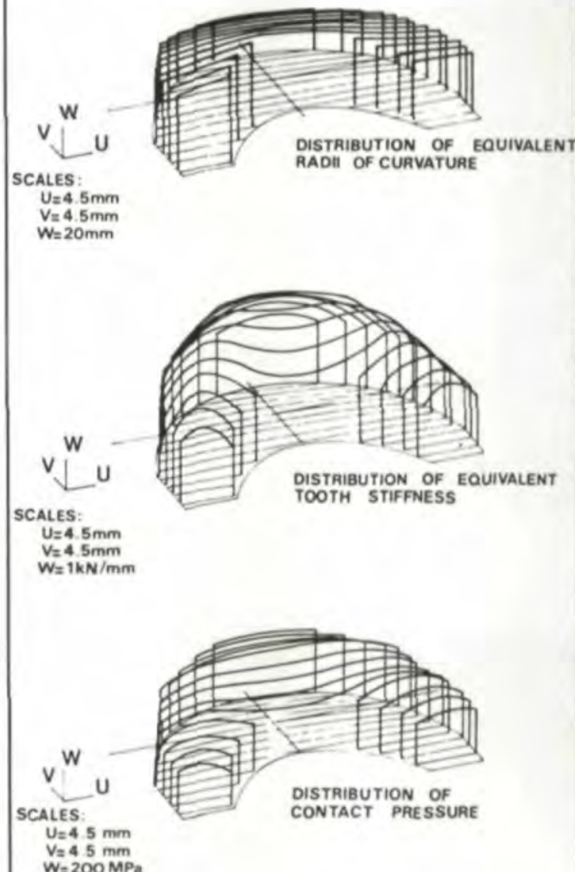


Fig. 7—Distribution along contact line.

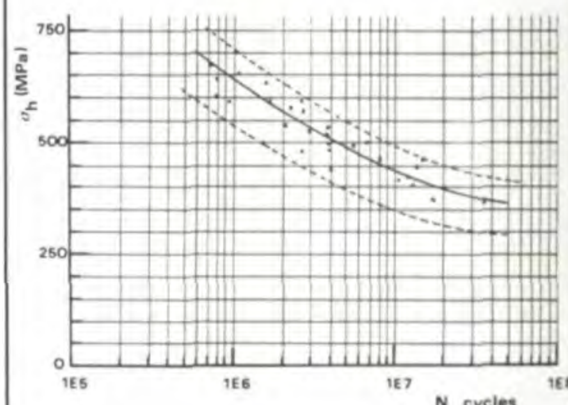


Fig. 8—Pressure cycles curves for UE12P bronze.

VIEWPOINT

Dear Editors:

The magazine *Gear Technology* is praiseworthy. I keep every issue for reference.

The May/June issue that arrived today contained some typos in the mathematical equations that will cause incorrect answers. I'm confident that Mr. Ilya Bass submitted (or has) the correct equations, because his sample calculation resulted in the correct value for M_s.

In Equation 1 on page 26 and the equation for M_s near the top of the second column of page 28, the subscript "n" does not belong with the term inv ϕ. As written, the equation yields a result for the sample calculation that errs by more than 0.0052 inches.

Sincerely,

Evan L. Jones.
Gear Engineer
Chrysler Motors Corporation

Mr. Bass' response;

I can understand Mr. Jones' confusion. When I defined the values for Equation 1, I perhaps should have specified that M_s is the *spanned* measurement in the *normal* plane. The article is correct as printed.

Conclusions

The experimental results are in good agreement with the results yielded by the analytical design method. Other tests are now in progress to validate this new approach to the design of worm gears. Other materials are also being tested on our disk-roller simulator.

Appendix

UE 12 P bronze is a chilled cast bronze with 12% of tin and less than 1% of phosphor. It is equivalent to SAE 65 bronze.

References:

1. HENRIOT, G. *Theoretical and Practical Treated in Gearing* T1 & T2 – Ed. DUNOD Paris, 1979 (In French)
2. OCTRUE, M. and M. DENIS *Geometry of Worm Gears* Technical Note 22 – CETIM 1982. (In French)
3. OCTRUE, M. Analytical Method for Rating Worm Gears. Doctoral Thesis. – 1985. (In French)
4. OCTRUE, M. Software CADOR–ROUVIS – CETIM 1988 (In French)
5. OCTRUE, M., M. DENIS, and M. FAURE – "Rating Method For Worm Gears" 2éme CME – Paris 1986. (In French)
6. BS 721 – Specification for worm gearing – 1984
7. WILKESMANN, H. "Calculation of cylindrical worm gear drives of different tooth profiles" – *ASME Mechanical Design*, 1981, Vol. 103.
8. WEBER, C. "The Deformations of Loaded Gears and the Effects on Their Load-Carrying Capacity." British Dept. of Sci. and Indust. Research – Report n° 3, 1949.

ACKNOWLEDGEMENT: *Reprinted with permission of the American Gear Manufacturers Association. The opinions, statements and conclusions presented in this paper are those of the Author and in no way represent the position or opinion of the AMERICAN GEAR MANUFACTURERS ASSOCIATION.*