On The Interference of Internal Gearing

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Introduction

Since size and efficiency are increasingly important considerations in modern machinery, the trend in gear design is to use planetary gearing instead of worm gearing and multistage gear boxes. Internal gearing is an important part of most of planetary gear assemblies. In external gearing, if the gears are standard (of no-modified addenda), interference rarely happens. But in an internal gearing, especially in some new types of planetary gears, such as the KHV planetary, the Y planetary, etc.,[1] various types of interference may occur. Therefore, avoiding interference is of significance for the design of internal gearing.

There are two categories of interference: cutting interference and meshing interference. The former is certainly related to the dimensions of the cutter. The latter is calculated through the dimensions of the meshing gears, which also bear relation to the cutter. Therefore, it is suggested that in calculating the geometrical dimensions and interferences of an internal gearing, the method of gear tooth generation and the parameters of the cutter to be used should be taken into account. However, this point of view has been neglected in most handbooks, textbooks and papers. For example, only one kind of interference is introduced and is based on the assumption that the gears are cut by a hob or a rack type cutter;[2] the formulae to determine the proportion of an internal gear tooth are borrowed from those for an external gear tooth;[3] and some standards, such as AGMA's, have not covered the internal gearing. Most internal and some external gears are cut by gear shaper cutters, not by hobs. The dimensions of a gear tooth cut by a shaper cutter are different from those cut by a hob. For designing an internal gearing, if we use the method based on hobbing and the formulae converted from those for external gearing, the data obtained seem to be correct, but practically, interferences may still exist, and sometimes the internal gear teeth cannot even be generated. Errors cannot be checked out because those formulae have no relation to the parameters of the cutter. Hence, for providing a correct calculation for geometrical dimensions and interferences, the methods of gear tooth generation and the parameters of the cutter should be discussed.

Methods for Gear Tooth Generation

Internal gears can be made by gear shaping, internal broaching, stamping, milling, etc. Some internal gears with large diameters can also be made by hobbing.[4] External gears can be made by hobbing, gear shaping, milling, rolling, etc. The most common method for generating internal gears is gear shaping, and for external gears is hobbing or gear shaping. In this article only these two methods will be discussed. For simplicity, "pinion" and "gear" are used for external gears and internal gears, respectively. The first thing that should be determined for designing an internal gearing is the method of generating gear teeth. There are two methods – shape-hobbed, wherein the gear is shaped and the pinion is hobbed, and double-shaped, where both the pinion and the gear are shaped.

Fig. 1 is the final position of cutting a pinion by a hob. M-M is the middle line on the hob. The root radius of the pinion cut by the hob is determined by this position; i.e.,

$$R_{f1} = 0.5N_1/P - a_h + X_1/P \quad (1)$$

where: R_{f1} – root radius of the pinion
N_1 – number of teeth of the pinion
P – diametral pitch of both the pinion and the hob
X_1 – addendum modification coefficient of the pinion
a_h – addendum of the hob.

Usually, $a_h = a + c$, where a is the standard addendum and c is the standard clearance.

The involute tooth profile is not an entire involute curve. It is composed of three different curves. The tip circle and the root circle are circular arcs. The active profile is an involute of a circle, which ends at the point K_1, where the tip of the hob intersects the line of contact pT. On the pinion from K_1 to the root, a curve is formed by the locus of the tip of the hob and is a modified involute of a circle (hidden line in Fig. 1). The pressure angle at the circle with radius $O_1 K_1$ is ϕ_{g1}, and

$$\text{TAN } \phi_{g1} = \text{TAN } \phi_o - 4(a_h P - X_1)/(N_1 \text{ SIN } 2\phi_o) \quad (2)$$

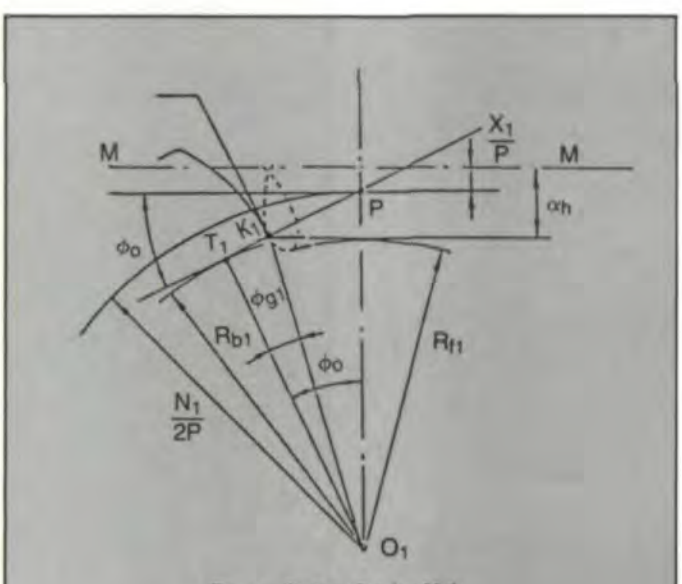

Fig. 1–Pinion Cut by Hob.

where ϕ_o is the standard pressure angle.

If the tip of the hob is rounded with a radius R_t, and $c > R_t(1 - \text{SIN } \phi_o)$, Equation 2 can still be used for calculating meshing interference.

Fig. 2 is the final position of cutting a pinion by a shaper cutter. The operating pressure angle between the cutter and the pinion at this position is ϕ_{1c}, and its involute function is

$$\text{INV } \phi_{1c} = 2(X_1 + X_c)\text{ TAN } \phi_o/(N_1 + N_c) + \text{INV } \phi_o \quad (3)$$

where: N_c – number of teeth of the cutter
X_c – addendum modification coefficient of the cutter.

The center distance between the cutter center and the pinion center at this position is

$$C_{1c} = 0.5(N_1 + N_c)\text{ COS } \phi_o/(P \text{ COS } \phi_{1c}) \quad (4)$$

The root radius of the pinion cut by the shaper cutter is determined by this final cutting position and the parameters of the cutter, or

$$R_{f1} = C_{1c} - R_{ac} \quad (5)$$

where R_{ac} is the radius of tip circle of the cutter.

The locus of the tip point K_1 on the cutter forms an epitrochoid on the fillet or the flank of the pinion (hidden line in Fig. 2). The pressure angle at the circle with radius $O_1 K_1$ is ϕ_{g1} and

$$\text{TAN } \phi_{g1} = (N_1 + N_c)\text{ TAN } \phi_{1c}/N_1 - N_c \text{ TAN } \phi_{ac}/N_1 \quad (6)$$

where ϕ_{ac} is the pressure angle at the tip circle of the cutter.

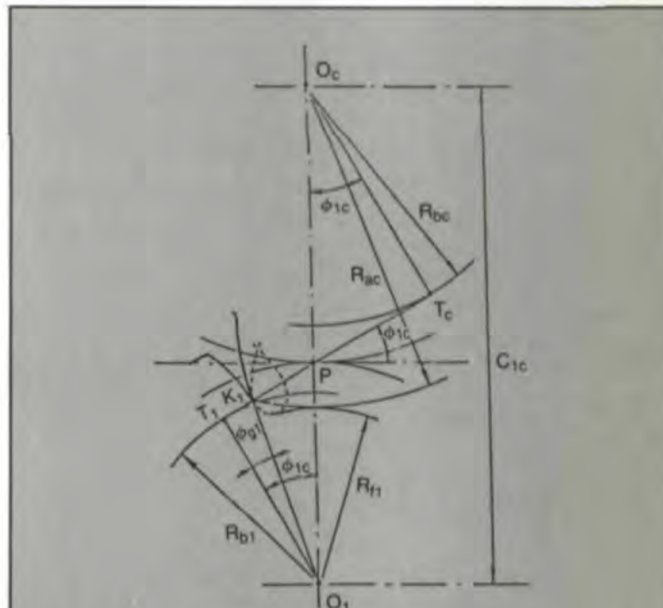

Fig. 2–Pinion Cut by Shaper Cutter.

Fig. 3 is the final position of cutting an internal gear by a shaper cutter. The operating pressure angle between the cutter and the gear at this position is ϕ_{2c}, and its involute function is

$$\text{INV } \phi_{2c} = 2\,(X_2 - X_c)\,\text{TAN } \phi_o/(N_2 - N_c) + \text{INV } \phi_o \quad (7)$$

where: X_2 – addendum modification coefficient of the gear
N_2 – number of teeth of the gear.

The center distance between the gear center and the cutter center (O_2 and O_c) is

$$C_{2c} = 0.5\,(N_2 - N_c)\,\text{COS } \phi_o/(P\,\text{COS } \phi_{2c}) \quad (8)$$

The root radius of the internal gear cut by the shaper cutter is determined by this position and the parameters of the cutter, or

$$R_{f2} = C_{2c} + R_{ac} \quad (9)$$

The locus of the tip point K_2 on the cutter forms a hypotrochoid on the flank or the fillet of the gear (hidden line in Fig. 3). The pressure angle at the circle with radius O_2K_2 is ϕ_{g2}, and

$$\text{TAN } \phi_{g2} = N_c\,\text{TAN } \phi_{ac}/N_2 + (N_2 - N_c)\,\text{TAN } \phi_{2c}/N_2 \quad (10)$$

From this, it is clear that the root radius of the gear, the root radius of the pinion and the tooth form are completely determined by the method of generating and the parameters of the cutter. The curve on the fillet portion of a tooth is a non-involute curve, such as hypotrochoid or modified involute or epitrochoid, and is called the transitional curve in this article.

Meshing Interference of Internal Gearing

An internal gearing has a much higher chance of interference than an external gearing. There are various meshing interferences, such as transitional interference, axial interference, radial interference, tip interference, inadequate clearance, etc., which will result in the failure of assembling or running or non-involute contact. During cutting, all the above interferences, except inadequate clearance, may occur too. Cutting interferences will result in undercut, trimming, etc.

Transitional Interference. If the tip of a gear falls into the region of the transitional curve on its mating gear, this pair of gears can not be assembled or there will be non-involute contact. Hence, it can not work, or the law of conjugation can not be satisfied. In many books (See Refs. 2, 5, 6, 7, 8), the point where the contact line is tangent to the base circle is taken as the end point of involute on the tooth. Therefore, interference can happen only "below the base circle". The calculation is simple, since there is no relation with the method of generation and the parameters of the cutters. However, it is incorrect because the ending point of the involute on the tooth is determined by the parameters of the cutter to be used, and usually this point is outside the base circle.

Fig. 4 shows an internal gearing during meshing. The center line is $O_1\,O_2$ and C is the center distance. $T_1\,T_2$ is the common tangent to the two base circles, R_{b1} and R_{b2}. The two tip circles R_{a1} and R_{a2} intersect $T_1\,T_2$ at B_2 and B_1, respectively. $B_1\,B_2$ is the interval of contact, and B_1 and B_2 are the beginning or ending point of contact. The pressure angle at the circle with radius $O_1\,B_1$ on the pinion is ϕ_{b1}, and the pressure angle at the circle with radius $O_2\,B_2$ on the gear is ϕ_{b2}. The operating pressure angle is ϕ, and ϕ_{a1} and ϕ_{a2} are the pressure angles at the two tip circles. Then

$$\text{TAN } \phi_{b1} = N_2\,\text{TAN } \phi_{a2}/N_1 - (N_2 - N_1)\,\text{TAN } \phi/N_1 \quad (11)$$

$$\text{TAN } \phi_{b2} = N_1\,\text{TAN } \phi_{a1}/N_2 + (N_2 - N_1)\,\text{TAN } \phi\,/N_2 \quad (12)$$

For avoiding the tip on the gear tooth interfering with the transitional curve on the pinion tooth, ϕ_{b1} should be larger than ϕ_{g1}, or

$$\text{TAN } \phi_{b1} > \text{TAN } \phi_{g1} \quad (13)$$

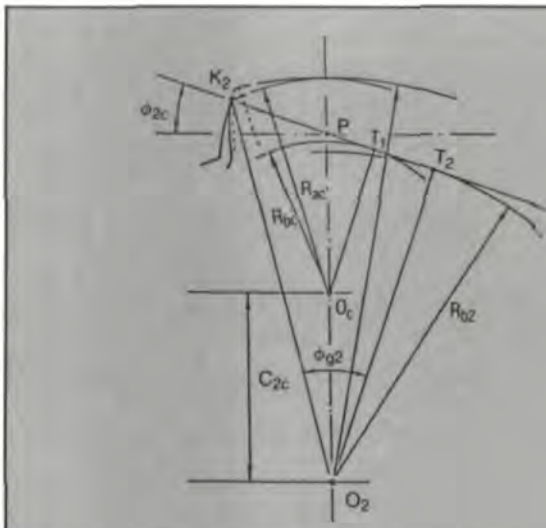

Fig. 3 – Internal Gear Cut by Shaper Cutter.

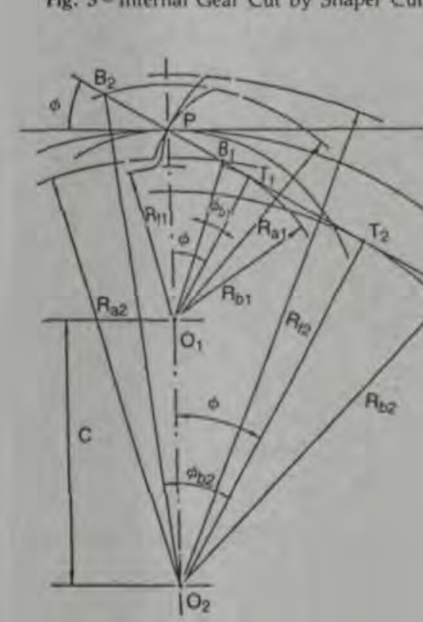

Fig. 4 – A Pair of Internal Gearing.

For shape-hobbed gearing and for double-shaped gearing, Eqs. 2 and 6, respectively, are used to calculate TAN ϕ_{g1}.

To avoid the tip of the pinion tooth interfering with the transitional curve on the gear, ϕ_{b2} should be smaller than ϕ_{g2}, or

$$\text{TAN } \phi_{b2} < \text{TAN } \phi_{g2} \qquad (14)$$

Tip Interference. As shown in Fig. 5, when the difference in numbers of teeth $N_2 - N_1$ is small, the tip of the pinion may contact the tip of the gear at some place opposite to the pitch point. The following relationship will avoid tip interference:

$$R_{a2} + C > R_{a1} \qquad (15)$$

Axial Interference. When the tooth difference $N_2 - N_1$ becomes small, after the involute meshing of a pair of teeth, the pinion tooth might contact the gear tooth again, which is called lap over, as shown in Fig. 6. Obviously, in such a case, the pinion can not be axially mounted into the gear. Therefore, this condition is called axial interference. For avoiding axial interference, the minimum tooth difference has been restricted to 10 or 12 for 20° pressure angle full depth teeth,[5, 9-11] without detail explanation and calculation formulae. But in some types of planetary gearing, for example, KHV planetary, the tooth difference is less than 6. In order to obtain a large speed ratio with a compact size and a high efficiency, use of the smallest tooth difference; i.e., $N_2 - N_1 = 1$ for the KHV planetary has been suggested.[12] This

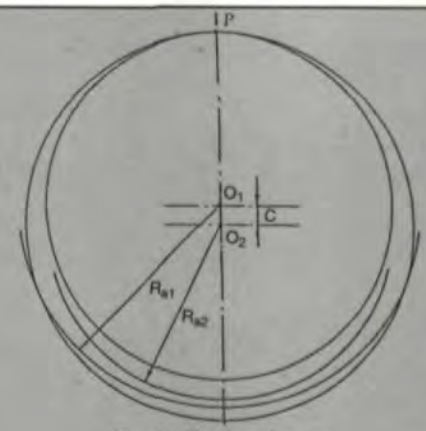

Fig. 5—Tip Interference.

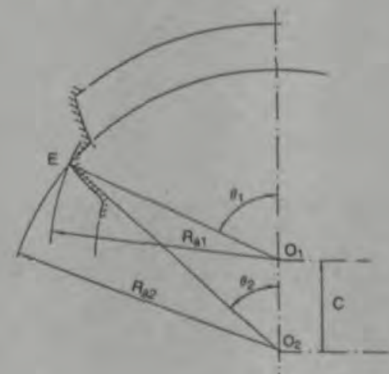

Fig. 6—Axial Interference.

might be considered impossible according to the restriction in the above references, however, the KHV gear boxes with $N_2 - N_1 = 1$ have been successfully tested, not only working smoothly without any interference, but also reaching a high efficiency of 92% when speed ratio is as high as 120.

The place where the two tip circles intersect (point E in Fig. 6) is most vulnerable to axial interference. The condition for avoiding axial interference is calculated based on this point and is expressed as follows:

$$N_1 (\theta_1 + \text{INV } \phi_{a1}) + (N_2 - N_1) \text{ INV } \phi > N_2 (\theta_2 + \text{INV } \phi_{a2}) \quad (16)$$

where:
$$\theta_1 = \text{ARC COS } [(R_{a2}^2 - R_{a1}^2 - C^2)/(2R_{a1} C)] \quad (17)$$
$$\theta_2 = \text{ARC COS } [(R_{a2}^2 - R_{a1}^2 + C^2)/(2R_{a2} C)] \quad (18)$$

Radial Interference. If the tooth difference is small, radial interference may occur and the pinion cannot be radially mounted into the gear. Suppose in Fig. 7, after contact at point p, the pinion rotates an angle of β_1, and the gear rotates an angle of $\beta_2 = \beta_1 N_1/N_2$. If at this position L_2 is greater than L_1, there will be no interference along the $O_1 O_2$, or the radial direction. We can obtain through Fig. 7,

$$L_1 = R_{a1} \text{ SIN } [\beta_1 - (\text{INV } \phi_{a1} - \text{INV } \phi)] \quad (19)$$

$$L_2 = R_{a2} \text{ SIN } [\beta_1 N_1/N_2 + (\text{INV } \phi - \text{INV } \phi_{a2})] \quad (20)$$

and
$$F(\beta_1) = L_2 - L_1 > 0 \quad (21)$$

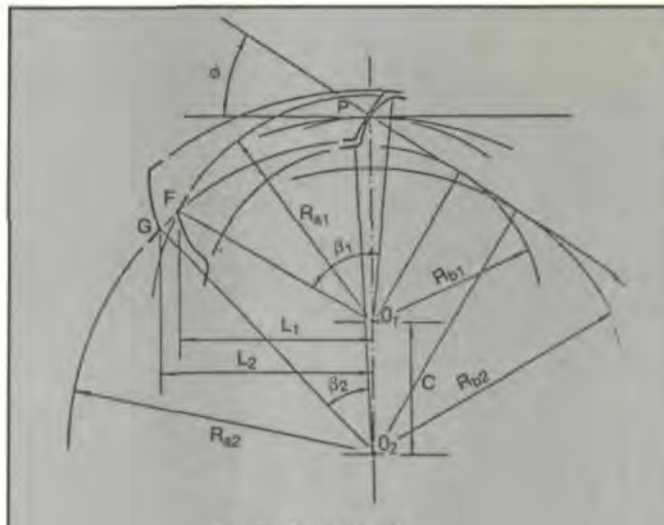

Fig. 7 – Radial Interference.

But this is for only one position of no interference. To avoid radial interference, all other positions should also satisfy Equation 21, including the minimum value of $F(\beta_1)$. Therefore, through Equation 21 and the differential of $F(\beta_1)$ equal to zero, we can eliminate β_1 and obtain the condition for avoiding radial interference as follows:

$$N_1 \{\text{ARC SIN } \sqrt{[1-(\text{COS } \phi_{a1}/\text{COS } \phi_{a2})^2]/[1-(N_1/N_2)^2]} + \text{INV } \phi_{a1} - \text{INV } \phi\} >$$

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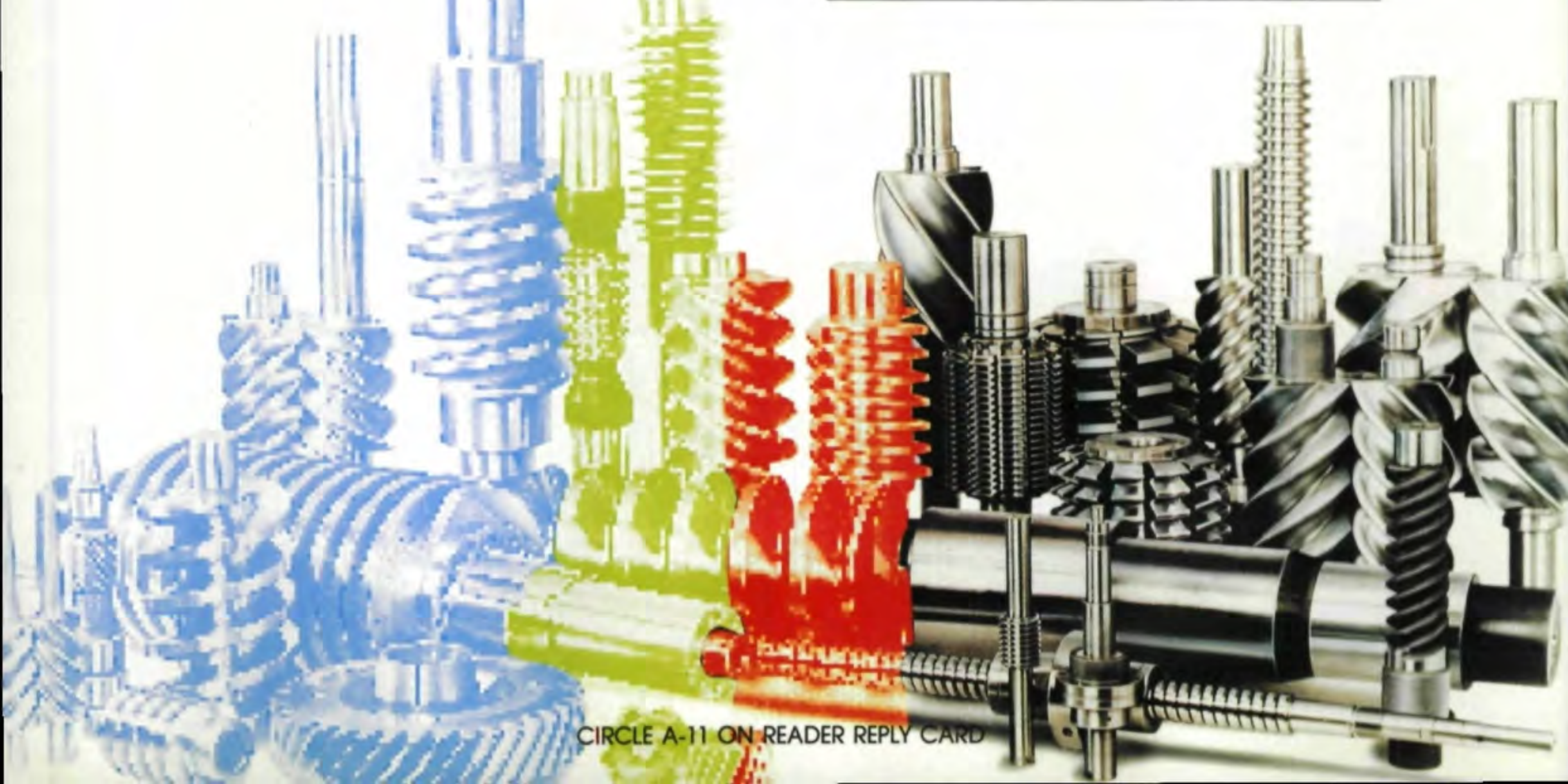

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$$N_2 \{ARC\ SIN \sqrt{[(COS\ \phi_{a2}/COS\ \phi_{a1})^2 - 1]/[(N_2/N_1)^2 - 1]} + INV\ \phi_{a2} - INV\ \phi\} \quad (22)$$

Inadequate Clearance. As shown in Fig. 4, the clearance between the tip of the pinion 1 and the root of the gear 2 is

$$C_{12} = R_{f2} - C - R_{a1} \quad (23)$$

The clearance between the tip of gear 2 and the root of the pinion 1 is

$$C_{21} = R_{a2} - C - R_{f1} \quad (24)$$

If $C_{12} < 0$ or $C_{21} < 0$, there will be interference. The value of clearance should be adequate. If it is too small, the lubricant stored in it will be insufficient. On the other hand, if it is too large, sometimes the contact ratio will be decreased.

Cutting Interference

Undercut. When the cutter extends into the base circle of the pinion being cut, there will be undercut. The conditions for avoiding undercut are as follows:

for hobbed pinion, in Equation 2, $\phi_{g1} > 0$

and for shaped pinion, in Equation 6, $\phi_{g1} > 0$

Transition Interference. The shaper cutter can be considered as a gear. The fillet portion on the cutter is a transitional curve as on a gear. The pressure angle at the circle, on which the ending point of the involute profile is located, is ϕ_{gc}. Similiarly in Equation 2, we can obtain

$$TAN\ \phi_{gc} = TAN\ \phi_o - 4\ (a_h\ P - X_c)\ /\ (N_c\ SIN\ 2\ \phi_o) \quad (25)$$

where: a_h – addendum of the rack form cutter or the grinding wheel for generating the shaper cutter

X_c – addendum modification coefficient of the shaper cutter.

When a gear or a pinion is cut by a shaper cutter, if the tip of the gear reaches the transitional curve portion of the cutter, the tip of the gear teeth will not be formed to an involute curve. In other words, the involute on the tip is trimmed off. The conditions for avoiding transitional interference during cutting are as follows:

for pinion, $(N_1 + N_c)\ TAN\ \phi_{1c}/N_c - N_1\ TAN\ \phi_{a1}/N_c > TAN\ \phi_{gc}$ (26)

for gear, $N_2\ TAN\ \phi_{a2}/N_c - (N_2 - N_c)\ TAN\ \phi_{2c}/N_c > TAN\ \phi_{gc}$ (27)

Radial Interference. When a shaper cutter is cutting an interal gear, the cutter has a radial movement or a radial feed. If there is radial interference, the tip of the gear tooth will be trimmed. Equation 22 can be used for checking the radial interference during cutting, except that the subsript "1" should be changed to "c", and "ϕ" should be changed to "ϕ_{2c}".

Slight trimming on the tip of the gear teeth may help the load distribution on the teeth and decrease the dynamic load. Large trimming will seriously affect the contact ratio and other meshing indices.

No Involute. The operating pressure angle during cutting an internal gear by a shaper cutter is ϕ_{2c} and is determined by Equation 7.

The teeth difference between the gear and the cutter $N_2 - N_c$ should be greater than a minimum value, for example, 18 as shown in Reference 13. But this may not be sufficient. As discussed above, attention should be paid not only to the number of teeth, but also to other parameters of the cutter. Otherwise, in some cases, the gear cannot cut into the involute tooth profile.

Example 1. An internal gear has the following parameters: pressure angle $\phi_o = 20°$, module m= 2 mm, number of teeth $N_2 = 77$, whole depth = 2.25 m, addendum modification coefficient $X_2 = 0$. It is a common gear with standard tooth form. A shaper cutter, which is also one in some standard, has the following data: $\phi_o = 20°$, m = 2mm, number of teeth $N_c = 50$, addendum modification coefficient $X_c = 0.577$, whole depth 2.25 m. If the gear is cut by the cutter, from Equation 7, then:

$$INV\ \phi_{2c} = 2(0 - 0.577)\ TAN\ 20°/(77 - 50) + 0.0149044 = -0.00652$$

or $\phi_{2c} < 0$.

Therefore, this gear cannot be cut by this cutter, another kind of interference during cutting. It is suggested that the operating angle during cutting ϕ_{2c} be greater than 7 to 10°.

Design by Maximum Dimension, Check by Minimum Dimension

The importance of the method of generating gear teeth and the parameters of the cutters is clear now. But each time the shaper cutter is sharpened, its outside diameter is decreased, and so is the addendum modification coefficent X_c. In other words, some important parameters of the cutter are changing. If using the parameters of a new cutter to design an internal pair of gears with no interference, there still might be interference if the gears are cut with a sharpened cutter. Therefore, the author suggests designing a gearing by the maximum dimensions of the cutter and checking the obtained data through the minimum dimensions. The maximum dimensions are the sizes of a new cutter or the measured sizes of a sharpened cutter to be used for this design. The minimum dimensions are defined as the minimum outside diameter and the minimum addendum modification coefficient of the cutter. The minimum sizes of the cutter are determined by strength and the correct tooth form. With such minimum sizes the cutter can still perform a normal cutting. If there is interference checked by the minimum dimensions, the cutter cannot be used to its minimum sizes, and the limited sizes should be specified, or another cutter should be chosen. Two more examples are given for further illustration.

Example 2. A pair of internal gearings has the following parameters: $P = 8\ 1/\text{in}$, $\phi_o = 20°$, $N_1 = 15$, $N_2 = 45$, $X_1 = 0.4425$, $X_2 = 1.328$, $C = 1.97''$.
The pinion is hobbed, and the gear is cut by a shaper cutter. The parameters of the new cutter are: $\phi_o = 20°$, $N_c = 24$, $X_c = 0.2564$, $R_{ac} = 1.6883$ in, addendum $a_c = 1.25/P$. The minimum sizes of the cutter are: $(R_{ac})_{min} = 1.6050$in, $(X_c)_{min} = -0.41$.

a). Design through the new cutter.

The root radii are determined by the cutters. Through Equations 1, 7-9, we can obtain: $R_{f1} = 0.8366''$, $R_{f2} = 3.1082''$.

The tip radii are preliminarily determined by contact ratio and clearances and are checked by transitional interferences. $R_{a1} = 1.1100''$, $R_{a2} = 2.8350''$, Contact ratio $m_t = 1.438$, $C_{12} = 0.0282'' = 0.226/P$, $C_{21} = 0.0284'' = 0.227/P$.
Whole depth $h_1 = R_{a1} - R_{f1} = 0.2743'' = 2.187/P$
$h_2 = R_{f2} - R_{a2} = 0.2732'' = 2.186/P$

TAN $\phi_{b1} = 0.1641953 >$ TAN $\phi_{g1} = 0.1326991$ (no interference)
TAN $\phi_{b2} = 0.5889354 <$ TAN $\phi_{g2} = 0.6178138$ (no interference)

b). Check by the minimum sizes.

The root radius is changed to $R'_{f2} = 3.0794''$. The clearance becomes $C'_{12} = R'_{f2} - C - R_{a1} = -0.0006''$ Since $C^1_{12} < 0$, there is interference. The cutter cannot be used to its minimum sizes.

c). Limited sizes.

Try $(R_{ac})_{min} = 1.6563''$ and $(X_c)_{min} = 0$.
Then $R'_{f2} = 3.0978''$ and $C'_{12} = 0.0178'' = 0.142\ /P$.
In Reference 9 the minimum clearance is 0.157/P. Practically, the minimum value of clearance has relation to velocity, lubricant, etc. The obtained clearance is small, therefore, the outside radius of the cutter should not be sharpened less than 1.6563".

The term of TAN $\phi'_{g2} = 0.611423$ is less than TAN $\phi_{g2} = 0.6178138$ (for the new cutter), but is still larger than TAN $\phi_{b2} = 0.5889354$. Therefore, there is no transitional interference.

Example 3. A pair of internal gearings have the parameters: $\phi_o = 20°$, $m = 3.5$mm, $N_1 = 27$, $N_2 = 75$, $X_1 = 0.26$, $X_2 = -0.255$, $C = 82$mm. Both pinion and gear are cut by the same shaper cutter with parameters: $\phi_o = 20°$, $m = 3.5$mm, $N_c = 28$, $a_c = 1.3$ m. The cutter is not a new one. The measured sizes are $R_{ac} = 54.33$mm and $X_c = 0.229$. The minimum sizes are provided from some standard, $(R_{ac})_{min} = 53$mm, and $(X_c)_{min} = -0.157$.

a). Design through measured dimensions

The root radii are determined by the parameters of the cutter; $R_{f1} = 43.515$mm, $R_{f2} = 134.737$mm.

The tip radius of the pinion is preliminarily determined by clearance $C_{12} > 0.3m = 1.05$mm, and the tip radius of the gear is determined by transitional interference TAN $\phi_{b1} >$ TAN ϕ_{g1}. Then
$R_{a1} = 51.5$ mm $R_{a2} = 127.5$mm
$C_{12} = 1.237\text{mm} = 0.35\text{m}$, $C_{21} = 1.985\text{mm} = 0.567\text{m}$
TAN $\phi_{b1} = 0.227724 >$ TAN $\phi_{g1} = 0.1912166$ (no interference)
TAN $\phi_{g2} = 0.413594 >$ TAN $\phi_{b2} = 0.3916624$ (no interference)

contact ratio: $m_t = 1.55$
whole depth: $h_1 = 7.985\text{mm} = 2.28\text{m}$.
$h_2 = 7.237\text{mm} = 2.07\text{m}$.

b). Check by minimum sizes
$R'_{f1} = 43.605$mm $R'_{f2} = 134.909$mm
$C'_{12} = 1.409\text{mm} = 0.403\text{m}$ $C'_{21} = 1.895\text{mm} = 0.541\text{m}$

TAN $\phi'_{g1} = 0.1733648 <$ TAN $\phi_{g1} = 0.1912166$

TAN $\phi'_{g2} = 0.4373599 >$ TAN $\phi_{g2} = 0.4135940$

For transitional interferences and clearance C_{12}, the gears cut by the sharpened cutter with minimum sizes are safer than those cut by the cutter with the measured sizes. The clearance C'_{21} is less than C_{21}, but is still greater than 0.3m. Therefore, the cutter can be used from the measured sizes to its minimum sizes for cutting this pair of internal gears.

(continued on page 43)

(continued from page 19)

CAD and Optimum Design

The design of an internal gearing is much more complicated than that of an external gearing. Computer aided design is recommended.[12] However, the following principles can also be used for external gearing design.

Mathematical Model. The first step is to analyze the specifications of the gearing and to construct a mathematical model that can simulate the problem to be solved. If the problem is restricted to geometrical dimensions only, usually efficiency or contact ratio can be chosen as the objective function. If the problem includes strength as well, the objective function can be chosen from one or two of the following variables: weight, size, output torque, efficiency, minimum stress, maximum strength, etc. The constraints are related to the data given. Sometimes the objectives and the constraints can be exchanged. For example, if the objective is the minimum size, the allowable stress or the maximum stress will be the constraint, and if the maximum strength is the objective, the given size or the minimum size will be the constraint. However, some parameters, such as different kinds of interference, may always be constraints. Both objective functions and constraint functions should be as simple as possible.

Optimizing Method. Since there are various parameters in a gearing design and the mathematic model is usually not a quadratic function or other well-behaved function, the author suggests not using indirect search techniques. These require both objective and constraint functions differentiable and continuous within the region of search, while the functions of a gearing are usually transcendental functions, the differentiation of which is sometimes very difficult. Therefore, it is recommended that the direct search techniques, such as the Hooke-Jeeves method, complex method, Rosenbrock method, all of which have been tested with good results in gear design by the author, be used. These methods have been introduced in many books.[14-15] However, they are analyzed as unconstrained problems, which might be common in mathematics; but most problems in engineering, including gear design, are constraint problems. Therefore, modifications of these methods should be made for a gear optimizing design, such as shown in Reference 12. Fig. 8 is a schemetic flow chart for a gear design. The program should be designed not for one cutter, but for a set of cutters. Different data can be obtained from different cutters from which the optimum result can be determined.

Conclusion

As a result of the above discussion and the illustrative examples, the following observations can be made.

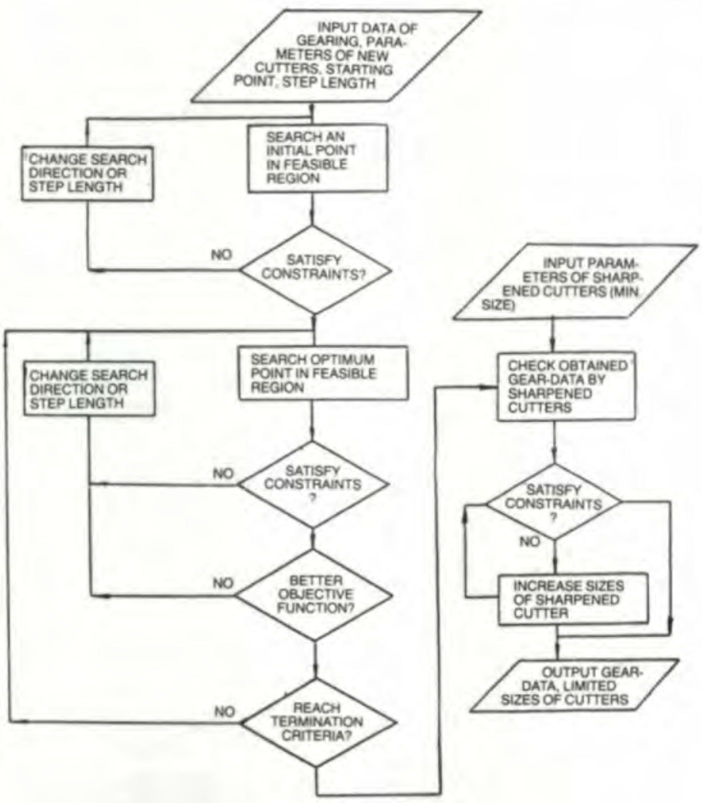
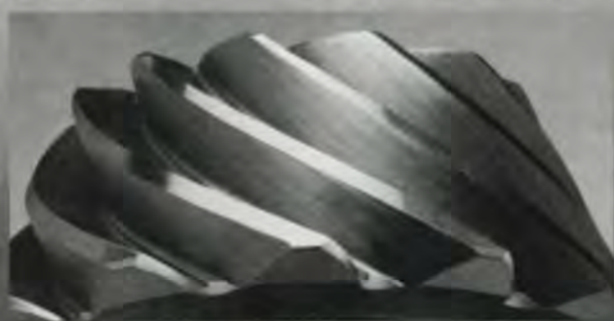


Fig. 8—Flow Chart for Gear Design.



1). The method of gear teeth generation and the parameters of the cutter, including interference calculation, should be taken into account in the design of the internal gearing. Otherwise, the design seems correct, but, practically, there might be interference and other problems.

2). After choosing the cutter, use the parameters of a new cutter or the measured sizes of the cutter to design the gearing. Then use the minimum sizes of the cutter to check the obtained data. Otherwise, even if the parameters of the new cutter are taken into account, there might be interference when the gears are cut by the sharpened cutter.

3). The root radius of a gear is directly related to the parameters of the cutter. The two tip radii of a pair of gears have relation to interference, contact ratio, sliding velocity, top land, clearance, whole depth, etc. Some of them are related to root radii too. Hence, the tip radii have indirect relations with the cutter as well. But in many books (References 3, 5, 10, 16 and 17), the formulae for calculating the root radii or the tip radii have no relationship to the parameters of the cutters. The relation between the tip radius and the root radius of the external gearing is mainly for keeping a standard whole depth and a standard clearance. This method has been introduced to internal gearing for making the whole depth and the clearance close to standard values. (See Reference 3.) In some books,[10] the root radius is calculated from a given dedendum or addendum plus clearance. However, the author considers those methods improper because interference, contact ratio and other meshing indices are more important than the standard or the given whole depth. Besides, the root radius is determined by the parameters of the cutter, and the whole depth and the dedendum should be calculated from the root radius. Moreover, there might be few cases where an internal gear cut by a shaper cutter can obtain the standard or given whole depth. From Equations 7-9, only when $(X_2 - X_c) = 0$ will it be possible to cut a gear with the standard dedendum or a dedendum equal to the addendum of the cutter. If the addendum of the gear is standard, then the whole depth can be standard. However, the X_c is changing after sharpening and even during cutting. Therefore, the standard whole depth and the standard clearance are really of no meaning for the internal gearing. In Reference 18, the clearance can have a range, 0.25/P to 0.35/P. It is better than one value, 0.25/P, but the range should be larger because the clearance is of less importance, and sometimes even if it is 0.35/P, there will still be interference. In the above examples, none of the clearances or the whole depths are standard. Nevertheless, these designs satisfy the required indices and have been tested. For the internal gearing in the KHV planetary systems, if $N_2 - N_1 = 1$, the clearance usually should be greater than 0.5/P, and sometimes as large as 0.7/P. Hence, the author suggests that for internal gearing, the root radii be determined by "cutting" and the tip radii be determined by "meshing". Cutting means the parameters of the cutter. Meshing means meshing indices which are determined not by a single gear, but by the two mating gears. According to different requirements of different types of planetary gearings, choose one or two meshing indices, for example, contact ratio or transitional interference or clearance, to determine preliminary tip radii; then check other required indices. Since this discussion is mainly of interference, the details of the design for internal gearing will be in another article.

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Acknowledgement: This article was first presented at the 2nd World Congress on Gearing, March 3-5, 1986, Paris, France.