Transmission Errors and Bearing Contact of Spur, Helical, and Spiral Bevel Gears

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Abstract:

An investigation of transmission errors and bearing contact of spur, helical, and spiral bevel gears was performed. Modified tooth surfaces for these gears have been proposed in order to absorb linear transmission errors caused by gear misalignment and to localize the bearing contact. Numerical examples for spur, helical, and spiral bevel gears are presented to illustrate the behavior of the modified gear surfaces with respect to misalignment and errors of assembly. The numerical results indicate that the modified surfaces will perform with a low level of transmission error in non-ideal operating environments.

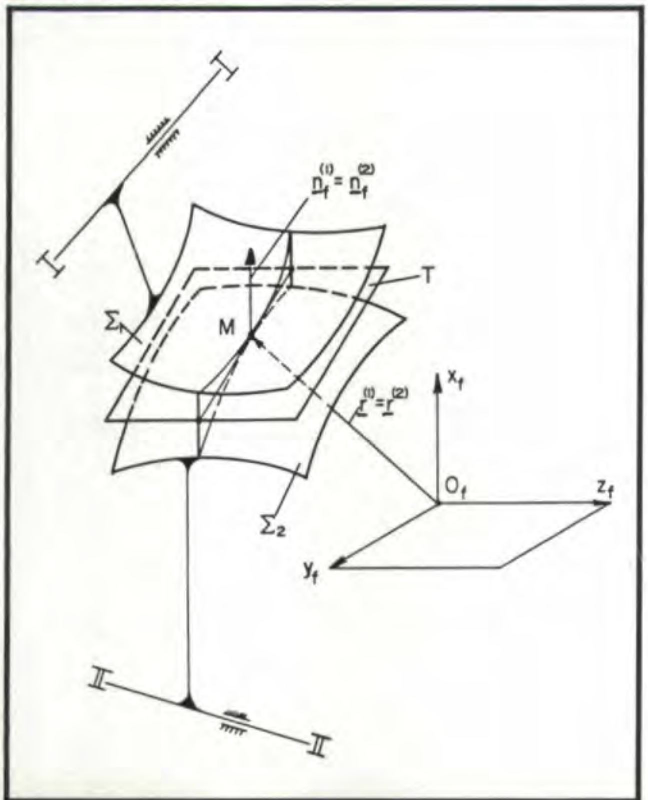

Fig. 1 – Contacting tooth surfaces.

Introduction

The most important criteria of gears are the level of their noise and bearing contact. A main source of gear noise is transmission errors. Traditional methods of gear synthesis that provide conjugate gear tooth surfaces with zero transmission errors and an instantaneous line of contact are not acceptable for the gearing industry due to errors of manufacturing and assembly. Taking into account such errors, the bearing contact is shifted to the edge of the tooth, and transmission errors of an unfavorable shape occur. The new trend of gear synthesis is to localize bearing contact and absorb the transmission errors. These goals may be achieved by a new topology of gear tooth surfaces, and this is the subject of this article.

Simulation of Meshing

The investigation of gear misalignment requires a numerical solution for the simulation of meshing and contact of gear tooth surfaces. The basic ideas of this method[2-3] are as follows:

(1) The meshing of gear tooth surfaces is considered in a fixed coordinate system, S_{F1}.

(2) Due to the continuous tangency of gear tooth surfaces, the position-vectors, $\underset{\sim}{r}^{(1)}$ and $\underset{\sim}{r}^{(2)}$, and the surface unit normals, $\underset{\sim}{n}^{(1)}$ and $\underset{\sim}{n}^{(2)}$, must be equal at every instant. (Fig. 1).

(3) Simulating the errors of manufacturing and assembly and considering that the gear tooth surfaces are in point contact, we may determine function ϕ_2 (ϕ_1) that relates the angles of rotation of the gears.

Then, the transmission errors will be determined from the equation

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \qquad (1)$$

where $\phi_2(\phi_1)$ is the function obtained numerically, with the aid of the developed computer program, and N_1 and N_2 are the numbers of teeth.

Simulation of Bearing Contact

The requirement that

$$\underline{r}^{(1)} = \underline{r}^{(2)} \text{ and } \underline{n}^{(1)} = \underline{n}^{(2)} \tag{2}$$

provides the information about the path of contact on the tooth surface. Due to the elastic approach of the gear tooth surfaces, their contact is spread over an elliptical area. It is assumed that the magnitude of the elastic approach is known from experiments or may be computed. Knowing in addition the principle curvatures and directions for two contacting surfaces at their point of contact, we may determine the dimensions and orientation of the contact ellipse.[2-3] The instantaneous point of contact is the center of symmetry of the contact ellipse. The set of contact ellipses represents the bearing contact. Simulating the errors of manufacturing and assembly, we are able to determine the real path of contact on the gear tooth surface and the real bearing contact.

Partial Compensation of Transmission Errors

The investigation of the effect of errors of assembly and machine tool settings shows that the real transmission function is a piece-wise linear or almost linear function (Fig. 2a). The transmission errors that are represented by Equation 1 are shown in Fig. 2b. Transmission errors of this type cause an instantaneous jump of the angular velocity of the driven gear at the transfer points, and vibration becomes inevitable. (At transfer points one pair of teeth in mesh is changed for next pair.) The proposed approach allows absorption of the transmission errors by modifying the gear tooth surfaces. The modification is directed to provide a predesigned parabolic function of transmission errors that will be able to absorb linear functions of transmission errors. The predesigned parabolic function of transmission errors will exist even for aligned gears. However, when the gears are misaligned, piece-wise linear transmission errors (Fig. 2b) will not occur. This is based on the possibility of a linear function being absorbed by a parabolic one.

Consider the interaction of a parabolic function given by (Fig. 3a)

$$\Delta\phi_2^{(1)} = -a\phi_1^2 \tag{3}$$

with a linear function represented by

$$\Delta\phi_2^{(2)} = b\phi_1 \tag{4}$$

The resulting function

$$\Delta\phi_2 = b\phi_1 - a\phi_1^2 \tag{5}$$

may be represented in a new coordinate system by (Fig. 3b):

$$\psi_2 = -a\psi_1^2 \tag{6}$$

where

$$\psi_2 = \Delta\phi_2 - \frac{b^2}{4a}, \; \psi_1 = \phi_1 - \frac{b}{2a} \tag{7}$$

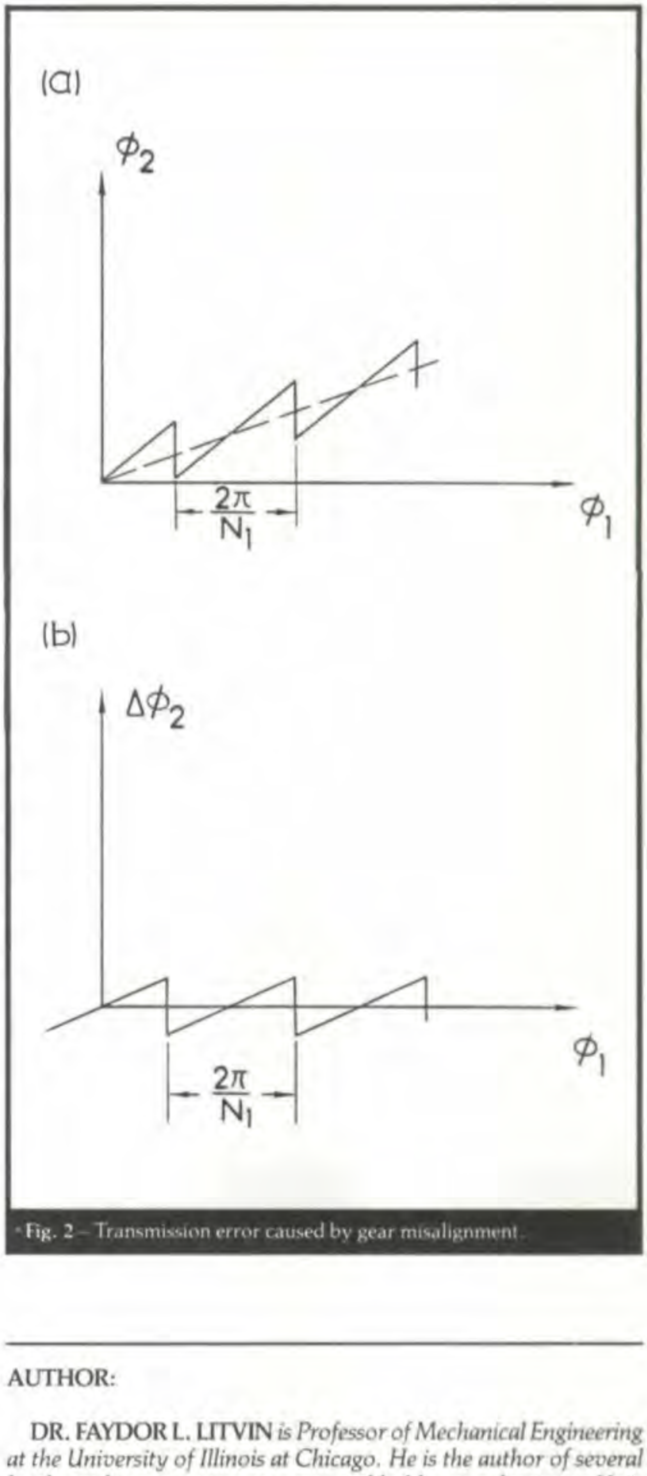

Fig. 2 – Transmission error caused by gear misalignment.

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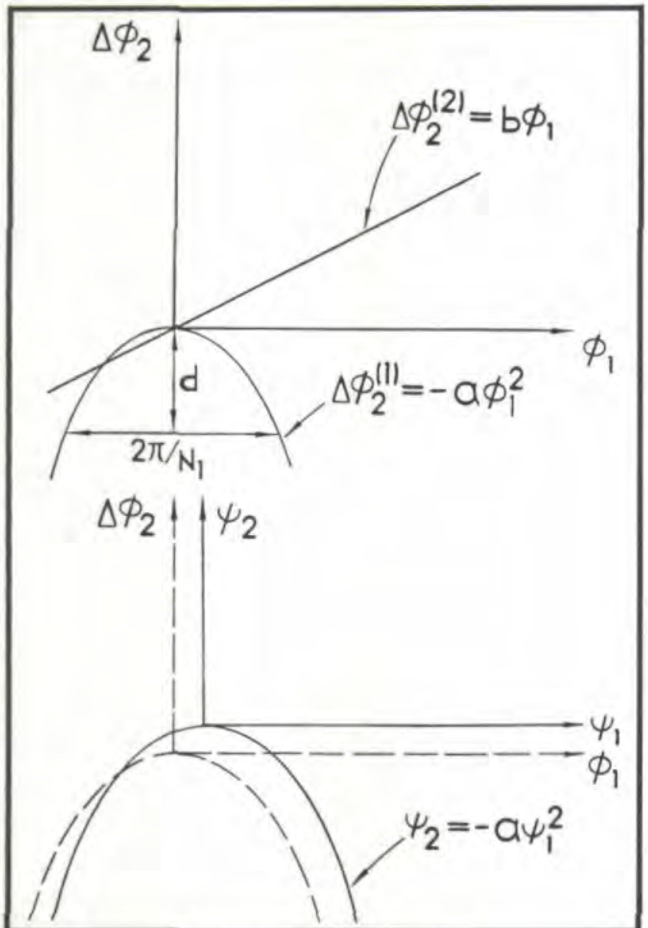

Fig. 3 – Interaction of parabolic and linear functions.

We consider that $\Delta\phi_2^{(1)} = -a\phi_1^2$ is the predesigned function that exists even if misalignments do not appear. The absorption of function $\Delta\phi_2^{(2)} = b\phi_1$ by the parabolic function $\Delta\phi_2^{(1)} = -a\phi_1^2$ means that gear misalignment does not change the predesigned parabolic function of transmission errors. Thus the resulting functions of transmission errors $\Delta\phi_2 = \Delta\phi_2^{(1)} + \Delta\phi_2^{(2)}$ will keep its shape as a parabolic function, although the gears are misaligned. The resulting function of transmission errors $\phi_2(\phi_1)$ may be obtained by translation of the parabolic function $\phi_2^{(1)}$.

The absorption of a linear function of transmission errors by a parabolic function is accompanied by the change of transfer points. The transfer points determine the position of the gears where one pair of teeth is going out of mesh and the next pair is coming into mesh. It is necessary to emphasize that theoretically the contact ratio of gears with transmission errors is one. The real contact ratio can be larger than one only due to elastic deformations. The change of transfer points is determined with $\Delta\phi_1 = \left|\frac{b}{2a}\right|$ and $\Delta\phi_2 = \frac{b^2}{4a}$. The cycle of meshing of one pair of teeth is $\phi_1 = \frac{2\pi}{N}$. It may happen that the absorption of a linear function by a parabolic function is accompanied by a change of transfer points that is too large. If this occurs, the transfer points and the resulting parabolic function of transmission errors, $\psi_2(\psi_1)$, will be represented as a discontinous function for one cycle of meshing (Fig. 4.) To avoid this, it is necessary to limit the tolerances for gear misalignment or increase the level of the predesigned parabolic function.

Modification of Pinion Tooth Surface for Spur Gears

The gear is provided with a regular involute surface. The pinion is provided with a crowned tooth surface, and two types of the crowned surface are considered. The surface of

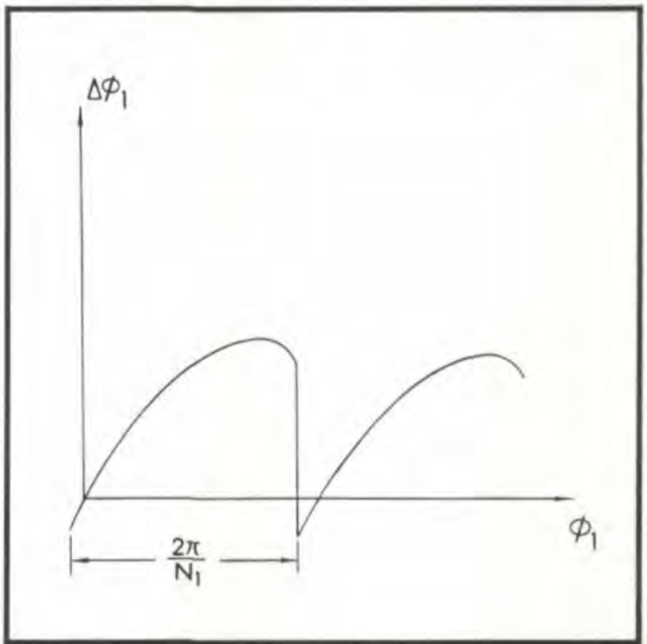

Fig. 4 – Discontinued parabolic function of transmission errors.

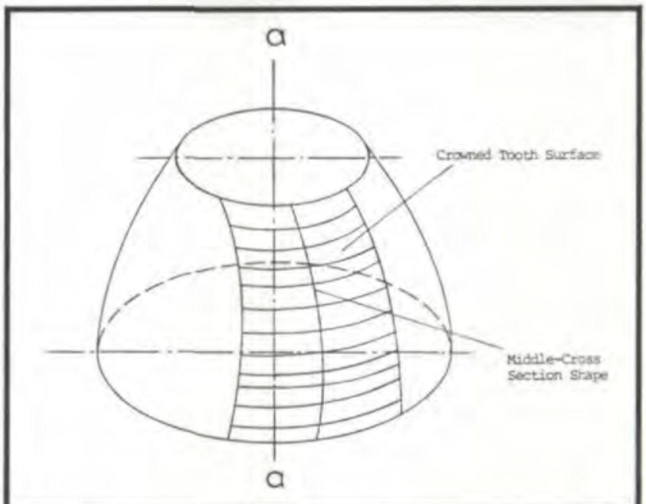

Fig. 5 – Crowned pinion tooth surface.

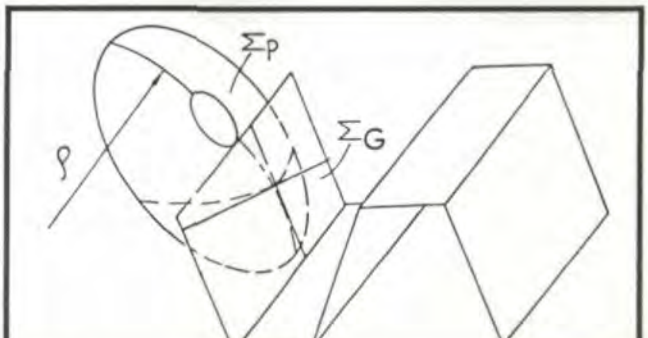

Fig 6 – Generating tool surface for helical pinion and gear.

the first type is considered as a surface of revolution (Fig. 5) whose axial sections slightly deviate from a regular involute curve. The surface of the second type is based on the method of generation that is presented in Fig. 6.

Pinion Tooth Surface as a Surface of Revolution. Consider first that the axial section of the pinion tooth surface is a regular involute curve. The pinion tooth surface, which is a surface of revolution, may be generated by rotation of the involute curve about the a-a axis (Fig. 5.) It is evident that the pinion and gear tooth surfaces will contact each other in the middle cross-section and transform rotation with a *constant* gear ratio if the gears are not misaligned. However, it is necessary to compensate for the transmission errors of misaligned gears. For this reason the shape of the pinion is to be synthesised as a curve that deviates from an involute curve to provide a predesigned parabolic function of transmission errors.

The sections of the pinion tooth surface that are perpendicular to axis a-a (Fig. 5) are circles. The radius of such a circle for the mean contact point may be determined from the requirements to the dimensions of the contact ellipse.

The proposed pinion tooth surface may be generated by form-grinding or by a computer controlled grinding or cutting machine.

Example 1. Given numbers of teeth $N_1 = 20$, $N_2 = 40$, diamentral pitch $P = 10 \frac{1}{in}$, then, pressure angle $\psi_c = 20°$. The pinion tooth surface is crowned as described above, and a parabolic function of transmission errors with the level of 2 arc seconds has been predesigned.

The influence of gear misalignment has been investigated with the developed TCA program for the following cases:

Table 1. Function of Transmission Errors.

$\phi_1^{(\circ)}$	-3	0	3	6	9	12	15
$\Delta\phi_2^{('')}$	-0.71	0.00	0.40	0.50	0.33	-0.09	-0.74

Table 2. Function of Transmission Errors.

$\phi_1^{(\circ)}$	-10	-7	-4	-1	0	2	5	8
$\Delta\phi_2^{('')}$	-2.02	-0.92	-0.24	0.01	0.00	-0.16	-0.75	-1.78

Case 1. The change of the center distance is $\frac{\Delta c}{c} = 1\%$; the gear axes are not parallel, but crossed, and the twist angle is 5 arc minutes. The resulting function of transmission errors is still a parabolic one with the maximum value of 1.2 arc seconds as shown in Table 1.

Case 2. The axes of the same gears are intersected and form an angle of 5 arc minutes. The resulting function of transmission errors is again a parabolic one with the maximal value of 2 arc seconds as shown in Table 2.

Generation of the Pinion Tooth Surface by Tool With a Surface of Revolution. Consider that two generating surfaces — a plane and a cone — are rigidly connected with each other and generate the gear tooth surface and the pinion crowned tooth surface, respectively. (Fig. 6.) The generating plane is the surface of a regular rack cutter. In the process of generation the rack cutter and the cone perform a translational motion, while the pinion and the gear rotate about their axes. (Fig. 7.) The rotation of the cone about its axis c-c is not related to other motions for the tooth surface generation, and the angular velocity of the cone depends only on the desired velocity of cutting. The tool for the crowning of the pinion can be designed as a grinding wheel or as a shaver. The opposite side of the pinion tooth is to be generated separately.

The described process of crowning of the pinion by a regular cone provides an involute shape of the pinion tooth surface in its middle section. The crowned pinion and the involute gear, if they are not aligned, can transform rotation without transmission errors. Also, their bearing contact is localized. But in reality, due to the misalignment of gears, they will transform rotation with a piece-wise linear function of transmission errors. (Fig. 2.) To absorb such errors it is necessary to predesign a parabolic function of transmission errors. This can be achieved if a tool surface of revolution that slightly deviates from the cone surface is used. (Fig. 8.) The curvature radius ϱ and the level of predesigned transmission errors of parabolic type are related. The deter-

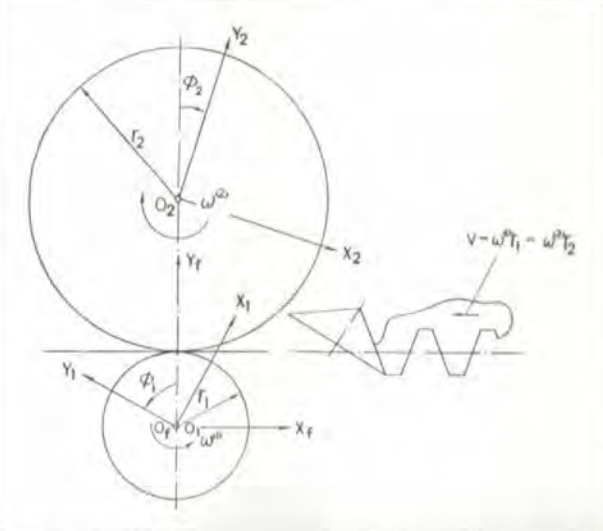

Fig 7 – Generation of gears.

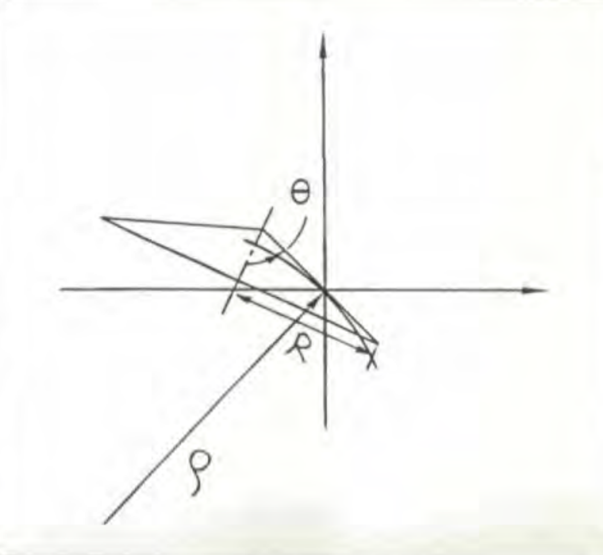

Fig 8 – Surface of revolution and its parameters.

mination of ϱ is the subject of synthesis of the pair of gears with the crowned pinion.

Example 2. The input data are the same as in Example 1. The parameters of the tool surface (Fig. 8) are: $\theta = 20°$, $\varrho = 500$ in., R = 10.6". The influence of misalignment has been investigated by the developed computer programs for two cases.

Case 1. The change of center distance is $\frac{\Delta C}{C} = 1\%$, and the twist angle of crossed axes is 10 arc minutes. The resulting function of transmission errors is of a parabolic type with the maximum value of 0.28 arc seconds as shown in Table 3.

Case 2. The gear axes are interesected and form an angle of 10 arc minutes. The resulting function of transmission errors is of a parabolic type with the maximum value of 0.34 arc seconds as shown in Table 4.

Table 3. Function of Transmission Errors.

$\phi_1^{(\circ)}$	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2^{('')}$	−0.26	−0.10	0.00	0.02	0.00	−0.02	−0.14	−0.33

Table 4. Function of Transmission Errors.

$\phi_1^{(\circ)}$	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2^{('')}$	−0.33	−0.15	−0.03	0.01	0.00	−0.02	−0.11	−0.20

Modification of Pinion Tooth Surface for Helical Gears with Parallel Axes

The crowning of the pinion of helical gears with a parallel axis is directed at the achievement of two goals: (i) to localize the bearing contact, and (ii) to reduce the level of transmission errors. These goals are contradictive, and in some cases a compromise solution has to be found. The following methods for crowning are based on: (i) generation of the pinion tooth surface by a tool with a surface of revolution, (Fig. 8.) (ii) generation of the pinion with circular arc teeth, and (iii) the change of the lead angle.

Generation of Helical Pinion Tooth Surface by a Surface of Revolution. The method of generation is based on the same ideas that have been developed for the generation of spur pinions. The only difference is the installment of the tool. Considering again two generating surfaces, (Fig. 6.) we have to require that the generating plane will be installed as a skew rack cutter and form with the gear axis an angle that is equal to or approximately equal to the helix angle on the pitch cylinder. By varying the installment angle, a slight change in the orientation of the bearing contact can be made. It is necessary to emphasize, however, that the path of contact will be in the middle section of the gear tooth or near this direction. (Fig. 9.) The results of investigation of helical gears with crowned pinion tooth surfaces are represented in the following example.

Example 3. Given: $N_1 = 20$, $N_2 = 40$, diametral pitch in the normal section $= P_n = 10$ in^{-1}, the pressure angle in the normal section is $\psi = 20°$, the helix angle on the pitch cylinder is $\beta = 15°$. Parameters of the surface of revolution (Fig. 8.) are $\theta = 20°$, $\varrho = 30$ in., R = 10.6 in. Helical gears with modified pinion tooth surface provide a parabolic type

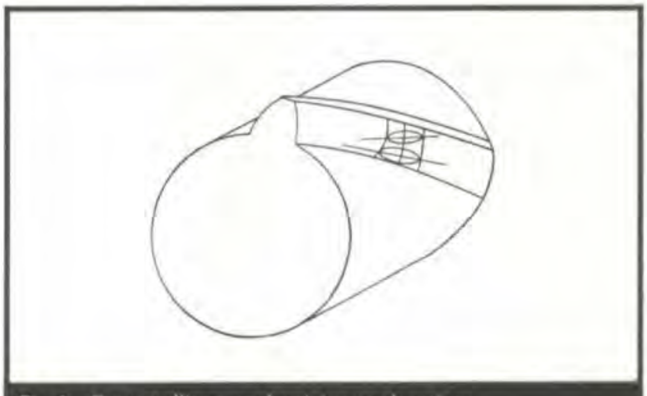

Fig. 9—Contact ellipses on the pinion tooth surface.

of predesigned transmission error with d = 6 arc seconds, (Fig. 3a.) and a path contact that is directed across the tooth surface. (Fig. 9.)

The influence of gear misalignment has been investigated by computer program, and the results of computation are represented in Tables 5 and 6 for crossed and intersected gear axes, respectively. The misalignment of gear axes is 5 arc minutes.

Table 5. Transmisson Errors of Crossed Helical Gears.

$\phi_1^{(\circ)}$	−14	−11	−8	−5	−2	1	4
$\Delta\phi_2^{('')}$	−4.99	−1.51	0.65	1.51	1.05	−1.75	−3.87

Table 6. Transmission Errors of Intersected Helical Gears.

$\phi_1^{(\circ)}$3	−11	−8	−5	−2	1	4	7
$\Delta\phi_2^{('')}$	−6.15	−2.72	−0.60	0.20	−0.32	−2.19	−5.40

The results of computation show that the resulting function of transmission errors is a parabolic one. Thus the linear function of transmission errors that was caused by gear misalignment has been absorbed by the predesigned parabolic function.

Change of the Pinion Lead. Helical gears in this case are designed as helical gears with crossed axes. The crossing angle γ is chosen with respect to the expected tolerances of the gear misalignment. (γ is in the range of 10 to 15 arc minutes.) The gear ratio for helical gears with crossed axes may be represented.[2]

$$m_{12} = \frac{\omega^{(1)}}{\omega^{(2)}} = \frac{r_{b2} \sin \lambda_{b2}}{r_{b1} \sin \lambda_{b1}} \quad (8)$$

where r_{bi} and λ_{bi} are the radius of the base cylinder, and the lead angle on this cylinder; i.e., 1,2. $|\lambda_{p2} - \lambda_{p1}| = \gamma$. Here: λ_{pi} is the lead angle on the pitch cylinder. The advantage of application of crossed helical gears is that the gear ratio is not changed by the misalignment (by the change of γ). The tooth surfaces contact each other at a point during meshing. The disadvantage of this type of surface deviation is that location of the bearing contact of the gears is very sensitive to gear misalignment. A slight change of the crossing angle causes shifting of the contact to the edge of the tooth. (Fig. 10.)

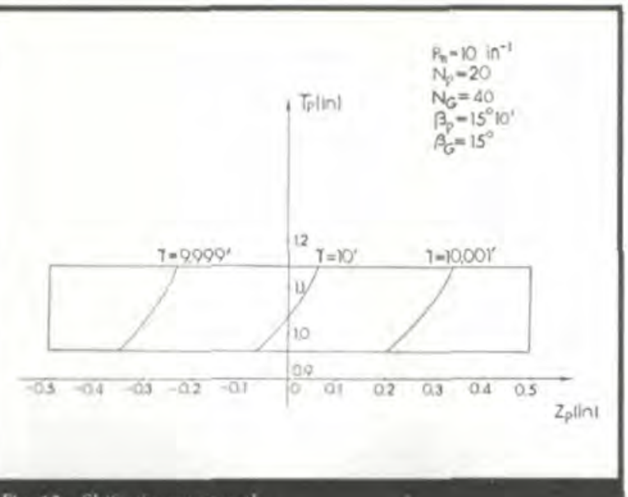

Fig. 10 – Shift of contact path.

The discussed type of surface deviation is reasonable to apply for manufacturing of expensive reducers of large dimensions when the lead of the pinion can be adjusted by regrinding. While changing by regrinding the parameters r_{b1} and λ_{b1}, it is required that the product $r_{b1} \sin \lambda_{b1}$ must be kept constant. Then, the gear ratio m_{21} will be of the prescribed value, and transmission errors caused by the crossing of axes will be zero.

Theoretically, transmission errors are inevitable if the axes of crossed helical gears become intersected. Actually, if gear misalignment is of the range of 5 to 10 arc minutes, the transmission errors are very small and may be neglected. The main problem for this type of misalignment is again the shift of the bearing contact to the edge. (Fig. 10.)

Crowned Helical Pinion with Longitudinal Path Contact. A longitudinal path of contact means that the gear tooth surfaces are in contact at a point at every instant, and the instantaneous contact ellipse moves *along* and not *across* the surface. (Fig. 11a.) This path can be achieved by crowning gear surfaces. It can be expected that this type of contact provides improved conditions of lubrication. Until now only the Novikov-Wildhaber gears[1] could provide a longitudinal path of contact. A disadvantage of this type of gearing is the sensitivity to the change of the center distance and the axis misalignment. The sensitivity to non-ideal orientation of the meshing gears causes a higher level of gear noise in comparison with regular involute helical gears. Litvin and Tsay[4] proposed a compromise type of nonconformal helical gears that may be placed between regular helical gears and Novikov-Wildhaber helical gears. The gears of the proposed gear train are the combination of a regular involute helical gear and a specially crowned helical pinion. The investigation of transmission errors for helical gears with a longitudinal path of contact shows that their good bearing contact is accompanied by an undesirable increased level of linear transmission error. To compensate for this disadvantage, a predesigned parabolic function of transmission errors that will absorb the linear function of transmission errors is proposed.

The generation of gears is based on the following idea:

Consider that two rigidly connected generating surfaces, Σ_g and Σ_p, are used for the generation of the gear and the pinion, respectively. (Fig. 11b.) Surface Σ_g is a plane and

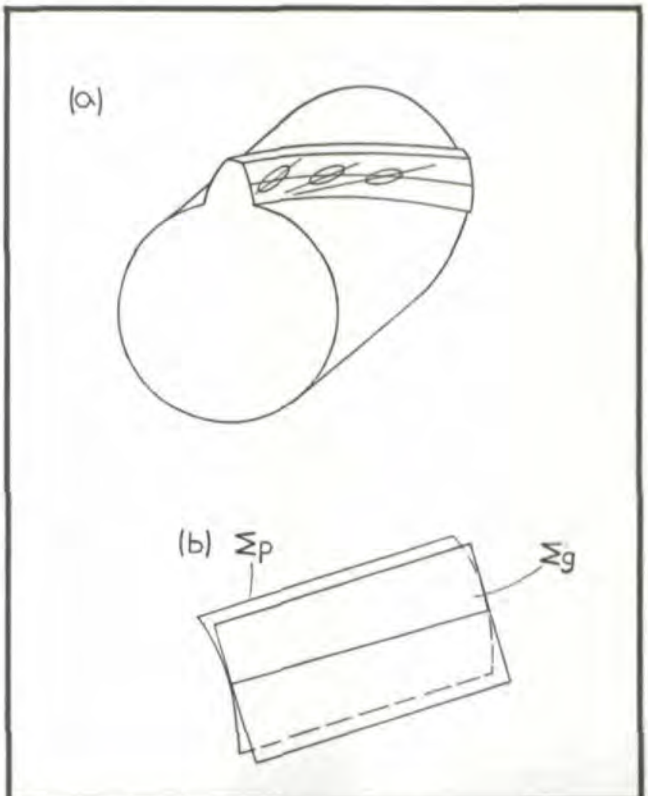

Fig. 11 – Helical gear and generating tool surface.

represents the surface of a regular rack cutter; surface Σ_p is a cylindrical surface whose cross-section is a circular arc. We may imagine that while surfaces Σ_g and Σ_p translate the pinion and the gear rotate about their axes. To provide the predesigned parabolic function of transmission errors, it is necessary to observe the following transmission functions by generation:

$$\frac{V}{\omega^{(2)}} = r_2 = \text{const}, \frac{V}{\omega^{(1)}} = r_2 \frac{N_1}{N_2} - 2a\phi_1 = f(\phi_1) \qquad (9)$$

Here $\omega^{(1)}$ and $\omega^{(2)}$ are the angular velocities of pinion and gear by cutting; V is the velocity of the rack cutter in translational motion; N_1 and N_2 are the gear and pinion tooth numbers; ϕ_1^1 is the angle of rotation of the pinion by cutting. The generated gears will be in point contact at every instant and transform rotation with the function

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 - a\phi_1^2 \; 0 < \phi_1 < \frac{2\pi}{N_1} \qquad (10)$$

This function relates the angles of rotation of the pinion and the gear, ϕ_1 and ϕ_2, respectively, for one cycle of meshing. The predesigned function of transmission errors is

$$\Delta\phi_2 = -a\phi_1^2 \qquad (11)$$

It is evident that after differentiation of Function (10), the gear ratio $\omega^{(2)}/\omega^{(1)}$ satisfies Equation (9).

To apply this method of generation in practice it is necessary to vary the angular velocity of the pinion in the

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TRANSMISSION ERRORS . . .
(continued from page 13)
process of its generation. That may be accomplished by a computer-controlled machine for cutting.

Synthesis of Spiral Bevel Gears with a Predesigned Parabolic Function of Transmission Errors

Generally, spiral bevel gears generated on Gleason machinery are designed and manufactured with non-conjugate tooth surfaces. By varying the machine tool settings, it is possible to obtain a lead function of transmission errors, a parabolic function with pinion lagging, or a parabolic function with gear lagging. Only a parabolic function with gear lagging is good for applications. The problem encountered is that it is very difficult to reduce the level of the parabolic function of transmission errors with gear lagging. Litvin et al.[5] proposed a method for generation of spiral bevel gears with conjugate tooth surfaces. Gears with such surfaces can transform rotation with constant gear ratio. Since the gear misalignment will cause a piece-wise linear function of transmission errors, it is necessary to predesign even for conjugate spiral bevel gears a parabolic function of transmission errors. Such a function will absorb the linear function of transmission errors. This goal can be achieved by modifying of the pinion tooth surface slightly. This is done by varying the machine tool settings for the conjugate gear tooth surfaces.

Rearranging (1), we can obtain

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 + \Delta\phi_2(\phi_1) \qquad (12)$$

Differentiating (9) by ϕ_1, we obtain

$$m_{21}(\phi_1) = \frac{N_1}{N_2} + \Delta\phi_2^1(\phi_1) \qquad (13)$$

where $m_{21}(\phi_1)$ is the instantaneous gear ratio.

Differentiating again (10) by ϕ_1, we receive

$$m_{21}'(\phi_1) = \Delta\phi_2''(\phi_1) \qquad (14)$$

To obtain a parabolic function with gear lagging, the value of m_{21}' must be negative.

The determination of machine tool settings is based on the local synthesis of the gears. This synthesis is based on the following ideas: (i) The gear tooth surfaces are in contact at the chosen mean point. (ii) The instantaneous gear ratio at the mean contact point is equal to the given value. (iii) The derivative of m_{21} is a negative that yields a predesigned parabolic function with gear lagging. (iv) The tangent to the path contact has the prescribed direction. (In the discussed case the above-mentioned tangent is directed along the surface.) (v) The principal curvatures and directions of the gear tooth surfaces must be related to satisfy the requirements to m_{21}, the direction of the tangent, and the dimensions of the contact ellipse. The meshing of the gears and their bearing contact in the region of meshing can be investigated by the TCA (Tooth Contact Analysis) Program.[6]

Example 4. The pinion is right-handed, $N_1 = 10$, $N_2 = 41$, diametral pitch is 5.559" in^{-1}, pinion root angle is 12°1', gear root angle is 72°25', mean spiral angle is 35°.

Table 7 shows the basic machine tool settings for conjugate gears and for gears with the predesigned parabolic function. The transmission errors for the gear convex side (drive side) when the gears are not misaligned are given in Table 8. Table 9 shows the resulting transmission errors for the gears with the crossing offset 0.002". Table 10 shows the resulting transmission errors for the gear convex side with the shift of the pinion axis 0.002". The transmission errors in both cases are represented by a parabolic function.

The bearing contact of the synthesized gears is shown in Fig. 12.

Conclusion

Modified tooth surfaces for spur, helical, and spiral bevel gears with a predesigned parabolic function of transmission errors have been proposed. The predesigned function allows absorption of a linear function of transmission errors that are caused by misalignment and errors of manufacturing. Methods for generation of the modified gears have been also developed.

Table 7. Machine-Tool Settings.

	Gear (LH)	Pinion (RH)	
	Both Sides	Concave Side	
	Both Cases	Conjugate Gears	Nonconjugate Gears
Cutter Radius	3.9571	3.8957	3.8953
Cutter Width	0.08	—	—
Blade Angle	20°	16.7979°	16.5919°
Machine Root Angle	72.4098°	12.0236	12.0236°
Radial	3.3775	3.3153	3.3031
Setting Angle	73.6837°	75.1337°	76.8170°
Machine Offset	0	0.00508	0.01391
Machine Ctr. to Back	0	−0.00041	−0.01130
Sliding Base	0	−0.00009	−0.00235
Ratio of Roll	0.9738	0.2375	0.2371
Tilt	0°	0°	0°
Swivel	0°	0°	0°

Table 8. Predesigned Function of Transmission Errors.

$\phi_1^{(°)}$	−17	−11	−5	1	7	13	19
$\Delta\phi_2^{('')}$	−7.66	−3.06	−0.61	−0.02	−1.10	−3.67	−7.60

Table 9. Function of Transmission Errors.

$\phi_1^{(°)}$	−32	−26	−20	−14	−8	−2	4
$\Delta\phi_2^{('')}$	−2.76	1.75	4.14	4.71	3.65	1.13	−2.69

Table 10. Function of Transmission Errors.

$\phi_1^{(°)}$	−24	−18	−12	−6	0	6	12
$\Delta\phi_2^{('')}$	−6.71	−1.89	0.62	1.16	0	−2.68	−6.71

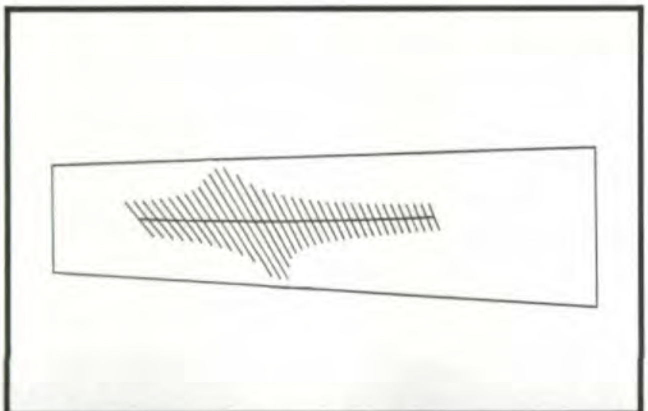

Fig. 12 – Bearing contact on the gear surface.

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