The European Rack Shift Coefficient "X" for Americans

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Introduction

The use of dimensionless factors to describe gear tooth geometry seems to have a strong appeal to gear engineers. The stress factors I and J, for instance, are well established in AGMA literature. The use of the rack shift coefficient "x" to describe nonstandard gear proportions is common in Europe, but is not as commonly used in the United States. When it is encountered in the European literature or in the operating manuals for imported machine tools, it can be a source of confusion to the American engineer.

What follows is intended to provide a source for the background and derivations of the "x" factor as used in European standards and papers. The addendum modification, rack shift, or profile shift factor has several mathematical definitions in the U.S. Most European documents use a specific definition, based on a theoretical "zero backlash" gear pair in tight mesh at the nominal center distance. (See McVittie, 86 FTM 1 for discussion.)

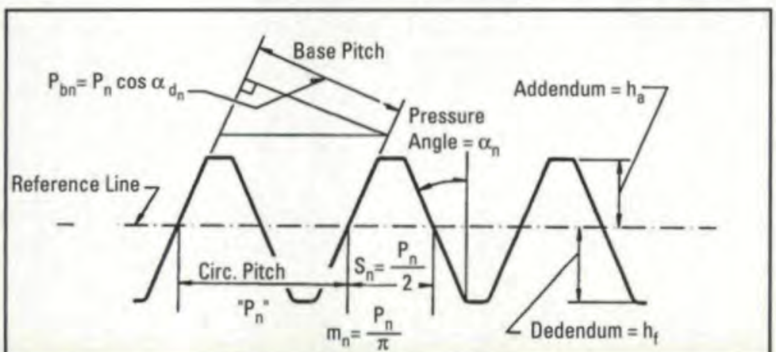

Fig. 1 — Basic rack definition — normal plane.

Basic Rack

The definitions and equations in this article are based on a "basic rack" in which addendum and dedendum are measured from a reference line located where the tooth thickness and the space width on the reference line are equal. The basic rack represents the tooth form in the normal plane of a gear with an infinite number of teeth. The normal module of the basic rack is equal to the normal circular pitch divided by π. The normal diametral pitch of the basic rack is equal to π divided by the normal circular pitch.

The basic rack represents the theoretical gear tooth form, not the form of the cutting tool. No allowance is made for backlash, finishing stock, or manufacturing method.

The standard 20° normal pressure angle basic rack of ISO 53 is commonly used. This document is valid for that basic rack and for any other basic rack which meets the criteria of Fig. 1.

Addendum Modification Factor

The addendum modification factor "x" (*Profilverschiebungsfaktor,* "profile shift factor" in German) represents the distance, in tight mesh, from the reference line of the basic rack to the reference circle of the gear (rack shift or profile shift) for normal module = 1.0 or normal diametral pitch = 1.0.

Sum of X Factors

The European practice is to define the sum of $x_1 + x_2$, (Σx) for a theoretical gear pair which operates in tight mesh (has no backlash) on the nominal operating center distance.

The basic equation can be derived from the basic tooth thickness involute geometry equations and the requirement that the sum of the transverse tooth thicknesses at the operating pitch diameter is equal to the transverse circular pitch at that diameter.

$$\Sigma x = \frac{z_1 + z_2}{2} \cdot \frac{\mathrm{inv}\,\alpha_{wt} + \mathrm{inv}\,\alpha_t}{\tan \alpha_n}$$

(1) DIN 3992 Eq 9

$$\tan \alpha_t = \frac{\tan \alpha_n}{\cos \beta}$$

(2) DIN 3992 Eq 6

$$a = m_t \cdot \frac{z_1 + z_2}{2} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}}$$

(3) DIN 3992 Eq 5

$$m_t = \frac{m_n}{\cos \beta} \qquad (4)$$

X Factor for Each Gear

The values of x for gear and pinion are chosen somewhat arbitrarily (See Maag, DIN 3992, and ISO/TR 4467 for further information on choice of x factors) according to the operating conditions and gear ratio, so that their total is equal to Σx. The theoretical addendum (tip) diameters and tooth thicknesses of the two gears in the gear pair are defined by their x factors.

$$d_a = d + 2 \cdot m_n (1+x) \qquad (5)$$

$$s_n = (\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_n) \cdot m_n$$

(6) ISO DTR 10064/2 Eq 6.4

The actual addendum diameters and tooth thicknesses are then adjusted (usually reduced) to control backlash and tip to root clearance.

Backlash Allowance

A common convention among gear manufacturers is to reduce the normal tooth thickness of each member by the same amount, which may be a value in µm or a function of module, such as $.024 \cdot m_n$. This maintains the same cutting depth for both members and maximizes contact ratio. The direction (normal, transverse, reference circle, or base tangent plane) in which the tooth thickness reduction is to be measured must be specified, since there is no recognized convention.

Working Group (WG)2 of ISO/TC60 is considering a draft technical report, DTR10064/2, containing tables which recommend that the tooth thinning for backlash, called "upper allowance of size", E_{ssn}, be a function of the pitch diameter of each part. The values are measured normal to the helix angle in the reference cylinder. The values can be converted as follows:

The transverse circular allowance, E_{sst}, is:

$$E_{sst} = \frac{E_{ssn}}{\cos \beta} \qquad (7)$$

The normal allowance in the base tangent plane, E_{bsn}, (normal to the tooth surface) is

$$E_{bsn} = E_{sst} \cdot \cos \alpha_t \cdot \cos \beta_b \qquad (8)$$

which can also be expressed as

$$E_{bsn} = E_{ssn} \cdot \cos \alpha_n \qquad (9)$$

The resulting transverse circular backlash at

Table of Symbols

ISO	AGMA	Definition
a	C	Center Distance
c		Clearance $(h_a - h_f)$
d	d	Diameter
E		Tooth Thinning for Backlash
h_a	a	Addendum
h_f		Dedendum
k		Tip Shortening Factor
j	B	Backlash
m	m	Module
p	p	Circular Pitch
s	t	Tooth Thickness
x		Addendum Modification Factor
α	ϕ	Pressure Angle
β	ψ	Helix Angle

Table of Subscripts

Subscript	Meaning
(none)	At Reference Diameter
a	At Addendum (Tip) Diameter
b	At Base Cylinder Diameter
f	At Root Diameter
n	Normal Plane
o	Tool Dimensions
t	Transverse Plane
w	At Working Diameter
y	At Any (Undefined) Diameter
1	Pinion
2	Gear or Rack

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the working diameter is a function of allowance, center distance, and tooth accuracy. (See AGMA 2002 for more information.)

Tip Shortening for Clearance

When $\Sigma x > 0$, the tips of external gear pairs should be shortened to maintain standard tip to root clearance. The reduction in clearance is often ignored for small values of Σx, but for larger values the addendum should be shortened by $k \cdot m_n$.

$$k = \frac{z_1 + z_2}{2} \cdot \left[\frac{\text{inv}\,\alpha_{wt} - \text{inv}\,\alpha_t}{\tan \alpha_n} - \frac{1}{\cos \beta} \cdot \left(\frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right) \right]$$

(10) Maag Eq 68

Tip diameters of internal gear pairs should be checked for clearance and interference with cutters and mates by calculation of actual cutting and mating conditions.

Actual Root Diameter and Clearance

The European method doesn't calculate the actual root diameter of gears which are thinned for backlash by feeding the cutter to greater depth. When the actual root diameters are calculated, the addendum diameter required for standard clearance can be calculated more accurately from Eq. 11.

$$d_{a1} = 2 \cdot (a - c) - d_{f2} \qquad (11)$$

The root diameter at maximum tooth thickness can be calculated as follows:

$$d_f = d - 2 \cdot \left(h_{ao} - x \cdot m_n + \frac{E_{ssn}}{2 \cdot \tan \alpha_t \cdot \cos \beta} \right) \qquad (12)$$

Equation 12 is based on the assumption that the cutter addendum, h_{ao}, is measured as shown in Fig. 1 for the basic rack. If the gear is to be finished in a second operation, as by shaving, skiving, or grinding, a more detailed study is required to estimate the finished root diameter. (See Appendix E, Sec. E6 of AGMA 218.01 for more information.)

Fig. 2 — Addendum modification

Convention for Signs

For external gears, the value of x is positive when the tooth thickness is increased and the value of Σx is positive when the center distance is greater than standard.

The same convention can be used for internal gears if the sign of the center distance is considered negative. "Long addendum" internal gears have a negative x. This convention is common, but is not universal.

Internal Gears

The equations in this article are arranged for external gears. With a few exceptions, they can be used for internal gears if the internal diameters, center distance, and number of teeth are made negative. The convention for signs must be checked carefully. One trap is division by a negative value to calculate an involute function, which must be positive. It is good programming practice to take the absolute value of the quotient before calculating the angle from the involute function. ■

Appendix — Derivations of Equation 9

$$E_{bsn} = E_{sst} \cdot \cos \alpha_t \cdot \cos \beta_b$$

$$= \frac{E_{ssn}}{\cos \beta} \cdot \cos \alpha_t \cdot \cos \beta_b \qquad (8)$$

$$\cos \beta = \frac{p_n}{p_t} \quad \cos \beta_b = \frac{p_{bn}}{p_{bt}}$$

$$\cos \alpha_n = \frac{p_{bn}}{p_n} \quad \cos \alpha_t = \frac{p_{bt}}{p_t}$$

$$E_{bsn} = E_{ssn} \quad \cos \alpha_n \qquad (9)$$

References:

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