Effects of Planetary Gear Ratio on Mean Service Life

M. Savage, K. L. Rubadeux & H. H. Coe

Nomenclature
Variables
a — gear addendum (mm, in) and bearing life adjustment factor
C — dynamic capacity (kN, lbs)
E — elastic modulus (MPa, psi)
f — face width (mm, in)
F — load (kN, lbs)
K_f — stress concentration factor
l — life (106 cycles and hours)
n — gear ratio relative to the arm, number of planets
n_a — actual transmission gear ratio
N — number of gear teeth
P_d — diametral pitch (1.0/inch)
R — gear radius (mm, in) and reliability
Y — Lewis Form Factor
ϕ — pressure angle (degrees, radians)
υ — Poisson's Ratio
ρ — radius of curvature (mm, in)
σ — bending stress (Pa, psi)
σ_H — Hertzian contact stress (Pa, psi)
ω — angular velocity, speed (rpm)
ω_b — bearing load cycle speed (rpm)
Σ — central angle between two adjacent planet center lines with the input shaft center (radians)

Subscripts	***Superscripts***
av — mean	b — Weibull slope exponent
d — dynamic	p — load-life exponent
o — output	
pl — planet	
r — ring gear	
s — sun gear	
1 — pinion	
2 — gear	
10 — 90% reliability	

Abstract

Planetary gear transmissions are compact, high-power speed reducers that use parallel load paths. The range of possible reduction ratios is bounded from below and above by limits on the relative size of the planet gears. For a single-plane transmission, the planet gear has no size at a ratio of two. As the ratio increases, so does the size of the planets relative to the size of the sun and ring. Which ratio is best for a planetary reduction can be resolved by studying a series of optimal designs. In this series, each design is obtained by maximizing the service life for a planetary transmission with a fixed size, gear ratio, input speed, power and materials. The planetary gear reduction service life is modeled as a function of the two-parameter Weibull distributed service lives of the bearings and gears in the reduction. Planet bearing life strongly influences the optimal reduction lives, which point to an optimal planetary reduction ratio in the neighborhood of four to five.

Introduction

Planetary gear transmissions offer the user a moderate gear reduction with a high power density. By carrying multiple planet gears on a rotating arm, load sharing is enabled among the planets. The symmetrical placement of the planets about the input sun gear provides radial load cancellation on the bearings that support the input sun and the output arm (Refs. 4 & 6). The fixed internal ring gear support also has no net radial load. With near-equal load sharing in medium-to-fine pitch gearing, a compact reduction results. Planetary reductions are often found in transportation power transmissions due to this weight and volumetric efficiency (Refs. 4 & 8).

Much of the published design literature for planetary gearing focuses on the kinematic proportioning of the unit to achieve one or more reductions through the use of clutches and brakes (Refs. 7 & 18).

Recent literature on planetary gears has focused on the dynamic loads in the transmission with measurements of load sharing and load variations in specific units (Refs. 3, 5, 8 & 10). Monitoring the dynamic loads in a planetary transmission has also been proposed as one method of determining the need for preventive maintenance in the transmission (Ref. 2).

While the reduction of dynamic loads in a planetary transmission is an important task, these studies do not indicate which ratio is best suited for a planetary transmission. Studies of rotating power in planetary transmissions have indicated that as the ratio is increased, the percent of rotating power in the unit decreases (Ref. 6). This suggests that the best ratio for a planetary reduction is the highest possible, which is reached with the largest planet gears. Addendum interference between the planets determines this limit. However, when one considers the size of a planetary reduction required to transmit a given power level at a given input speed, the loading on the gears and bearings in the reduction become an important factor, as do the component lives under load (Refs. 11 & 15).

Since aircraft and automotive transmissions can see service in excess of their nominal design lives, periodic maintenance is provided throughout their lives (Refs. 2 & 12). The service life of a transmission between maintenances is a design variable that one would like to maximize for a given size and power.

Programs have been written to optimize transmissions for service life (Refs. 9, 13 & 14). The service life of the transmission is modeled as a function of the service lives of the components that have a two-parameter Weibull distribution. The critical components for this calculation are the bearings and the gears in the transmission. A mean life of the transmission is determined from the mean lives of the critical components under load.

In this article, the influence of speed reduction magnitude on the service life of a planetary gear reduction is investigated for reductions with similar components. An optimal gear reduction for a planetary gear set is sought considering the size and capacities of the components. For a fixed power level and transmission size, the life is charted versus the reduction ratios for a fixed input speed and three, four and five planets.

Planetary Constraints

In comparing the lives of similar transmissions, one needs to specify the conditions of similarity. The planetary gear reductions considered in this work are single-plane reductions with input sun gears, fixed ring gears and multiple planet gears. The planet gears are placed symmetrically about the concentric input and output shafts as shown in Fig. 1. Each planet of a reduction is connected to the output arm through a single ball bearing at its center. Since the input sun gear and fixed ring gear mesh with all the planet gears, a single diametral pitch or module is used for all gears in a reduction, as is a single face width.

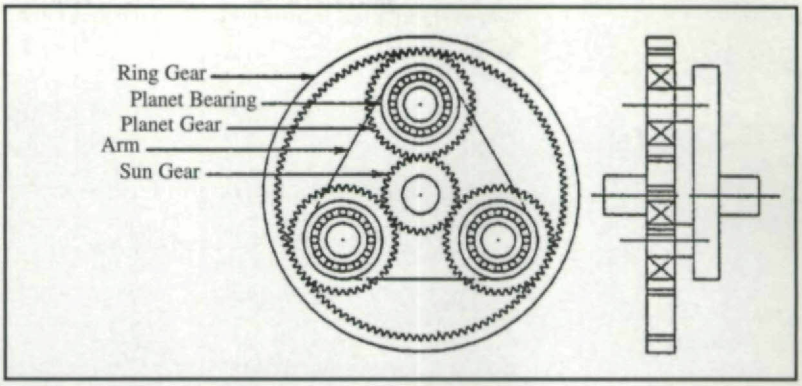

Fig. 1 — Single-plane planetary transmission.

No bearings are included on the input or output shaft since the internal loads in the planetary transmission are balanced on these shafts due to the symmetric placement of the planets. Bearings are needed on these shafts, but their placement and loading are based on external considerations.

All transmissions carry the same power and have the same outside diameter, which provides a radial ring thickness outside the ring gear teeth of 1.5 times the tooth height.

In this comparison, the input speed and torque are fixed as the ratio is varied. For each design, the planetary system life is maximized subject to the above constraints in addition to constraints on the stresses in the gear teeth and on assembly clearances. The parameters that define each design are the number of teeth on the sun gear, N_s, the face width of the gears, f, and the diametral pitch of the gears, P_d.

Kinematics

In a planetary gear train, the planetary gear ratio is the ratio of the speeds of the input and output shafts. To determine this ratio, one first needs to calculate the gear ratio of each gear mesh in terms of the number of teeth on each gear. The gear ratio of the sun gear mesh with the arm fixed is

$$n_1 = -\frac{N_{pl}}{N_s} \tag{1}$$

and the gear ratio of the ring gear mesh with the arm fixed is

$$n_2 = \frac{N_r}{N_{pl}} \tag{2}$$

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where the overall transmission gear ratio relative to the arm is

$$n = n_1 \bullet n_2 \qquad (3)$$

and the speed of the output arm relative to the fixed ring is

$$\omega_o = \frac{\omega_o}{(1 - n)} \qquad (4)$$

So the planetary gear ratio is

$$n_a = 1 - n \qquad (5)$$

And the speed of each planet gear is

$$\omega_{pl} = \omega_s \left(\frac{n - n_1}{n_1 (1-n)}\right) = \omega_s \left(\frac{1 - n_2}{1-n}\right) \qquad (6)$$

The planet bearing load cycle speed is the speed of the planet with respect to the arm:

$$\omega_b = \omega_{pl} \left(\frac{n}{n - n_1}\right) \qquad (7)$$

For each transmission studied, the planetary gear ratio, n_a, is fixed, and the number of teeth on the sun gear is an independent design parameter. Values that maximize the service life for a given transmission size are found for the number of teeth on the sun gear, the gear face width and the diametral pitch. This requires the number of teeth on the ring gear, N_r, and on each planet gear, N_{pl}, to be found in terms of n_a and N_s.

The number of teeth on the ring gear is related to the number of teeth on the sun by the gear ratio relative to the arm, since the planets become idlers in this inversion.

$$N_r = -nN_s = (n_a - 1)\, N_s \qquad (8)$$

Since the diameter of the ring gear is equal to the diameter of the sun gear plus twice the diameter of the planet gear, the number of teeth on each planet gear can be calculated by

$$N_{pl} = \frac{N_r - N_s}{2} = \frac{(n_a - 1 - 1)N_s}{2} = \frac{(n_a - 2)N_s}{2} \qquad (9)$$

To keep the number of planet teeth positive, the transmission gear ratio, n_a, must have a value greater than 2. At 2, the planet gears have no size, and the planetary reduction ceases to exist.

To prevent interference among gear teeth of adjacent planet gears, sufficient circumferential clearance must be provided. Requiring that the distance between the axes of two adjacent planets be greater than the outside diameter of the planet gear by twice the tooth addendum will accomplish this:

$$2 \bullet (R_s + R_{pt}) \bullet \sin\left(\frac{\Sigma}{2}\right) > 2 \bullet (R_{pl} + 2 \bullet a) \qquad (10)$$

where Σ is the central angle between two adjacent planet center lines and a is the addendum of the planet gears.

One additional constraint is needed to allow the planets to be positioned symmetrically around the sun gear. The sum of the number of teeth on the sun and on the ring divided by the number of planets must produce an integer.

$$\frac{N_s + N_r}{n_{pl}} = I \qquad (11)$$

Tooth Strength

The AGMA model for gear tooth bending uses the Lewis form factor and a stress concentration factor to determine the stress in the tooth for a load at the highest point of single tooth contact (Ref. 1). The bending stress model is

$$\sigma = \frac{F_d \bullet P_d \bullet K_f}{f \bullet Y} \qquad (12)$$

where F_d is the tangential dynamic load on the tooth, K_f is the stress concentration factor and Y is the Lewis form factor based on the geometry of the tooth. Since the Lewis form factor is a function of the tooth shape, it is dependent on the number of teeth on the gear, as is the stress concentration factor.

Large localized stresses occur in the fillets of gear teeth due to the change in the cross section of the tooth. Although the maximum stress is located closer to the root circle than predicted by Lewis' parabola, the distance between the two locations of maximum stress is relatively small, and the stress concentration factor accurately compares the maximum stress in the tooth to the Lewis stress (Ref. 1). This method of rapid calculation of bending stress for external gear teeth is extended to include the bending stress in the internal gear teeth of the ring gear (Ref. 17).

In addition to bending stresses, surface contact stresses can contribute to gear tooth failure. The Hertzian pressure model closely predicts these contact pressures:

$$\sigma_H = \left(\frac{F_d}{\pi \bullet f \bullet \cos\phi}\left(\frac{\frac{1}{\rho_1} + \frac{1}{\rho_2}}{\frac{1 - \upsilon_1^2}{E_1} + \frac{1 - \upsilon_2^2}{E_2}}\right)\right)^{1/2} \qquad (13)$$

where ϕ is the normal pressure angle of the gear mesh, ρ_1 and ρ_2 are the radii of curvature of the pinion and gear tooth surface at the point of contact, ν_1 and ν_2 are the Poisson ratios and E_1 and E_2 are the moduli of the material elasticity for the two gears.

Contact pressure near the pitch point leads to gear tooth pitting, which limits the life of the gear tooth. Gear tip scoring is another type of failure that is affected by the contact pressure at the gear tooth tip. One model for gear tip scoring includes the pressure times velocity factor, where the sliding velocity at the gear tip is tangent to the tooth surfaces.

Service Life

Surface pitting due to fatigue is the basis for the life model for the bearings, gears and transmission. Fatigue due to this mode of failure has *no endurance limit, but has a service life* described by a straight line on the log stress versus log cycle S-N curve. This life-to-load relationship can be written for a specific load, F, at which the 90% reliability life is l_{10} and which is related to the component dynamic capacity, C, as:

$$l_{10} = a\left(\frac{C}{F}\right)^p \qquad (14)$$

Here the component dynamic capacity, C, is defined as the load that produces a life of one million cycles with a reliability of 90%, and a is the life adjustment factor. The power, p, is the load-life exponent, which is determined experimentally.

Complementing this load-life relationship is the two-parameter Weibull distribution for the scatter in life. In this distribution, the reliability, R, is related to the life, l, as:

$$Ln\left(\frac{1}{R}\right) = Ln\left(\frac{1}{0.9}\right) \bullet \left(\frac{l}{l_{10}}\right)^b \qquad (15)$$

A meaningful estimation of service time is the mean time between overhauls. The mean life for a two-parameter Weibull distribution can be expressed in terms of the gamma function, Γ, as:

$$l_{av} = \frac{l_{10} \bullet 10^6 \bullet \Gamma\left(1 + \frac{1}{b}\right)}{60 \bullet \omega_o \bullet \left[Ln\left(\frac{1}{0.9}\right)\right]^{\frac{1}{b}}} \qquad (16)$$

including the conversion from million cycles to hours, where ω_o is the output speed in rpm.

If the repairs are component repairs, rather than full replacements, then the mean life between overhauls is based directly on the mean lives of the individual components. In this case, the transmission repair rate, which is the reciprocal of the mean life, is the sum of the individual component repair rates. Thus, the transmission mean service life is estimated as the reciprocal of the repair rate:

$$l_{av,s} = \frac{1}{\Sigma \frac{1}{l_{av,i}}} \qquad (17)$$

Table 1 — Planetary Design Inequality Constraints

Constraint	*Value*	*Unit*	*Type*
Bending stress: sun-planet	40,000.000	psi	upper
Full load Hertz stress: sun-planet	180,000.000	psi	upper
Gear tip Hertz pressure: sun-planet	180,000.000	psi	upper
PV factor of sun-planet teeth	50.000	10^6 psi-ft/min	upper
Flash temp of sun-planet teeth	200.000	deg. F	upper
Sun involute interference	0.001	radians	lower
Sun face width to diameter	0.750	ratio	upper
Bending stress: planet-ring	40,000.000	psi	upper
Full load Hertz stress: planet-ring	180,000.000	psi	upper
Gear tip Hertz pressure: planet-ring	180,000.000	psi	upper
PV factor of planet-ring teeth	50.000	10^6 psi-ft/min	upper
Flash temp of planet-ring teeth	200.000	deg. F	upper
Involute interference: planet-ring	0.001	radians	lower
Planet circumference clearance	0.100	in	lower
Bearing diameter	0.400	in	lower
Diameter of ring gear	12.000	in	upper
Volume of transmission	1,000.000	in^3	upper

Planetary Designs

In considering the effects of the gear ratio on the mean transmission life, the input speed and power were held constant. The input speed was 2,000 rpm for all transmissions, which carried a power of 51 hp with a fixed input torque of 1,600 lb-in. Each transmission has a maximum ring gear outside diameter of 12". The sun gear mesh and the ring gear mesh both had a normal pressure angle of 20° and the same diametral pitch. All gears were made of high strength steel with a surface material strength of 220 ksi. The Hertzian contact pressure was limited to less than 180 psi, and the tooth bending stresses were limited to less than 40 ksi. These limits include a total load design factor of 1.5 to adjust the nominal stress calculations of Eqs. 12 and 13 to code levels. The PV factor was limited to less than 50 million psi-ft/min, and the gear tooth flash temperature was limited to less than 200°F. The Weibull slope of the sun gear, the three planet gears and the ring gear was 2.5. The load-life factor of all five gears was 8.93. The planet bearings were 300 series, single-row ball bearings, with a Weibull slope of 1.1, a load-life factor of 3.0 and a life adjustment factor of 6.

Table II — Design Service Lives									
Planet	Ratio	Tooth Numbers			Face Width	Pitch	Life	Pitch	Life
	n_a	N_s	N_{pl}	N_r	f in	P_d in^{-1}	l_{av} hrs	P_d1 in^{-1}	$l_{av}1$ hrs
3	3.0	60	30	120	1.0	11	1040	10.68	1320
	3.5	48	36	120	1.0	11	2430	10.7	3000
	4.0	45	45	135	1.25	12	4720	11.95	4870
	4.5	40	50	140	1.5	13	3880	12.4	5500
	5.0	36	54	144	1.5	13	4940	12.7	5870
	5.5	36	63	162	1.5	15	4300	14.2	6230
	6.0	30	60	150	1.5	14	3600	13.2	5590
	6.5	24	54	132	1.5	12	3740	11.68	4560
	7.0	24	60	144	1.5	13	3870	12.7	4600
	7.5	24	66	156	1.5	14	3900	13.7	4580
	8.0	24	72	168	1.5	15	3810	14.7	4450
4	3.0	60	30	120	1.0	11	1850	10.68	2340
	3.5	40	30	100	1.0	10	1640	9.02	3680
	4.0	40	40	120	1.25	11	5880	10.7	7240
	4.5	40	50	140	1.5	13	6900	12.35	10100
	5.0	36	54	144	1.5	13	8780	12.68	10560
5	3.0	60	30	120	1.0	11	2890	10.68	3660
	3.5	40	30	100	1.0	10	2570	9.05	5600
	4.0	40	40	120	1.25	11	9180	11.7	11300

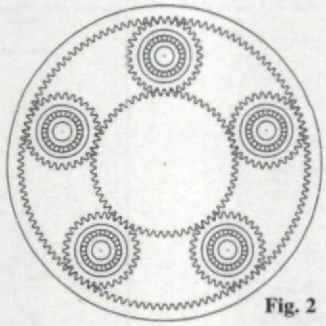

Fig. 2

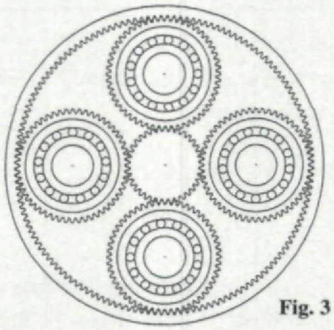

Fig. 3

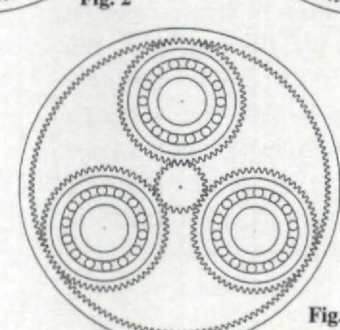

Fig. 5

Fig. 2 — Planetary transmission with a reduction ratio of 3.0

Fig. 3 — Planetary transmission with a reduction ratio of 5.0

Fig. 4 — Planetary transmission with a reduction ratio of 7.0

Table II lists the obtained designs with the numbers of teeth on the sun, planet and ring gears, the gear face width and the diametral pitch for each ratio. These teeth numbers are discrete values that produce the required planetary ratio and allow symmetric placement of the planets for radial load cancellation. After the diametral pitch, the mean service life of the transmission is listed for component replacement at repair. This life corresponds to the integer diametral pitch listed before it. It also corresponds to a somewhat smaller transmission as dictated by the integer pitch. The last two columns show larger lives, which vary more continuously, and the fractional diametral pitch required to obtain these lives by allowing the transmission to have the full 12" outside ring diameter. The table includes blocks of data for three-, four- and five-planet designs.

Even higher lives would be possible with fine pitch gearing, since the outside diameter limit includes the ring gear dedendum and the rim height outside the ring gear pitch diameter. Both distances are proportional to the tooth height. However, the diametral pitch is limited to 16 or less to maintain overload tooth bending strength.

The results show that as the gear ratio was increased, the size of the sun gear decreased, and the size of the planet gears increases. Figs. 2–4 show planetary transmission designs for speed reduction ratios of three, five and seven.

The effect of the gear ratio on the mean life of the transmission is plotted in Fig. 5. For the integer diametral pitch designs with three planets, the mean service life, plotted as a series of crosses, increased from 1,040 hours for a gear ratio of three to 4,940 hours for a gear ratio of five, and then decreased to 3,600 hours for a gear ratio of six, with a final life of 3,810 hours for a gear ratio of eight. Higher lives that varied more continuously were available with uneven pitches and are plotted as a life limit line above the found design lives. This line corresponds to the primed pitches and lives of Table II and is also jagged due to the discrete nature of the numbers of teeth.

Similar data are plotted with circles for integer pitch designs with four planets and with squares for designs with five planets. For the four-planet designs, the integer pitch design lives ranged from 1,850 hours for a gear ratio of three to a maximum of 8,780 hours for a gear ratio of five. And for the five-planet designs, the mean service lives varied from 2,890 hours for a gear ratio of three to 9,180 hours for a gear ratio of four. Similar life limit designs are plotted above these points for designs with the full 12" outside diameter and non-integer diametral pitches.

At low planetary ratios, the planet and planet bearing sizes were small. At a ratio of three, the smallest bearings for the optimal designs were selected, causing the low life designs for each number of planets. As the planetary ratio was increased, the size of the planets and the planet bearings increased, which increased the life of the transmissions. With more planets to share the load, the four- and five-planet designs had greater lives than the three-planet designs. However, circumferential planet interference limited the five-planet designs to a maximum ratio of four and the four-planet designs to a maximum ratio of five. At ratios above 5.5, the life of the three-planet designs dropped due to the increase in the output torque. Once again, the lower transmission life was attributable to lower planet bearing life. At a gear ratio of eight, the pitch diameter of the sun gear had decreased to 1.6" with a face width of 1.5". Larger ratios would have decreased this length-to-diameter ratio even further and would have increased the bending stress in the sun gear teeth above the 21 ksi present in the eight-to-one gear ratio design. So the table and graph were cut off at this gear ratio even though designs are possible at higher ratios with three planets.

Conclusions

The effect of the gear ratio on the life of the transmission was examined. Of interest is the possibility of an optimal planetary gear reduction from a life standpoint. In this study the overall size of the transmission was held constant, its strengths were maintained and the ratio was varied for the three-, four- and five-planet arrangements. Each optimal design was defined by the number of teeth on the sun gear, the gear face width and the diametral pitch of the gears. For the comparison, the transmission input speed and power were held constant. The results show that as the gear ratio increased, the size of the sun gear decreased, and the size of the planet gears increased. At a ratio of three, the planet bearings were reduced in size relative to the transmission sufficiently to limit the transmission life. Five-planet designs had a maximum ratio of four with no planet interference, and four-planet designs could be obtained with ratios up to five. Above five and a half, the lives of the three planet designs fell off due to the higher output torques. The optimal design exists for a transmission with a gear ratio of approximately four to five.

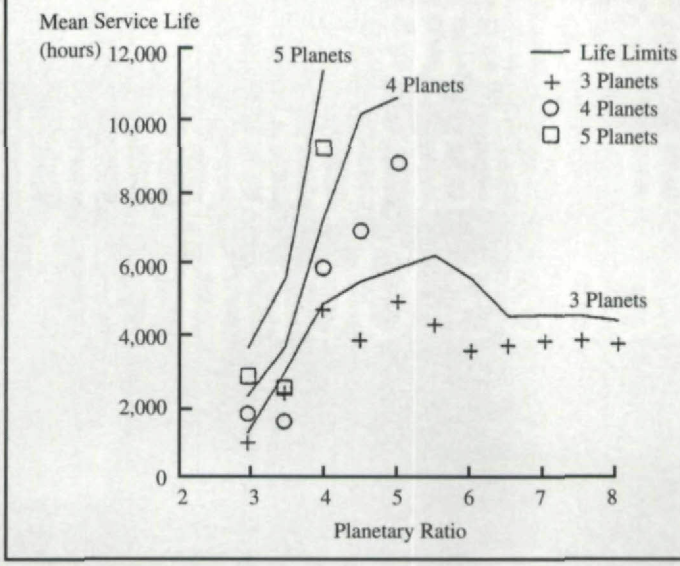

Fig. 5 — Mean transmission service life versus speed reduction ratio with constant input speed and torque.

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