Effects of Axle Deflection and Tooth Flank Modification on Hypoid Gear Stress Distribution and Contact Fatigue Life

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Management Summary

As is well known in involute gearing, "perfect" involute gears never work perfectly in the real world. Flank modifications are often made to overcome the influences of errors coming from manufacturing and assembly processes as well as deflections of the system. The same discipline applies to hypoid gears. This paper, first of all, presents an approach on validating FEA-predicted axle system deflections. Next, influences of axle deflections and typical flank modifications (lengthwise crowning, profile crowning and twist) on contact pattern and stress distribution of the hypoid gear set are simulated by using a face-hobbed hypoid gear design. Finally, two groups of experimental hypoid gear sets are made with two different designed flank modifications. Actual tooth surface topographies are examined by using a CMM to assure the desired flank modifications are achieved. These experimental gear sets are tested to investigate the impact of flank modifications on actual gear life cycles. Test results of the sample gears are reported to illustrate the effect of tooth flank modifications on contact fatigue life cycles.

Introduction

Hypoid gears are widely used in many applications, such as axles and four-wheel drive transmissions for on- and off-highway vehicles. Recent studies on hypoid gearing are found in the subject of tooth surface generation and contact analysis of hypoid gears manufactured by the face hobbing process (Refs. 1–4), noise (Ref. 5) and dynamics (Ref. 6), friction and efficiency (Refs. 7–8), wear (Refs. 9–10), lubrication (Ref. 11), as well as lapping and superfinishing (Ref. 12). For hypoid gear drives applied in heavy vehicle axles, durability has been the primary concern. Axle deflections have a significant impact on gear tooth strength (Ref. 13) while axle deflection data are typically obtained through experimental measurements of a loaded axle under certain controlled conditions (Ref. 14). Unexpected deflections can cause severe edge loading that is detrimental to gear surface life as well as noise performance. For heavily loaded hypoid gears, flank modifications are particularly critical to achieve required durability performance under relatively large axle deflections (Ref. 4). There are basically three types of flank modifications—profile crowning, lengthwise crowning and twist (Ref. 15). For hypoid gears, in addition to tool geometry and basic machine settings, higher-order machine settings are also very important to achieve desired flank modifications (Refs. 15–16).

The objective of this paper is to study the impact of axle deflection and tooth flank modification on hypoid gear stress distribution and contact fatigue life. First, an approach to validate axle system deflections will be proposed. Then, by using computer programs and an example of a face-hobbed hypoid gear design, influences of axle deflections and typical flank modifications on contact pattern and stress distribution of the hypoid gear set will be simulated. And finally, several experimental hypoid gear sets are made with two differently designed flank modifications. These samples are tested under the high-cycle fatigue drive side in an axle assembly to investigate the impact of flank modifications on actual gear life cycles.

Methodology

Axle deflection. Axle deflection is one of the most critical issues in the design and analysis of hypoid and spiral

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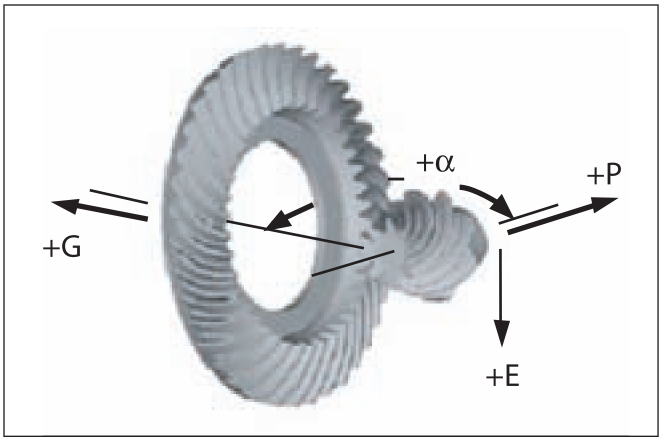

Figure 1—Definition of E, P, G and α.

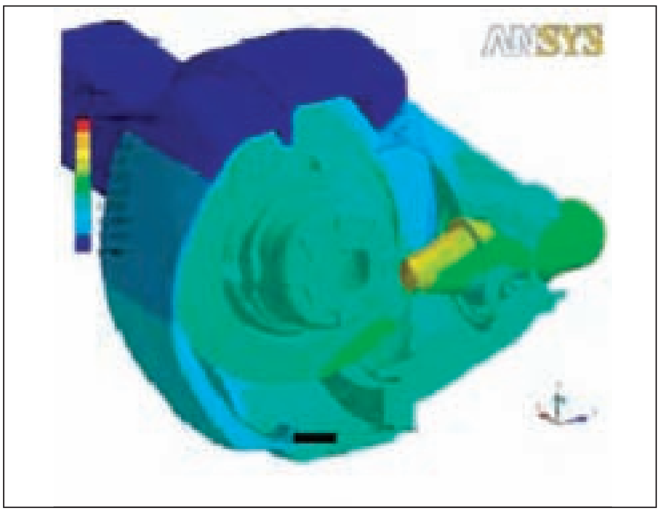

Figure 2—FEA model of an axle deflection analysis.

Figure 3—Experimental measurement of an axle deflection.

bevel gear drives. Axle deflection is commonly defined, as shown in Figure 1, in terms of E, P, G and α, which are normally derived from experimental measurements. Positive E, P, G and α represent deflections under certain load conditions that would enlarge the pinion offset, pinion mounting distance, gear mounting distance and shaft angle, respectively. Countless gear design and analysis software packages have been developed and are commercially available today that take a predefined set of E, P, G and α to describe the rigid body motions and incorporate them into gear contact analysis to simulate the loaded contact characteristics of the gear drives.

Without knowing the deflections of the axle for which the gears are designed, an ideal gear design on paper or a perfect development showing on a loose gear set may possibly lead to poor performances in axle assembly, such as excessive gear tooth wear, low fatigue or impact life cycles and unacceptable noise and dynamic behaviors. Before a physical axle is built, the numerical or analytical method is the only way to estimate the deflections. In this study, an FEA model is created using *ANSYS* as shown in Figure 2. The predicted deflection data need to be validated or corrected.

Experimental measurement of the axle deflection is the conventional approach to validate the FEA predictions. With the availability of a physical axle, actual measurement can be conducted. In this study, measurements are taken based on the procedure that is similar to Gleason's procedure (Ref. 14). Figure 3 illustrates the setup used in this measurement.

However, this type of measurement is very costly and time-consuming. Furthermore, measured results for a particular axle do not apply to other axle models or the same model with different configurations. It is not practical, and perhaps impossible, to repeat the measurement across different axle models or different configurations. To overcome these disadvantages, in this study, an indirect approach is proposed to validate the FEA-predicted deflections.

It is known that axle deflections will alter gear contact patterns in terms of contact area size, contact path, positions and shapes on tooth surfaces as well as transmission errors. In other words, under certain controlled conditions, variations of contact patterns for a gear set are the direct responses from axle deflections. This led to the idea to use computer simulations with FEA-predicted deflections to compare simulated contact patterns to the ones from actual loaded contact tests. If, under the same load condition, the simulated contact patterns agree with the actual patterns obtained from loaded contact tests, then one can consider that the deflection data predicted from FEA are valid. If not, the FEA model needs to be refined. This is also a development process to achieve a valid FEA model for axle deflection analysis. In this study, the validation is carried out following the procedures described below:

- Conduct loaded contact test per Dana Commercial Vehicle System procedure. In this study, loaded contact tests were performed at five loading conditions, i.e.—no load; 25% of full load; 50% of full load; 75% of full load; and 100% of full load. Pictures of the contact patterns under these loads are recorded as shown in Figure 4 (a'–e');

- Use a computer program *Loaded Tooth Contact Analysis* (*LTCA*) to simulate the contact patterns under the same five levels of loads. Deflections predicted by the FEA analysis at these five loading conditions are used;
- Compare respective patterns between simulated and actual ones;
- If the comparisons show good agreement between calculated patterns and actual patterns, then the FEA-predicted deflections are validated. Otherwise, refine the FEA model and repeat steps 2 and 3.

In this study, the resulting contact patterns from computer simulation with FEA- predicted deflections are shown in Figure 4 (a–e). At all five of these loading conditions, the *LTCA*-simulated patterns agree well with the actual patterns. Thus the deflections predicted by the FEA program are considered valid. The validated deflections can then be used for further gear analysis and design improvement. This approach can be used for other axle models, provided the loaded contact patterns are available. With the development of this process, the FEA model can be further refined and trained so that one can rely on the FEA-predicted deflections with good confidence for future axle/gear design and development.

Flank modifications. Flank modifications to a conjugate gear pair are normally designed to avoid edge contact and stress concentration, which could result from deflections under load, manufacturing errors and misalignment, etc. Flank modifications are also desired for ease of manufacturability. There are basically three types of gear tooth flank modifications—lengthwise crowning, profile crowning and longitudinal twist.

For hypoid and spiral bevel gears, lengthwise crowning is mainly a result of cutter radius change. Lengthwise crowning can also be achieved by cutter head tilt in conjunction with blade angle modification (Ref. 15). Figure 5 (a)

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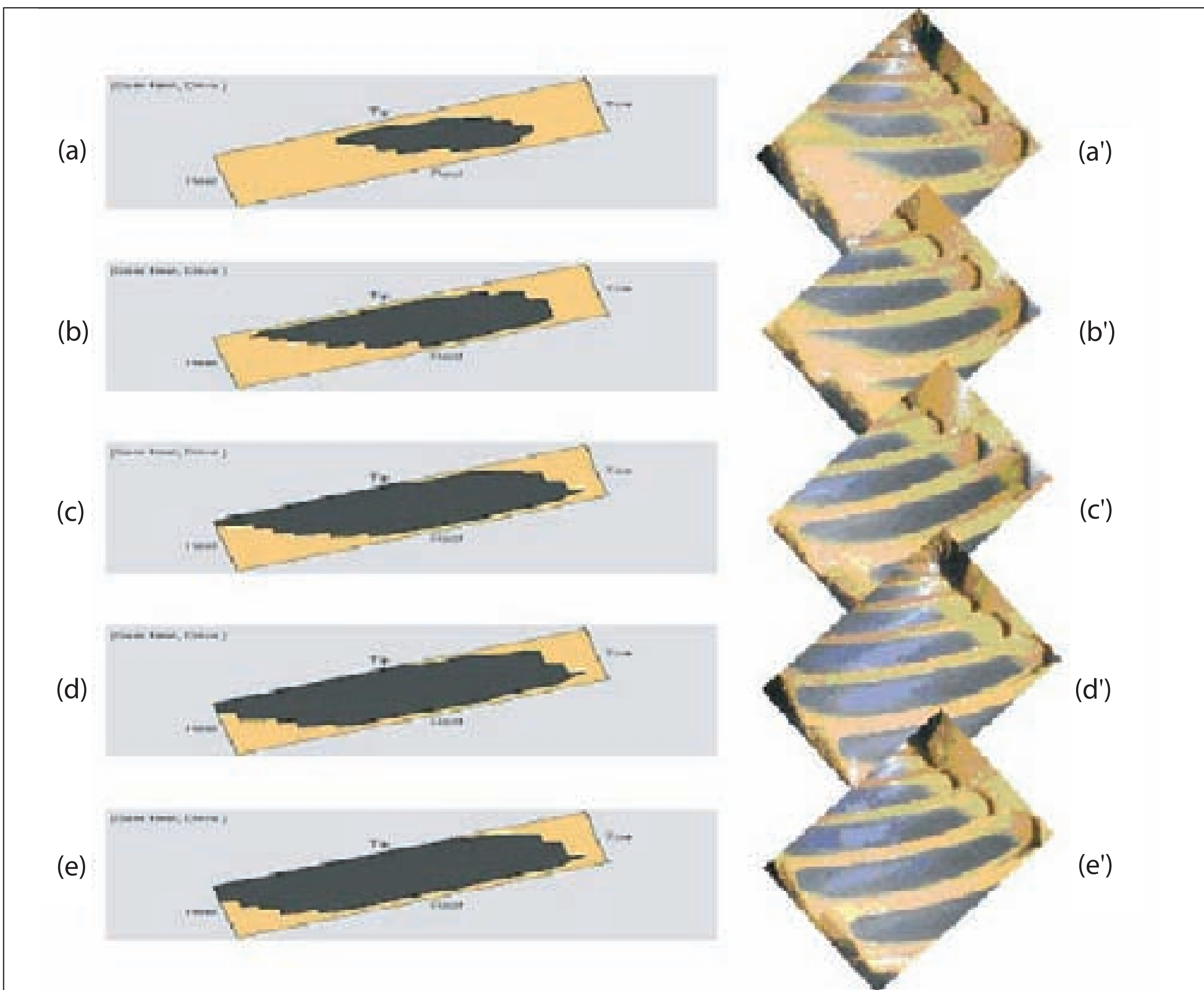

Figure 4—Comparison between simulated contact pattern (a–e) and actual painted pattern (a'–e'): (a–a') no load; (b–b') 25% of full load; (c–c') 50% of full load; (d–d') 75% of full load; (e–e') 100% of full load.

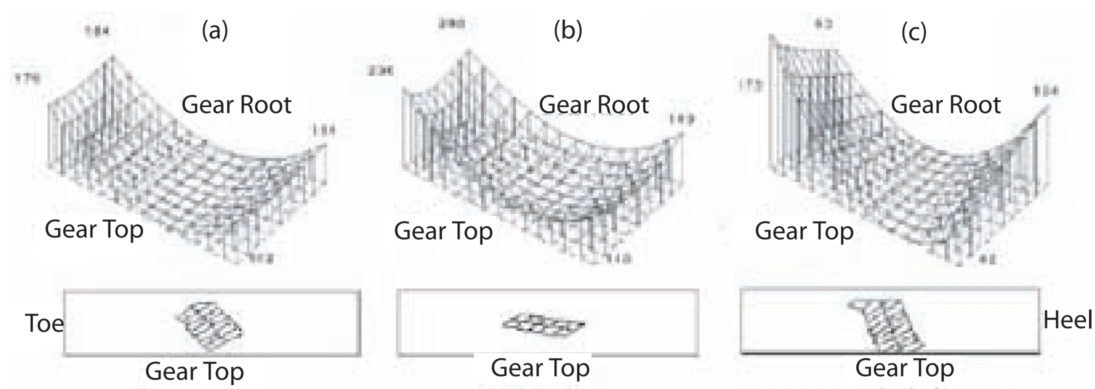

Figure 5—Illustration of flank modification and resulting contact patterns.

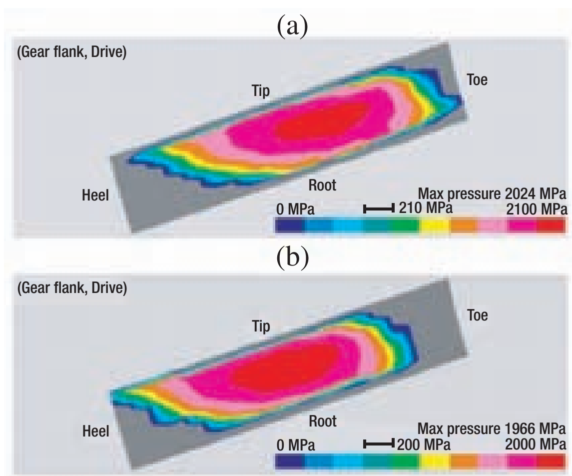

Figure 6—Contact patterns and stress distribution analyzed (a) without deflections; and (b) with deflections.

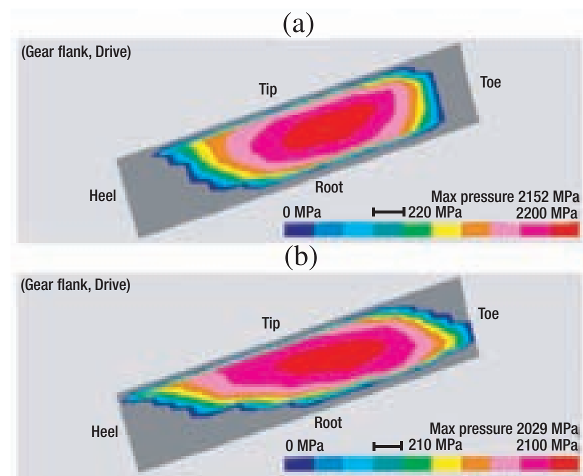

Figure 8—Contact patterns and stress distribution analyzed with pinion lengthwise crowning (a) 0.007 inches; and (b) 0.005 inches.

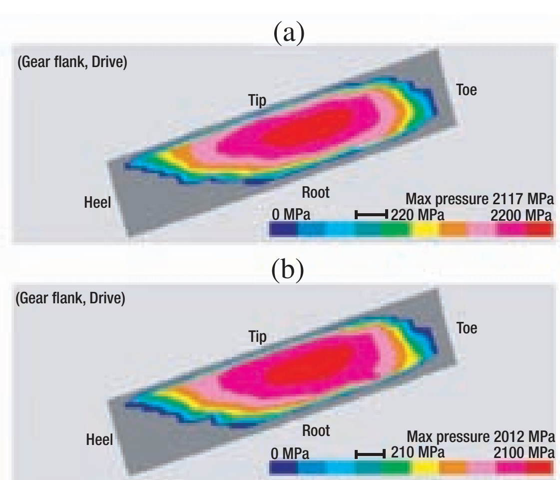

Figure 7—Contact patterns and stress distribution analyzed with pinion blade profile radii of curvature (a) 20 inches; and (b) 100 inches.

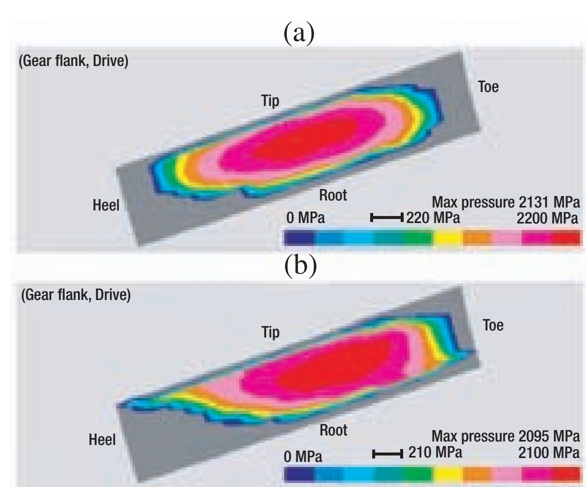

Figure 9—Contact patterns and stress distribution analyzed with pinion longitudinal twist (a) 0 degrees; and (b) 1.5 degrees.

illustrates a typical ease-off (also called mismatch, a term used to describe the deviations of the actual tooth surfaces to theoretical conjugate surfaces) and TCA (Tooth Contact Analysis) as a result of lengthwise crowning to the pinion member. Profile crowning normally results from blade profile curvature. It can also be achieved by a mechanism called modified roll (Ref. 15). Figure 5 (b) shows the ease-off and TCA as a result of profile crowning with added blade profile curvature. Shown in Figure 5 (c) is an example of ease-off and TCA as a result of tooth twist. Tooth twist—also known as bias—can be achieved by cutter tilt or using higher order machine motions, such as higher orders of helical motion and modified roll. These three types of flank modifications are often combined to achieve the desired contact pattern, ease-off and transmission error. Flank modifications can be made to both pinion and gear surfaces, although in practice, they are often designed for pinion surface only, for hypoid and spiral bevel gears.

In the following section, the impact of flank modifications on hypoid gear contact pattern and stress distribution will be simulated by a computer program. In this program, ease-off is characterized by using a two-dimensional quadratic model as a function of spiral angle difference, pressure angle difference, lengthwise crowning, profile crowning and twist (Ref. 17). The resulting coefficients from a least square approximation for lengthwise crowning, profile crowning and twist of the quadratic model are defined as the amount of respective flank modifications that will be referred to in the next section.

Computer simulation of the impacts of deflections and flank modifications on hypoid gear contact pattern and stress distribution. Given the validated axle deflections—E, P, G and α—*LTCA* is used to investigate the impacts of axle deflections and flank modifications on gear contact pattern and stress distribution. In this paper, a hypoid gear designed with the face hobbing process is used as an example. Figure 6 (a) shows a loaded contact pattern and contact stress distribution simulated by the computer program without considering any deflections. From this figure, one may be concerned about the edge contact at toe while having no concern at heel. However, performing the same analysis—but with the consideration of the axle deflections—Figure 6 (b) clearly indicates this design may have edge contact at heel with no concern at toe. Based on Figure 6 (a), one may consider making a reasonable adjustment to shift the pattern toward heel, which will, in fact—as is evident in Figure 6 (b)—adversely worsen the contact by having severe edge contact at heel that could lead to early contact failure. Thus the gear contact pattern must be designed and developed appropriately, based on the axle deflection characteristics to avoid the unfavorable contact.

Figures 7–9 show the results from *LTCA* with axle deflections and with different amounts of flank modifications. In this paper, flank modifications are made to the pinion only and the ring gear is unchanged. In this study, profile-crowning variations are controlled by using different pinion blade curvatures, while all other parameters remain unchanged. Figures 7 (a) and 7 (b) show the contact pattern and stress distribution with 20- and 100-inch radii of curvature on the pinion blade cutting side, respectively.

It is observed that while the overall contact patterns do not differ significantly under load, the computed maximum contact stress does vary from 2,117 to 2,012 MPa—more than a 5% reduction with 100-inch radius of curvature on the pinion blade.

In this study, lengthwise crowning is achieved by changing pinion machine settings, including basic and higher-order motions with no blade geometry change. Figures 8 (a) and 8 (b) show the results with 0.007-inch and 0.005-inch lengthwise crowning designed on the pinion tooth surface, respectively. It is evident that with 0.005-inch lengthwise crowning, the contact pattern is much longer and the maximum contact stress is about 6% lower.

Figures 9 (a) and 9 (b) show the results with 0-degree and 1.5-degree longitudinal tooth twist designed on the pinion tooth surface, respectively. It is obvious that with 1.5-degree longitudinal twist, the contact pattern is longer in the diagonal direction, with less than 2% reduction on the maximum contact stress. In this study, longitudinal twist is obtained by changing basic and higher-order machine settings while locking the blade geometry. It should be mentioned that in the comparisons above—in Figure 9, for example—as a result of longitudinal twist there is also some amount of lengthwise and profile crowning change as calculated by the computer program. For the comparisons in this section, care has been taken to keep the differences—other than the targeted modification—as small as possible. Meanwhile, for hypoid and spiral bevel gears, the flank modifications can be achieved through approaches other than those used in this paper, which may result in some different tooth topography. Thus, the simulated results should not be taken as exclusive.

Experimental Study of the Impact of Flank Modifications on Hypoid Gear Contact Fatigue Life Cycles

Design and test samples. Based on the study in the previous section, both axle deflections and tooth flank modifications can evidently impact the contact pattern and stress distribution. With the given deflection data from FEA analysis, two designs are made with the same macro geometry, i.e., same tooth combination, pitch diameters, tooth depth proportions, pressure angles, spiral angle, face width, cone distances and angles, etc. Ring gears for these two designs are kept exactly the same. But these two designs have different flank modifications on the pinion tooth surface that is often called microgeometry change. Seven experimental gear sets are developed, four samples for Design 1 and three samples for Design 2. Figure 10 (a) shows the representative no-load contact pattern for Design 1, and Figure 10 (b) shows the representative no-load pattern for Design 2. The theoretical

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Figure 10—Representative contact patterns from samples of (a) Design 1; and (b) Design 2.

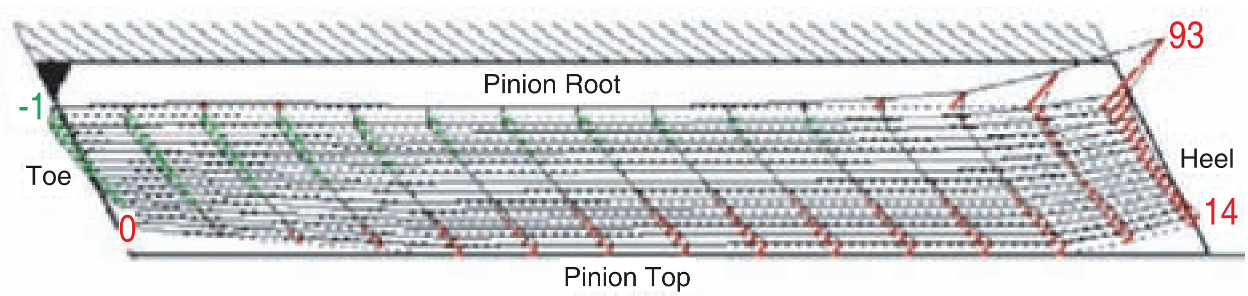

Figure 11—Comparison of theoretical pinion drive side topography: Design 1 (base) and Design 2.

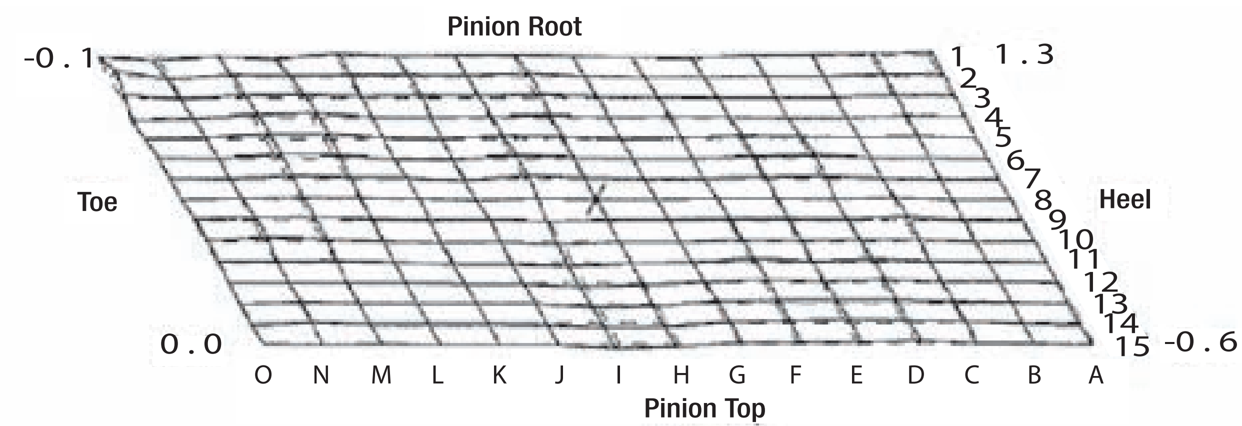

Figure 12—CMM measurement of Design 1: actual pinion drive side topography versus designed Theory 1.

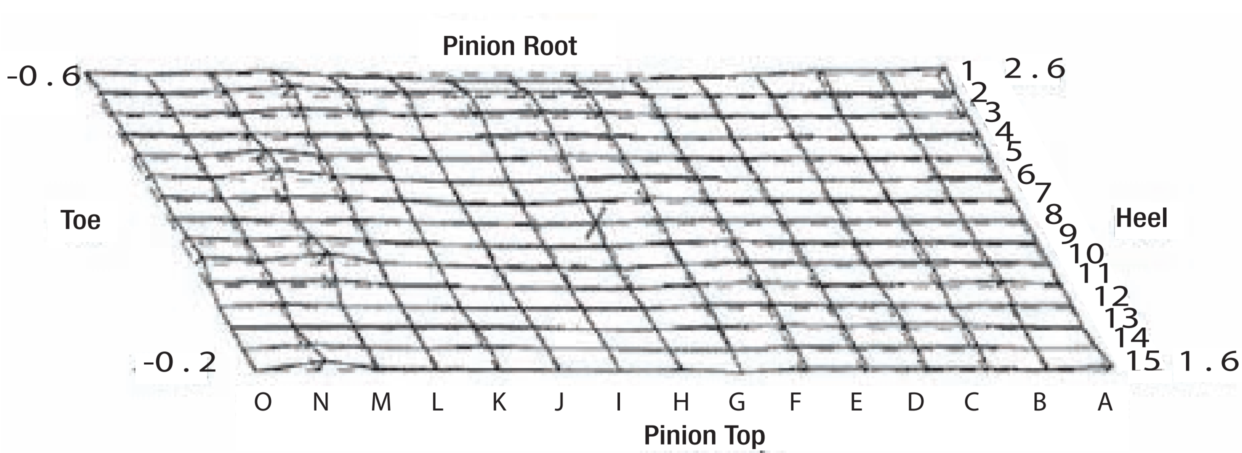

Figure 13—CMM measurement of Design 2: actual pinion drive side topography versus designed Theory 2.

pinion drive side (concave side) tooth topography of Design 1 and Design 2 are compared in Figure 11. The numbers in Figure 10 are in the unit of 0.0001 inch. It is evident that the main differences between these two designs are the ease-off at the pinion heel root. Compared to Design 1, Design 2 has a 0.0093-inch larger relief at the pinion heel root, an additional amount of flank modification designed to avoid edge contact under heavy load. Other differences between these two designs that are calculated by the computer program are: spiral angle difference of 0.03 degree; pressure angle difference of 0.14 degree; lengthwise crowning of 0.0005 inch; profile crowning of 0.0017 inch; and longitudinal twist of 0.555 degree.

A CMM is used to ensure that the actual tooth surface topography agrees with the designed theory. In this study, four teeth are measured, and the results are averaged to eliminate tooth-to-tooth variations. A total 225 points are measured by a 15 x 15 grid setup as shown in Figures 12 and 13. In this paper, only the concave side of the pinion surface is reported. The outputs shown in Figures 12 and 13 are also in the unit of 0.0001 inch. Figure 12 shows the measured pinion concave side topography against the theory of Design 1. As indicated in this figure, the actual surface topography matches the theory very well, with a Sum of Squared Errors of 210 (0.0001 in) ^{2}. This measurement also reported 31 seconds and 0 seconds in pressure angle error and spiral angle error, respectively. The same conclusion can be drawn from Figure 13, which shows the actual pinion surface of Design 2 against the theory of Design 2. In the case of Design 2, the measurement reported 276 (0.0001 in)^2 in Sum of Squared Errors, 14 seconds in pressure angle error and again 0 seconds in spiral angle error.

Test Results

The four samples from Design 1 and three samples from Design 2 are tested for drive-side-only, high cycle fatigue per Dana Heavy Vehicle Systems Group's standard testing procedure. All tests resulted in the pinion as the primary failed component and pitting failure as the primary failure mode. A representative failure of these samples on the pinion is shown in Figure 14. The test results are summarized in Figure 15, which clearly shows that all the samples from Design 2 with larger pinion heel root relief consistently yielded significantly better fatigue lives than samples from

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Figure 14—Representative pinion failures of the seven test samples.

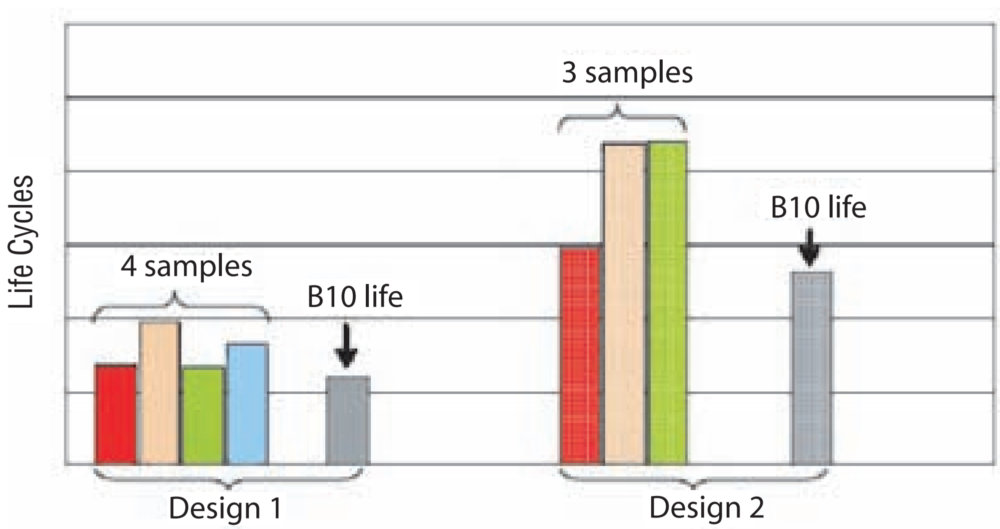

Figure 15—Drive side only high cycle fatigue test results.

Design 1. Resulting B10 life for Design 2 is more than double Design 1's.

Conclusions

This paper presents a study of the impact of axle deflection and tooth flank modification on hypoid gear stress distribution and contact fatigue life. An approach to validate axle deflections has been proposed that eliminates the cost and complexity of actual measurements. By using a face-hobbed hypoid gear design as an example, the influences of axle deflections and typical flank modifications on contact pattern and stress distribution have been simulated. High-cycle fatigue test results of several experimentally made hypoid gears show that appropriate flank modifications can significantly improve gear surface fatigue life.

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