# Manufacturing Method of Large-Sized Spiral Bevel Gears in Cyclo-Palloid System Using Multi-Axis Control and Multi-Tasking Machine Tool 

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## Management Summary

The large-sized spiral bevel gears in a Klingelnberg cyclo-palloid system are manufactured using multi-axis control and a multi-tasking machine tool. This manufacturing method has its advantages, such as arbitrary modification of the tooth surface and machining of the part minus the tooth surface. The pitch circular diameter of the gear treated in this study is more than $1,000 \mathrm{~mm}$ (approx. $40^{\prime \prime}$ ). For this study, we first calculated the numerical coordinates on the tooth surfaces of the spiral bevel gears and then modeled the tooth profiles using a 3-D CAD system. We then manufactured the large-sized spiral bevel gears based on a CAM process using multi-axis control and multi-tasking machine tooling. After rough cutting, the workpiece was heat treated and finished by swarf cutting (Ed.'s note: The removal and cutting of metal in which the axis of the cutting tool is varied with respect to the part being machined) using a radius end mill. The real tooth surfaces were measured using a coordinate measuring machine and the tooth flank form errors were detected using the measured coordinates. Moreover, the gears were meshed with each other and the tooth contact patterns were investigated. As a result, the validity of this manufacturing method was confirmed.

## Introduction

Large-sized spiral bevel gears are often used for power transmission/ thermal power generation applications (pulverizing, etc.). Due to the increase of energy demand in the world, the demand for large-sized spiral bevel gears has increased accordingly and may continue so for some time. These gears are usually manufactured based on a cyclo-palloid system, which pro-
duces equi-depth teeth, but can also be produced using a face hobbing system, which produces tapered teeth (Refs. $1-3)$. The spiral bevel gears in this system are usually generated by a contin-uous-cutting procedure using special gear generating machines. However, the availability of those generators for this use has declined recently, while production costs have not. Therefore, the demand for high-precsion machin-
ing of large-sized spiral bevel gears has grown.

This article discusses the manufacture of large-sized spiral bevel gears in the Klingelnberg cyclo-palloid system using multi-axis control and multitasking machine tooling. The material of the workpiece was 17 CrNiMo 06 and was machined using a coated carbide end mill. As a result, the detected tooth flank form errors were small.

Moreover, the tooth contact patterns of the manufactured large-sized spiral bevel gears were observed and those positions were good.

## Tooth Surfaces of Spiral Bevel Gears

The generator and cutter heads that Klingelnberg manufactures are typically utilized in spiral bevel gear cutting in the cyclo-palloid system. The equi-depth teeth of the complementary crown gear are produced one after another by the rotating and turning motions of the cutter in this methodi.e., the tooth trace of the complementary crown gear is an extended epicycloid. Therefore, the spiral bevel gears in this system are generated by a continuous cutting procedure.

Figure 1 shows the basic concept that produces an extended epicycloid. O-xyz is the coordinate system fixed to the crown gear and the $z$ axis is the crown gear axis. $O_{c}$ is the center of both the rolling circle $R$ and the cutter. The cutter fixed to the rolling circle $R$ rotates under the situation. $\mathrm{OO}_{c}$ is the machine distance and is denoted by $M_{d}$. When the rolling circle $R$ of radius $r(M d-q)$ rolls on the base circle $Q$ of radius $q$, the locus on the pitch surface described by the point $P$ which is a point fixed to the circle $R$ is an extended epicycloid. When the spiral bevel gear is generated for hard cutting on the special generator after heat treatment, a cutter with circular-arc cutting edges is used. These circular-arc cutting edges provide a profile modification to the tooth surfaces of the generated gear. Therefore, a cutter with circular-arc cutting edges is considered in this article.

Figure 2 shows the cutter with circular cutting edges. $\mathrm{O}_{c}-x_{c} y_{c} z_{c}$ is the coordinate system fixed to the cutter. $\mathrm{O}_{c}$ is the cutter center; $z_{c}$ is the cutter axis; $r_{c}$ is the cutter radius; $\gamma$ is the pressure angle of the inner cutting edge of the cutter; $\rho$ is the radius of the curvature of circular arc cutting edge; $y_{c i}$, $z_{c i}$ are the coordinates of the center of curvature of circular arc in plane $x_{c}=$ 0 , and are expressed as a function of $\gamma$ and $\rho$ (Ref. 7); $\theta$ is the parameter
which represents inner curved line. The inner cutting edge $X_{c}$ is expressed on plane $y_{c} z_{c}$ in $\mathrm{O} c-x_{c} y_{c} z_{c}$ by the following equation:

$$
\boldsymbol{X}_{c}(\theta)=\left[\begin{array}{c}
0  \tag{1}\\
y_{c i}+r_{c} \cos \theta \\
z_{c i}-r_{c} \sin \theta
\end{array}\right]
$$

The surface of the locus described by $X_{c}$ in $\mathrm{O}-x y z$ is expressed as:

$$
\begin{equation*}
\boldsymbol{X}(v, \theta)=\boldsymbol{C}\left(\theta_{1}\right) \boldsymbol{X}_{c}(\theta)+\boldsymbol{D}(v) \tag{2}
\end{equation*}
$$

where $C$ is the coordinate transformation matrix for the rotation about $z$ axis:

$$
\begin{align*}
& \boldsymbol{C}\left(\theta_{1}\right)=\left[\begin{array}{ccc}
\cos \theta_{1} & -\sin \theta_{1} & 0 \\
\sin \theta_{1} & \cos \theta_{1} & 0 \\
0 & 0 & 1
\end{array}\right] \\
& \theta_{1}(v)=\frac{M_{d} v}{r}+\Theta_{0} \\
& \cos \Theta_{0}=\frac{R_{m}^{2}+r_{c}^{2}-M_{d}^{2}}{2 R_{m} r_{c}}  \tag{3}\\
& \boldsymbol{D}(v)=\left[\begin{array}{c}
-M_{d} \sin \left(v-\theta_{0}\right) \\
M_{d} \cos \left(v-\theta_{0}\right) \\
0
\end{array}\right] \\
& \cos \theta_{0}=\frac{M_{d}^{2}+R_{m}^{2}-r_{c}^{2}}{2 M_{d} R_{m}}
\end{align*}
$$

In Equations 2 and 3, $v$ is a parameter which represents the rotation angle of the cutter about the $z$ axis, and $R_{m}$ is the mean cone distance (Fig. 3). $X$ expresses the equation of the tooth (tool) surface of the complementary crown gear. The unit normal of $X$ is expressed by $N$.

The complementary crown gear is rotated about the $z$ axis by angle $\psi$ and generates the tooth surface of the spiral bevel gear. We call this rotation angle $\psi$ of the crown gear the generating angle. When the generating angle is $\psi, X$ and $N$ are rewritten as $X \psi$ and $N \psi$ in $\mathrm{O}-X Y Z$ assuming that the coordinate system $\mathrm{O}-x y z$ is rotated about the z axis by $\psi$ in the coordinate system $\mathrm{O}-X Y Z$ fixed in space. When $\psi$ is zero, O-XYZ coincides with O-xyz.

Assuming the relative velocity
$W(X \psi)$ between crown gear and generated gear at the moment when generating angle is $\psi$, the equation of meshing between the two gears is as follows (Refs. 8-9):

$$
\begin{equation*}
\boldsymbol{N}_{\psi}(v, \theta ; \psi) \cdot \boldsymbol{W}(v, \theta ; \psi)=0 \tag{4}
\end{equation*}
$$

From Equation 4 we have $\theta=\theta(\nu$, $\psi)$. Substituting $\theta(v, \psi)$ into $X \psi$ and $\mathrm{N} \psi$, any point on the tool surface of the crown gear and its unit normal are
continued


Figure 1—Extended epicycloid.


Figure 2-Cutting edges of cutter.


Figure 3-Locus of cutting edge.


Figure 4-Tooth profile of gear modeled using 3-D CAD system.


Figure 5-Tooth profile of pinion modeled using 3-D CAD system.


Figure 6-Gear workpiece on multitasking machine.
defined by a combination of $(v, \psi)$, respectively (Ed.'s note: Or normal vector-the normal to a surface is a vector perpendicular to it. The normal unit vector is often desired, sometimes known as the "unit normal."). When the tool surface of the complementary crown gear in $\mathrm{O}-X Y Z$ is transformed into the coordinate system fixed to the generated gear, the convex tooth surface is expressed. A similar expression is applied to the concave tooth surface. In this case, the difference of the turning radius between inner and outer cutting edges $E_{x b}$ that provides a crowning to the tooth surface of the generated gear should be considered. The convex and concave tooth surfaces of the gear are expressed as $X_{g}$ and $X_{g}{ }^{\prime}$, respectively. The concave and convex tooth surfaces of the pinion are expressed as $X_{p}$ and $X_{p}^{\prime}$ respectively. Moreover, the unit nomals of $X_{g} X_{g}^{\prime} X_{p}$ and $X_{p}^{\prime}$ are expressed as $N_{g} N_{g}{ }^{\prime} N_{p}$ and $N_{p}{ }^{\prime}$.

## CAD/CAM System

The numerical coordinates on the tooth surfaces $X_{g} X_{g}^{\prime} X_{p}$ and $X_{p}^{\prime}$ of the spiral bevel gears were calculated based on the concept in the previous section. Moreover, those unit normals $N_{g} N_{g}{ }^{\prime} N_{p}{ }^{\prime}$ and $N_{p}$ were also calculated. Table 1 shows the dimensions of the

| Table 1—Dimensions of spiral bevel gear. |  |  |
| :--- | :---: | :---: |
|  | Pinion | Gear |
| Number of teeth | 16 | 40 |
| Pitch circle diameter | 540.0 mm | $1,350.0 \mathrm{~mm}$ |
| Pitch cone angle | 21.801 deg. | 68.199 deg. |
| Hand of spiral | Right | Left |
| Normal module | 24.9799 |  |
| Shaft angle | 90 deg.$$ |  |
| Spiral angle | 32 deg. |  |
| Pressure angle | 20 deg.$$ |  |
| Mean cone distance |  | 727.0 mm |
| Face width |  | 185.0 mm |
| Whole depth |  | 56.21 mm |

Table 2 - Cutter specifications and machine settings.

| Cutter radius | $r_{C}$ | 450.0 mm |
| :--- | :---: | :---: |
| Radius difference | $E_{x b}$ | 4.5 mm |
| Radius of curvature of circular arc | $\rho,\left(\rho^{\prime}\right)$ | $3,500 \mathrm{~mm}$ |
| Cutter blade module |  | 23.0 mm |
| Pressure angle | $q$ | 20 deg. |
| Base circle radius | $M_{d}$ | 546.9441 mm |
| Machine distance | 610.4189 mm |  |

spiral bevel gears. Table 2 shows the cutter specifications and machine settings in the calculation of the design; the PCD (pitch circle diameter) of the gear is $1,350 \mathrm{~mm}$ (approx. 53").

The determined coordinates are changed by the phase of one pitch after the tooth surfaces $\mathrm{X}_{\mathrm{g}} \mathrm{X}_{\mathrm{g}}{ }^{\prime}, \mathrm{X}_{\mathrm{p}}$ and $\mathrm{X}_{\mathrm{p}}{ }^{\prime}$ are calculated. This process is repeated and produces the numerical coordinates on other convex and concave tooth surfaces. When the range-of-existence of the workpiece that is composed of the root cone, face cone, heel and toe, etc., is indicated, the spiral bevel gear is modeled.

Figures 4 and 5 show the tooth profiles of the gear and pinion modeled using a 3-D CAD system based on the calculated numerical coordinates. The tool pass is calculated automatically after checking tool interferences, choosing a tool and indicating cutting conditions. In this way the CAM process is realized; when the numerical coordinates of the tooth surfaces are calculated, the tooth surfaces are estimated by the smoothing of a sequence of points, removal of the profile of undercutting, offset of tool radius and generation of NURBS (non-uniform rational basis-spline) surface (generated from a series of curves). Moreover, by virtue of calculations of intersecting curved lines of convex and concave tooth surfaces-and sectional curved line-an approximation of straight line is conducted. This approach "escape" is added in order to avoid the interference. When the attitude of the tool and coordinate transformation is conducted, NC data and IGES (initial graphics exchange specification) data for the machining and display are obtained.

## Manufacturing of

## Large-Sized Spiral Bevel Gears

The gears were manufactured based on CAD/CAM system mentioned above. The manufacturing processes were divided into three parts-roughing, semi-finishing and finishing machining.

Manufacturing of gear. The gear was machined by a ball end mill utilizing a vertical, three-axis machining center. However, the gear could
not be machined efficiently due to the machining using only one point on the end mill. This manufacturing method was not suitable for the largesized gear with a PCD of more than $1,000 \mathrm{~mm}$. Moreover, the accuracy of machining was lacking. Therefore, a five-axis control machine (DMG Co., Ltd. DMU210P) was utilized. In this case, the plural surfaces-but not the installation surface-can be machined and a tool approach from an optimal direction can be realized using multiaxis control, as the structure of the two axes of the inclination and rotation, in addition to 3 axes of straight line, are added. It is therefore possible to use a thicker tool. This is expected to reduce the machining time and to obtain better roughness values. Cemented carbide radius end mills for hard cutting were used in the machining of the tooth surface. The number of edges was 12 , and the diameters of end mills were 20 mm and 10 mm , respectively. Ball end mills were used in the machining of the tooth bottom. The number of edges was again 12, and the diameters of the end mills were 10 mm and 5 mm , respectively. The gear blank made out of 17 CrNiMo 06 was prepared. The tool pass was 1 mm for the large-sized gear. First, the gear blank was rough-cut and heat treated. The gear was then semifinished with the machining allowance of 0.2 mm after heat treatment. Finally, the gear was finished with the machining allowance of 0.05 mm by swarf cutting that is machined using the side of the end mill. Machining utilizing the advantages of multi-axis control and multi-tasking machine tooling in swarf cutting should deliver high accuracy and high efficiency.

Table 3 shows the conditions for semi-finishing and finishing in gear machining. Figure 6 shows the gear workpiece on the multi-axis control and multi-tasking machine; Figure 7 shows the cutting of the gear. The machining time of one side in roughcutting is about six hours; and with semi-finishing and finishing, about seven hours. The machining was completed with no complications.

Manufacturing of pinion. A five-
axis control machine (Mori Seiki Co., Ltd. NT6600) was utilizied for pinion machining. The radius end mills made of cemented carbide for a hard cutting tool were used in machining the tooth surface. The number of edges was 12 , and the diameters of end mills were 20 mm and 16 mm , respectively. Ball end mills were used in the machining of the tooth bottom. The number of edges was 12 and the diameters of end mills were 10 mm and 5 mm , respectively. The material of the pinion was the same as that of the gear. The pinion blank was rough-cut and heat treated. The pinion was then semi-finished with the machining allowance of 0.2 mm after heat treatment. Moreover, the pinion was finished with the machining allowance of 0.05 mm by swarf cutting. Table 4 shows the conditions for semi-finishing and finishing in pinion machining. Figure 8 shows the pinion on the multi-axis control and multitasking machine. The machining time of one side in rough cutting was about eight hours, and with semi-finishing and finishing, about 32 hours. The machining was again finished without trouble.

Tooth flank form error and tooth contact pattern. The real gear and pinion tooth surfaces were measured using a coordinate measuring machine and compared with nominal data using the coordinates and the unit surface normals (Refs. 10-13). A Sigma M\&M 3000 developed by Gleason Works was utilized. This measuring machine cor-
responds to large-sized spiral bevel gears. Figure 9 shows the measured result of the gear and Figure 10 shows that of the pinion. The tooth flank form errors are no more than about $\pm$ 0.06 mm and pitch accuracy is Class-1 JIS (Japanese Industrial Standards) for both the cases of the gear and pinion. We de not believe that these errors will have an influence on the tooth contact patterns for large-sized spiral bevel gears.

The gears were set on a gear meshing tester and the experimental tooth
continued


Figure 7-Swarf cutting of gear.


Figure 8-Pinion workpiece on multitasking machine.

| Table 3 - Conditions of gear machining. |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Processes | Diameter <br> of end mill <br> (mm) | Revolution of <br> main spindle <br> (rpm) | Feed <br> $(\mathrm{mm} / \mathrm{min})$. | Depth of cut <br> $(\mathrm{mm})$ | Time/one <br> side (min.) |
| Semi- <br> finishing | 20.0 | 2,000 | 1,150 | 0.3 | 110 |
| Finishing | 20.0 | 2,200 | 1,100 | 0.05 | 310 |


| Table 4 - Conditions of pinion machining. |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Processes | Diameter <br> of end mill <br> $(\mathrm{mm})$ | Revolution of <br> main spindle <br> (rpm) | Feed <br> $(\mathrm{mm} / \mathrm{min})$. | Depth of cut <br> $(\mathrm{mm})$ | Time/one <br> side (min.) |
| Semi- <br> finishing | 16.0 | 2,800 | 1,100 | 0.2 | 480 |
| Finishing | 16.0 | 3,300 | 1,100 | 0.05 | 1,440 |

contact patterns were investigated. Figures 11 and 12 show the results of the tooth contact patterns on the gear tooth surfaces of the drive and coast sides, respectively. The tooth contact pattern is positioned at the center of the tooth surface and its length is about $50 \%$ of the tooth length, based on the analysis of the tooth contact pattern. The experimental tooth contact patterns are positioned around the center of the tooth surfaces of both drive and coast
sides, respectively, although the length of the tooth contact pattern on the drive side is somewhat smaller. From these results the validity of the manufacturing method using multi-axis control and multi-tasking machine tooling was confirmed.

## Summary/Conclusions

Large-sized spiral bevel gears are usually manufactured based on a cyclopalloid system by a continuous cutting procedure using a special generator.


Figure 9—Measured result of gear ( $\mu \mathrm{m}$ ).


Figure 10-Measured result of pinion ( $\mu \mathrm{m}$ ).


Figure 11-Tooth contact pattern of drive side.


Figure 12-Tooth contact pattern of coast side.

However, production of the machine tools corresponding to the largesized spiral bevel gears has recently decreased and the machines themselves are expensive.

In this paper, a manufacturing method of large-sized spiral bevel gears in the Klingelnberg cyclo-palloid system using multi-axis control and multi-tasking machine tooling was proposed. For this study, first the numerical coordinates on the tooth surfaces of the spiral bevel gears were calculated and the tooth profiles were modeled using a 3-D CAD system. The large gears were manufactured based on a CAM process using multi-axis control and multi-tasking machine tooling. After rough cutting, the workpiece was heat treated and finished by swarf cutting using radius end mills. The real tooth surfaces were measured using a coordinate measuring machine and the tooth flank form errors were detected using the measured coordinates. Moreover, the gears meshed well and the tooth contact patterns were investigated. As a result, the validity of this manufacturing method was confirmed.
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