# FE-Based Approaches for Tip Relief Design

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## Introduction

The deformation of the gear teeth due to load conditions may cause premature tooth meshing. This irregular tooth contact causes increased stress on the tooth flank. These adverse effects can be avoided by using defined flank modifications, designed by means of FE-based tooth contact analysis.

The deformation of the gear teeth due to load conditions may cause premature tooth meshing. The purpose of a tip relief is to prevent premature tooth meshing and corresponding negative effects on the load distribution. The increase of tooth flank load carrying capacity by tip reliefs has been proven by Haslinger and is explained by an unloading of areas with high negative slip (Ref. 1). The pitting fatigue strength of profile-corrected gears differs depending on the design of the tip relief. Furthermore, the location of initial pitting damages depends on the design of tip reliefs. In this context, locally reduced radii of curvature in the transition region between the corrected and uncorrected involute have a major influence.

In order to achieve a lightweight design and, therefore, a high power density, the increase of load carrying capacity by means of pressure-optimized profile corrections has to be exhausted in the best possible way. For this purpose it is necessary to take the influence of the tip relief design on the tooth flank stress into account while designing the gears, including the local radii of curvature, the amount and the length of tip reliefs.

Therefore, this paper aims for the development of a FE-based method for designing pressure- optimized profile corrections for spur gears. By means of the existing FE-based tooth contact analysis, the load on the tooth surface is calculated in form of line loads. Following this, an analytical approach is presented to calculate the Hertzian pressure under consideration of the local radii of curvature. Using the developed method, a parameter study is performed to investigate the influence of different geometries of profile modifications on the Hertzian pressure distribution along the path of contact, and to derive a recommendation to determine a pressure-optimized geometry. Here the focus is on an objective method for evaluating the change of tooth flank stress, depending on the tip relief design. Furthermore, an FE-based approach for the definition of amount and length of a tip relief is presented in order to consider the influence of local stiffness, combined modifications, and their tolerances on the premature tooth meshing.

## State of the Art

Manufacturing deviations, as well as deformation under load of teeth, gear body and other power transmitting parts of the gearbox affect the excitation behavior and the load carrying capacity of gears. The deformation of teeth under load can provoke a premature tooth meshing, which can have a negative effect on the tooth flank load capacity. Due to this, profile modifications — typically in the form of tip relief — are applied. In the following chapters the influence of tip reliefs on the running behavior of spur and helical gears and the approaches for the design of tip reliefs are discussed.

***Influence of tip reliefs on the running behavior of spur and helical gears.*** The influence of tip reliefs on the running behavior of gears is has been under scientific investigation since the 1960s. The focus of the existing research is mostly on the acoustic behavior, dynamic tooth forces, and scuffing load capacity (Refs. 2–7).

Toppe (Ref. 2) shows in his work a generally positive influence of tip reliefs on the excitation behavior of gears. By calculation of the tooth under load within the single contact area, a tip relief can be derived and a premature tooth meshing can be avoided. According to Toppe, an especially long tip relief design according to Niemann/Winter (Ref. 8) can decrease the excitation of gears (Refs. 2 and 8). The following works of Tesch, Baethge and Knabel confirm the overall positive influence of tip reliefs on the excitation behavior (Refs. 3–5). Concerning the optimal length of a tip relief, there is no clear result. Tesch and Baethge indicate that the design of a tip relief is dependent on the gear geometry (Refs. 3–4).

The influence of the amount and length of a tip relief on the pitting load capacity is systematically investigated by Haslinger (Ref. 1). A positive influence of tip reliefs on the pitting load capacity is established and explained by an unload of the areas with high negative slip (Ref. 1). Further investigation shows an influence of the form of the tip relief and especially of the design of the transition area between modified and unmodified involute (Refs. 9–12).

Nazifi focuses on the influence of locally reduced radii of curvature on the Hertzian pressure during tooth flank contact. He compares gears with a linear and a parabolic tip relief. The Hertzian pressure within the transition area between modified and un-modified involute is increased by the linear tip relief by 110%, while the pressure is only increased by 8% for the parabolic tip relief. Nazifi concludes that the locally reduced radii of curvature in the transition area can reduce the pitting load capacity. Due to this, a parabolic tip relief should be preferred compared to a linear tip relief in order to achieve a maximum load capacity (Ref. 10).

The research of Luetzig also indicates a correlation between local radii of curvature and the tooth flank stress. He focuses

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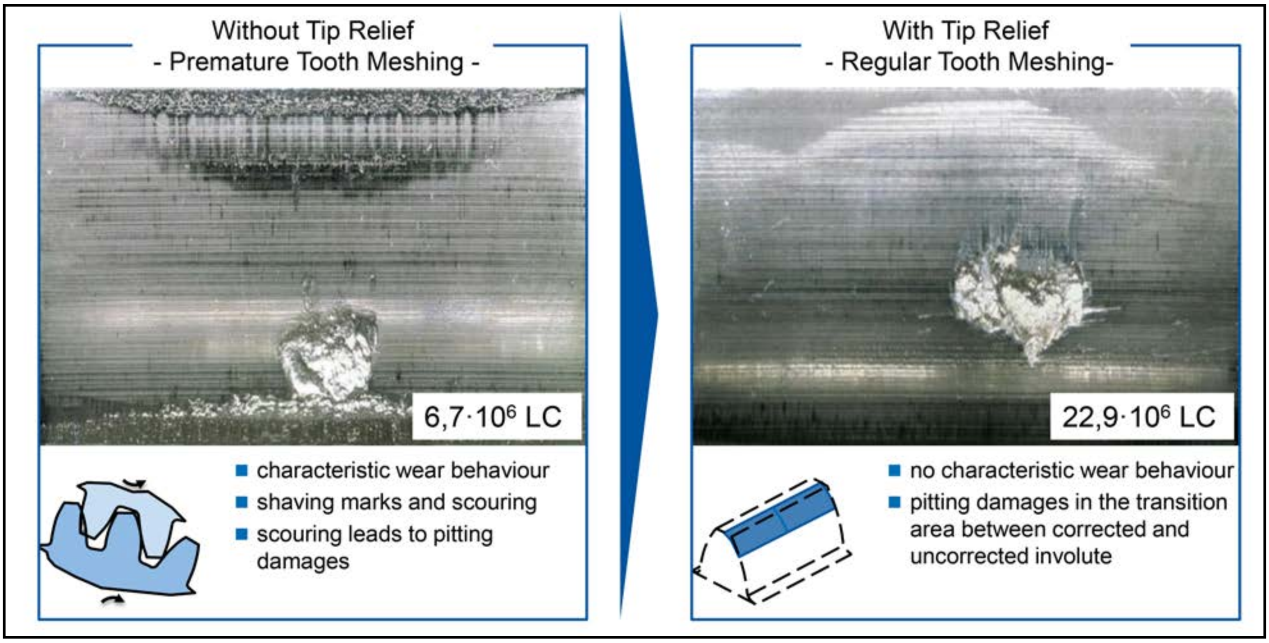


**Figure 1 Influence of tip reliefs on the tooth flank load capacity.**

on the wear in the tooth root flank area of modified and unmodified gears. For the modified gears a correlation between the location of maximum wear and minimum radii of curvature can be found (Ref. 9).

Brecher et al analyze the tooth flank stress of modified and unmodified spur gears by means of a high resolution FEM-simulation. At the same time experimental tests are performed by means of a back-to-back test rig. During the experimental tests modified and non-modified spur gears are compared concerning the resulting tooth flank load capacity. All in all, the modified gears show an increased load capacity. The increase of load capacity by the application of a tip relief is strongly dependent on the design of the tip relief. For the modified variants, pitting damages occur mostly in the transition area with locally reduced radii of curvature (Fig. 1). The FEM analysis confirms an increase of pressure in these regions (Ref. 11).

***Approaches for tip relief design.*** The premature tooth meshing is caused by the deformation of teeth under load. The aim of a tip relief is compensation of the deformation (Refs. 3–4). Existing research has reached the conclusion that the amount and length of a tip relief should be derived by the analyzation of a theoretical penetration at the tooth, which comes into contact (Refs. 1, 3–4). This recommendation is derived by the analyzation of spur gears and has not been confirmed for helical gears.

The deformation of a tooth under load is dependent on applied load and tooth stiffness (Ref. 2). Therefore a tip relief can only be designed for a specific load torque (Ref. 8). A widely spread approach for the design of tip reliefs is the approach according to Niemann/Winter (Ref. 8); this approach separates short and long tip reliefs. The long tip relief starts/ends at the beginning/end of the single tooth contact area. For the short tip relief, the length of the unmodified part of the involute results in a profile contact ratio of $e_\alpha = 1$. The amount of the tip relief is calculated according to Equation 1. A separation of spur and helical gears is mainly realized by use of different tooth stiffness.

$$C_{\alpha a} = \frac{F_{bt}}{c' \cdot b} + f_p \tag{1}$$

| | | |
|---|---|---|
| $b$ | [mm] | Tooth Width |
| $C_{\alpha a}$ | [µm] | Amount of Tip Relief |
| $F_{bt}$ | [N] | Normal Force |
| $c'$ | [N/(mm·µm)] | Maximum Tooth Stiffness |
| $f_p$ | [µm] | Pitch Deviation |

Although the Niemann/Winter approach is generally acknowledged, the results are often combined with experienced-based knowledge. Reasons for this proceeding are the missing consideration of local stiffness behavior and combined modifications. Furthermore, manufacturing-induced tolerance fields are not taken into account (Ref. 8).

An alternative approach for the design of tip relief is the FE-based, TCA approach; it is based on the substitutional spring model of Neupert (Ref. 13). This method is already used for designing modifications such as lead crowning under consideration of tolerance fields and load-induced deformations (Ref. 14). The FE-based TCA is used in the works of Wittke in order to estimate the theoretical penetration of the following tooth and its effects on the excitation behavior (Ref. 15). Wittke simply takes the influence of tip relief into account and does not consider other modifications or their tolerance fields.

***Conclusion.*** The state of the art shows a principally positive influence of tip reliefs on the excitation behavior, as well as on the tooth flank load capacity of gears. For the design of the tip relief amount, it is recommended to estimate the theoretical penetration of the following tooth caused by the deformation under load. This recommendation is applied for both optimal excitation behavior and for optimal load capacity as well. The excitation behavior in particular is mainly influenced by the length of the tip relief, for which different recommendations exist. The research works of Tesch and Baethge show that optimal tip relief design depends on the geometry of the gear stage (Refs. 3–4). There exists no

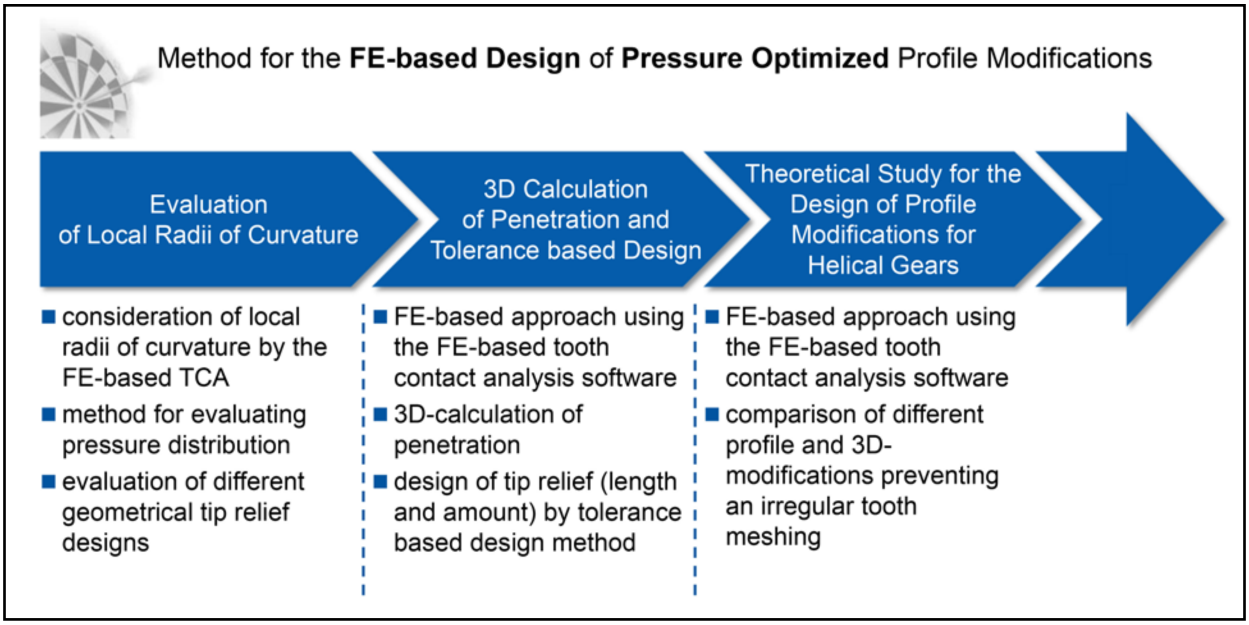


**Figure 2 Objective and approach.**

recommendation for estimation of tolerances for the tip relief design parameters.

The exact geometry of a gear and the resulting local stiffness is not considered by the methods usually used for the design of tip reliefs. This deficit is compensated by the experience and knowledge of the designer. Furthermore, the difference of spur and helical gears is only considered concerning the overall tooth stiffness. The different local contact characteristics are not taken into account. A promising alternative method is presented by the FE-based TCA. The approach of Wittke could be further developed in order to calculate the resulting theoretical penetration and by this design a tip relief. In addition, the FE-based TCA enables consideration of local radii of curvature and by this the evaluation of different tip relief geometries (Ref. 12). By implementing an approach for the consideration of local radii of curvature in the FE-based TCA, an integrated FE-based design of tip relief can be performed and the potential increase of the tooth flank load carrying capacity can be achieved.

## Objective and Approach

Premature tooth meshing influences the tooth flank load carrying capacity and the excitation behavior of gears. In order to avoid the adverse effects of premature tooth meshing, tip reliefs are applied. The design of the tip relief concerning the amount, length and form of the modification, influences the increase of the tooth flank load carrying capacity and the excitation behavior of gears. Design methods used in practice do not consider local stiffness, influences of combined tooth flank modifications, tolerances and local radii of curvature. By using an alternative design approach, i.e. — FE-based TCA — these effects can be considered and the potential increase of tooth flank load carrying capacity and the potential enhancement of excitation behavior can be exploited.

Therefore this paper aims at an FE-based design method for pressure-optimized profile modifications. The objective and approach are depicted in Figure 2. Based on the FE-based TCA, a method for regarding the local radii of curvature is developed and a parameter study is performed in order to determine advantageous geometry forms for profile corrections concerning the tooth flank stress. Therefore an evaluation method is described to differentiate the different profile corrections. For the second step, the FE-based TCA is extended by a three-dimensional calculation of penetration. In this way the effectiveness of profile corrections concerning premature tooth meshing can be evaluated.

- Consideration of local radii of curvature by the FE-based TCA
- Method for evaluating pressure distribution
- Evaluation of different geometrical tip relief designs
- FE-based approach using FE-based tooth contact analysis software
- Comparison of different profile and 3-D modifications preventing an irregular tooth meshing
- FE-based approach using the FE-based tooth contact analysis software
- 3-D calculation of penetration
- Design of tip relief (length and amount) by tolerance-based design method

By an integration of the calculation of penetration in the variational calculus approach of the FE-based TCA, a tolerance based design is enabled. By means of a weighted grading algorithm, the tip relief design can be further adapted to the current case of application. The development and application of the FE-based design method for tip relief is demonstrated for an industrial use case. Finally, a study for an adapted design of profile modifications for helical gears is presented.

## Influence of Profile Modification Form on the Tooth Flank Stress

***Modification of the FE-based tooth contact analysis.*** The FE-based TCA enables the local calculation of stress and deformation with regard to micro geometrical modifications including different geometrical forms of profile corrections. However, regarding tip reliefs, only linear and circular forms can be analyzed. Other geometrical forms such as logarithmic, exponential or polynomial tip reliefs cannot be evaluated.

The micro-geometrical modifications are considered as a change of distance within the spring-based replacement system according to Neupert (Ref. 13). In this way the influence of the modifications on the distribution of normal forces is taken into account. Subsequently, the derived line loads and the radii of curvature of the uncorrected involute are used for the pressure calculation. Up to now, locally reduced radii of curvature in the transition area of uncorrected and corrected involute of profile-corrected gears are not considered.

Based upon these aspects, two modifications of the FE-based TCA must be implemented in order to evaluate the influence of different profile correction forms on the tooth flank stress. On one hand, the consideration of further tip relief forms has to be enabled. On the other hand, the local radii of curvature have to be taken into account for pressure calculation.

Regarding the various tip relief forms, the import function of the FE-based TCA for topography measurement files can be used. As all geometrical tip relief forms can be described through mathematic functions, synthetic topography measurement files can be calculated and subsequently imported into the FE-based TCA. For the consideration of local radii of curvature, an analytical calculation approach is used in combination with the FE-based TCA. While the line loads and the resulting distribution of normal forces are calculated by means of the FE-based TCA, the resulting pressure is calculated analytically according to Hertz. For this the geometry of the tooth flank is described by means of the involute function, and profile modifications are considered by means of mathematic formulas. Thus the geometry of the flank can be described analytically and the calculation of radii of curvature is not linked to the number of elements of the FE model. For the calculation of local radii of curvature, the circular function is used with the coordinates of three neighboring points of the tooth flank. The described calculation method is validated by comparing the calculated pressure with the results of a high-resolution, general FE- method calculation that shows good comparability.

***Evaluation of different profile modification forms.*** The calculation method is also used for a parameter study, analyzing the influence of different profile modification forms on tooth flank stress. The study is performed for a test gear set with a number of teeth $z_{1/2} = 17/18$, and a normal module of $m_n = 5$ mm. The amount and length of the tip relief are calculated according to Niemann/Winter (Ref. 8) for a torque of $M_1 = 550$ Nm. To avoid effects of premature tooth meshing, the amount of the profile correction is defined as $C_a = 70$ µm. The tip reliefs are applied to pinion and gear. For every tip relief form, the short and the long length according to (Ref. 8) are considered. The parameter study includes linear, exponential, logarithmic and polynomial tip reliefs. In addition, a circular, symmetric profile crowning is regarded. For a valid comparison the profile crowning is applied only to the gear and an amount is chosen equal to the tip relief ($C_\alpha = C_a$); the parameters vary for each geometrical form of profile correction. Concerning the linear tip relief, the length of the transition area between the corrected and uncorrected involute $\Delta l_{Ca}$ is varied. For the exponential forms, the base B, and for the polynomial form, the exponent $e$ is modified. The logarithmic tip relief and the profile crowning are fully defined by the amount and length of the correction. In total, 58 variants are investigated.

For the evaluation of the tooth flank stress, the behavior of the calculated pressure along the path of contact is analyzed. In Figure 3 the calculated pressure for a linear tip relief with a transition length of $\Delta l_{Ca} = 200$ µm is depicted. Compared to the calculated pressure for an uncorrected gear set, the pressure calculated for the *corrected* gear set is continuously rising — from the beginning of contact (A) to the beginning of double tooth contact (B) — and continuously falling from the end of double tooth contact (D) to the end of contact (E). Hence, the areas of high slip are unloaded. Another difference between the uncorrected and the

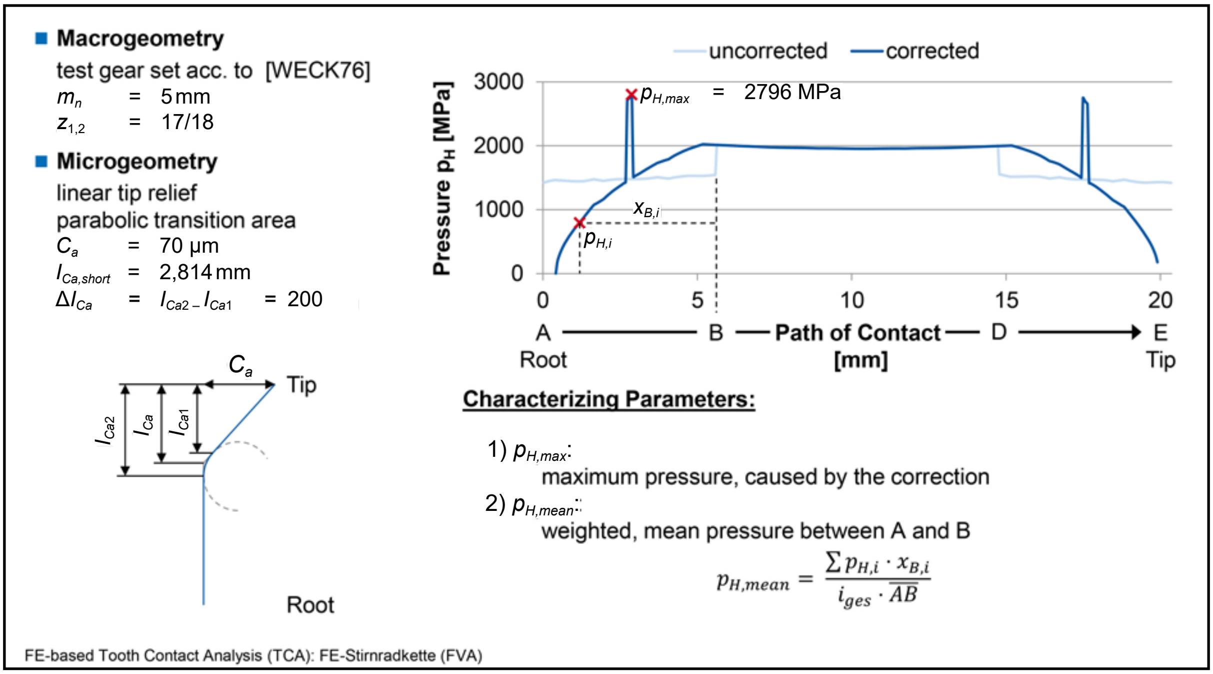


**Figure 3** Characterizing parameters for the tooth flank stress.

corrected gear set are high pressures calculated for the transition area between uncorrected and corrected involute. This is caused by locally reduced radii of curvature, which can be reduced by up to 90% ($r_{uncorrected} = 31\,\text{mm} > r_{transition} = 3\,\text{mm}$). The amount of the increased pressure depends on the length of the transition area; for a longer transition area, the pressure is decreasing. But at the same time, the highly stressed volume is increasing. For evaluation of pressure behavior, more than maximum pressure must be considered — i.e., combination of pressure and *sliding speed* must be taken into account. And so, two characterizing parameters are derived for evaluating the potential increase of tooth flank load carrying capacity of each profile correction form (Fig. 3). Besides the maximum pressure caused by the profile correction $p_{H,max}$, a weighted, mean pressure in the area of high negative slip (A-B) $p_{H,mean}$ is introduced, as the high sliding speed in this area leads to high risk of pitting damages.

The described characterizing parameters are calculated for each variant of the parameter study. For an objective comparison of the variants, the two parameters are used as coordinates of each variant and plotted in a diagram (Fig. 4).

The linear tip reliefs mainly differ concerning the maximum pressure $p_{H,max}$. Due to the Hertzian model, the amount of $p_{H.max}$ is linked to the square root of the length of the transition area. Hence, the extension of the transition area only makes sense to a certain length.

For the focused test gear, a length of

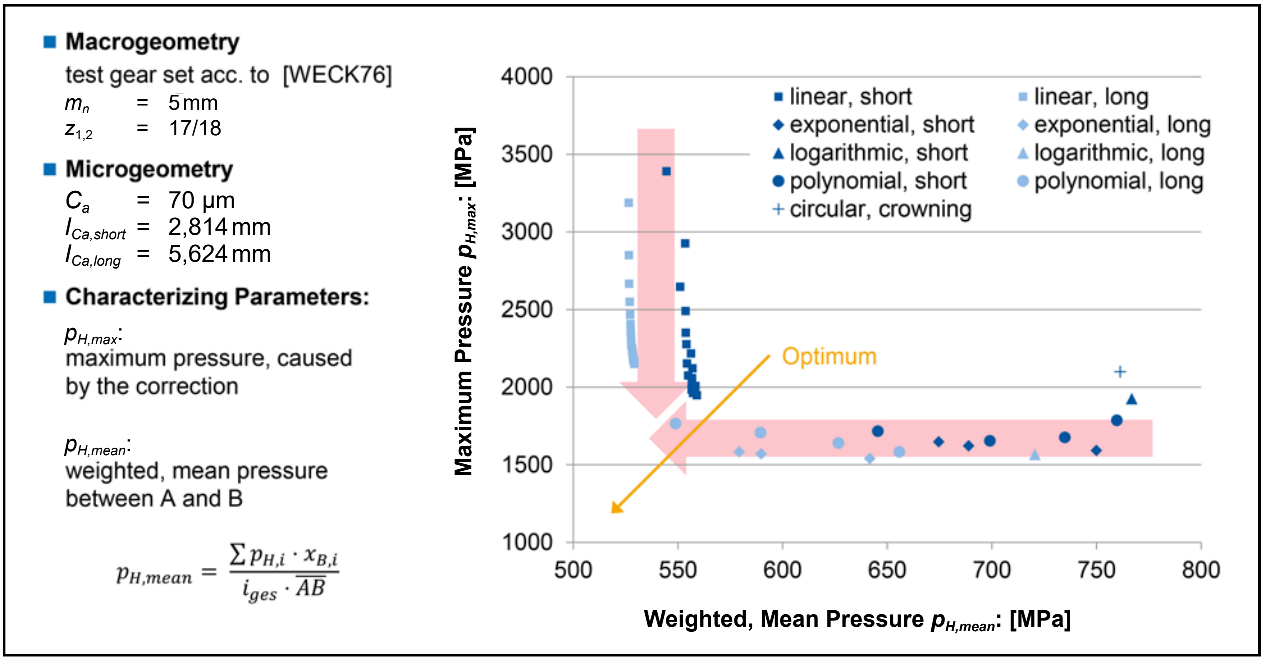


**Figure 4** Result of tooth flank stress evaluation.

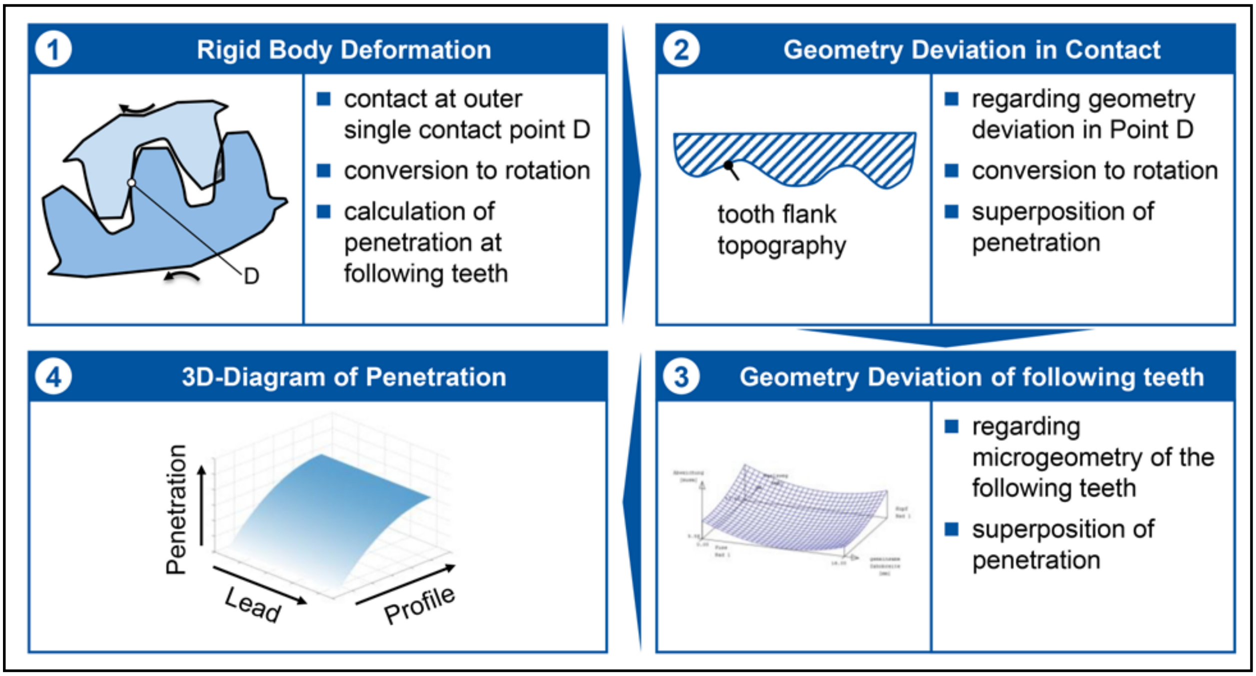


**Figure 5** Approach for the calculation of penetration.

the transition area of 15% of the relief length is reasonable. The other tip relief forms mainly differ in the weighted, mean pressure $p_{H,mean}$. Especially the polynomial tip relief (2nd order, circular) shows a low tooth flank stress. The logarithmic tip relief and the profile crowning show the highest tooth flank stress. Based on these results — the linear tip relief, with a transition area length of 15%, and the circular tip relief — are the focus for future work.

## Method for the Design of Tip Reliefs for Spur Gears

***FE-based calculation of penetration.*** The concept and approach of the FE-based calculation of penetration is shown (Fig. 5). The calculation model can be divided into different parts that consider different influences on the penetration.

As a first step, the rigid body deformation is calculated for the point of contact, where the single tooth contact ends (D). This point of contact is the critical one for the theoretical penetration at the following pair of teeth. The calculated deformation is transformed in a relative rotation of the gears. Based on this, the theoretical penetration of the following tooth is calculated. In the following steps further influences on the penetration caused by geometrical deviations are considered. On the one hand, the geometry deviations at the point of contact (D) caused by desired modifications or undesired manufacturing tolerances lead to a further relative rotation of the gears. On the other hand, applied profile modifications at the following tooth have a direct influence on the calculated penetration. Finally, the resulting theoretical penetration at the following tooth can be calculated with regard to the local stiffness and the geometrical deviations caused by modifications and manufacturing tolerances.

The described calculation approach is further validated by means of contact patterns. For this a test gear set is used, which exists with two differently applied tip reliefs (Fig. 6).

In the upper part of the figure the calculated maximum penetration is shown. For the smaller tip relief ($C_a = 30\,\mu m$), a penetration is avoided up to a torque of $M_1 = 1{,}000$ Nm. For the larger tip relief ($C_a = 75\,\mu m$) the penetration is avoided up to a torque of $M_1 = 2{,}500$ Nm. For both gear sets ($C_a = 30/75\,\mu m$), contact patterns are depicted in the lower part of the figure for differently applied torques. For the smaller tip relief ($C_a = 30\,\mu m$) and an applied torque of $M_1 = 1{,}000$ Nm, there is still a small amount of contact compound left. For an applied torque of $M_1 = 1{,}250$ Nm, there is no compound left and the tooth is loaded along the complete height. Thus, for higher torques a premature tooth meshing and, therefore, a theoretical penetration, have to exist. This result shows good correlation with the calculated penetration. The larger tip reliefs avoid a premature tooth meshing up to a torque of $M_1 = 2{,}500$ Nm, according to the contact patterns. This result shows good correlation with the calculation results, as well.

***Tolerance-based design of tip reliefs.*** The described calculation approach for the theoretical penetration is also used for a tolerance-based design of tip reliefs. By a full factorial variational calculus in the FE-based TCA, a significant number of possible tip relief lengths and tip relief amounts can be evaluated in a short time. By means of a weighted algorithm the possible variants can be evaluated; e.g., concerning tooth flank stress, excitation behavior and penetration. By means of weighting, important load cases and output parameter can be focused and the tip relief design can be optimized for each application. The results are further united in possible tolerance fields that can be described by a worst-case/best-case sce-

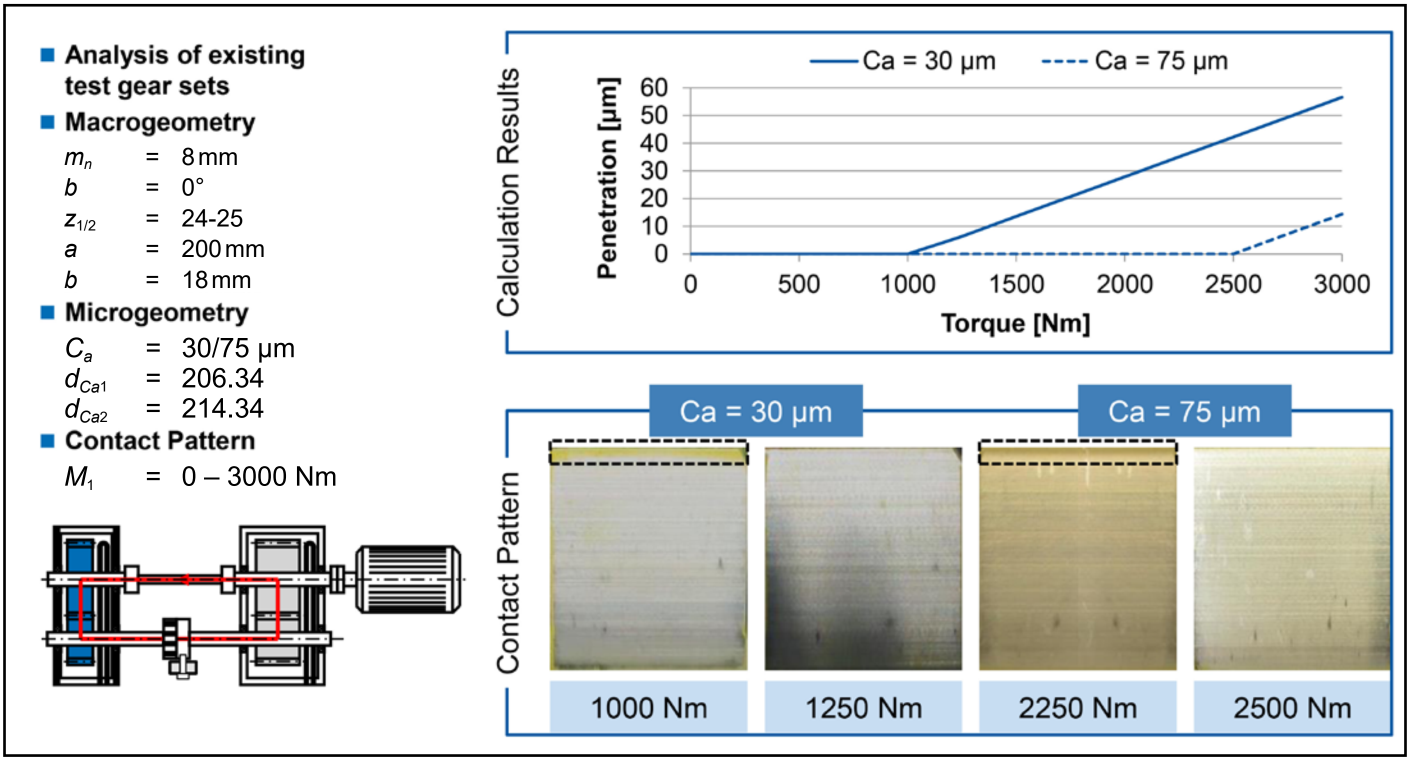


**Figure 6** Validation of penetration calculation by means of contact pattern.

nario and target behavior.

Based on the results of the previous chapters, a linear and a circular tip relief are designed for an industrial gear set by means of the tolerance-based design method. The resulting tip reliefs are compared to analytically designed tip reliefs; the tip relief parameters are shown (Table 1).

For an objective comparison, characterizing parameters are needed. In Figure 7 (top), worst-case/best-case and target value of the penetration and the first gear mesh order of transmission error are shown, depending on the applied torque for the linear analytical design. For the comparison of the designed variants, the nominal torque of $M_1 = 18.5$ kNm is analyzed and the values of the worst-case/best-case and target behavior are derived (Fig. 7, bottom).

The described values are derived for each design variant so that an objective comparison is possible; the result of this approach is shown (Fig. 8, top).

The analytically designed tip reliefs, independent on the geometrical form, are not preventing a penetration, although the nominal torque of $M_1 = 18.5$ kNm has been used for the analytical design approach. In this connection the circular tip relief causes a higher penetration than the linear tip relief. The FE-based-and-designed variants are able to avoid a penetration, independent on the tip relief geometry. Therefore the FE-based design of the tip relief can efficiently prevent a premature tooth meshing and its consequences — such as higher tooth flank stress. In addition to that, the FE-based designed variants show an improved excitation behavior in comparison to the analytically designed tip reliefs. Compared to the analytical variants, the first gear mesh order is significantly lower for the FE-based variants. Comparing the tip relief geometry, the linear tip relief shows the most promising results.

Finally, the differently designed tip reliefs are compared concerning the caused tooth flank stress by means of the characterizing parameters introduced previously in this paper. Comparing the FE-based tip reliefs with the **analytically designed**, the weighted, mean pressure *pH,mean* is up to 35% lower for the FE-based variants. However, the above-described promising excitation behavior of the FE-based variants is realized by high lengths of the modifications. Thus the transition area and the locally reduced radii of curvature are nearer to the single tooth contact, which is connected to high normal forces. This is the reason for the higher maximum pressure *pH,max* of the FE-based variants. Nevertheless, the increase of the maximum pressure is not as high as the reduction of the weighted, mean pressure (19% increase of *pH,max* vs. 35% decrease of *pH,mean*). Also, the analytically designed tip reliefs do not prevent a premature tooth meshing, which increases the tooth flank stress further.

## Theoretical Study for the Design of Profile Modifications for Helical Gears

***Irregular tooth mesh of helical gears.*** The contact conditions during irregular tooth mesh of helical gears have not been analyzed in detail up to now. Occurring tooth flank damage — indicative of an irregular tooth mesh — has been encountered by tip reliefs. Such a damage pattern is presented (Fig. 9, top). The region of the tooth root flank shows strong material fatigue in form of a pitting line, similar to spur gears with a premature tooth mesh. However, the pitting line is orient-

**Table 1 Parameters of designed tip reliefs**

| Approach | Analytical | | FE-based | |
|---|---|---|---|---|
| Geometry | Linear | Circular | Linear | Circular |
| Amount Pinion | 35 ± 5 µm | | 50 ± 5 µm | 55 ± 5 µm |
| Diameter Pinion | 223,95 ± 1 mm | | 221 ± 1 mm | 221 ± 1 mm |
| Amount Gear | 35 ± 5 µm | | 55 ± 5 µm | 55 ± 5 µm |
| Diameter Gear | 371,55 ± 1 mm | | 369 ± 1 mm | 369 ± 1 mm |
| Transition Area | 15% of Length | – | 15% of Length | – |

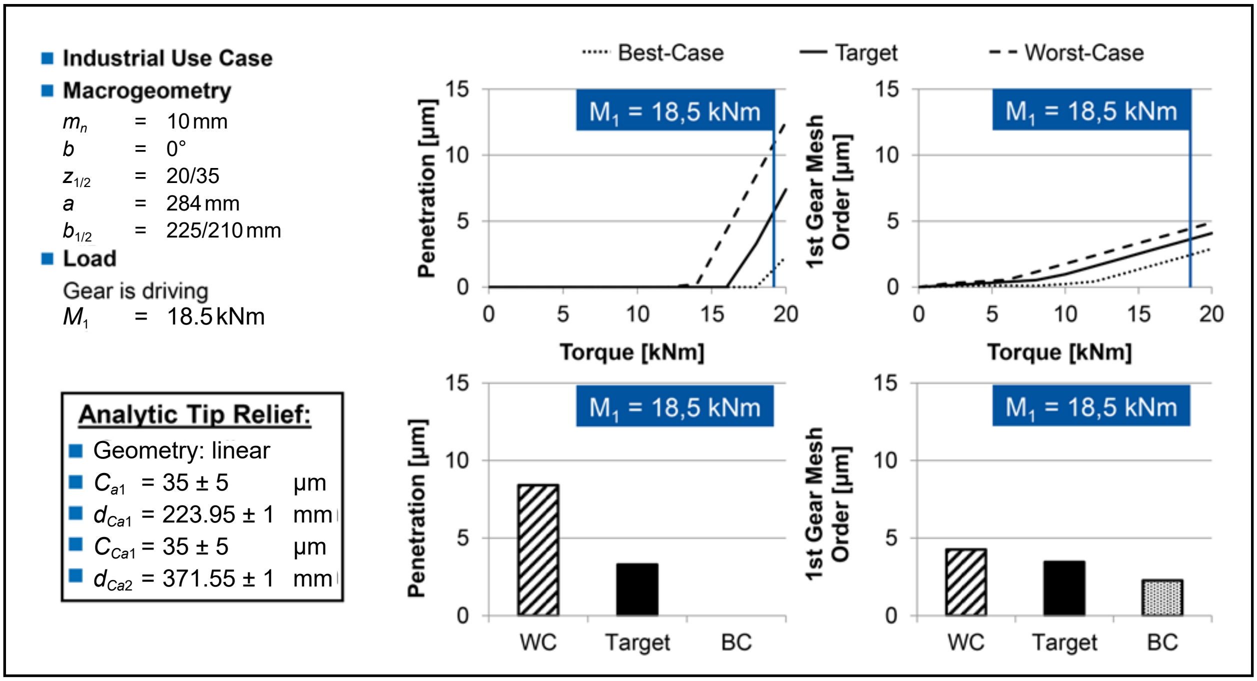


Figure 7 Approach for objective comparison of design variants.

ed in the direction of the flank lead — not along the path of contact. A premature tooth mesh of a helical gear can only take place at the start and end points of meshing — A and E (Fig. 9). As a result, there must be another reason for high tooth flank stress in the tooth root flank area of helical gears.

In order to explain the damage pattern (Fig. 9), a detailed FEM analysis is performed. In (Fig. 9, middle), it can be seen that the first point of contact is at the left side of the tooth root flank area of the driving gear. For this point of time, increased pressure is calculated and, against the theory of the gear contact, the point of contact first moves down in the direction of the tooth root. This behavior is similar to a premature tooth mesh of spur gears, but with the difference that it only takes place in a very small region of the flank of helical gears.

During further contact the line of contact moves across the driving flank from the left to the right side. It is noticeable that the maximum pressure of each contact line is at the point nearest to the tooth root. Furthermore, the maximum pressure decreases while the contact line moves from left to right. After crossing the middle of the tooth flank, the maximum pressure is at the tip region and increases from the left to the right.

A possible explanation for this behavior is the de-central application of force of helical gears. It is known for tooth root stress that the maximum is not middle-of-the-tooth width (Ref. 16). The reason for this is the inhomogeneous load along

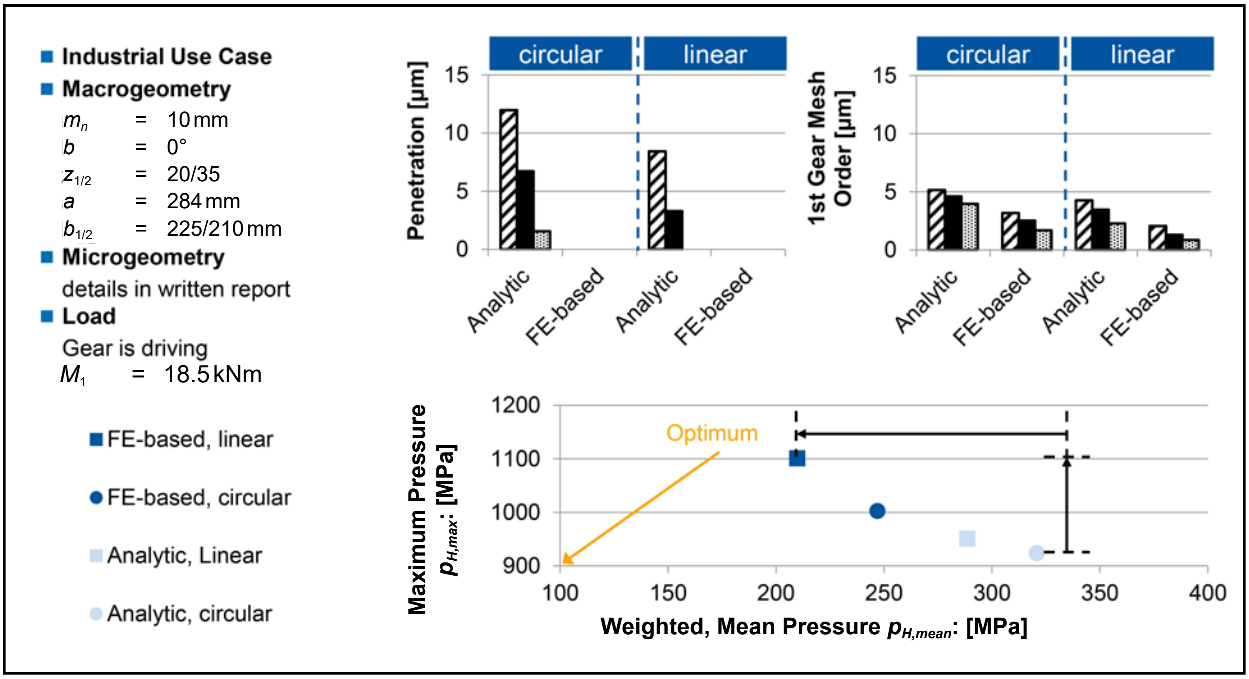


**Figure 8** Comparison of design variants.

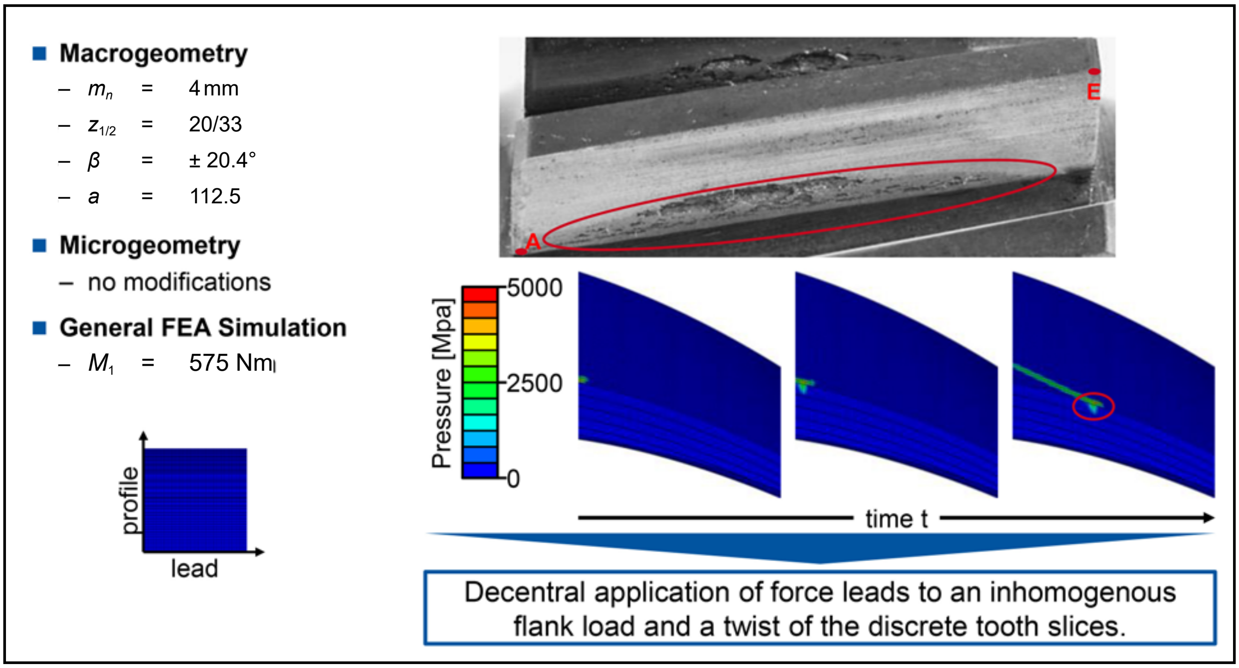


**Figure 9** Damage pattern of helical gear with irregular tooth mesh.

the tooth width of helical gears. This inhomogeneous load application can lead to a twist of the flank, so that the effect of increased pressure in the tooth root and tooth tip region can be explained by a kind of premature tooth mesh of each tooth slide. As the increase of pressure is noticeable for almost every line of contact, and as its amount is dependent on the place of line of contact, it has to be investigated whether modifications other than a tip relief, e.g. — multi-dimensional modifications — are more efficient in order to decrease the overall flank stress.

***Approach for design of tip reliefs for helical gears.*** The previously described effect, which leads to an increased pressure in the tooth root and tooth tip flank region of the driving gear, is caused by a twist of the loaded flank. The influence of this effect on the pressure distribution can be analyzed by means of the FE-based TCA. In this work the focus is on the suitability of different flank modifications concerning the prevention of pressure increase.

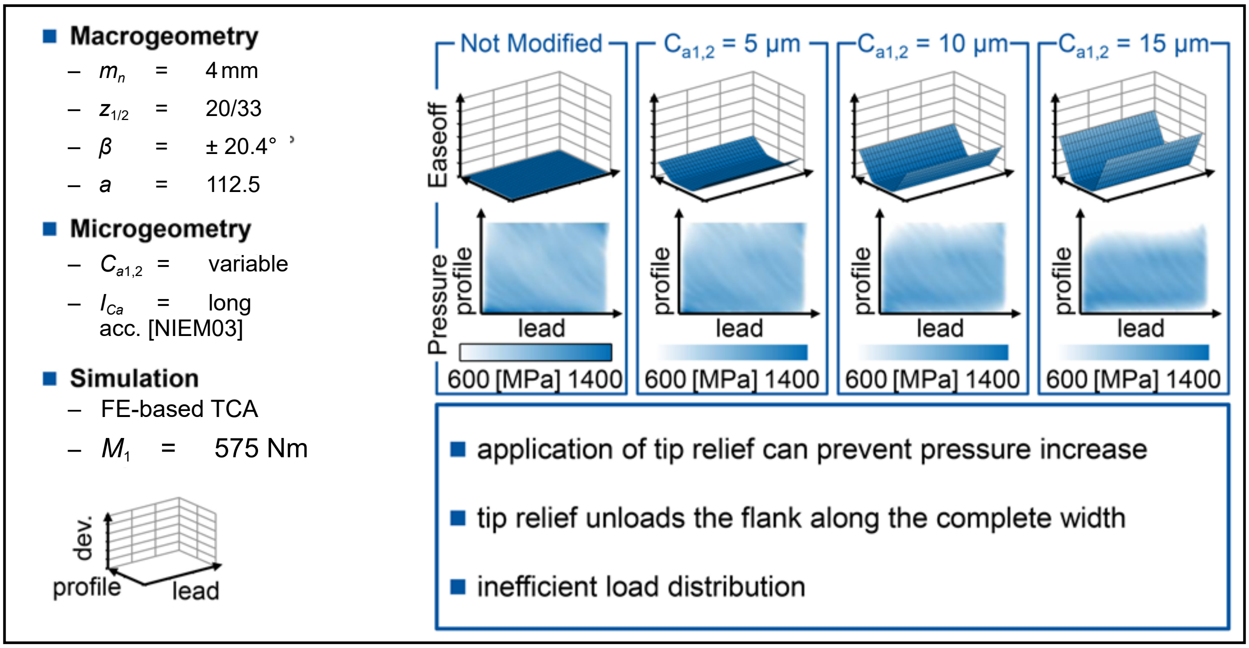
In a first step, the influence of tip reliefs on the pressure distribution is examined (Fig. 10). For the previously analyzed macro-geometry, different amounts of tip relief for the driving and driven gear are investigated. The length of the tip relief is always a long tip relief, according to NIEMANN/WINTER (Ref. 8). In Figure 10 (top) the resulting Ease-Off of the modifications is shown; below that is the resulting pressure distribution on the flank. For the unmodified gearset, the line of increased pressure, orientated in tooth width direction, can be seen in the tooth root flank region on the left side of the flank, and in the tooth tip flank region for the right side of the flank. By the application of a tip relief, the flank is unloaded in the tooth root and tooth tip flank region and along the complete tooth width. As the added pressure is decreasing from the face side to the middle of the flank, a constant unloading of the flank along the direction of the tooth width leads to an inhomogeneous pressure distribution. The flank regions at the upper left and lower right position are much lower loaded than the flank regions of the lower left and upper right. What's more, overdoing tip relief leads to a new line of increased pressure, which is then located more in the flank middle region. In terms of a high load capacity, this is not desirable.

As the increased pressure is dependent on the location along the tooth width, the influence of a specific flank twist on the pressure distribution of helical gears is examined. In order to simplify the interpretation, the twist is only applied for the driving gear (Fig. 11).

Choosing a twist with the wrong orientation leads to an amplification of the studied effect (Fig. 11, top-left). The flank regions with the higher flank load are loaded even higher, in turn leading to a worsening of the pressure distribution. A twist with the correct orientation decreases the load in these regions, and increases the load in the regions of the upper-left and lower-right flank. Thus a homogeneous pressure distribution along the tooth width can be realized. Nonetheless, the line of increased pressure caused by the effect previously addressed cannot be avoided. Therefore, a combination of flank twist and tip relief is analyzed. The twist leads to a homogeneous load distribution along the flank width; the tip relief prevents the pressure increase. And so a uniform load of the complete tooth flank is achieved, which is desirable for a high-load capacity.

## Summary and Outlook

The purpose of tip relief is to prevent premature tooth meshing and corresponding, negative effects regarding load carrying capacity and excitation behavior. In existing research projects a positive influence of tip reliefs on the excitation behavior and the tooth flank load carrying capacity can be observed. This positive influence is dependent on the design of tip relief — especially the definition of geometry, amount, and length of tip reliefs. These design parameters are today calculated by means of analytical approximate formula. A disadvantage of this design method is the missing regard to local radii of curvature, local stiffness, and the influence of further tooth flank



■ **Macrogeometry**
- $m_n$ = 4 mm
- $z_{1/2}$ = 20/33
- $\beta$ = ± 20.4°
- $a$ = 112.5

■ **Microgeometry**
- $C_{a1,2}$ = variable
- $l_{Ca}$ = long acc. [NIEM03]

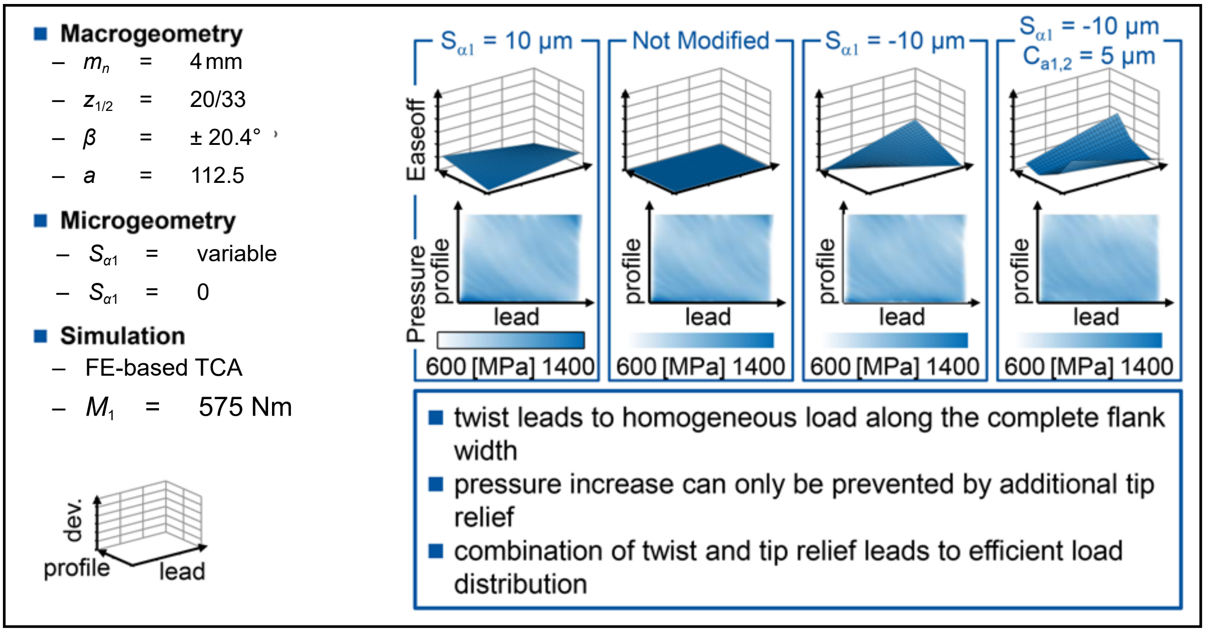
■ **Simulation**
- FE-based TCA
- $M_1$ = 575 Nm

■ application of tip relief can prevent pressure increase

■ tip relief unloads the flank along the complete width

■ inefficient load distribution

**Figure 10 Influence of tip reliefs on the pressure distribution of helical gears.**



- **Macrogeometry**
  - $m_n$ = 4 mm
  - $z_{1/2}$ = 20/33
  - $\beta$ = ± 20.4°
  - $a$ = 112.5
- **Microgeometry**
  - $S_{\alpha 1}$ = variable
  - $S_{\alpha 1}$ = 0
- **Simulation**
  - FE-based TCA
  - $M_1$ = 575 Nm

- twist leads to homogeneous load along the complete flank width
- pressure increase can only be prevented by additional tip relief
- combination of twist and tip relief leads to efficient load distribution

**Figure 11** Influence of twist on the pressure distribution of helical gears.

modifications. Furthermore, a design method for appropriate tolerance fields is missing.

Therefore, this work focuses on the development of an FE-based method for the design of pressure optimized tip reliefs. For this purpose a method considering local radii of curvature in the pressure calculation, based on the existing FE-based tooth contact analysis, is presented and a parameter study is carried out to investigate the potential increase of tooth flank load carrying capacity by profile modifications. Furthermore, the theoretical penetration of the following teeth is calculated und used for the tolerance-based design of tip reliefs.

The FE-based designed tip relief is compared to an analytically designed tip relief concerning excitation behavior and tooth flank stress. Finally, a theoretical study focuses on the design of profile modifications for helical gears.

Due to the large number of design variants, two characteristics are used for an objective evaluation of the potential increase of tooth flank load carrying capacity. On the one hand, the maximum pressure caused by the profile modification is used. On the other hand, a weighted mean pressure is presented. In this way the location of the pressure is also considered, as well as its magnitude. The results of the parameter study reveal a different behavior of linear tip reliefs when compared to other relief geometries. What all variants of the linear relief have in common is that the weighted, mean pressure is on a comparable level. In general, a high increase of tooth flank load carrying capacity can be expected from linear and polynomial (2nd order, circular) tip reliefs. Based on these results the FE-based calculation of penetration developed in this paper is used for the design of the amount and length of tip reliefs, including appropriate tolerance fields. It can be shown that the analytical design approach is not suitable for preventing a premature tooth meshing. By using the FE-based design approach, a theoretical penetration and, by this, a premature tooth meshing is avoided. Along with the prevention of premature tooth meshing, the weighted, mean pressure and, by this, the pitting risk, can be reduced significantly for the FE-based variants. Furthermore, the excitation behavior, represented by the 1st gear mesh order, can be improved by the FE-based design. Summarizing, the FE-based design approach has a high potential for the improved, application-oriented design of tip reliefs.

By means of detailed FEM analysis, a premature tooth mesh is analyzed for helical gears. In contrast to spur gears, the premature tooth mesh for helical gears leads only for a small flank region to increased stress. The effect is limited to the points of the beginning and the end of the contact — A and E. In addition, another effect can be observed, i.e. — the decentral load application of helical gears — which is known from works focusing on tooth root stress — leads to a twist of the flank. In this way increased contact pressure occurs in the tooth flank root and tip area during the complete tooth contact. Further analyzation of damage pattern shows that this effect is even more critical, concerning the load capacity, than a premature tooth mesh. The effect is caused by the decentral load application and the local stiffness behavior that can be mapped by the FE-based TCA. Using the FE-based TCA, the influence of different flank modifications is discussed. For the focused test gear set, a tip relief combined with a specific flank twist shows the most promising results concerning a homogenous load distribution and, in turn, a high load capacity.

In the future the derived design method for tip reliefs has to be validated in further test rig investigations. Besides promising variants, unfavorable variants also must be tested by means of running tests to validate the design method presented in this paper.

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