Face Gears: An Interesting Alternative for Special Applications—Calculation, Production and Use

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Introduction

Crown gearings are not a new type of gear system. On the contrary, they have been in use since very early times for various tasks. Their earliest form is that of the driving sprocket, found in ancient Roman watermills or Dutch windmills. The first principles of gear geometry and simple methods of production (shaper cutting) were developed in the 1940s. In the 1950s, however, crown gears' importance declined. Their tasks were, for example, taken over by bevel gears, which were easier to manufacture and could transmit greater power. Current subject literature accordingly contains very little information on crown gears, directed mainly to pointing out their limitations (Ref. 1).

That literature contrasts with studies and development work carried out independently in the United States and the Netherlands over the last 10 years. The research in those two countries laid the foundations for the design and calculation of crown gear trains and, particularly, developed new, competitive production processes. Overall, those trends indicate that crown gear trains again represent an interesting solution for an increasing number of applications.

Applications for Crown Gear Systems

The range of applications for crown gears is extremely wide and is practically unlimited because of all the gears' possible sizes, materials and qualities. The application of crown gears is most practical when their special properties and characteristics can be exploited systematically. The examples and pictures of the Cylkro® face gears shown in this article have been made available by Crown Gear B.V. of the Netherlands.

Angular crown gear transmissions with shaft angles from 0°–135° (Fig. 1) are used in milling machines, for example, for the positioning of the milling head, or as power is sent to the tool. Another example is the use of 17°-shaft-angle crown gears in conical mixers (Fig. 2).

Gear ratios of up to 15 can be realized with spur pinions. The use of helical pinions offers the possibility of even higher gear ratios, as seen in wheel chair drives, using a gear ratio of 19.5 (Fig. 3). Further improvements are studied and tested, in which pinions with a small number of teeth and high helix angles, like worms, are being used.

The axial freedom of the pinion is particularly useful in applications that require low backlash. The axial mounting position of the pinion does not affect the backlash. In applications where the posi-

Figure 1—45°-shaft-angle crown gears and a 135°-shaft-angle crown gear for milling machines.

Figure 2—Conical mixer with shaft angle of 17°.

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Figure 3—High crown gear ratios in one stage.

Figure 4—Crown gears for a radar antenna with low backlash.

tion of the axes should be very accurate—in robot drives or radar antenna drives (Fig. 4)—crown gear sets with low or zero backlash are used.

Another field of application is the area of electrical hand tools. An advantage in that application is the direct mounting of the pinion on the motor shaft, whereby the axial movements of the motor shaft do not affect the contact pattern. Since the number of crown gears used in the application—as well as in other household appliances—is usually very large, alternative production methods, such as sintering, injection molding and metal injection molding, are often used (Figs. 5 and 6).

A growing number of applications can be found in the automotive area. Those applications can be divided into low power drives, such as mirror actuators, windshield wipers and starter motors, and high power drives. The high power drives can be rear axle drives, using skived or ground helical and offset crown gears, differential gears or camshaft drives. Thanks to the cylindrical pinions, a differential using crown gears can be constructed significantly smaller in height and weight than a typical bevel gear differential of the same power. The differential used in a 4X4 vehicle, for instance (Fig. 7), is made of a small crown gear and a large crown gear with several pinions between them. By driving the pinions, the torque

Figure 5—Sintered crown gears.

Figure 6—Injection-molded crown gears.

is not divided equally between the front and the rear axle, but the relation is proportional to the diameters of the crown gears.

An additional advantage, closely linked with free adjustment, is the simplicity of crown gear applications for multiple machine drives, power take-offs and power splitting, transmissions whereby after splitting the torque in the first stage, the torque comes together again in the next stage. Several of those power-split designs have been developed and patented for helicopter transmissions. One patented power split by Crown Gear B.V. (Fig. 8) shows an input pinion driving the first output crown gear. With help from a set of parallel gears (not intermediate gears), a second pinion drives the second output crown gear. The two crown gears are floating axially and are mounted together at the back face or are one part. Although more parts are needed in that design, the advantages lie in a greater system power density (gear size, bearings, shaft diameters).

Special Features and Properties of Crown Gears

Crown gears are essentially angular gear systems in which the pinion, a normal cylindrical gear, meshes with a rack-gear, the crown gear. The shaft angle may vary across a continuous range from 0° (cylindrical gear pair) to 135° (Fig. 9) (Ref. 2). The most common application is, however, a shaft angle of 90°. The following discussion will also focus on that application.

As is evident from Figure 9, the geometry of the crown gear is determined by its shaft angle, its required gear ratio and—decisively—by its pinion geometry. The pinion is a cylindrical gear. It can be a spur or helical gear and may be offset to the crown gear.

A particular characteristic of spur pinions is that no axial forces act on the pinion bearings; such systems can therefore be smaller and lower-priced. Helical pinions achieve higher overlaps and are subject to axial loads that are smaller than for spiral bevel gears.

Shaft offsets are used principally when the input and output shafts cross, for example, in a multidrive system or a power take-off. Combined with helical teeth, shaft offsets allow extreme downward gear ratios (i = 100) in a relatively small space (Ref. 2).

The most important attribute of crown gears is doubtless the axial freedom of the pinion (Ref. 3). The pinion can be displaced axially at random without altering the contact parameters or flank play (Fig. 9). That results in a number of advantages. First, assembly is greatly simplified; the pinion can be positioned with relative freedom along its shaft axis. The pinion also can be slid into place sideways during assembly. Second, the "free adjustment" of

the pinion shaft simplifies the interaction of a number of pinions with a crown gear or the use of a single pinion to drive two crown gears.

Geometry of a Crown Gear System

The most obvious external attribute of a crown gear system is the shape of its teeth. The shape changes continuously across the width of the tooth. Studies aimed at a complete description of all crown gear variants—that is, gear systems with spur, helical or offset shaft pinions at various shaft angles, have been conducted only recently in the United States (Ref. 4) and the Netherlands (Ref. 5). Numerical simulation software was developed in the Netherlands for numerical contact analysis and production simulation, laying the foundations for the use of crown gears in power gear systems.

The following sections describe the origins and characteristics of crown gear geometry. The simplest and most common case of a pinion with spur teeth and shafts intersecting at an angle of 90° is considered.

Kinematic principle. In simplified form, a crown gear may be imagined as the interaction of a spur-toothed cylindrical gear with a curved rack. Figure 10 (left side) depicts the face section of such a rack-and-pinion combination. In operation, the pinion drives the rack at a constant angular velocity, ω_1. That results in a constant peripheral velocity, v_{b1}, at the base circle diameter. From that, the translatory speed of the rack, v_{hor}, may be calculated via the rack pressure angle, α_2.

In the case of a straight rack, v_{hor} is constant. Because the crown gear revolves about its axis of rotation, however, a variable radius-dependent horizontal velocity v_{hor} results across the face width of the tooth. The velocity at the outside diameter of the crown gear must accordingly be greater than that at the inside diameter. In order for tooth contact to exist over the entire face width of the tooth, the pressure angle, α_2, of the crown gear must consequently change over the diameter of the gear. The right-hand section of Figure 10 shows the resulting tooth shape for the crown gear. The changing pressure angle is shown clearly in the section. α_2 is smallest at the inside diameter; it increases towards greater diameters.

Usable tooth flank, contact path curve and overlap. The fact that the crown gear pressure angle α_2 is variable means that the height of the tooth flank that can be used actively for transmitting rotation also varies over the diameter.

Apart from determining the usable tooth height, the pressure angle α_2 decides the feasible face width of the crown gear. Theoretically, the smallest radius d_{i2} of the crown gear is found when the pres-

sure angle $\alpha_{i2} = 0°$. In practice, α_{i2} is restricted to higher values ($\alpha_{i2} \geq 10°$, Ref. 5) dependent on the ratio and the number of pinion teeth, to prevent undercut.

The largest pressure angle occurs at the outside diameter d_{a2} of the crown gear. It limits the outside diameter of the teeth in such a way that the tooth crests become pointed or overcut from a certain pressure angle α_{a2} onwards (Fig. 10). In practice, therefore, a maximum value of $\alpha_{a2} = 45°$ is not exceeded (Ref. 5).

The newly developed numerical contact simulations allow the tooth face of crown gear teeth to be extended further than was previously possible (Ref. 6). A greater tooth face permits a more favorable load distribution and is advantageous in terms of higher transmittable torques, especially at low gear ratios.

Crown Gear B.V. (Ref. 5) achieves a greater tooth face by applying continuous addendum modifications on the crown gear teeth, on the outside diameter to prevent pointy teeth (case hardening), and on the inside diameter to prevent contact errors

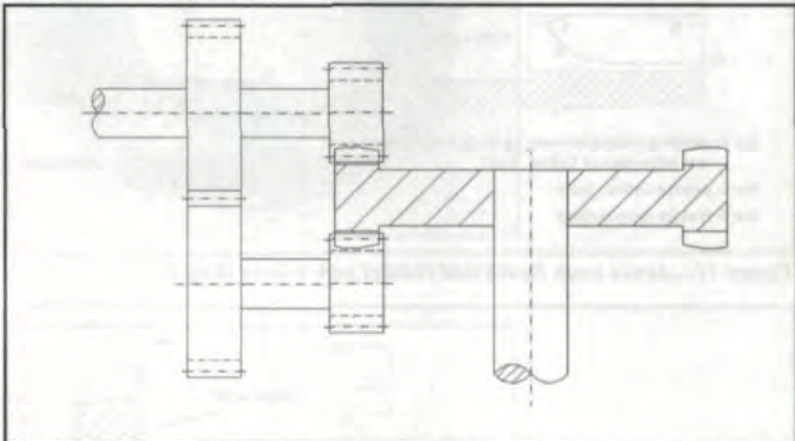*Figure 7—Interaxial crown gear differential for a four-wheel driven car.*

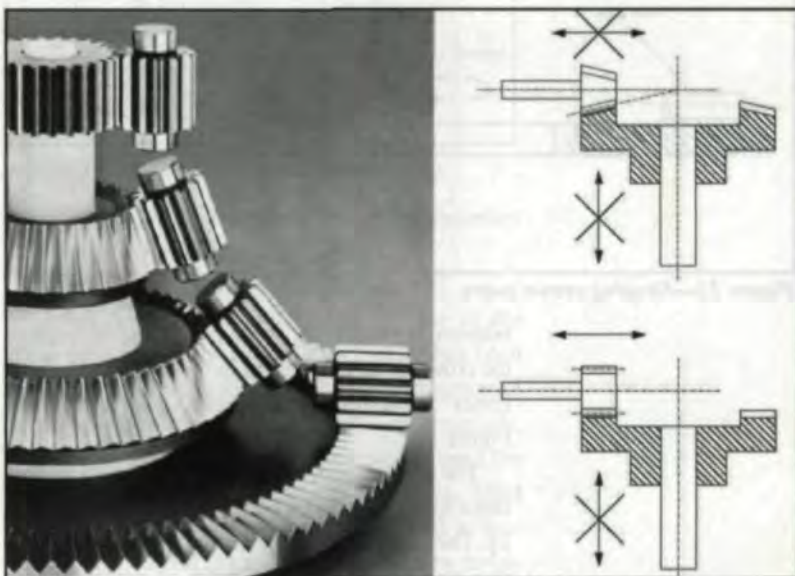*Figure 8—Patented power split from Crown Gear B.V.*

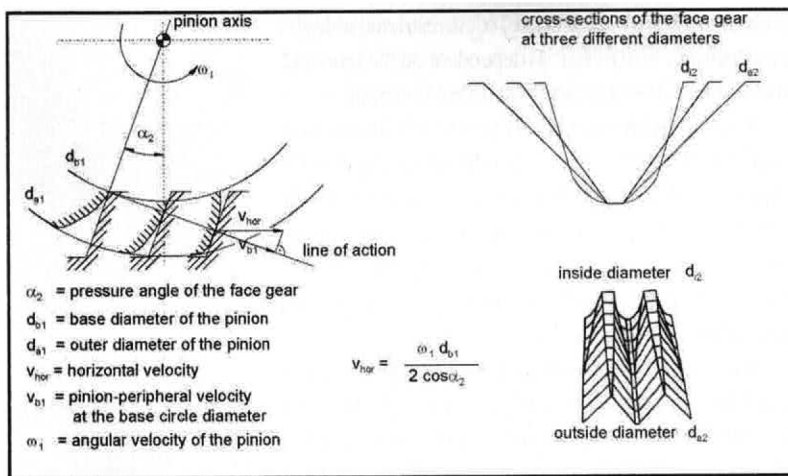*Figure 9—Crown gear system and axial pinion freedom (Ref. 2).*
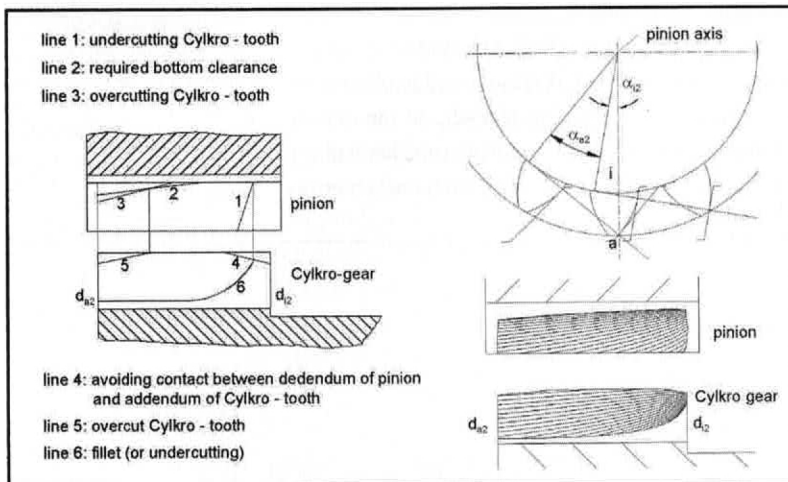
Figure 10—Path of contact and tooth shape.

α₂ = pressure angle of the face gear
d_{b1} = base diameter of the pinion
d_{a1} = outer diameter of the pinion
v_{hor} = horizontal velocity
v_{b1} = pinion-peripheral velocity at the base circle diameter
ω₁ = angular velocity of the pinion

$$v_{hor} = \frac{\omega_1 \, d_{b1}}{2 \, \cos \alpha_2}$$

inside diameter d_{i2}
outside diameter d_{a2}

line 1: undercutting Cylkro - tooth
line 2: required bottom clearance
line 3: overcutting Cylkro - tooth

line 4: avoiding contact between dedendum of pinion and addendum of Cylkro - tooth
line 5: overcut Cylkro - tooth
line 6: fillet (or undercutting)

Figure 11—Active tooth flanks and contact path curves (Ref. 2).
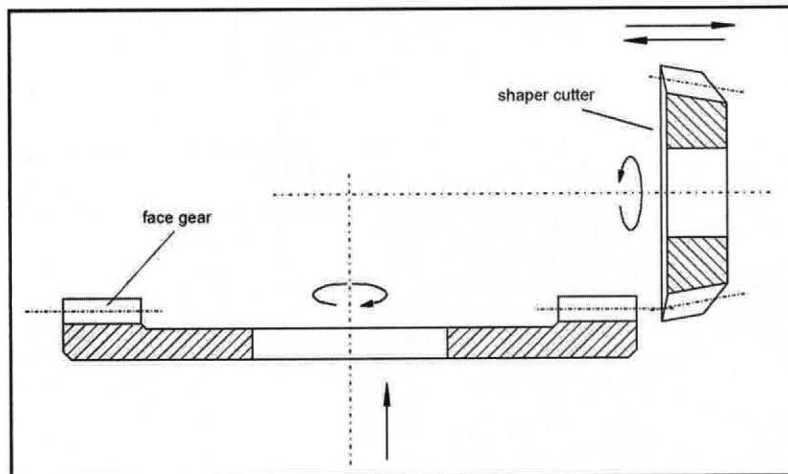

Figure 12—Shaping crown gears.

between the tooth root of the pinion and the crest of the crown gear. That results in the active tooth flank zones for pinion and crown gear shown in Figure 11.

The contact path curves of a gear are of importance for its overlap ratios and load-carrying capacity. The driving pinion of a crown gear system is a spur-toothed gear, but the paths of contact across the tooth flanks of the pinion and crown gear are helical or curved, due to the continuously varying pressure angle α₂ (Fig. 11).

The change in profile overlap across the face width and the differing values for total overlap, ε_γ, are a result of the inclined contact path curves. The helical contact paths result in an "apparent skip overlap ε_β" of the gear. As a result, higher total overlaps are attained, affecting load-carrying capacity and running behavior positively, as in the case of helical gears.

Load-Carrying Capacity

Reliable methods of determining and checking the load-carrying capacity of crown gears became necessary in order to allow their use in power gear systems. The literature identifies the pinion as the weaker element, which also determines the load-carrying capacity of the crown gear train (Refs. 5, 7 and 8).

As the pinion is a conventional cylindrical gear, its load-carrying capacity can be determined by the usual computational methods, for example, according to DIN 3990 (Ref. 9).

The calculation method developed by Crown Gear B.V. allows for the special contact paths and overlap ratios of crown gear systems. The computation is based on DIN 3990. Some of the corrective factors in the DIN standard are modified in the case of crown gears (Ref. 5). The validity of the computational method was confirmed in service life tests (Ref. 5). The following sections on tooth root stress and tooth contact stress briefly outline that method of computation.

Computing the tooth root stress. It is known from FEM simulations that the bending stresses at the tooth root of the pinion are roughly 40% greater than those on the crown gear (Ref. 5). Given the same materials for pinion and crown gear, the bending stresses at the tooth root of the pinion are therefore the deciding factor.

According to DIN 3990, the rated tooth root stress, σ_{F0}, may be calculated as follows:

$$\sigma_{F0} = [F_t/(b \cdot m)] \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\varepsilon \cdot Y_\beta$$

The factors Y_{Fa} and Y_{Sa} are unchanged for a cylindrical gear pair. Owing to the contact line curves and overlap ratios indicated in *Usable tooth flank, contact path curve and overlap*, the overlap factor, Y_ε, and the helix factor, Y_β, for the pinion of the crown gear must be corrected accordingly.

The real tooth root stress, σ_F, can be calculated analogously with DIN as:

$$\sigma_F = \sigma_{F0} \cdot K_A \cdot K_V \cdot K_{F\alpha} \cdot K_{F\beta}$$

The application factor, K_A, and the dynamics factor, K_V, are assumed to be equal to those for helical cylindrical gears. The face factor, $K_{F\alpha}$, and the width factor, $K_{F\beta}$, were determined in running tests.

Calculating the tooth contact stress. The nominal tooth contact stress, σ_{H0}—that is, the stress exerted by the static rated torque in a perfect gearing—is calculated according to the formula given in DIN 3990:

$$\sigma_{H0} = Z_E \cdot Z_H \cdot Z_\varepsilon \cdot Z_\beta \sqrt{F_t/(b \cdot d_1)}$$

The elasticity factor, Z_E, which allows for the material, may be adopted unchanged. The zone factor, Z_H, is determined by the radii of curvature. A simplified assumption is made that the radius of curvature of the pinion is decisive for the calculation, the radius of curvature of the crown gear—considered as a curved rack—being infinite. The overlap factor, Z_ε, and the helix factor, Z_β, are again changed as in *Kinematic principle,* according to the specific contact line situation.

The real tooth contact stress, σ_H, is calculated by analogy with the data given in DIN 3990, the factors $K_{H\alpha}$ and $K_{H\beta}$ being matched to the geometry:

$$\sigma_H = \sigma_{H0} \cdot Z_B \sqrt{K_A \cdot K_V \cdot K_{H\alpha} \cdot K_{H\beta}}$$

Studies of load-carrying capacity and running behavior. There are few existing studies and findings related to the operational behavior of crown gears. An older edition of A. Müller's "Kronradgetriebe" quantifies load-carrying capacity for shaped crown gears as roughly 80%–90% of that for bevel gears with the same tooth width (Ref. 3). He presumes surface hardening of the teeth.

Crown Gear B.V. quotes the same order of magnitude for gears with spur-toothed pinions and a gear ratio greater than three as for bevel gear systems of the same size. The crown gear is again case-hardened and also has been hard-finished. Using the same pinion, the load-carrying capacity of the crown gearing falls at low gear ratios, because the usable tooth width becomes smaller.

More recent load-carrying capacity studies have been carried out in the United States on behalf of the helicopter industry (Refs. 7 and 8). Service life tests confirm the feasibility of crown gears for power gear systems. Using crown gears for a new helicopter drive concept brings about a weight reduction of 40% as compared to other types of drives (Ref. 8). It is claimed that the excitation of vibrations and noise emission are also reduced. On the service life test stand, the tooth contact frequency and higher harmonic of the crown gear system were
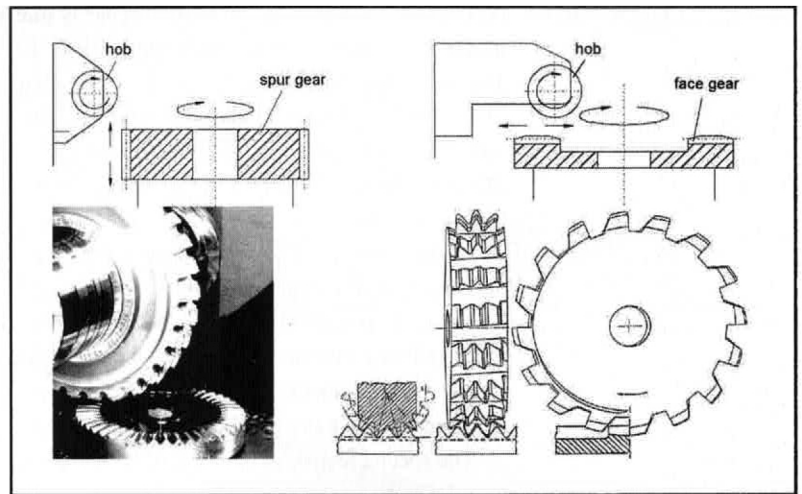

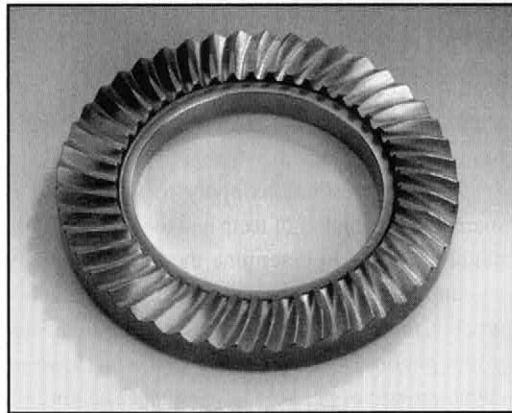Figure 13—Hobbing crown gears.

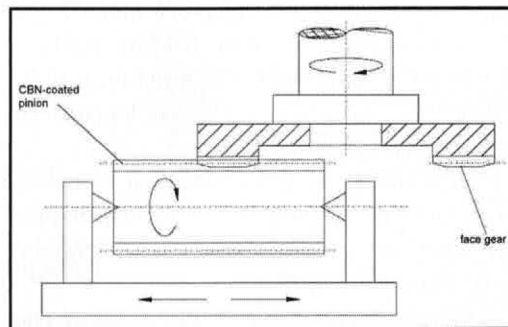Figure 14—Helical offset ground crown gear for a rear axle drive.

Figure 15—Principle of crown gear honing.

significantly lower than those of other frequency components (Ref. 7).

Manufacture of Crown Gears

Apart from load-carrying capacity, one of the main obstacles to the use of crown gears has been the problem of manufacture. With the exception of some special production methods—for example, sintering in mass production—shaping has been the only realistic industrial production process. Hard finishing after hardening has been confined to lapping. That area also has seen a number of new developments or innovations in recent years, extending the range of applications for crown gear systems.

Soft gear cutting. In the traditional shaping

process for crown gears, the crown gear is manufactured by means of a shaper cutter (Fig. 12). The shaper cutter must possess the tooth shape of the pinion. Ideally, it should have the same number of teeth, but the number of teeth may be increased to generate longitudinal crowning on the crown gear flank.

Crown Gear B.V. has developed hobbing as a new production process for crown gears (Ref. 5). The process is carried out on a slightly modified series hobbing machine (Fig. 13). Recent developments allow production on standard CNC hobbing machines without any modifications necessary.

The special feature of that type of hobbing is the toroidal hob employed (Fig. 13). The generating motion of a pinion tooth in a tooth gap on the crown gear is modelled in the face geometry of the hob. The successive cutter teeth correspond to the tooth profile of the pinion in discrete hobbing positions. After one revolution of the cutter, the crown gear has turned by one pitch.

The same restrictions apply to hobbing as to shaping. The number of teeth and tooth shape of the pinion continue to determine the cutter geometry. By simulating a rotating pinion in the production process, the hob's geometry must be closely related to the pinion used later in the gearbox. However, slight variations can be compensated, enabling the use of a tool for a larger class of pinions. Frankly, due to the kinematics, a shifting of the tool is not possible. On the other hand, production costs can be reduced by using series hobbing machines, which can be additionally employed for hard finishing the crown gear and, of course, for producing conventional cylindrical gears.

Hard finishing. Tooth flanks can be finished after case-hardening on a normal hobbing machine, as in the case of soft cutting (Ref. 5). A carbide-coated cutter is used.

Skive-hobbing achieves extremely good pitch accuracies (DIN quality 3–4) and surface roughnesses (R_a = 0.2–0.4 μm) (Ref. 5). Other finishing processes, like honing or grinding, must be used when quality requirements are greater. Crown Gear B.V. developed the continuous grinding process, using either dressable or non-dressable grinding worms. In close cooperation with a European manufacturer of grinding machines, a helical offset ground crown gear was produced (Fig. 14) as a replacement for a hypoid spiral bevel gear set for a rear axle drive. Honing with a CBN-coated pinion as the tool is already being done. Figure 15 shows the principle underlying the honing process.

During honing, material is removed due to the relative sliding motion of the flanks in the profile axis. For non-offset pinions, an additional axial motion in the pinion axis is necessary to achieve machining at the pitch cone. That is not necessary in the case of offset pinions. The surface quality of the gear can be improved by honing; an improvement in pitch accuracy is not possible.

Conclusion

The use and application of crown gears is currently in transition. Newly developed computational methods, including numerical contact simulation, provide decisive support for design and production. For the first time, numerical design and strength calculation methods are available, opening the way to applications in power gear systems.

Alongside those developments, innovative production processes provide a basis for cost-effective production of crown gear trains. Systematic exploitation of the special design features and properties of crown gears provides interesting solutions for specific applications, offering an alternative to other gear concepts in many cases.

Acknowledgments

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