Local 3-D Flank Form Optimizations for Bevel Gears

Hartmuth Müller

Abstract

Optimizing the running behavior of bevel and hypoid gears means improving both noise behavior and load carrying capacity. Since load deflections change the relative position of pinion and ring gear, the position of the contact pattern will depend on the torque. Different contact positions require local 3-D flank form optimizations for improving a gear set.

This paper presents methods on flank form modifications for spiral bevel or hypoid gears. Flank form modifications applied to generated gears are based on additional machining motions superimposed on the rolling motion. Additional motions are modified roll, helical motion, vertical motion, and horizontal motion. These motions are represented by a Taylor series approach. A Taylor series is a mathematical development of a function using polynomial functions: $f(x) = \sum_i a_i \cdot (x - x_0)^i$. The independent variable is the cradle angle.

In case of nongenerated gears, flank form modifications are performed by machining different plunging positions continuously. A plunging position is defined by cradle angle, radial distance, offset, machine root angle and sliding base. According to the approach used for generated gears, the motions are described as Taylor series depending on an imaginary strictly limited cradle movement.

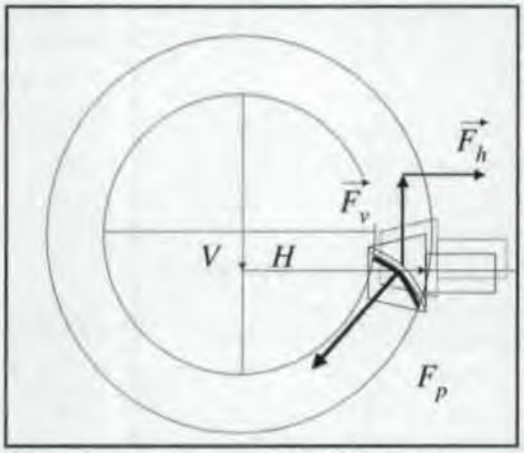

Figure 1—Tooth forces in a hypoid gear set.

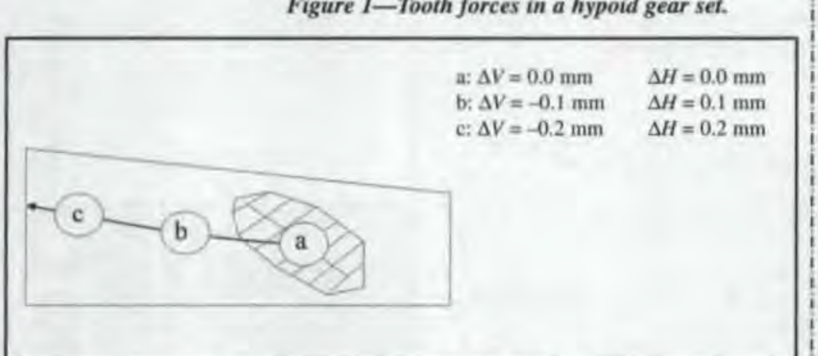

Figure 2— Location of the contact pattern for ΔV and ΔH changes.

Introduction

Designing spiral bevel and hypoid gears covers a lot of different aspects. In general, the aim is to minimize gear noise and gear volume as well as to maximize load carrying capacity. Minimizing gear volume and maximizing load carrying capacity are especially conflicted aims. Nevertheless it is important to design a well-balanced compromise within these aims.

For practical applications, it is necessary to keep the characteristics of the running behavior in a bandwidth defined by assembly tolerances.

Everything related to the running behavior of gears originates in the meshing operation of the teeth. Therefore a detailed look into the meshing operation with respect to influences of housing and bearings will be necessary.

Noise Behavior and Crownings

In the last few years, several approaches have been published dealing with noise optimizations (Refs. 1 and 2). The result of all these efforts concentrated on the transmission error and a reasonable pitch quality. If the transmission error exceeds a certain limit, the noise behavior will not be acceptable. The influence of the pitch quality is not very drastic. Gears with spacing better than DIN 5 will not show significant influences on the pitch error.

The influence of the shape of the transmission error is not yet fully clear. One possibility is to perform a Fourier transform in order to separate the first mesh harmonic from higher orders (Ref. 3).

A Fourier transform is any periodical function that can be expressed by a series of sinusoidal functions: $f(x) = a_0/2 + \sum_i [a_i \cdot \cos(i \cdot x) + b_i \cdot \sin(i \cdot x)]$. This principle performs a harmonic analysis. The coefficients a_i and b_i are called Fourier coefficients.

Since very small surface defects on the flanks of the gears have a great influence on higher orders of the Fourier transform, this area will be of great interest in the future (Ref. 3).

With a conjugate gear set, the unloaded contact area at each instant in time will be a line and the transmission error will be zero. As the gear pair rolls through mesh, these instantaneous line contacts result in a full contact pattern covering the whole active flank. Meshing conditions like this are ideal for minimum noise. However, even a small displacement of the pinion or ring gear will change the full contact pattern into a small line at the border of the teeth and would result in a noisy gear set.

Therefore every gear design requires reasonable crownings in profile and lengthwise direction of the teeth. The bigger the crownings are, the smaller the transmission error will change due to displacements, but the higher the transmission error will be. The smaller the crownings are, the smoother the gear will run, but the more sensitive it will be to displacements.

Gear Load and Displacements

In general, load applied to a bevel or hypoid gear set influences housings, bearings, gear bodies, shafts and teeth. For a better understanding, this paper separates two effects of the load. The first effect is a displacement of pinion and ring gear caused by tooth forces deflecting the pinion shaft and the ring gear according to the stiffness of bearings, housing and gear bodies. The second effect is the deflection of the flanks due to the load, resulting in surface stress and root bending stress.

Although this approach simplifies the real situation, it helps in understanding the effects of load. In Figure 1, a schematic view of a hypoid gear set is shown.

When the pinion rotates counterclockwise, the concave pinion flank will drive the convex ring gear flank. The force F_p of the pinion's flank will create an answering force $F_g = F_v + F_h$ of the ring gear. The vertical component F_v will decrease the hypoid offset V, and the horizontal component F_h will increase the mounting distance H. Increasing the mounting distance will push the pinion into the bearings. Therefore this direction of rotation is called the drive side.

The different locations of the contact pattern depending on ΔV and ΔH are shown in Figure 2. This figure schematically represents the characteristics for the drive side. The contact pattern in the ring gear is shown. The toe is on the right side, the heel on the left and the root on the bottom.

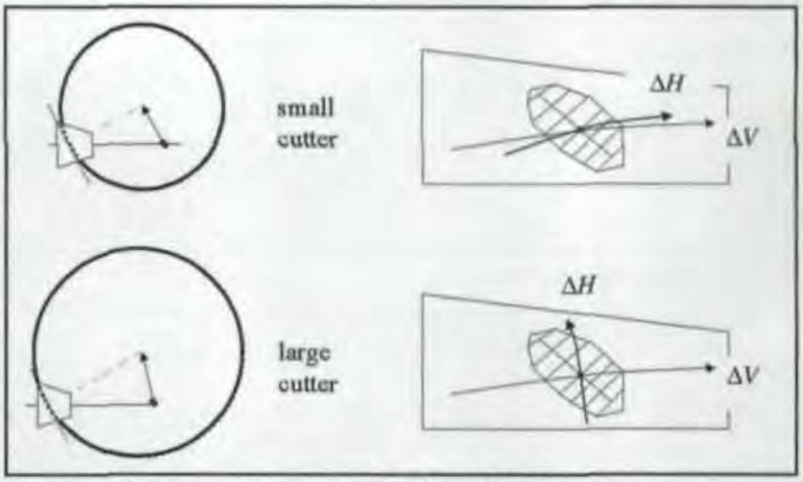

Figure 3—Contact pattern reactions to ΔV and ΔH changes.

This figure shows a bias-in condition, since the contact moves towards the tip-heel area of the tooth with $\Delta H = -\Delta V$. If the contact moved towards the root-heel area, the condition would be called bias-out.

The other direction of rotation is called the coast side and shows forces in the opposite direction. For the coast side, the tooth forces will increase the hypoid offset V and decrease the mounting distance H.

Macrogeometry Conditions

Even when the deflections are small numbers, the position of the contact pattern will change drastically. Depending on the diameter of the cutter, we get changes in the position of the contact pattern as shown in Figure 3.

The influence of the cutter diameter can be used to design a gear set insensitive to deflections. Since the tooth forces decrease the offset and increase the pinion mounting distance, an optimum for the cutter diameter is found when the trajectories for ΔV and ΔH are as close to the same as possible.

The optimal cutter diameter is given by $D_{cutter} = 2 \cdot R_m \cdot \sin(\beta_m)$, where R_m is the mean cone distance and β_m is the mean spiral angle.

Ease-Off Design

Besides the macrogeometry, the shape of the flanks is the most effective part optimizing a gear set. Noise optimization is done by reducing crownings for a smooth meshing operation. Load optimization needs to increase crownings in order to avoid edge contact and peaks in the root bending stress.

As shown earlier, the position of the contact pattern changes due to load deflections. Since the contact moves towards the heel area with increasing load, it is obvious that the unloaded contact

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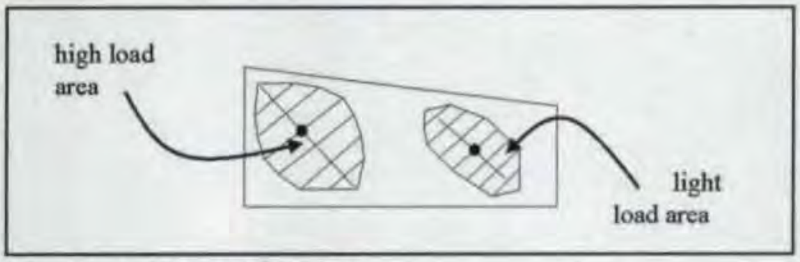

Figure 4—Load defined flank areas.

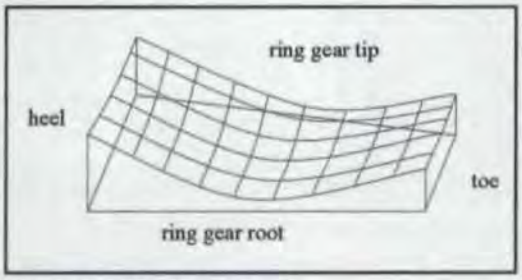

Figure 5—Standard ease-off.

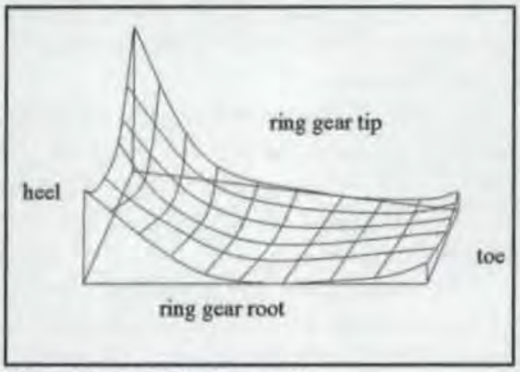

Figure 6—Optimized ease-off.

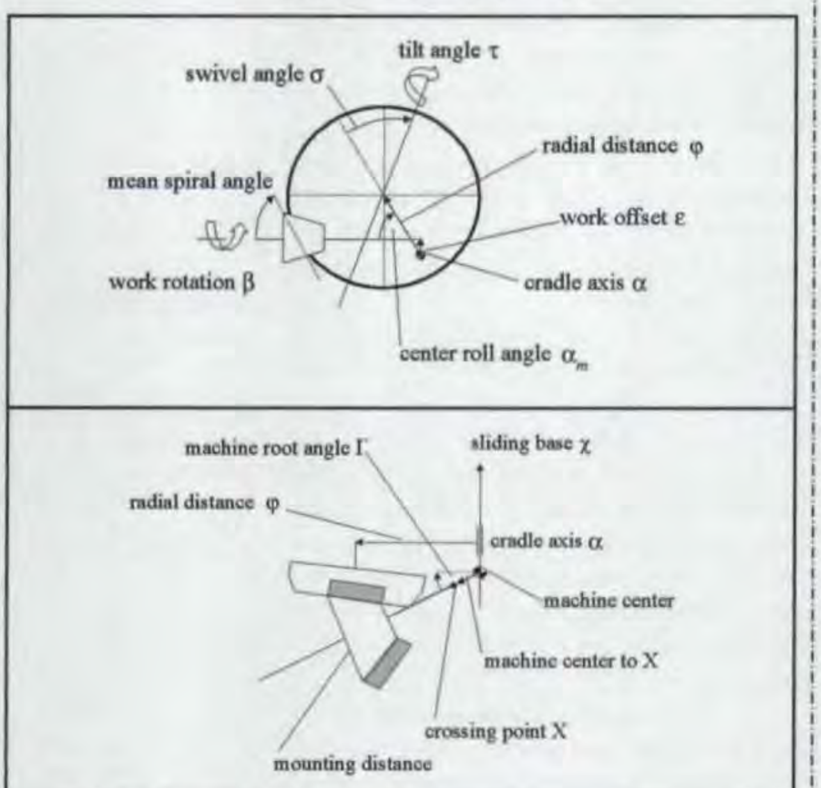

Figure 7—Machining parameters.

should be moved more toward the toe area. The phenomena of a load depending position of the contact pattern is used to define different load areas, shown in Figure 4.

The meshing topography is represented by the ease-off. The ease-off is the contact distance between two flanks meshing, ideally without transmission error. Figure 5 shows a typical standard ease-off. All other characteristics, like transmission error and size and location of the unloaded contact pattern, can be derived from the ease-off. A conjugated gear set has an ease-off which equals zero all over the flank.

Optimizing the noise behavior is more critical under light load conditions. The more load is applied, the more the surface of the flanks will be deflected resulting in a lower level of the transmission error. The lower the load is, the more important the flank's shape is for noise emission. Noise optimization will be achieved by introducing locally reduced crowning in the toe area. For practical use of this idea, the ease-off must guarantee a small unloaded transmission error within a pinion mounting distance variation of ±0.05 mm. This corresponds to practical tolerances for the assembly.

Optimizing the load carrying capacity can be achieved by introducing locally increased high crowning in the heel area (Figure 6). The load will deflect the meshing flanks. This can be seen by comparing the unloaded transmission with the loaded transmission, applying the same displacement to pinion and ring gear. The high crownings will help to avoid edge contact. Edge contact would result in peaks in the root bending stress and in peaks of the Hertzian pressure distribution.

Modified Motion

Modified motion is a principle of changing the flank's shape locally. It can be applied to pinions and generated ring gears. Plunged ring gears need a different approach, which is presented later. Since the principle of making all spiral bevel and hypoid gears is based on rolling a plane gear with a workpiece, it is completely sufficient to consider a virtual machine having a cradle. Modern 5- or 6-axis CNC machines just emulate a kinematically unlimited cradle-style machine.

The principle is to use small motions superimposed on the rolling motion. Figure 7 shows the principle of generating a pinion. The upper image is a front view in the direction of the cradle axis, the lower is a top view with the cradle axis in the drawing plane.

The standard rolling motion for a pinion or a generated ring gear is given by:

$\beta = \text{const} \cdot (\alpha - \alpha_m)$, $\Gamma = \text{const}$, $\tau = \text{const}$,
$\sigma = \text{const}$, $\varepsilon = \text{const}$, $\chi = \text{const} \cdot (\alpha - \alpha_m)$

Free form motions are introduced by developing a Taylor series of the sixth order around the center roll angle α_m. With this approach, the machining motion looks like:

Modified Roll:
$$\beta = a_\beta + b_\beta(\alpha - \alpha_m) + c_\beta(\alpha - \alpha_m)^2 + \ldots + g_\beta(\alpha - \alpha_m)^6$$

Angular Motion:
$$\Gamma = a_\Gamma + b_\Gamma(\alpha - \alpha_m) + c_\Gamma(\alpha - \alpha_m)^2 + \ldots + g_\Gamma(\alpha - \alpha_m)^6$$

Helical Motion:
$$\chi = a_\chi + b_\chi(\alpha - \alpha_m) + c_\chi(\alpha - \alpha_m)^2 + \ldots + g_\chi(\alpha - \alpha_m)^6$$

The principle of the superimposed additional movements is shown in Figure 8. The lines of contact between tool and flank are diagonal over the flanks. Each line has a corresponding cradle angle α.

When applying an additional movement by using a high order coefficient, we obtain a change of the flank's form along the line of rolling. In case of modified roll, the additional angular movement of the workpiece will affect both sides with opposite signs. This is shown in Figure 9 for the coefficients from the second order c_β up to the sixth order g_β. The upper left drawing shows the reference flank form without any modified roll coefficient.

For helical motion, the reaction on the flank depends on the flank angle of the tool. The bigger the tool's flank angle is, the more affected the flank form will be. Usual completing designs use a flank angle of about 30° on the inside blades and about 10° for the outside blades. This will result in nearly no effect on the concave side and big effects on the convex side. Using modified roll will affect both sides nearly the same. By combining modified roll and helical motion, the ease-off for the drive side and the ease-off for the coast side can be designed separately for completing designs. Figure 10 shows the effect of helical motion coefficients from the second order c_χ up to the sixth order g_χ. The tool's flank angle on the inside blades is 32°, the outside blade angle is 8° (Ref. 4).

Modified Crowning

All principles shown in the previous section need a rolling motion to allow flank form modifications. All these modifications work diagonally over the flank. Changing the ease-off values along

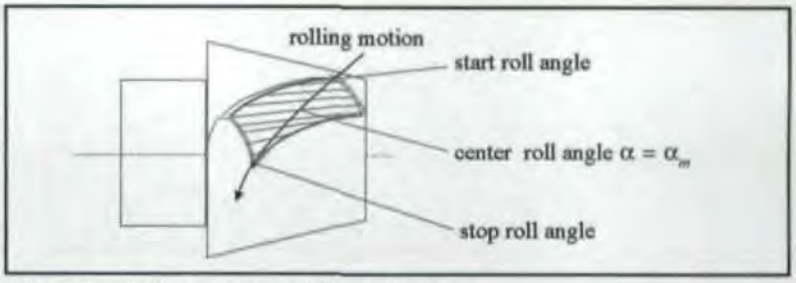

Figure 8—Rolling motion and lines of contact.

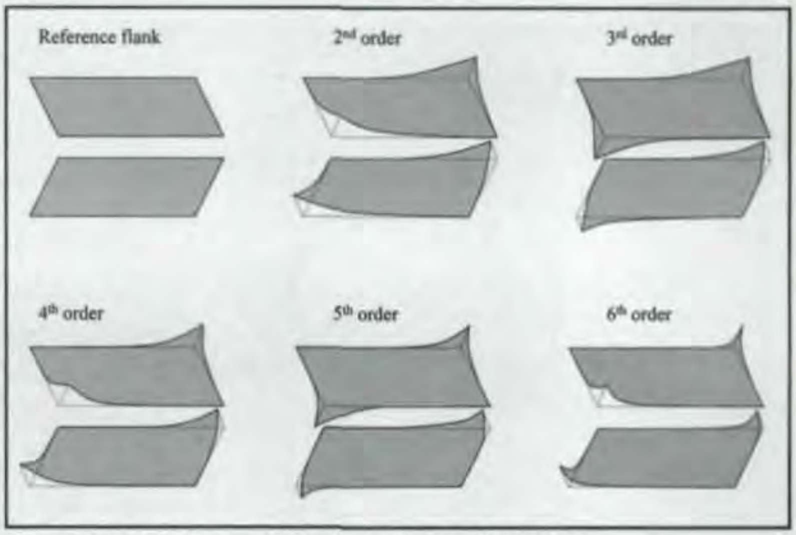

Figure 9—Flank form modifications using modified roll.

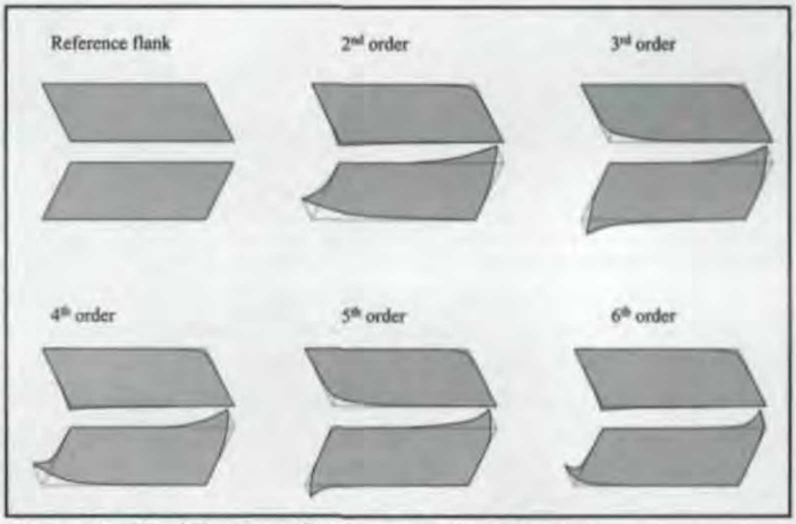

Figure 10—Flank form modifications using helical motion.

the other diagonal line can only be done by changing the tool's diameter.

If the gear ratio is high enough, the ring gears are usually made in a plunging operation. In this case, no rolling operation is performed. The shape of the gap corresponds to the shape of the tool. Changing the flank's form can be done using the Flared Cup® grinding principle (Ref. 5). The idea is to move a grinding wheel which has line contact to the gear flank along the face width. This movement permits defining of Taylor coefficients and superimposing of additional movements when

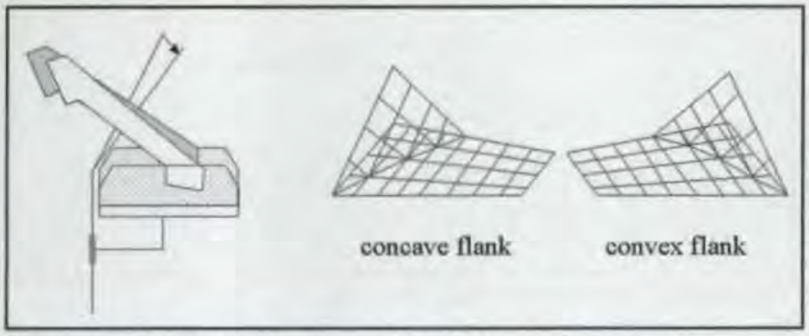

Figure 11—Machining a ring gear with two different plunging positions.

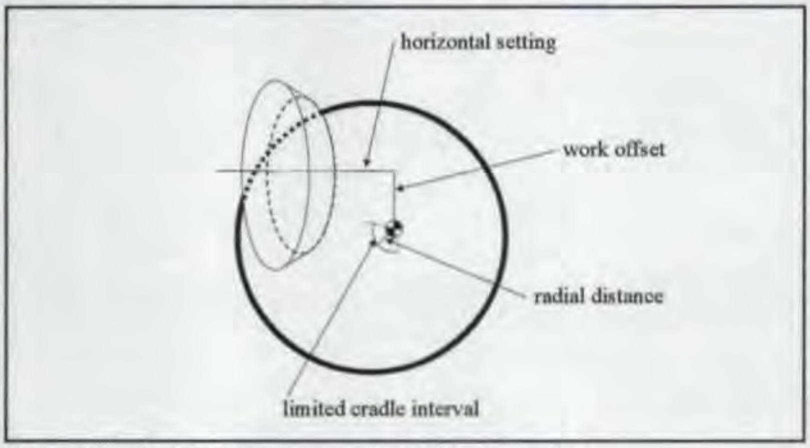

Figure 12—Machining parameters for modified crowning.

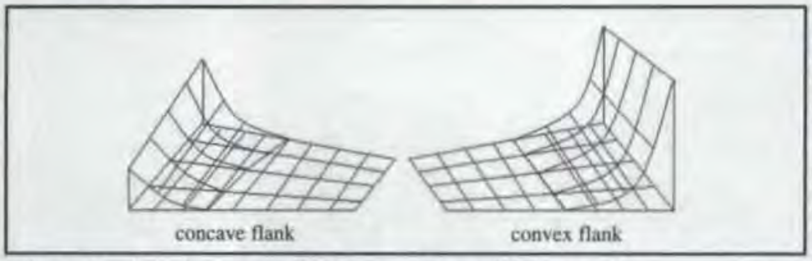

Figure 13—Flank form modifications using modified crowning.

going through the gap.

Another approach is modified crowning. The basic idea is to use different plunging positions to locally modify limited areas of the tooth. Figure 11 shows the effect of plunging a ring gear with two different machine root angles.

Machining a ring gear with the first setting will result in a flank form corresponding to a Formate® ring gear. In Figure 11, this is represented by the thin grid. When the second machine setting is applied to this ring gear, we will get a stock removal in the heel-tip area shown in Figure 11 with thick lines. When using a modification in the radial distance and the angular position of the workpiece, the triangular area of modification can become a square. Modifications in the toe area can be obtained by changing the machine root angle in the other direction.

Affecting both the heel and toe areas will be done by introducing three different plunging positions. For achieving a smooth transition between the two areas, it is sufficient to perform a smooth machining motion between the different plunging positions. Getting a well-conditioned numerical system, a small cradle movement with a very small radial distance is introduced, as shown in Figure 12.

With the principle of introducing a very small radial distance and a corresponding work offset and horizontal setting, we have the possibility of using the Taylor series approach known from generated gears. There is no generating process even if we introduced a limited cradle interval. Any cradle angle outside the limited interval will result in false ring gears.

Figure 13 shows the local flank form modification using two plunging positions for modifying the heel area on both flanks in a completing process (Ref. 6).

Conclusions

Noise optimization and load carrying capacity have been divergent aims. Compared to bevel gears, the contact conditions for spur and helical gears do not change drastically when load deflects the gearbox itself. The load-caused change in contact conditions for bevel gears needs to cover the displacement of ring gear and pinion as well as the deflection of the teeth themselves. When applying local 3-D flank form modifications to different areas of the gear's flank, we are able to improve both the load carrying capacity and the noise emission of a bevel or hypoid gear set. ○

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