Size and Material Influence on the Tooth Root, Pitting, Scuffing and Wear Load-Carrying Capacity of Fine-Module Gears

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Management Summary

A definite trend exists towards the miniaturization of actuators in the field of drivetrain engineering. Therefore, gears with smaller module size ($m_n \leq 1$ mm) are increasingly in demand.

But since neither results of load-carrying-capacity tests or specific calculation methods of tooth-root bending, pitting and scuffing load-carrying capacity for this gear size are given, the general calculation methods according to DIN 3990/ ISO 3663 (Refs. 2–3) have been used. In this study, limiting values for the load-carrying-capacity of fine-module gears within the module range 0.3–1.0 mm were determined and evaluated by comprehensive, experimental investigations that employed technical, manufacturing and material influence parameters. These limiting values exceed the load-carrying-capacity values—as expected—according to DIN 3990/ISO 3663 (Refs. 2–3) gear material quality MQ (maximum quality).

Introduction

The scope of fine-module gears has been constantly increasing in recent years; Figure 1 summarizes some of the most important applications of miniaturized actuators.

The existing calculation standards—according to DIN/ ISO (Refs. 2–3)—do not limit their validity for small-size gears and are therefore also used to calculate the load-carrying-capacity of fine-module gears. The experimental coverage of these calculation methods is based primarily on the module range between 2–20 mm; the reference gear of DIN/ISO has a module of 5 mm. Most durability tests are conducted on gears within the module range 3–5 mm.

The use of these standards for small-size gears has been validated on a limited number of experiments and is based mainly on the general-size influence according to materials strength. Although there are no limitations for use of DIN/ ISO in the module range smaller than 1 mm, their usage may be problematic and risky without reliable verification.

As miniaturized drives are increasingly used in space technology, robotics and medicine, a comprehensive, experimental study for proving the load-carrying-capacity of fine-module gears is urgently needed. The aim of this work is to prove the usability of DIN/ISO on smaller gears and to expand the experimental coverage (Fig. 2)—thus providing a reliable dimensioning of small-size gears.

Theoretical Study

The following chapter presents some basic relations regarding the size influence based on an example of geometrically similar gears—i.e., a gear with the same number of teeth and same addendum modification coefficient. The face width changes proportionally to the module. Furthermore, a symmetrically mounted shaft is assumed, and bearing clearance and bearing deformations are ignored. All values mentioned in Figures 3 and 4 refer to the test gear designed for this work.

Relations Based on Gear Load-Carrying Capacity

Tooth root-carrying capacity. Potential load peaks notwithstanding, the nominal tooth root stress can be calculated according to DIN/ISO as:

$$\sigma_{F0} = \frac{F_t}{b \cdot m_n} \cdot Y_{FS} \cdot Y_\beta \cdot Y_\varepsilon \leq \sigma_{FP} \qquad (1)$$

$$\text{i.e.: } \sigma_{FP} \sim \frac{F_{tP}}{b \cdot m_n}$$

as Y_{FS}, Y_β and Y_ε are dimensionless and independent from the size.

Assuming an equal, allowable tooth root stress for all gear sizes, the maximum allowable/permissible (index *P*) load-per-face-width depends on the module as:

$$\left(\frac{F_t}{b}\right)_P \sim m_n \qquad (2)$$

With $b \sim m_n$, the allowable tooth load depends quadratically on the module—i.e., gear size:

$$F_{tP} \sim m_n^2 \qquad (3)$$

The relationship between the maximum allowable torque and the gear size can be derived:

$$T_p = F_{tp} \bullet d$$

Because $d = z \bullet m_n$, it follows that:

$$T_P \sim m_n^3 \qquad (4)$$

Pitting durability. Absent any possible load peaks, the occurring contact stress can be calculated according to DIN/ISO (Refs. 2–3) as:

$$\sigma_{H0} = \sqrt{\frac{F_t}{b \cdot d_1} \cdot \frac{u+1}{u}} \cdot Z_H \cdot Z_E \cdot Z_\varepsilon \cdot Z_\beta \leq \sigma_{HP}$$

$$\text{i.e.: } \sigma_{H0P} \sim \sqrt{\frac{F_{tP}}{b \cdot d_1}} \qquad (5)$$

as Z_H, Z_ε and Z_β are dimensionless and—including Z_E—are independent of the gear size. Assuming an equally allowable contact stress for all gear sizes, the maximum allowable load per face width depends on the module also as:

$$\left(\frac{F_t}{b}\right)_P \sim \sigma_{HP}^2 \cdot d_1 \quad \text{with} \quad d_1 = z_1 \cdot m_n \;\Rightarrow\; \left(\frac{F_t}{b}\right)_P \sim m_n \qquad (6)$$

Consequently, the allowable load-per-face-width and the allowable torque for tooth root and pitting-carrying-capacity have an equal dependence on the gear size. Figure 3 illustrates the dependence of the gear size for fixed, nominal tooth root stress σ_{F0} and nominal contact stress σ_{H0}.

Dynamic performance of spur gears. The related rotational speed of a gear can be calculated as follows:

$$N = \frac{n_1}{n_{E1}} = \frac{n_1 \cdot \pi \cdot z_1}{30000} \cdot \sqrt{\frac{m_{red}}{c_{\gamma\alpha} \cdot b}} \qquad (7)$$

With $c_{\gamma\alpha}$ = const., $m_{red} \sim m_n^3$ and $b \sim m_n$, it follows that:

$$\Rightarrow N \sim m_n \cdot n_1 \qquad (8)$$

so that…

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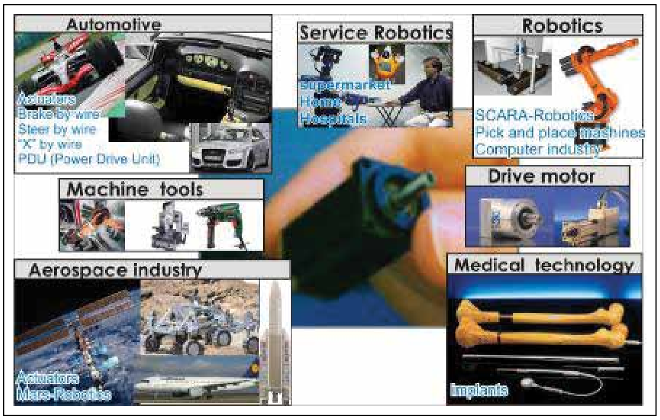

Figure 1—Applications of fine-module gears (*Source: Alpha Getriebebau GmbH and own elaboration*).

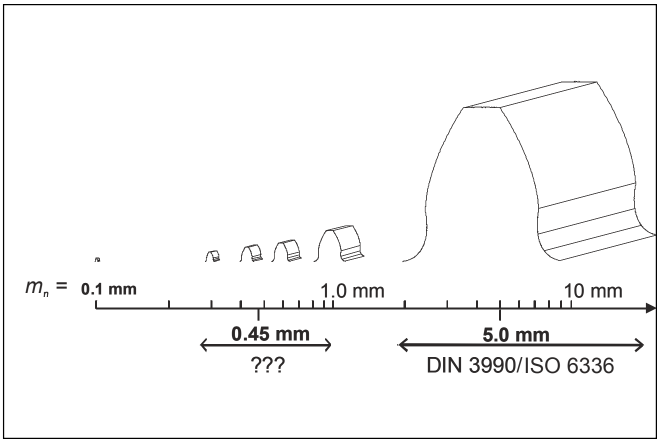

Figure 2—Research area of the present work in comparison to DIN 3990/ISO 6336.

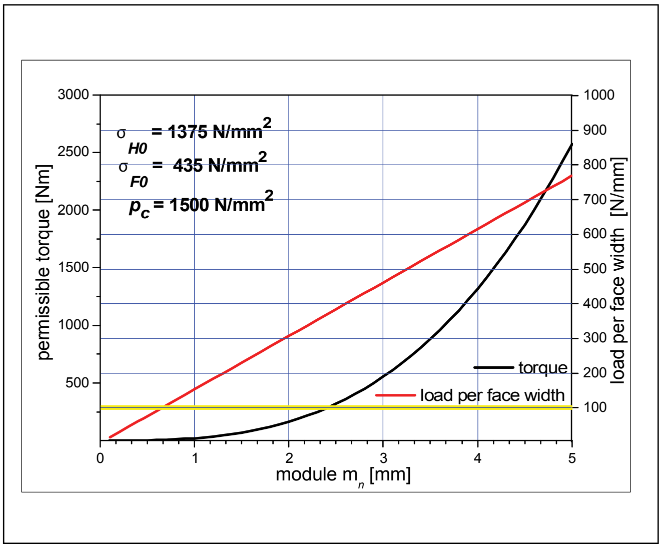

Figure 3—Influence of gear size on allowable load per face width and torque concerning pitting and tooth root bending for b/m_n = 15.

$$\text{So that} \quad n_1 \sim \frac{N}{m_n} \quad \text{for a given } N. \tag{9}$$

Figure 4 shows clearly the dependence of the related rotational speed on the gear size for resonance and pre-resonance (N = 1/2, 1/3, 1/4) ratio. According to these relationships it can be summarized that reducing the gear size increases the speed range under the resonance ratio. It therefore follows that by decreasing the gear size, and for a constant N, higher rotational speeds are acceptable.

Influence of size on notch sensitivity. By calculating the load-carrying capacity according to the general theory of

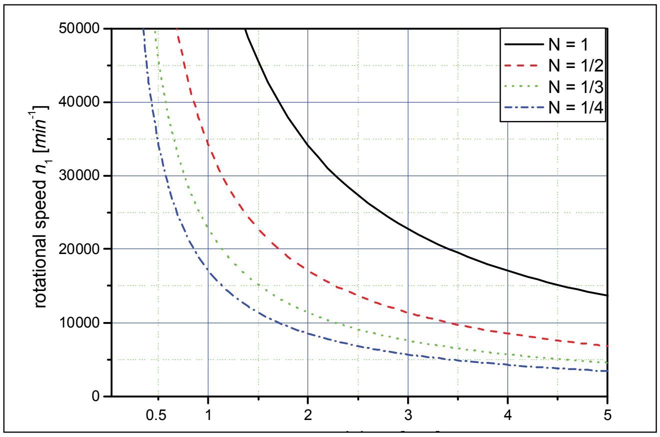

Figure 4—Dependence of related rotational speed on gear size.

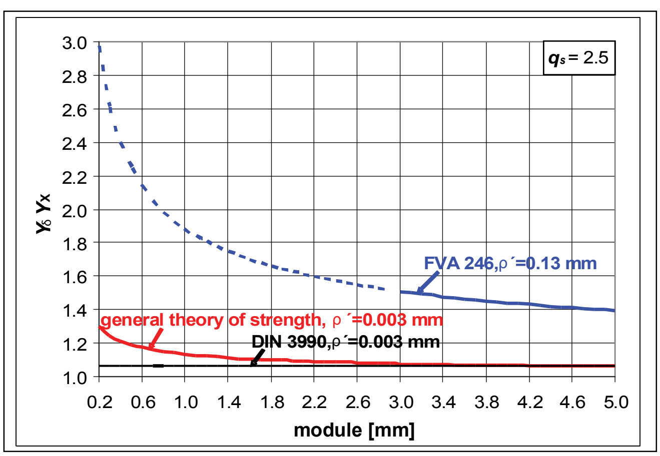

Figure 5—Dependence of gear size on the supporting effect of materials according to DIN 3990/ISO6336, the general theory of strength and FVA Research Project 246.

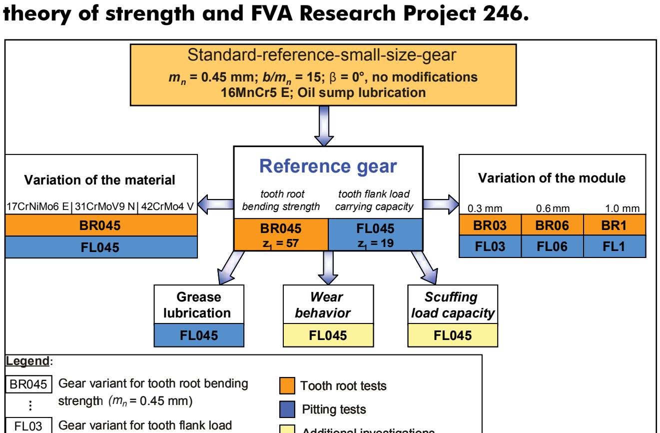

Figure 6—Research program.

material strength (Ref. 5), the notch sensitivity factor β_K is a measure of the supporting effect of the material; it is determined from the ratio of notch-stress-concentration factor α_K and notch-sensitivity nx. At the same time, for the geometrically similar gear, the stress concentration factor depends only on the notch geometry.

Related to the approaches of Petersen (Ref. 6), Siebel (Ref. 8) and Neuber (Ref. 4), the notch sensitivity can be calculated as a function of one constant dependent on the material and the stress gradient on the notch with maximum stress:

$$n_\chi = 1 + \sqrt{\rho' \cdot \chi^*} \tag{10}$$

As a function of the material, the slip-layer thickness is independent of the size. In contrast the relative stress gradient (Ref. 8) depends on the gear size:

$$\chi^* = \frac{1}{\sigma_{max}} \cdot \left(\frac{d\sigma}{dy}\right) \sim \frac{1}{m_n} \tag{11}$$

The supporting effect of material—DIN 3990/ISO 6336 (Refs. 2–3)—is included in the relative-notch-sensitivity factor $Y_{\delta\ rel\ T} = f(q_s)$ and the size factor Y_X. The relative-notch-sensitivity factor $Y_{\delta\ rel\ T}$ is the quotient of the notch-sensitivity factor of the calculated gear Y_δ divided by the standard test gear factor $Y_{\delta\ T}$. From this purely geometrical dependence can be concluded that the relative-notch-sensitivity factor is independent of the size. It enables only the influence of the notch sensitivity of the material to be taken into account.

The size effect on the stress gradient is taken into account in the size factor Y_X. If the module of the gear is smaller or equal to 5 mm, the size factor—DIN/ISO—amounts to 1.0. Therefore an increasing, supporting effect of material—thereby increasing load-carrying-capacity for a geometrically similar gear (m_n < 5 mm)—is not implied in this factor (Fig. 5).

The notch-sensitivity of gears with variable tooth root geometry was an important topic of FVA Research Project No. 246 (Ref. 7). As a result of many extensive experiments during the project, a size influence for gears smaller than 5 mm was found—resulting in a slip-layer thickness ρ´ for case-hardened gears amounting to 0.13 being used.

In comparing the three discussed calculation methods (Fig. 5), there is no increase of the load-carrying capacity by decreasing the size from m_n = 5 mm to m_n = 0.45 mm (DIN/ISO). Otherwise—according to the general theory of material strength—there is a load-capacity increase of 15%. By appropriate extrapolation of the results of FVA Research Project 246, a 60% load-carrying capacity increase is expected.

Similar to the calculation method of the tooth root load-carrying capacity, there is also no increase expected for the pitting-carrying capacity (DIN/ISO) by reducing the gear size. For gears smaller than m_n = 10 mm, the size factor Z_X amounts to 1.0; for much smaller gears (m_n << 10 mm) there is no increased support effect implied.

Research Program: Test Gearing and Test Rig

Test gearing. The performed research program (Fig.

6) included experimental investigations of the influence of gear size upon the gear load-carrying capacity of case-hardened spur gears within a module range of 0.3–1.0 mm. Furthermore, additional investigations of the material influence—case-hardened, nitrided and through-hardened steels—on the load-carrying capacity and random tests to appraise scuffing-load capacity and wear behavior of the reference small gear were done; a variation with a grease lubricant was also provided.

To investigate different kinds of damage in the same test rig, two different, standard-reference gears were defined, allowing a systematic investigation of the tooth root bending strength (BR) and the tooth flank load-carrying capacity (FL).

The standard reference spur gear has a module m_n = 0.45 mm with a ratio between tooth width and module of b/m_n = 15; it is case-hardened (16MnCr5) (Table 1).

For the module variation, no dimensionless, geometric gear parameters have been changed. Center distance, tip diameter and face width vary proportionally with the module. After heat treatment all tooth flanks were ground; the gears had no flank modifications. During this study adequate test conditions and adapted methods for load-carrying capacity investigations of small-size gears were defined.

When establishing the test conditions, attention was paid to a good transferability of the results to the standard gear size with module ≈ 5 mm. The tests and evaluation of the results are also based on the experience of earlier studies on the standard DIN reference gears.

Test rigs. A suitable test rig was developed for the needs of this study, taking into account the special circumstances of small gears. Figure 7 shows the FZG small-gear test rig that was especially developed for testing gears with a center distance between 7.5–65 mm. The test rig has a speed range of 50 rpm–10,000 rpm and a range-of-test torque of 0.5 Nm–200 Nm. These rig parameters allow the testing of gears in the module range between 0.3–2 mm.

Inspection of the test gears is possible without disassembling the test gear box (Fig. 7); this saves time and capacity during the testing. The memory function of the test rig also enables the parallel testing of three different test gearboxes without losing time for inspection and documentation of the test gears.

The studies on tooth bending strength were carried out mainly in a high-frequency resonance pulsator with the wheel of the gearing z_1/z_2 = 19/29. In order to confirm the results of the pulsator rig, additional gear-running tests on the FZG small-size test rig for each of the test variants were performed.

Experimental Results

Regarding the aim of this study, it is important to mention here that the investigations had been made on real gears produced according to the current state of technology. The test gears were milled, hardened and grind-finished. As such, the milling, grinding and heat treatment process achieve the technical limits of realization with decreasing gear size.

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Figure 7—FZG small-size gear test rig.

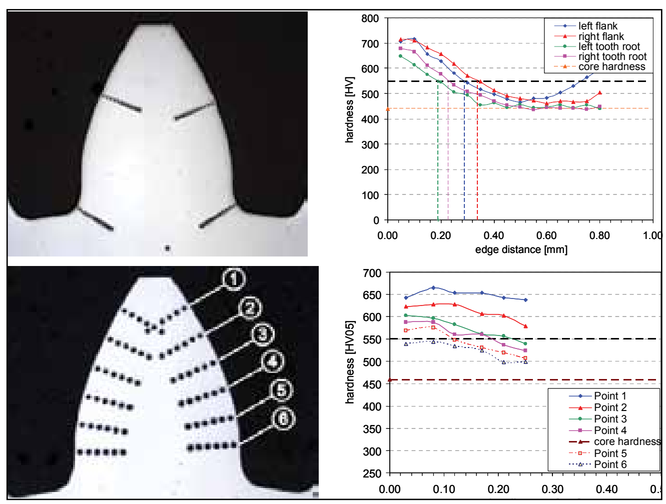

Figure 8—Measurement of the case depth of flank variant test gear (FL06: z = 19, above) and reference gear (FL045: z = 19, below) along tooth profile. Case depth graphic of FL045 is an example of one right flank.

Table 1—Basic geometry parameters of the reference test gears.

Description		FL045	BR045	Description		FL045	BR045
module	m_n [mm]	0.45	0.45	number of teeth	z	19/29	57/58
normal pressure angle	α_n	20°	20°	addendum modification coefficient	x_1	0.450	0.903
pressure angle at the pitch cylinder	α_{wt}	20°	20°		x_2	0.689	0.500
helix angle	β	0°	0°	transverse contact ratio	ε_α	1.40	1.23
center distance	a[mm]	7.50	26.46	tooth width	b[mm]	6.75	6.75

All test gears were measured for geometry and hardness. Up to the gear size m_n = 0.6 mm, the test gears had no noticeable metallographic problems (Fig. 8). In contrast, the case depth profile of the gear series module m_n = 0.45 mm (Fig. 8) was not constant along the tooth profile. The case depth near the tooth root fillet was smaller than the case depth at the tooth tip. This fact can be explained by the limits of heat treatment for small gears.

As a consequence, the gear series m_n = 0.3 mm was manufactured in a different way. These gears were ground into the hardened material; these heat treatment results and manufacturing conditions have to be considered for the evaluation of the load-carrying capacity of the test gears m_n

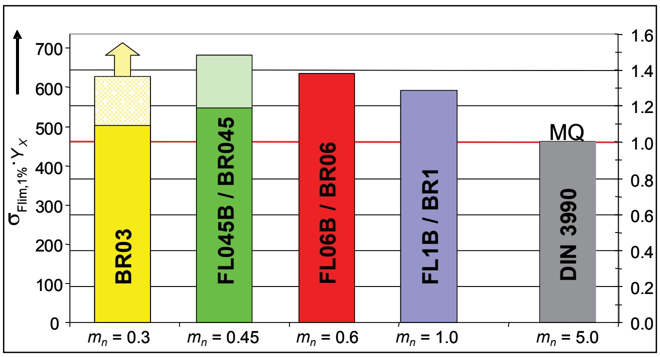

Figure 9—Results for the tooth root load-carrying capacity of different gear sizes.

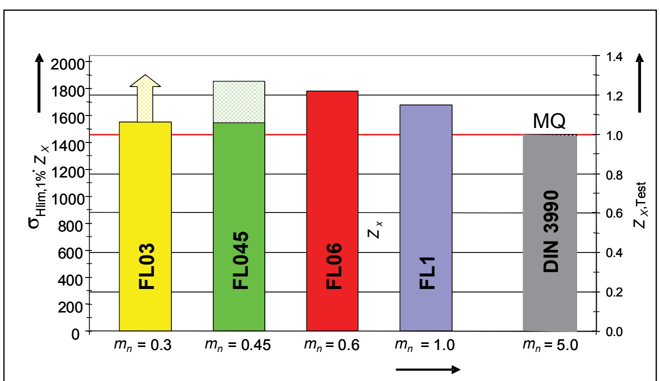

Figure 10—Results for the pitting load-carrying capacity of the different gear sizes.

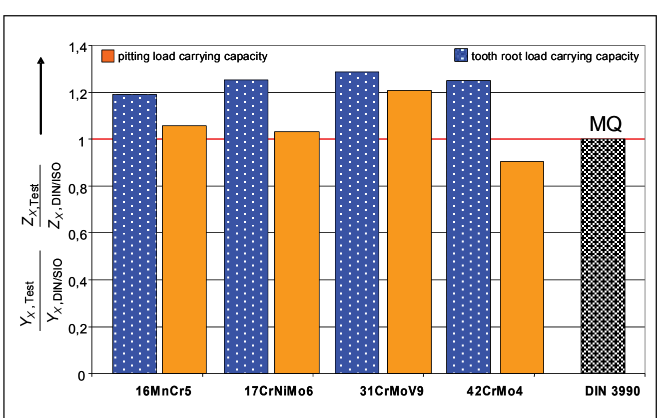

Figure 11—Material influence on the pitting and tooth root load-carrying capacity determined on the reference gear (m_n = 0.45 mm).

= 0.45 mm and m_n = 0.3 mm.

Influence of module (gear size) and tooth root load-carrying capacity. Figure 9 compares the experimentally determined load-carrying capacity values with those values derived from the DIN 3990/ISO 6336 (Refs. 2–3). The size factor according to DIN/ISO is already taken into account. The chart shows that if we pay attention to the existing gear quality in the tooth root area, we will find a proven relationship between the theoretical study and the experiments. The tooth root load-carrying capacity for m_n = 0.6 mm is 40% higher than what is documented by DIN/ISO. Yet despite the technical limits, the tooth root load-carrying capacity for m_n = 0.45 mm is 20% higher compared to DIN/ISO—with a greater potential of approximately 50%. The potential can be estimated with the approach that the strength profile over the material depth is a function of the local hardness profile $\sigma_{permissable}(y) \approx c \bullet HV(y)$ with $c = const.$

Metallographic examination of the wheel BR03 reveals no existing case depth in its tooth root area. This gear size was manufactured differently from those of the other test gear series; therefore, this variant cannot be directly compared to the other gear sizes. And yet, there are some load-carrying capacity reserves visible.

Pitting load-carrying capacity. The right ordinate axis of Figure 10 illustrates the evaluated allowable stress number $\sigma_{Hlim'Test1\%}$ by the tests related to the allowable stress number σ_{Hlim} (DIN 3990/ISO 6336) (Refs. 2–3). This corresponds to an experimentally determined size factor $Z_{X,\ Test.}$ It is clear that the pitting load-carrying capacity increases up to module 0.6 mm for approximately 20% when compared to the reference value in DIN/ISO. The values of the reference gear m_n = 0.45 mm are 5% higher than the values in DIN/ISO (the dark bar). If the reference gears are manufactured with equivalence to the experience with higher-module gears, there is an additional load-carrying potential—approximately 20% higher permissible contact stress than the existing experimental results (bright bar).

The experiments with a gear size m_n = 0.3 mm were prepared by a different manufacturing process and cannot be directly compared. All gears of this size have failed because of a broken tooth from the tooth flank. If these gears were to be manufactured like higher-module gears, there is also an additional load-carrying capacity potential; additional research is needed in this area.

Influence of material**: **The evaluated material influence related to the existing values of the materials specified in DIN 3990/ISO 6336 (Fig. 11). It is obvious that there is no clear difference between the case-hardened gear materials. The case-hardened steel 17CrNiMo6 (*Note: since publication of DIN EN 10084, the case-hardened steel 17CrNiMo6 was substituted with 18CrNiMo7–6*) has a slightly higher root-carrying capacity than 16MnCr5; both materials possess equal pitting load-carrying capacity values.

The through-hardened gears also show a higher tooth root-carrying capacity than the values contained in DIN/ISO.

The *load-carrying capacity increase* of the nitrided gear series 31CrMoV9 must be emphasized. This result is explained by the fact that, for smaller gear sizes, the nitrided depth profile appears similar to that of the case depth profile of the case-hardened gears.

This means that because of the obviously better-managed heat treatment, the nitriding process may become much more important for smaller gears.

Additional Investigations

Scuffing load-carrying capacity. By calculating the safety factors against scuffing for the reference gear series (m_n = 0.45 mm) under the defined test conditions, both calculating methods in DIN 3990/ISO 6336 obtain different results (Table 2).

Although using FVA No. 2 oil without EP additives, no scuffing damage had been found during the tests. In using the integral-temperature method (DIN 3990) safety factors $S_{int\ S}$, safety factors smaller than 1 are allowed if the flash temperature method yields safety factors larger than 1.4. The calculation results show that the usage of the calculation methods included in DIN/ISO is safely and reliably applicable to small-size gears.

Wear behavior. Figure 12 describes via (inventor and metallurgist John T.) Plewes diagram the experimentally determined wear coefficient of the reference gear series—investigated at different speed and temperature conditions, and constant torque.

The linear wear coefficients obtained by the tests on small-size gears with FVA oil No. 2 are significantly below the reference values for mineral oils without EP additives. The wear behavior remains nearly constant for the same oil temperature and different peripheral speed.

By increasing the temperature, the linear wear coefficient is changing to areas with greater wear.

Comparison of Test Results with State-of-the-Art Standard

Figure 13 shows a comparison between the experimentally determined size factors $Y_{X,\ Test}$ (tooth root bending) for the case-hardened gears (Fig. 9) and the size factor according to DIN 3990/ISO 6336. Also described are experimentally determined size factors that were evaluated in other research projects at the FZG Gear Research Center on gear modules m_n = 1.75 mm; m_n = 3.0 mm; m_n = 5.0 mm; m_n = 8.0 mm; m_n = 10.0 mm; m_n = 16.0 mm; and m_n = 20.0 mm. A clear trend towards increasing size factor for gears smaller than 5 mm is noted.

The trend line illustrates the experimentally evaluated size factor $Y_{X,\ Test}$ for the case-hardened steel 16MnCr5. Because of its better hardenability, steels like 17CrNiMo6 and 17NiCrMo14 are generally used for larger-module-size gears. Test results with these steels are also included in Figure 13; one can see that DIN 3990/ISO 6336 (Refs. 2–3) easily covers the size factor for these steels in the module

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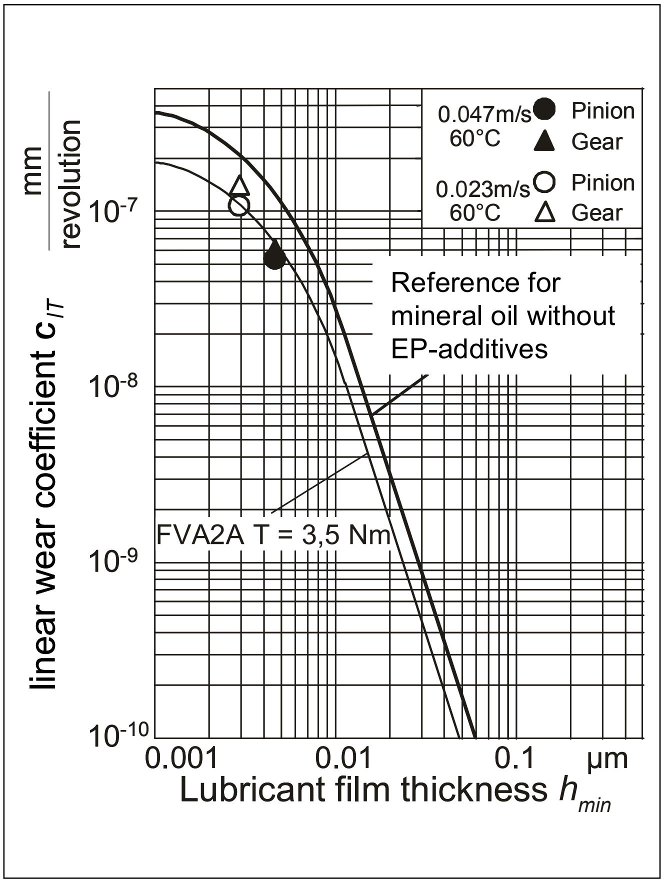

Figure 12—Wear coefficients obtained on FL045 for T_1 = 2.85 Nm.

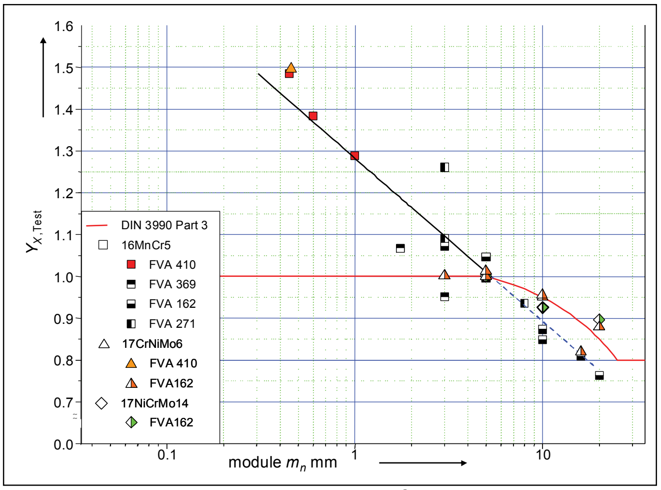

Figure 13—Experiment-derived size factors $Y_{X,\ Test}$ against $Y_{X,\ DIN/ISO}$ (tooth root bending) for case-hardened steel.

Table 2—Calculated safety factors against scuffing for the reference gear (FL45) Refs 2-3.

T_1 [N_m]	$\theta_{Start\ S}$ [˚C]	$\theta_{int\ S}$ [˚C]	$S_{int\ S}$ [-]	$\theta_{B\ max}$ [C]	S_B [-]
3.5	60˚	120.9	0.96	127.9	2.53
3.5	90˚	168.4	0.69	177.0	1.63
4.2	60˚	130.6	0.89	139.0	2.18
4.2	90˚	180.4	0.64	190.8	1.41

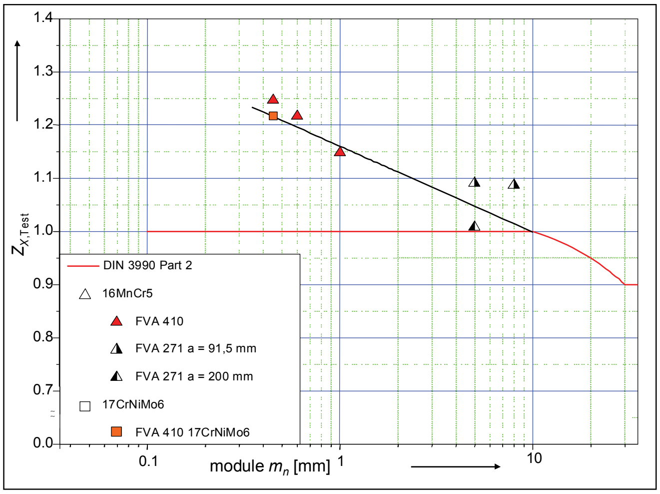

Figure 14—Experiment-derived size factors $Z_{X,\ Test}$ against $Z_{X,\ DIN/ISO}$ (pitting) for case-hardened steel.

range over 10 mm.

Analogous to Figure 13, Figure 14 includes a comparison between the experimentally determined size factors $Z_{X\ Test}$ (pitting) for the case-hardened gears (Fig. 10) and the size factor according to DIN 3990/ISO 6336. Experimental results (Ref. 9) for modules m_n = 5.0 mm and m_n = 8.0 mm (similar number of teeth) are also attached. The trend line illustrates the experimentally evaluated size factor Z_X, $_{Test}$ for the case-hardened 16MnCr5 steel. Here one can also recognize a trend towards increasing size factor for modules smaller than 5 mm. It should be noted that there are few experimental results known within the module range 1 mm < m_n < 3 mm. Consequently, further investigation of this module range is needed.

Summary

This work presents fundamentals in the load-carrying capacity research field of small-size gears and provides for reliable dimensioning of gears within the module range 0.3–1.0 mm. The results for the tooth root and pitting load capacity show that with a decreasing gear size, the load capacity related to the torque load increases up to 1.5 times of the DIN reference gear, i.e.:

(module 5 mm); i.e., Y_X (m_n = 0.45 mm) ≈ 1.5 respectively: Z_X (m_n = 0.45 mm) ≈ 1.5.

The variation of material for small-size gears shows no disadvantages in using 17CrNiMo6 steel when compared to 16MnCr5. The performance of the nitrided gears in this gear-size range is emphasized.

The definition of adequate test conditions and methods—as well as the design and successful operation of a specific test rig for small-size gears during this work—provide the groundwork and basic knowledge for further scientific investigation in the research of small-size gears.

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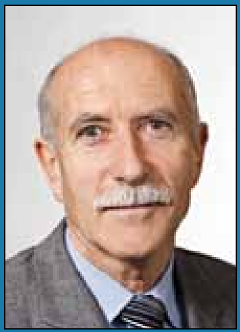

Dr. Bernd-Robert Höhn *studied mechanical engineering at the Technical University Darmstadt (1965-1970) and served as an assistant lecturer (1970-1973) at the Institute for Machine Elements and Gears at the Technical University Darmstadt prior to becoming an assistant professor at the university (1973-1979); in 1978, he received his PhD (Dr. Ing.) in mechanical engineering. In early April, 1979 Höhn worked as a technical designer in the department for gear development of the Audi, and by 1982 was head of the department for gear research and design for the automaker. In 1986 Audi named Höhn department head for both gear research and testing of automotive transmissions, until his departure in 1989 to become head of both the Institute of Machine Elements at the Technical University and of the Gear Research Centre (FZG). Höhn has served since 2004 as vice president for VDI for research and development and since 1996 has led the working group 6 and 15 for ISO TC 60—calculation of gears.*

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