History of the AGMA JFACTOR

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I felt a tap on my shoulder. Turning, I saw the chief draftsman who said, "You're in charge of gears."

And he walked away.

Dumbfounded, I stared at the back of his head, and sat down at my drafting board. It was November, 1963, shortly after JFK was assassinated, and after I was discharged from the U.S. Army in October, 1963. I rejoined the company that I had worked for before being drafted into the Army (in those days companies were required to rehire veterans). Safety Electrical, in Hamden, Connecticut, was so named because their main product was electrical generators for passenger trains that were first introduced to replace the gas lamps that illuminated passenger cars. The generator was installed below the passenger car and was driven by an axle-mounted gearbox that was manufactured by a division of the Dana Corporation. In the 1960s, JFK inspired resurgence in mass transportation by funding expansion of the nation's subway systems. Dana upgraded the gearbox used for driving the Safety Electrical generator and used it to drive subway rail cars. It became a booming business because many of the major subway systems in the U.S. purchased Dana gearboxes. Subsequently, however, Dana decided to offer Safety Electrical a gearbox business at a price that couldn't be refused. And so, overnight, I was indeed "in charge of gears."

Fortunately for me, the deal included an agreement whereby Safety Electrical acquired Dana's chief engineer of the rail car gearbox division who was assigned to train me in all aspects of the design and manufacture of rail car gearboxes. This was a very exciting time for me — and a wonderful learning experience. However, the Dana engineer was on loan for only three years, and he returned to Dana in 1966.

By then, I had determined to specialize in gear design, but there were only electrical engineers at Safety Electrical, and no mechanical engineers to continue my training. Therefore, I joined Sier Bath Gear Company in North Bergen, New Jersey in 1966. Sier Bath was a very innovative company who specialized in high-accuracy carburized and ground gears. But most importantly for me, they regularly used consultants for gear design, including then-industry living legends Eliot Buckingham, Darle Dudley, and Ernest Wildhaber. What's more, my gear knowledge grew exponentially with mentoring from Sier Bath's chief engineer Jack Pearson and working with Sier Bath's consultants. Dudley showed me how to determine the AGMA gear strength geometry factor (JFACTOR) by graphical layout, which is a very tedious and not especially accurate method to determine JFACTORs. However, the JFACTOR determines a gear's bending strength, and an accurate value is needed for the design of every gear. Realizing that the graphical layout was not practical, I searched the literature for a computer method. I discovered Wadhwa's (Ref. 1) analytic method and programmed it in the *BASIC* ("*Beginner's All-purpose Symbolic Instruction Code*") language using Sier Bath's connection to a GE time share computer. I found that Wadhwa's method worked well for most gears, but failed to solve certain gear geometries. Wadhwa was an IBM programmer who chose a difficult gear geometry issue to demonstrate the capability of computers; however, he was not a gear engineer. Therefore, he turned to Buckingham's classic book (Ref. 2) for the equations of a trochoid. Unfortunately, as will be shown, Wadhwa's choice of using trochoid equations and selecting θ_T as the independent parameter introduced a numerical problem. (Note: There is no closed-form solution for the JFACTOR, and numerical iteration is required to find the critical stress point defined by the Lewis parabola (Ref. 3). Wadhwa's analytic method fails to converge to the correct solution for certain gear geometry.)

Figure 1 shows Wadhwa's analytic method where the independent parameter is a coordinate of the primary trochoid, θ_T. After solving for point S on the primary trochoid, point F on the secondary trochoid (on the root fillet) is found by matching the slope, ψ_T at point S.

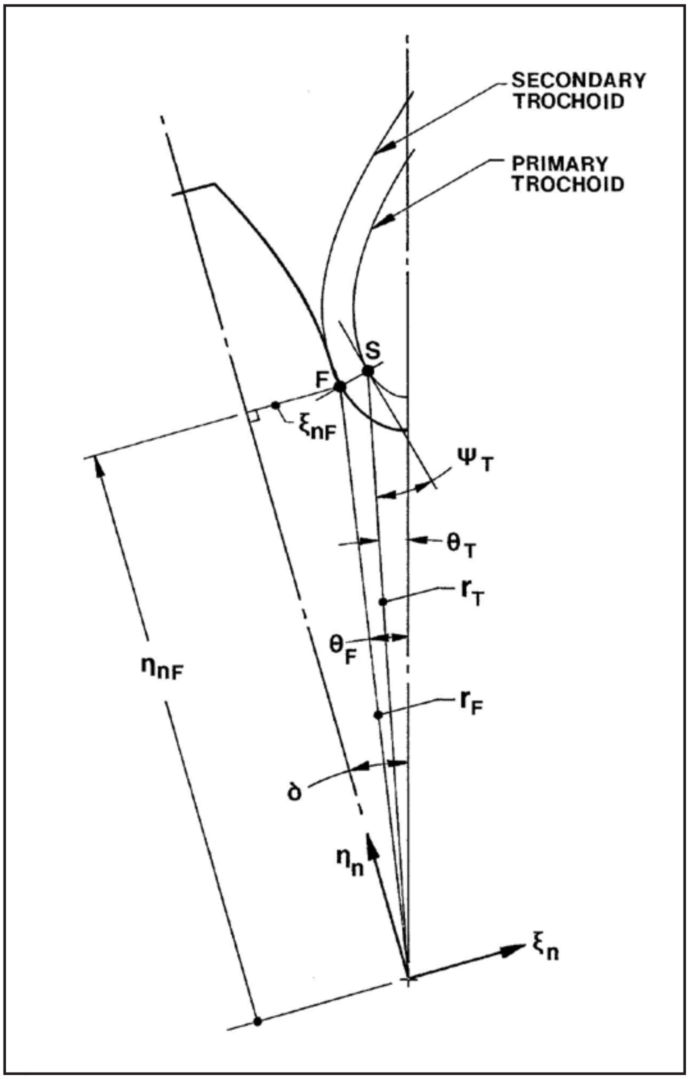

Figure 1 Wadhwa analytic method.

Knowing that Wadhwa's analytic method had a numerical problem, I asked Wildhaber (A brilliant geometrician with over 300 patents (Ref. 4) with applications to gears. In 1924, he began a long career at the Gleason Works where he invented the hypoid gear and the Revacycle cutting process that enabled a high production rate for straight bevel gears. As a consultant to Sier Bath, he helped to develop the Vari-Crown gear coupling.) to derive the equations for the AGMA JFACTOR (I didn't mention Wadhwa's paper because I wanted an independent analysis). Wildhaber developed what he termed a "kinematic" method, whereby the gear tooth geometry was generated from the kinematics of the generating tool and gear workpiece.

Fundamentally, it was based on the law of gearing, which requires that the normal to the cutting point, F, must pass through the pitch point, P, at all points of contact. Figure 2 shows Wildhaber's kinematic method where the independent parameter is the inclination of the contact normal, α_n.

Figures 3 and 4 show graphical depictions of the numerical problem that plagued Wadhwa's analytic method. Figure 3 shows there are two roots to the solution of the equations for the JFACTOR. The correct root is marked "ROOT" (Fig. 3). The incorrect root lies closer to $\alpha_n = 0$ (Fig. 3).

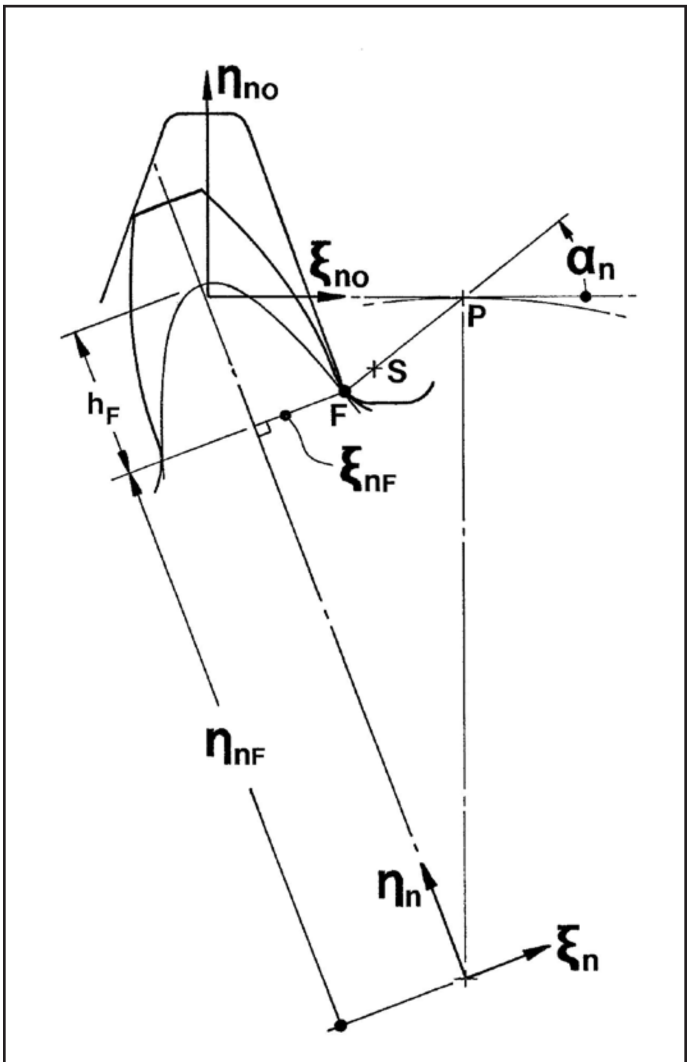

Figure 2 **Wildhaber kinematic method.**

Figure 4 shows the incorrect root corresponds to an inverted Lewis parabola that opens upward, rather than downward.

Hence, Wadhwa's "analytic" method converged to the incorrect root for gears with a large number of teeth or large profile shift. Happily, I found that Wildhaber's "kinematic" method resolved the numerical problem that exists in Wadhwa's "analytic" method.

In 1971, I finally tired of going to night school (I had spent nine years trying to complete a BSME degree) and decided to leave Sier Bath to attend full-time at the University of California in Berkeley (UCB). I graduated from Berkeley's Mechanical Engineering Department with BSME and MSME degrees in

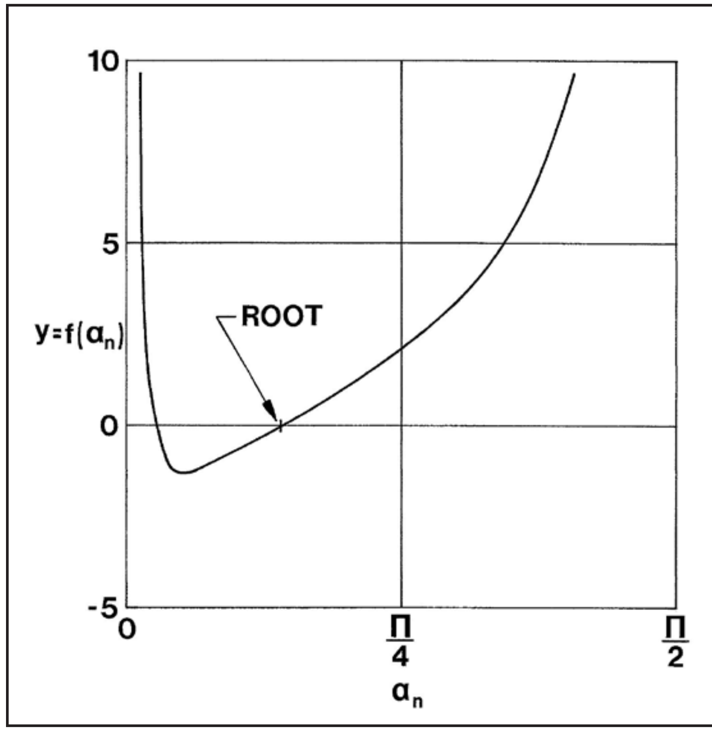

Figure 3 **Graph showing there are two roots to the solution.**

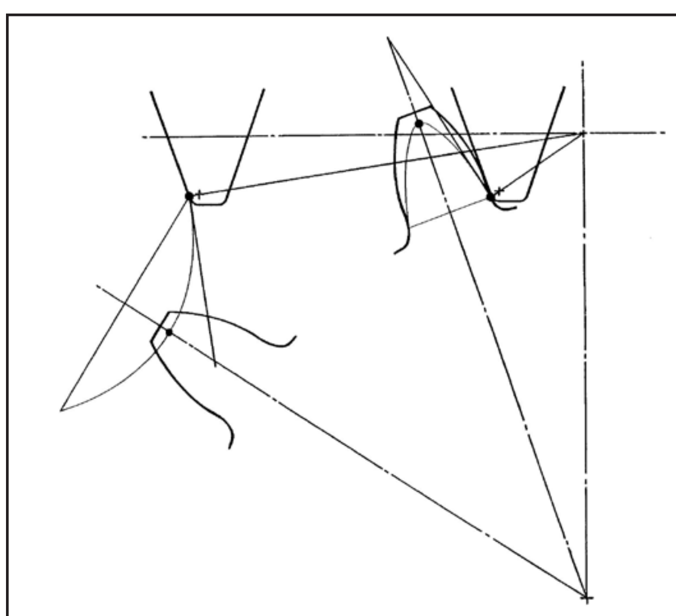

Figure 4 **Showing correct and incorrect roots.**

1975. I then — from 1975 through 1976 — worked briefly for Western Gear in Lynwood, California. During my brief time there I became interested in structural dynamics. I therefore decided to go back to UCB to study earthquake engineering and graduated with a Master of Engineering in Structural Dynamics in 1978.

In 1978, I founded GEARTECH as a consulting firm. At the time, the only personal computer available was the Apple computer — but it was expensive and required some assembly. So I decided to program the JFACTOR algorithm on a Texas Instruments TI-59 calculator. I soon found that the iteration method I had used for Wildhaber's kinematic method was much too slow using the TI-59 calculator; it took hours to solve for one JFACTOR. I knew Newton's method of iteration was very efficient, but it required calculation of the derivative from a complex set of equations. Nevertheless, I bit the bullet, and slogged through the algebra. Months and reams of paper later, I arrived at a simple equation for the derivative — so simple, in fact, I didn't believe that it could be correct. However, when I programmed the algorithm on the TI-59 calculator, it converged in three iterations within seconds (Fig. 5). Figure 5 shows that Newton's method of iteration converges to the correct root in as little as three iterations.

I published the algorithm in a paper (Ref. 5) that was presented at the 1981 International Symposium on Gearing & Power Transmissions in Tokyo.

Sier Bath encouraged me to participate in AGMA technical meetings, and in 1967 I started to attend the face-to-face meetings of the Helical Gear Rating Committee (HGRC). The HGRC began the first draft of AGMA 218.01 (Ref. 6) in 1973. During one of the meetings, committee members expressed the desire that the current graphical layout method for determining the JFACTOR be replaced with a computer algorithm, and asked for a volunteer to contribute their computer program. Although there were representatives from all the major gear manufacturers on the HGRC, no member was willing to contribute their program because they knew their programs were not reliable.

I then immediately recognized their problem, i.e. — they had programmed Wadhwa's method (Ref. 1). I explained this to the committee and offered my algorithm — and stated that I had resolved the numerical problem. However, the committee decided that the best approach would be to publish my algorithm as an AGMA technical paper, and then ask the AGMA membership to test the algorithm to validate that it was reliable. I agreed with the committee's recommendation, and I published the algorithm in a paper (Ref. 7) and presented it at the 1981 AGMA Fall Technical Meeting in Toronto. During the Q&A following my presentation, several attendees expressed an interest in extending the algorithm to encompass external spur and helical gears generated by pinion-type shaper cutters. Thus I began applying Wildhaber's kinematic method to shaper-cut gears. I recalled that Wells Coleman — a member of the `HGRC — mentioned that it was possible to write an algorithm for pinion-type shaper cutters that would also be applicable to rack-type cutters (hobs, rack cutters, and generating grinding wheels) by simply inputting a pinion-type cutter with a large number of teeth. When Wells first mentioned this notion, I didn't follow his recommendation because I was overwhelmed with the task of developing an algorithm for rack-type cutters. However, after

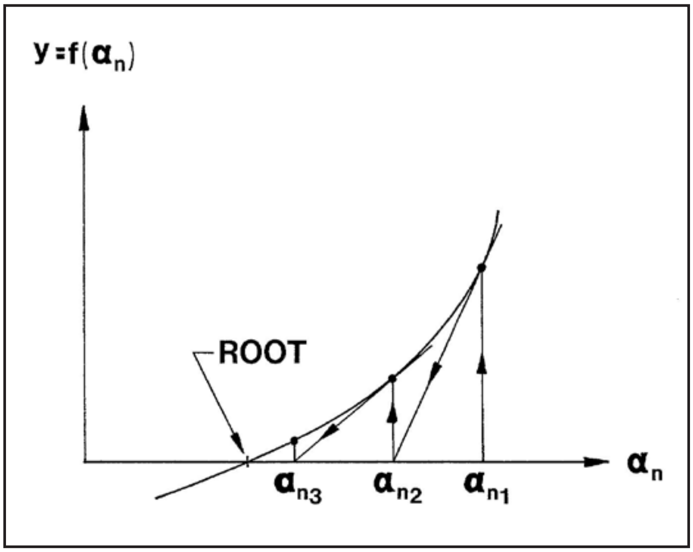

Figure 5 Iteration by Newton's method.

I developed the algorithm for pinion-type cutters, I tried inputting a pinion-type cutter with 10,000 teeth in the algorithm and it successfully duplicated the JFACTOR that was generated by a rack-type cutter. No question — Coleman's recommendation was correct. I published the algorithm for pinion-type cutters in a paper (Ref. 8) that was presented at the 1983 AGMA Fall Technical Meeting in Montreal.

The algorithm (Ref. 8) was tested for several years by the AGMA membership and was finally accepted as the official AGMA method for calculating JFACTORS and was published in AGMA 908-B89 (Ref. 9).

Next, the HGRC developed the information sheet — AGMA 918-A93 (Ref. 10) — to assist designers in the proper use and interpretation of AGMA 908-B89 and to assist in the development of computer programs for calculating geometry factors for pitting resistance *I* and bending strength *J*. AGMA 918-A93 includes a flow chart and several numerical examples.

For more information.

Questions or comments regarding this paper? Contact Robert Errichello at *rlegears@mt.net.*

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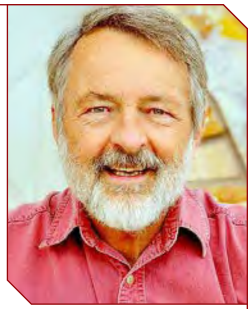

Robert Errichello is founder of GEARTECH. He has over 50 years of industrial experience and has taught courses in material science, fracture mechanics, vibration, and machine design at San Francisco State University and the University of California at Berkeley. He presented seminars on design, analysis, lubrication, and failure analysis of gears and bearings to professional societies, technical schools, and gear, bearing, and lubrication industries. He is a graduate of the University of California at Berkeley and holds BS and MS degrees in Mechanical Engineering and a Master of Engineering degree in structural dynamics. Bob is a member of the AGMA Gear Rating Committee, AGMA/AWEA Wind Turbine Committee, ASM International, ASME Power Transmission and Gearing Committee, STLE, NREL and GRC. Bob has published over 100 articles on design, analysis, and application of gears. He is technical editor for Gear Technology magazine and STLE Tribology Transactions. He is recipient of the AGMA TDEC Award, AGMA E.P. Connell Award, AGMA Lifetime Achievement Award, STLE Wilbur Deutch Memorial Award, STLE Edmond E. Bisson Award, and AWEA Technical Achievement Award.

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