The Interrelationship of Tooth Thickness Measurements as Evaluated by Various Measuring Techniques

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Abstract

Measured tooth thickness as established by measurements made by conventional gear measuring techniques: over pins, the span measurement, or with a gear tooth vernier caliper, do not always agree with the "effective tooth thickness," (the value "seen" by the mating gear). Methods of adjusting the specified value of measured tooth thickness to assure that the required value of effective tooth thickness will not be exceeded are discussed.

Introduction

The first commandment for gears reads "Gears must have backlash!" When gear teeth are operated without adequate backlash, any of several problems may occur, some of which may lead to disaster. As the teeth try to force their way through mesh, excessive separating forces are created which may cause bearing failures. These same forces also produce a wedging action between the teeth with resulting high loads on the teeth. Such loads often lead to pitting and to other failures related to surface fatigue, and in some cases, bending failures.

If, however, the mesh contains excessive backlash, certain applications, particularly those in which the direction of loading reverses, will exhibit rough running and poor performance. It is, therefore, very important that the tooth thickness and the center distance, both of which govern backlash, be correctly designed, properly specified and accurately controlled.

In most cases, when problems relating to backlash do occur, it is not because the basic design values used by the gear designer were too small; rather, the problem is that the maximum values for the tooth thickness measurements specified on the drawings could actually yield effective tooth thicknesses greater than the designer anticipated as "seen" by the mating gear.

The objective of this article is to show how to determine drawing specifications for conventional tooth thickness measuring techniques that will yield gears with the desired effective tooth thicknesses. The relationships between the various types of tooth thickness are considred. The methods used by the AGMA to specify the allowable variations for each given quality number of accuracy, spacing, profile, runout and lead are examined. The way in which each of these allowable variations enter into each tooth thickness measuring method are considered. Finally, a simplified method of relating the measured value of tooth thickness to the effective thickness is shown.

Types of Tooth Thickness

In order to establish proper tooth thickness specifications, four types of tooth thicknesses should be recognized:

- Design tooth thickness.
- Effective tooth thickness.
- Functional tooth thickness.
- Measured tooth thickness.

Design tooth thickness is the arc distance measured along a specific circle, usually the standard pitch circle, (See Equation 2.) between the involute curves defining each side of a gear tooth.

It is a theoretical value, usually established by engineering considerations. AGMA Standards 218, 219, 360 and 370 offer guidance in establishing design tooth thickness.

The maximum limits on design tooth thickness for each member of a pair is typically established on the basis of minimum operating center distance, considerations of thermal differential expansion due to temperature extremes in the gears and mountings, the internal runouts and clearances within the bearings supporting the gears and the minimum allowable backlash.

The maximum design tooth thickness may be interpreted as a maximum metal condition on all of the active surfaces

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of the teeth in a gear or pinion. (See Fig. 1.)

Effective tooth thickness is the envelope of tooth thickness as "seen" by the mating gear in an operating set of gearing. In most cases, the numerical value of the maximum effective tooth thickness, T", is established equal to the maximum design tooth thickness, T'.

Functional tooth thickness is a measured value of the effective thickness of a gear as "seen" by a master gear. It is determined by means of a properly designed and calibrated master gear operating on a gear rolling fixture.

The maximum functional tooth thickness of a work gear may be obtained from Equation 7. The value of C_{max} to use in the equation is the largest value of instantaneous center distance that was observed when all teeth of the work gear had passed through mesh.

Measured tooth thickness of a gear is the arc distance from a point on one side of a tooth to a similar point on the other side at a specific diameter. It may be determined by means of a gear tooth vernier, a measurement over 1, 2 or 3 wires

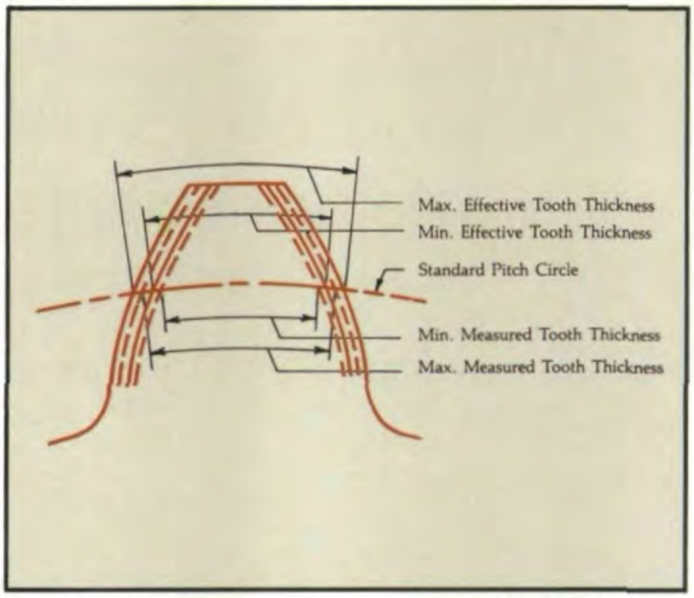

Fig. 1 – The relationship of effective and measured tooth thickness.

NOMENCLATURE

a	Radial distance from measuring circle (chordal tooth thickness measurement) to outside circle
a_c	Chordal addendum setting on a tooth vernier caliper
C	Standard center distance
C'	Operating center distance
C_M	Center distance, work gear and master gear (use $C_M = C_{max}$ for largest value)
C_{max}	Largest observed center distance master and work on a gear rolling fixture
ΔC	Change in center distance, (runout)
D	Standard pitch diameter
D_b	Base circle diameter
D_M	Pitch diameter, master gear
d_w	Measuring wire diameter
d_M	Diameter of measuring circle
M_N	Testing center distance specification for gear rolling fixture
M_1	Measurement over 1 wire
M_2	Measurement over 2 wires
N_P	Number of teeth in pinion
N, N_G	Number of teeth in gear
N_M	Number of teeth in master gear
n	Number of teeth in span
P_{nd}	Diametral pitch, normal
P_d	Diametral pitch, transverse
R_W	Radius of circle through center of measuring wire
s	Space width on standard pitch circle
T_E	Tooth thickness, effective
T_M	Tooth thickness, master gear
T_{Mc}	Tooth thickness, chordal tooth thickness measurement
T_{Ms}	Tooth thickness, span measurement
T_{M1}	Tooth thickness, over wires measurement, (1 wire)
T_{M2}	Tooth thickness, over wires measurement, (2 wires)
T_W	Tooth thickness, functional, maximum
T'	Tooth thickness, maximum design
T"	Tooth thickness, maximum effective
t	Tooth thickness on the standard pitch circle
t_b	Base tooth thickness
tc	Chordal tooth thickness specification
Δt	Change in arc tooth thickness
Δt_1	Tooth thickness adjustment factor (measurement over 1 wire)
Δt_2	Tooth thickness adjustment factor (measurement over 2 wires)
Δt_c	Tooth thickness adjustment factor (Chordal tooth thickness measurement)
Δt_s	Tooth thickenss adjustment factor (Span measurement)
v_L	Lead variation, allowable
v_P	Profile variation, allowable
v_S	Spacing variation, allowable
v_R	Runout variation, allowable
v_{TR}	Runout variation, in pitch plane, allowable
Φ	Standard pressure angle, profile
Φ'	Operating pressure angle
Φ''	Operating pressure angle, work gear with master gear
Φ_M	Pressure angle at center of wire
Ψ	Helix angle

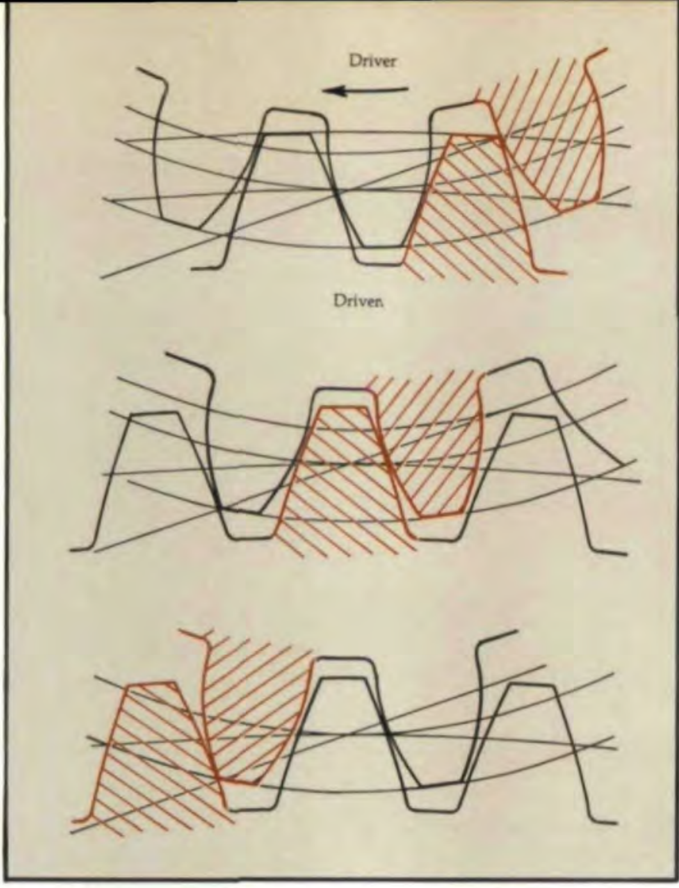

Fig. 2—Meshing sequence of a pair of theoretically perfect gear teeth viewed from the transverse plane.

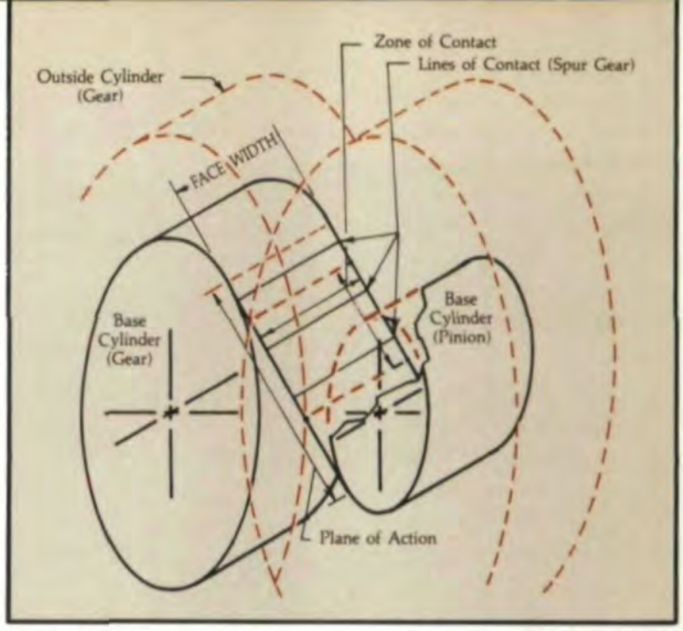

Fig. 3—Lines of contact in the plane of action for the same set of theoretically perfect gear teeth.

(pins), a span measurement or by means of a master gear and a gear rolling fixture.

Geometric Considerations

Fig. 2 shows the sequence of events that takes place in a transverse plane as a pair of theoretically perfect gear teeth go through mesh. All contact between involute teeth takes place along a line of action defined by the base circles of the meshing gears. The line of action is actually an edge view of the plane of action. The plane of action is a surface tangent to each of the base cylinders of the gears. In the plane of action, illustrated in Fig. 3, the lines of contact are shown. These lines show the actual contact across the length of the teeth. At any instant, in perfect gears, contact occurs throughout the entire length of all of the lines bounded by the zone of contact. The zone of contact is defined by the sides of the gears (ends of the teeth) and the outside diameters of each member.

Contact between meshing gears can also be studied in a meshing plane, which is the developed surface of either of the pitch cylinders of the pair. (See Fig. 4.) The cross hatched sections indicate the arc thickness of the teeth of each member. The allowable variations in spacing, the allowable lead and the radial component of runout, as shown in the *AGMA Handbook* are specified in this plane.

Actual gear teeth are not perfect. They have variations in spacing, profile and lead. Also there are low and high areas relative to the theoretical surfaces of the teeth. A "high" area will be "seen" by a meshing tooth as a thicker part of the tooth. A "low" area, however, is not so likely to be "seen" by the mating tooth, since it may be bridged over by the overall length of the line of contact. Thus, a mating gear "sees" only a maximum metal condition.

Measurement of Tooth Thickness

Functional tooth thickness is measured by means of a calibrated master gear, a gear rolling fixture and the *AGMA Handbook, 390.03.* (See Fig. 5.) This method of measurement determines the maximum metal conditions on the work gear, since the master gear has the potential of contacting all parts of the active profiles of each tooth. For those gears that can be checked by the functional check, it is generally satisfactory to specify the value of maximum allowable testing center distance, $C_{T\,max}$, which is based on the value of maximum design tooth thickness, T′. (See Equation 9.)

From both the gear user's and the gear manufacturer's standpoint, it would be ideal if a single method of measurement that would determine if gears would be suitable for their intended service could be applied prior to shipment. The functional check comes close to this ideal, but its application is limited to a somewhat restricted range of gear sizes. It is the only method of gear inspection in common use that directly evaluates the effective tooth thickness of a gear.

The *AGMA Handbook* describes the functional check and the methods of calibrating the gear rolling fixture and the master gear when tooth thickness measurements are to be made. For gears that cannot be conveniently measured by the functional check, it is necessary to use a more indirect method to determine the effective tooth thickness of a gear. Three steps are involved in this process. First, the measured tooth thickness is obtained by means of a measurement over wires, a chordal tooth thickness measurement or by a span measurement. From a practical standpoint, each of these measurements actually evaluate only a small local area of the tooth.

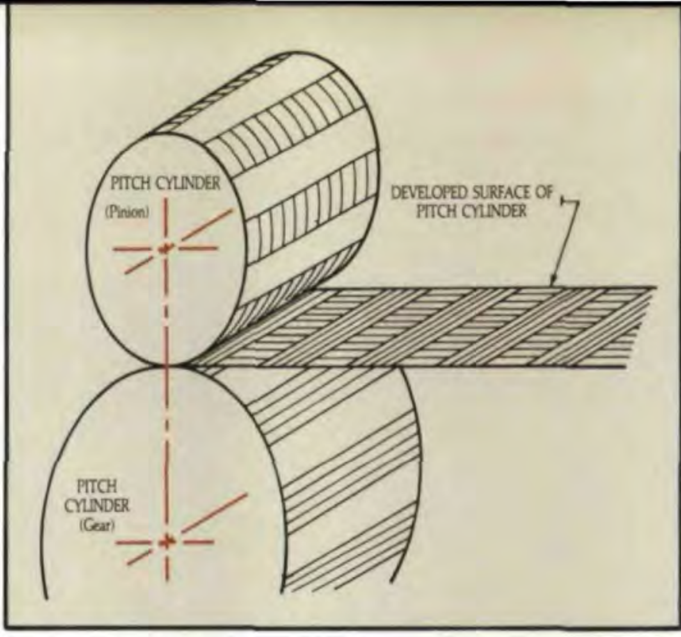

Fig. 4 – Developed surface of pitch cylinders containing cross sections of the meshing teeth.

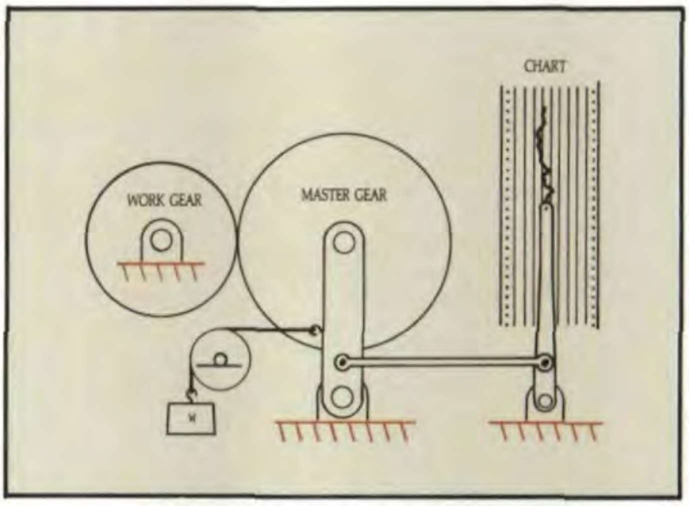

Fig. 5 – Schematic diagram of a gear rolling fixture.

Next, the gear is evaluated to determine its quality by means of measurements of its individual tooth element variations. The *AGMA Handbook* recognizes four specific types of tooth element variations. These are lead, pitch, profile and runout. Allowable tolerances are given for each AGMA Quality Number based on the pitch diameter and diametral pitch of the specific gear.

Finally, the effective tooth thickness is determined by adding to the measured tooth thickness the amount that each of the individual elements, lead, pitch, profile and runout, contribute to the effective tooth thickness of the gear.

When specifying gear tooth dimensions, the gear designer should specify a value of measured tooth thickness which is

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less than the value of effective tooth thickness. The amount by which the effective tooth thickness should be reduced is a function of the combined statistical effects of each of the individual allowable tooth element variations.

Effects of Tooth Element Variations on Effective Tooth Thickness

The effects that the various individual element variations have on effective tooth thickness can be illustrated by means of the meshing plane, which is a developed surface of the pitch cylinders of the meshing gears. (See Fig. 4.)

Lead. Fig. 6 shows a cross section of a tooth of Gear B lying in the space between two teeth of Gear A in a meshing plane. To the left is shown the situation of perfect teeth; neither member has a lead variation. The tooth of Gear B lies parallel to the space of Gear A. In this case, the measured tooth thickness is the same as the effective tooth thickness. This produces the backlash shown. The backlash is the difference between the effective space width and the measured tooth thickness. To the right is shown an example in which the teeth of Gear A have no lead variation. (The effective space width is the same as in the left illustration.) Gear Tooth B, however, shown in the right illustration, has the same measured tooth thickness as Gear Tooth B in the left illustration. It also has a lead variation which produces an effective tooth thickness that is larger than the measured tooth thickness. The resulting backlash is a smaller value.

The method of measuring lead and the way in which tolerance on lead is specified shows why the effective tooth thickness will generally exceed measured tooth thickness.

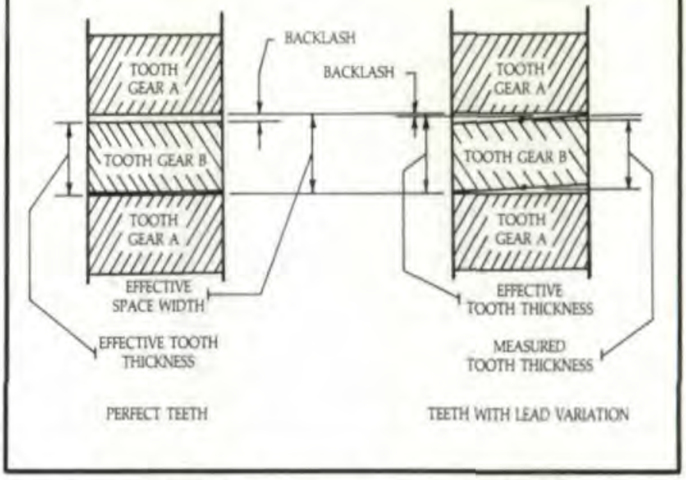

Fig. 6 – Effect of lead variation on measured tooth thickness.

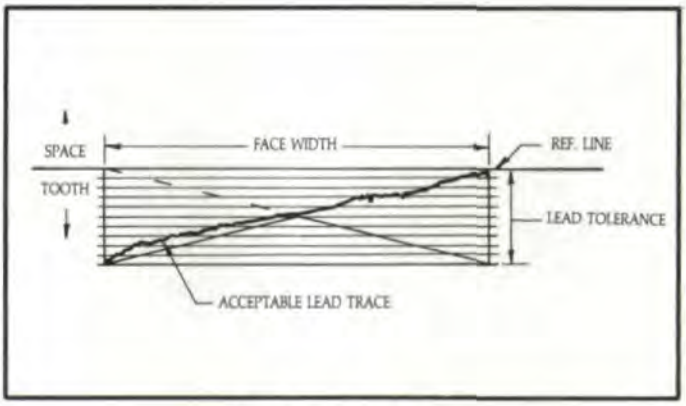

Fig. 7 – Lead "K" Chart.

Lead may be considered as the amount that a point on an active profile at one end of a tooth is ahead or behind the position of a similar point on the same active profile at the other end of the tooth. Lead is generally measured on a gear measuring machine called a lead checker. A stylus is moved along the length of the tooth surface to determine the amount that the surface departs from a theoretical helix at that given diameter.

A typical lead checking machine produces a trace on a recording chart which is a record of the lead variations found in the tooth being measured. The allowable magnitude of lead variations is usually specified by means of a lead "K" chart. (See Fig. 7.)

For a given AGMA quality class, any lead trace recorded by the lead measuring machine that will fit within the shaded area of the lead chart is acceptable. A point at one end of a given helix can be ahead or behind a similar point at the other end of the same helix by the amount of the lead tolerance shown by the "K" chart. This also applies to the helix on the other side of the tooth. Thus, a condition recognized as taper is permitted. The "K" chart indicates that the allowable deviation from the theoretical lead at the middle of the tooth is 1/2 of the allowable lead variation for the given AGMA quality level.

As shown previously, the effective tooth thickness is an envelope value of tooth thickness. It is the maximum metal condition for each specific gear. The measured tooth thickness, as obtained by means of a measurement over wires, a chordal tooth measurement or a span measurement, is only a local value. It is a linear dimension between two similar points on a given tooth. These points represent only a very small part of the total surface of the tooth. Also, these points are evaluated near the mid-height of the tooth and usually at the middle of the length of the tooth. Each of the measuring methods involve only one or two of the tooth element tolerances, lead, pitch, profile or runout, in various combinations.

Spacing. Fig. 8, left illustration, shows a cross section of two teeth of a perfect pair of gears in mesh in their meshing plane. The teeth are shown at the phase of the meshing cycle when there are two pair of teeth in contact. (See Fig. 2, top and middle illustrations.) Fig. 8, left illustration, shows the teeth of a perfect pair of gears in mesh. The right illustration shows the effect of a spacing variation. When one tooth of Gear A is in contact with Gear B, the next tooth is displaced by the magnitude of the spacing variation, thus, increasing the effective tooth thickness. Spacing (pitch) may be viewed as the amount that a given tooth is ahead or behind its correct position along its pitch circle relative to its adjacent tooth; thus, spacing variations over a group of teeth can be cumulative.

Profile. Profile is generally measured by means of a profile checking machine. The machine traverses a stylus along

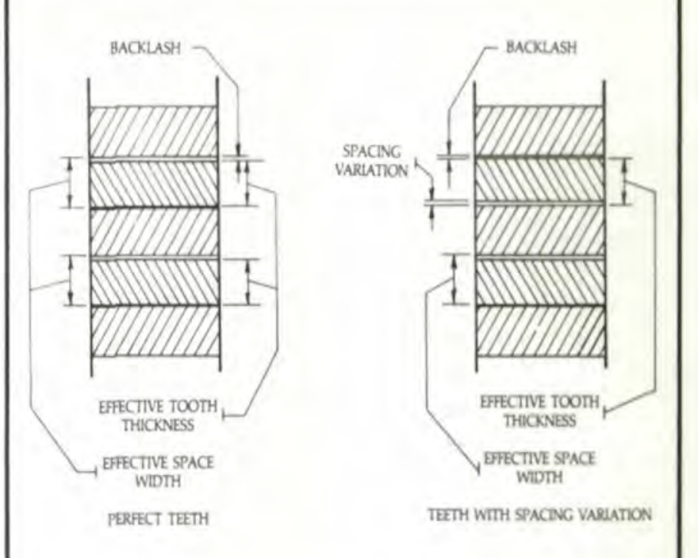

Fig. 8 – Effect of spacing variation on measured tooth thickness.

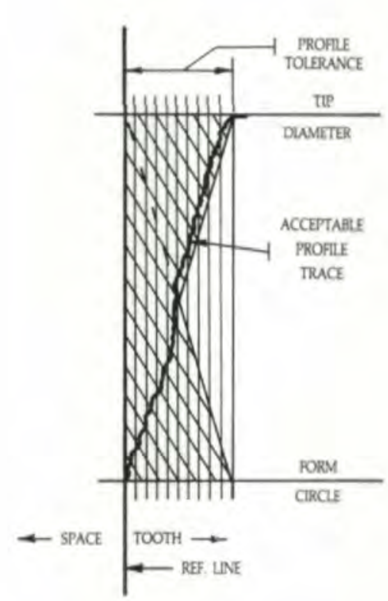

Fig. 9 – Profile "K" chart.

the active face of the tooth from root to tip. A chart is produced which shows the departure of the actual profile from a theoretical involute profile for the tooth being evaluated.

The allowable magnitude of profile variation is usually specified by means of profile "K" chart. (See Fig. 9.) For any given AGMA quality class, any profile trace that will fit within the shaded area of the profile chart is acceptable. The chart indicates that the allowable deviation from the theoretical profile at mid-height is 1/2 of the allowable variation for each quality level. For example, for a gear of a size and AGMA quality number permitting a .002" profile tolerance, the maximum metal condition could exceed the mid-height value by .001". In such a case, the effective tooth thickness would exceed the measured tooth thickness by .001" on each side of the tooth, a total of .002".

Runout. Runout may be measured by placing a measuring wire in each successive tooth space as the gear is rotated about its axis under an indicator. A radial reading is taken at each position. Runout is the maximum variation from high to low readings of all of the tooth spaces in the gear. It is due to an eccentric condition between the circle on which the teeth were cut and the axis of rotation, or due to an out-of-roundness of the gear. Runout in a gear as "seen" by its mating gear appears as variations in tooth thickness. Equation 18 shows the relationship among space width, tooth thickness and circular pitch on the standard pitch circle. The relationship between radial measurement of out-of-roundness variation (runout), ΔC, and the tooth thickness variation is given in Equation 19.

Tooth Thickness Adjustment Factor

In order to determine the drawing specification value of tooth thickness measurement for each of the measuring methods, over wires, chordal tooth thickness or span measurement, it is necessary to determine the value of tooth thickness adjustment factor for the method to be specified. This value is subtracted from the calculated value of effective tooth thickness to achieve a value of measured tooth thickness. The value of measured tooth thickness is then used to calculate the specific drawing dimensions required by the chosen measuring technique.

The magnitude of the tooth thickness adjustment factor, Δt, depends on the specific method of tooth thickness measurement to be specified. In each of the tooth thickness measuring methods there is a unique combination of tooth element variations that enter into the calculations, providing a stack-up of tolerances. Since it is reasonable to assume that these tolerances will exhibit a normal distribution, the root mean square of all of the individual element tolerances that enter into each specific measurement is used. This gives better than a 95% assurance that the effective tooth thickness will not be exceeded.

Measurement Over A Single Wire. The measured tooth thickness of a gear is obtained most directly by a measurement over a single wire. This measurement provides the chordal distance between similar points on the profiles on each side of a tooth space at a specific distance from the axis of rotation. A gear measuring wire, made to precise tolerances on roundness and diameter, is placed in a tooth space, and a radial measurement is made from the axis of rotation of the gear to the top of the wire. The wire typically contacts the tooth at about mid-height. The quality variations that enter into this measurement are lead (each side), profile (each side) and runout. According to the profile "K" chart, the tip or the root of a tooth can be plus metal by up to 1/2 of the profile tolerance. This is also true of lead. It is also reasonable to assume that with two measurements taken at random, the mid-point of runout will be measured.

Tooth Thickness Adjustment Factor

$$\Delta t_1 = [2\,(\tfrac{1}{2}v_P)^2 + 2(\tfrac{1}{2}v_L)^2 + \tfrac{1}{4}v_{TR}^2 + v_S^2]^{1/2}$$

where

$$v_{TR} = 2 \tan \Phi \; v_R$$

The measured tooth thickness, T_{M1}, on which the drawing values of measurement over one pin are based, may be calculated as follows:

$$T_{M1} = T_E - \Delta t_1$$

The value of measured tooth thickness is used to calculate the specified measurement over one wire. (See Equation 12.)

Gear Tooth Vernier Measurement. The gear tooth vernier caliper measures the normal chordal distance between the profiles of a tooth at a specified distance from the top land (outside diameter) of the tooth. A gear tooth vernier is a type of vernier caliper which includes a special blade that can be pre-set to a drawing value of chordal addendum to establish the distance from the top land to the point on the teeth where the measurement is to be taken. Equation 13 may be used to calculate the chordal addendum setting. The distance between the jaws is read as they make contact with the sides of the tooth. This is the chordal tooth thickness. This value can be converted to measured arc tooth thickness by Equation 14. Contact between the sides of the tooth and the jaws of the tooth vernier caliper occurs over a relatively small area of the tooth in most cases. Thus, the measurement does not include either the profile or the lead variations. Since only a single tooth is evaluated at one measurement, spacing is not included. Also, the diameter at which the jaws contact the tooth is fixed by the outside diameter, which is usually machined independantly of the teeth; thus,the full effect of


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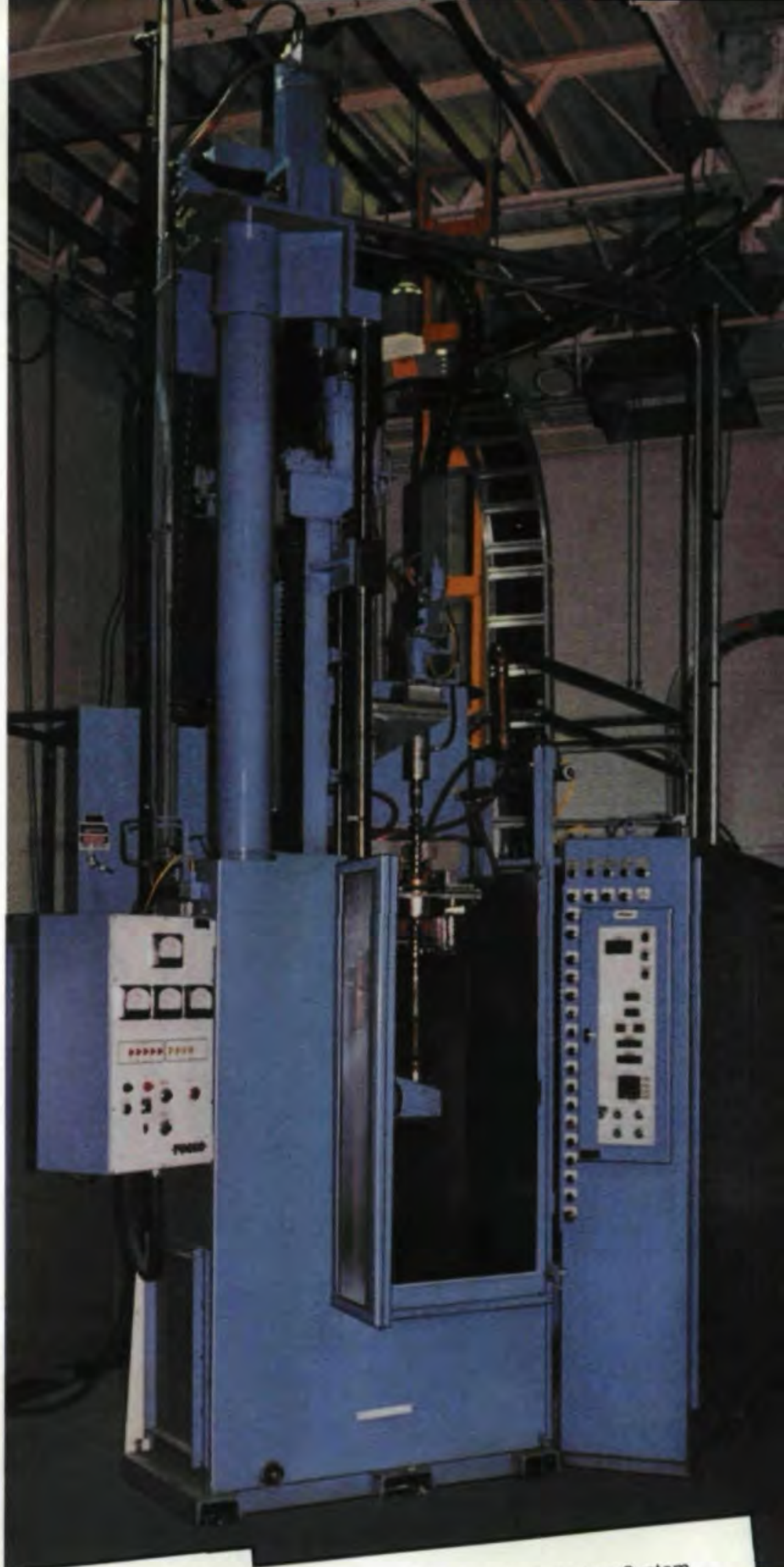

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runout may be present.

Tooth Thickness Adjustment Factor

$$\Delta t_c = [2(\tfrac{1}{2}v_P)^2 + 2(\tfrac{1}{2}v_L)^2 + \tfrac{1}{2}v_{TR}^2 + v_s^2]^{1/2}$$

The measured tooth thickness, T_{Mc}, on which the drawing values of chordal tooth thickness measurement are based, may be calculated:

$$T_{Mc} = T_E - \Delta t_c$$

The vernier caliper setting for chordal tooth thickness may be obtained from Equation 14.

Measurement Over Two Wires. In this technique, a gear measuring wire is placed in each of two tooth spaces near or at opposite ends of a diameter, and a measurement made across the tops of the wires. The wire diameters are chosen such that the wires will contact the tooth surfaces near their mid-height. Contact conditions are similar to the over one wire measurement, except that the measurement is not related to the gear axis.

Tooth Thickness Adjustment Factor

$$\Delta t_2 = [2(\tfrac{1}{2}v_P)^2 + 2(\tfrac{1}{2}v_L)^2 + v_S^2 + V_{TR}^2]^{1/2}$$

The measured tooth thickness, T_{M2}, on which drawing values of measurement over two pins are based, may be calculated from:

$$T_{M2} = T_E - \Delta t_2$$

The value of measured tooth thickness is used to calculate the specified measurement over two wires. (See Equations 15 or 16.)

Span Measurement. In this technique, a measurement is made over a group (span or block) of teeth using a conventional vernier (dial) caliper. The number of teeth included within the span will determine where the contact will take place on the teeth. In most cases, a number of teeth to be included in the span is selected which will provide contact near the mid-height of the teeth. In addition to the effects of profile and lead on the sides of two teeth, there is the effect of the pitch accumulation produced by the number of teeth within the span. The full effect of spacing is included in the measurement.

Tooth Thickness Adjustment Factor

$$\Delta t_s = [2(\tfrac{1}{2}v_P)^2 + 2(\tfrac{1}{2}v_L)^2 + v_{TR}^2]^{1/2}$$

The measured tooth thickness, T_{MS}, on which the drawing values of span measurement are based may be calculated as follows:

$$T_{MS} = T_E - \Delta t_s.$$

Appendix A
General Equations for Tooth Thickness
(Parallel Shaft Gearing)

Standard Center Distance

$$C = (N_P + N_G) / (2\ P_d) \quad (1)$$

Standard Pitch Diameter

$$D = N/P_d \quad (2)$$

Operating Pressure Angle

$$\Phi' = \cos^{-1}(C \cos\Phi / C') \quad (3)$$

Transverse Diametral Pitch

$$P_d = P_{nd} \cos\Psi \quad (4)$$

Pitch Diameter of Master Gear

$$D_M = N_M / P_d \quad (5)$$

Operating Pressure Angle
(On a Gear Rolling Fixture)

$$\Phi'' = \cos^{-1} (C \cos\Phi / C_M) \quad (6)$$

Tooth Thickness of Work Gear
(From a Gear Rolling Fixture Measurement)

$$T_W = [(\mathrm{inv}\Phi'' - \mathrm{inv}\Phi)(D_M(N + N_M) + \pi D_M) / N_M] - T_M \quad (7)$$

Operating Pressure Angle
(Gear Rolling Fixture)

$$\Phi'' = \mathrm{inv}^{-1}\ \mathrm{inv}\Phi + [N_M(T_M + T_W) - \pi D_M] / [D_M(N_M + N)] \quad (8)$$

Testing Center Distance Specification
(From Value of Functional Tooth Thickness)

$$M_M = C \cos\Phi \ / \ \cos\Phi'' \quad (9)$$

Pressure Angle To Center of Wire
(Measurement over 1 wire)

$$\Phi_2 = \mathrm{inv}^{-1} [(T_{M1}/D_w) + \mathrm{inv}\Phi + (d_w/D_b) - (\pi N)] \quad (10)$$

Radius to Center of Wire

$$R_W = D_b/\cos\Phi_2 \quad (11)$$

(continued on page 36)

THE INTERRELATIONSHIP OF TOOTH . . .
(continued from page 34)

TERMS

The following is a list of terms and definitions as used in Equation (1) through (11).

Ainv – Perform the arcinvolute function.
Acos – Perform the arccosine function.
Asin – Perform the arcsine function.
Sin – Perform the sine function.
Cos – Perform the cosine function.
Inv – Perform the involute function.
BD – Diameter of the gear base circle.
BTN – Normal tooth thickness at the gear base circle.
CP – Contact point of the ball and the tooth side.
D – Exact ball diameter to contact the tooth at the midpoint between the outside circle and the form point.
DBALL – Dimension over the balls.
DD – Diameter of the circle to any designated point on the involute surface.
DS – Diameter of the standard ball.
FD – Diameter of the gear form circle.
HADD – Helix angle at diameter DD.
K – Designator to determine external or internal – K is + 1 for external gears; K is – 1 for internal gears.
OD – Diameter of the gear outside circle.
PACB – Transverse pressure angle at the center of the ball.
PACP – Transverse pressure angle at the ball contact point.
RCB – Radius to the center of the ball.
RCP – Radius to the contact point.
ROB – Radius over the ball.
RUB – Radius under the ball.
TTDD – Normal tooth thicknes at diameter DD.
Z – Number of teeth on the gear.

$$PACB = K * \left[\frac{\pi}{Z} - \left[\frac{(\ BTN}{Cos\ (BHA) * BD} \right] + \right. \tag{3}$$

If Z is even:

$$DBALL = 2 * RCB + K * DS \tag{11}$$

Measurement Over 1 Wire

$$M_1 - R_W + (d_w/2) \tag{12}$$

Chordal Addendum Specification

$$a_c - a + (T_{Mc}{}^2 \cos^2\Psi) \ / \ (4\ d_M) \tag{13}$$

Chordal Tooth Thickness Specification

$$t_c - T_{Mc} - (T_{Mc}{}^3 \cos^4) \ / \ (6\ d_M{}^2) \tag{14}$$

Measurement Over 2 Wires
(Even Number of Teeth)

$$M_2 - D_w + (d_w/2) \tag{15}$$

(Odd Number of Teeth)

$$M_2 - 2\ R_w[\cos(90/N)] + d_W/2 \tag{16}$$

Span Measurement Specification

$$M_s - D \cos\Phi[\pi/(2N) + \mathrm{inv}\Phi] + (n - 1)(\pi/P_d \cos\Phi) - (T_{Ms}) \cos\Phi \tag{17}$$

Tooth Thickness and Space Width

$$\pi/P_d - t + s \tag{18}$$

Change in Arc Tooth Thickness vs. Change in Center Distance

$$\Delta t - 2 \tan \Phi \Delta C \tag{19}$$

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