BACK TO BASICS...

Involutometry

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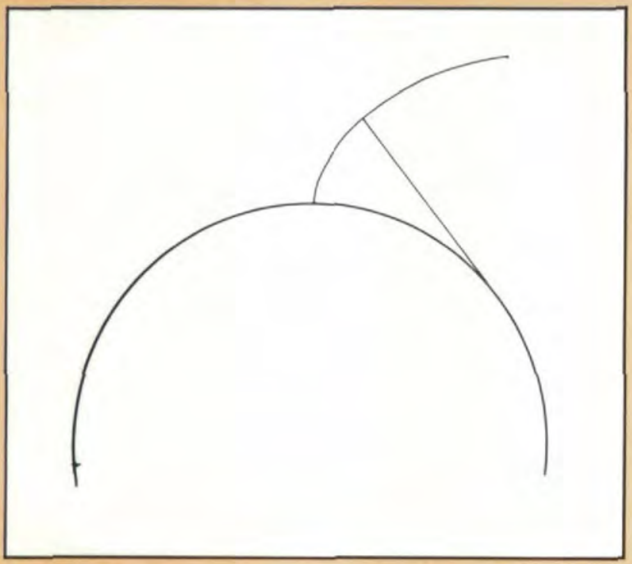

Fig. 1 – Involute curve.

Involute Curve Fundamentals

Over the years many different curves have been considered for the profile of a gear tooth. Today nearly every gear tooth uses an involute profile. The involute curve may be described as the curve generated by the end of a string that is unwrapped from a cylinder. (See Fig. 1.) The circumference of the cylinder is called the base circle.

Following are specific features of the involute curve.

- A perpendicular to the involute surface is always tangent to the base circle.
- The involute surface is a uniform rise cam (equal rise per increment of rotation).
- The radius of curvature of an involute surface is equal to the length of the tangent to the base circle.
- The same force applied perpendicular to the involute surface of a gear tooth at different tooth radii results in the same torque on the gear.

Because the involute surface is a uniform rise cam, any involute surface imparts uniform angular motion when operating against any other involute surface. To illustrate further, consider two pulleys with a crossed-belt drive as shown in Fig. 2. If we place knots in the string, then the knots will generate involute curves as they leave one circle and approach the other circle as shown in Fig. 3. If the knots are spaced close enough, there will always be knots on an involute surface when they are not on the circle. These generated curves can then be considered as teeth on the gear.

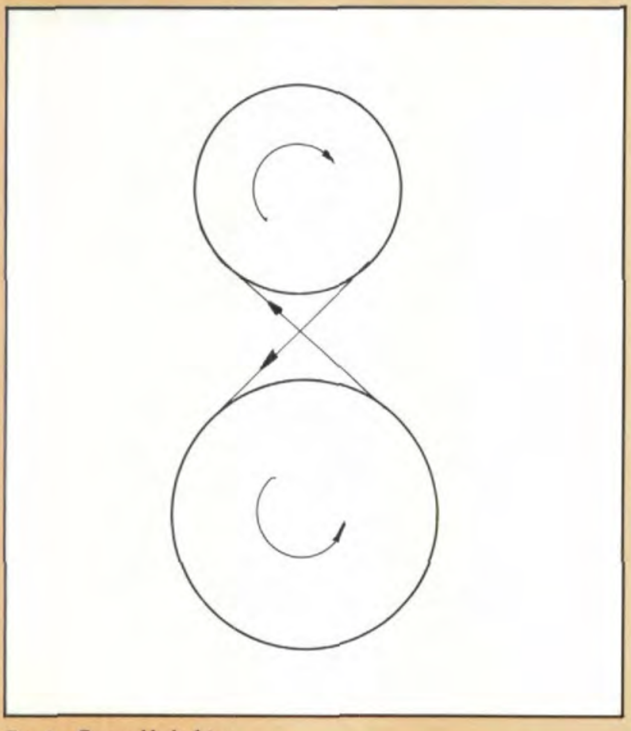

Fig. 2 – Crossed belt drive.

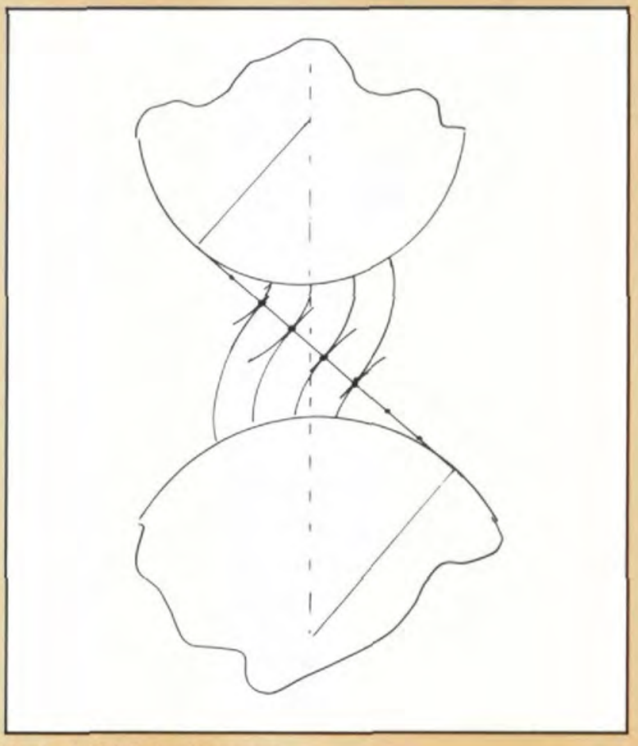

Fig. 3 – Multiple involute curves.

Of course, there are other considerations. The distance between knots must add up to an integer around the base circle, and the knots must be far enough apart to provide a structure for the tooth. When these criteria are met, the distance between knots is called the base pitch. Any two spur gears that have the same base pitch can be meshed together. (See Fig. 4.)

Just as the diameters of the two pulleys in the crossed-belt drive establish the ratio of the two shafts, likewise, the diameters of the base circles establish the ratio of the two gears.

The contact between two involute surfaces of two gears always occurs on a line that is tangent to the two base circles. This tangent line is called the "line of action". (See Fig. 5.) This phenomenon results from the from fact that a perpendicular to the involute surface is tangent to the base circle.

The line of action will cross the center line between the two gears. This intersection is called the pitch point. If we draw circles on each gear using the gear centers and passing through the pitch point, we have pitch circles. These pitch circles may be considered as cylinders which would roll with each other by friction with no slippage. The distance between the center lines of gear teeth on either pitch circle is called the circular pitch. If we divide the circumference of the pitch circle by the number of teeth in the gear, we get the circular pitch. These circles are important in the calculation of tooth thicknesses because it is here that the two tooth thicknesses, plus the backlash, must add up to the circular pitch.

Each gear has two distinct pitch circles to be considered. The gear will have one pitch circle with its cutter and another pitch circle with the mating gear. If it is the middle gear in a string of three gears, it may have a different pitch circle with each mating gear.

The pitch circles of mating gears will change as the center distance is changed. If the center distance is increased, the pitch circles will get larger, and the circular pitch will increase with the same tooth thicknesses as before. The backlash will also increase. A gear does not have an operating pitch diameter until its mating gear and center distance are specified.

Since gear teeth must make contact along the line of action, and since the contacting forces are parallel to this line, there is a force which acts to push the two gears' centers apart. The angle the line of action makes with a perpendicular to the center line of the gears is called the pressure angle. The pressure angle is defined as

$$\text{Pressure angle } \alpha = \text{Acos} \frac{\text{Base radius}}{\text{Pitch radius}} \qquad (1)$$

Just as the pitch radius with another gear is unique, so the pressure angle with another gear will be unique.

This may be confusing until one is reminded of the crossed-belt drive. The cylinders would rotate in the same manner, and the knots in the string would generate the same involutes independent of the center distance between the two cylinders. The crossover point of the line of action across the center line will result in different pitch circles, and the pressure angle will change as the center distance is changed. The force along the line of action will remain the same; it is the torque divided by the base circle radius. As we will see later, there are considerations of tooth thickness and contact ratio which restrict the

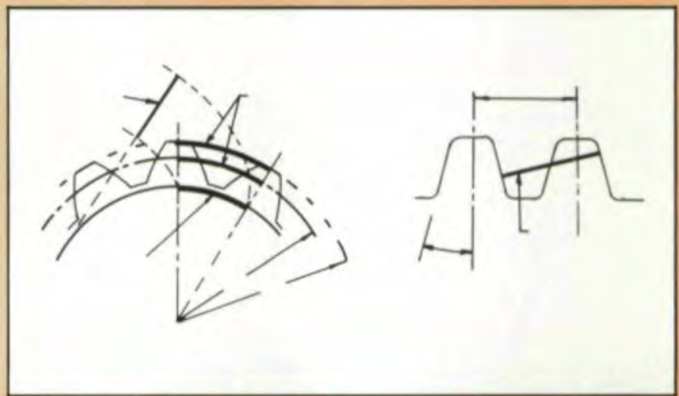

Fig. 4 – Base pitch of a gear and a rack.

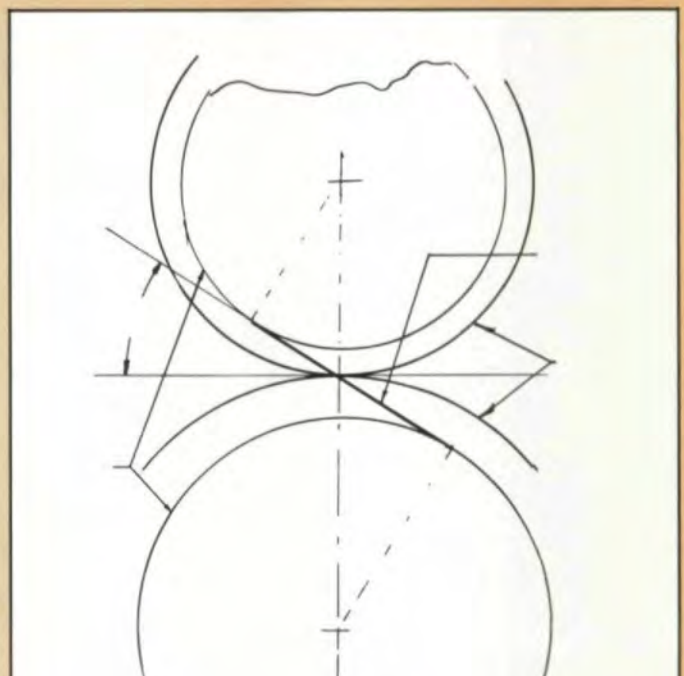

Fig. 5 – Line of action and pressure angle.

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range that the center distance may be changed.

The insensitivity of the involute action to the changing of the center distance of the gears is a distinct advantage over other tooth profiles considered in the past. As shafts deflect due to changes in load, the involute action is unchanged, assuming sufficient contact ratio still exists.

The involute profile is the path of the end of a string unwound from a base circle. The design of the gear involves establishing the ends of this curve. At the beginning of the curve there must be some sort of fillet blend to the root and the involute of the adjacent tooth. It is not good practice to plan on using the involute near the base circle because its radius of curvature is very small and changing rapidly. It is very difficult to manufacture the tooth accurately near the base circle. However, this end of the involute profile must be defined accurately to avoid problems of interference with the tip of the mating gear tooth. The "form diameter" is used to specify the root end of the involute profile.

The tooth tip end of the involute profile can either extend to the outside diameter of the gear, or if a chamfer is provided, it ends with the start of the chamfer. A chamfer is used by many manufacturers to prevent damage to the involute surface from nicks due to handling.

Helical Gears

More and more new machines use helical gears. The principal reason for this is the demand for quieter gears. Not only do we have an increased emphasis upon quiet machines, but also most machines operate at higher speeds and higher loads, both of which increase the noise made by spur gears. A pitch line velocity of 2000 feet per minute is considered by many to be the top limit for using spur gears in non-aircraft applications. The writers have observed spur gears operating quietly at pitch line velocities of 6000 feet per minute, but these were probably very accurate gears. With the normal industrial production tolerances, we can expect spur gears to be noisy above pitch line velocities of 2000 feet per minute. If this speed must be exceeded and quietness is preferred, then the design should incorporate helical gears.

We will introduce several new terms to describe physical characteristics of helical gears. The first of these is the helix angle. The helix angle is the inclination of the tooth in a lengthwise direction. The helix angle results in the gear having end thrust or an axial force during operation, and the support bearings must be able to resist this force. There is no rule for the best helix angle. It may be up to 45°.

The helix angle of the tooth depends upon the radius under consideration. Normally we quote a helix angle associated with the generating helix angle. This relates to the pressure angle of the cutter and the generating pitch radius. It is more important to work with the helix angle at the base circle. For two gears to mesh properly and operate on parallel axes, they must have the same normal base pitch and the same base helix angle.

To determine the different helix angles on a gear requires a familiarity with the term "lead". The lead of a gear may be defined as the width of a gear required for a tooth to wind one revolution around the gear. A large helix angle will result in a small lead. A spur gear may be considered as having an "infinite" lead.

In very large power applications common practice dictates the use of two helical gears, side by side, with opposite angles of the helix (hand) so that the end thrust is neutralized. Usually there is a small space between the gears to facilitate manufacture and also to give the tooth flexibility for load sharing.

Helical gears have involute tooth profiles similar to those in spur gears. Each gear will have a base circle and a line of action like a spur gear. If we observe the gear from an axial position, the gear contacts will occur along the line of action. However, an entire tooth will not make contact with the mating gear instantaneously, as does a spur gear tooth. The contact may start at the tip of one end of the tooth and proceed along the length of the tooth, with the first point of contact moving down the tooth before the other end of the tooth makes contact. The instantaneous load contact will be a diagonal line across the tooth face with a different radius as we follow the contact across the tooth. For very wide gears the pattern may be repeated several times across the gear as several teeth, with their helix angles, contact on the line of action.

With helical gears we have a second contact ratio to consider. The contact ratio expressed by the length of the zone of action divided by the base pitch is still used. The second, called face contact ratio, is a function of the helix angle and the width of the gear. For smooth action both contact ratios should exceed one.

The teeth on a helical gear have a slight "twist" in the radial direction. If a "pin" is placed in the tooth space, as is done when measuring the distance across pins to determine tooth thickness, the pin will sit on a high spot and "rock". A ball must be used for making this measurement with helical gears, which have an odd number of teeth.

To study the tooth geometry we have to consider two planes, the transverse plane and the normal plane. The gear teeth operate in the transverse plane, but the teeth are generated in the normal plane. The tooth thickness and generating pressure angle usually refer to the normal plane. The base circles, outside radius, root diameter and contact ratio are determined in the transverse plane.

To determine the tooth thickness at a second radius requires the conversion of the given tooth thickness in the normal plane to the transverse plane. The tooth thickness at the desired radius is calculated in the transverse plane. It can then be converted to the normal plane again, taking into account the change of helix angle at the desired radius.

Helical gears may be cut with the same hob that is used to cut spur gears. The hobbing machine has the ability to position the hob at an angle corresponding to the helix angle.

For gears that must be cut with a shaper because there is no clearance for hob runout, the shaper machine must be equipped with a "guide". The guide provides rotation of the cutter as it generates the teeth. Each guide has a specific lead, and the helix angle generated depends upon the diameter of the shaper cutter. A large diameter cutter will generate a larger helix angle than a small diameter. The shaper cutter must have nearly the same lead; "nearly" because the shaper cutter may still be usable even though its helix angle is up to one degree different than the helix angle being generated. The cutter has enough side relief to permit this. This should only be done for small lots.

When selecting shapers for a helical gear, the availability of a guide is most important because the cost of a guide is many times that of a shaper.

The mathematical formulae for helical gears are nearly the same as for spur gears, except for the inclusion of a function of the helix angle in the equation. The formulae are basically set up to work in the transverse plane (plane of rotation); therefore, if the tooth thickness and pressure angle are in the normal plane, they must be converted to the transverse plane.

Gear Design Mathematics

The tooth geometry of a spur or helical gear may be described by seven pieces of data. These are number of teeth, base diameter, outside diameter, form diameter, root diameter, tooth thickness at base diameter and lead (and hand).

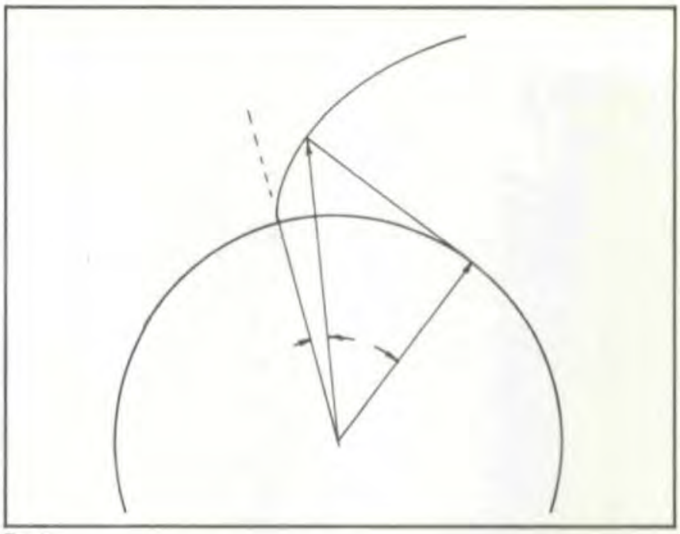
The design of a gear involves the use of mathematical equations to obtain the above seven numbers. The mathematics involves extensive use of trigonometry. The designer must be familiar with the use of angles in radians and the involute function.

As mentioned previously, the involute curve is the path of the end of a string unwound from a cylinder.
The following relationships hold.

$$\operatorname{Tan}\alpha = \frac{\sqrt{R^2 - R_b^2}}{R_b} = \frac{R_b(\alpha + \theta)}{R_b}$$

$$\operatorname{Tan}\alpha = \alpha + \theta$$

$$\theta = \operatorname{Tan}\alpha - \alpha = \operatorname{inv}\alpha \qquad (2)$$

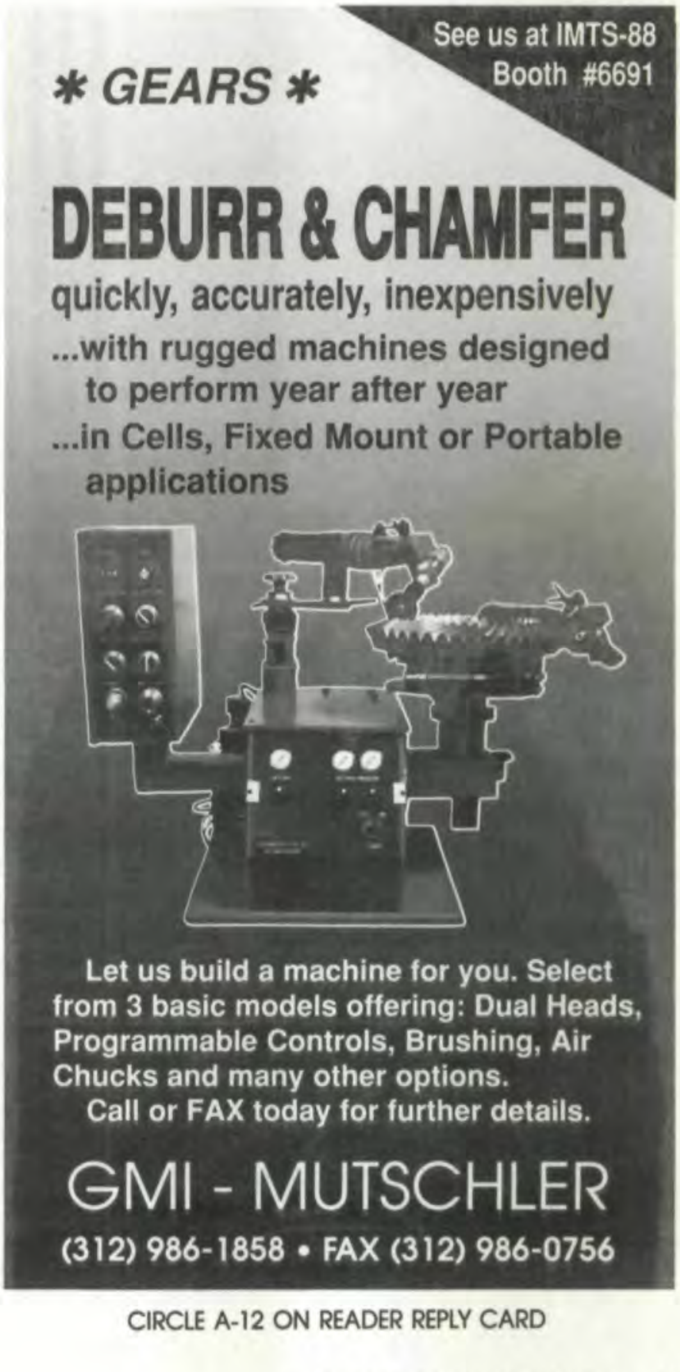


Eq. 2

The involute function is the angular displacement of a point on the tooth from its point of intersection with the base circle. This relationship is useful for calculating the circular thickness of a tooth. When the thickness at a given diameter is desired, there is a corresponding angle or "pressure angle" of the tooth at that radius. By determining the pressure angle at a point on a tooth and then taking the involute of this angle, the angular shift of the curve from where it left the base circle is available. Now by multiplying this angle by the new radius, we have the length of arc of the shift of the tooth profile. By subtracting two times this angle from the angle on the base circle for the entire tooth, and multiplying this resultant angle by the desired radius, we get the circular tooth thickness at the desired radius. (See Equation 3.)

$$T_R = 2R\left\{\frac{T_b}{2 \cdot R_b} - \operatorname{inv}\alpha\right\} \qquad \alpha = \operatorname{Acos}\left\{\frac{R_b}{R}\right\} \qquad (3)$$

Good practice suggests use of the tooth base thickness as a fundamental dimension on the gear because finding the thickness at any other radius only requires finding one involute. If the known thickness is at some other diameter, then it is necessary to have the involute at this radius, and then use the difference in involutes to determine the difference in known tooth thickness and the desired tooth thickness.



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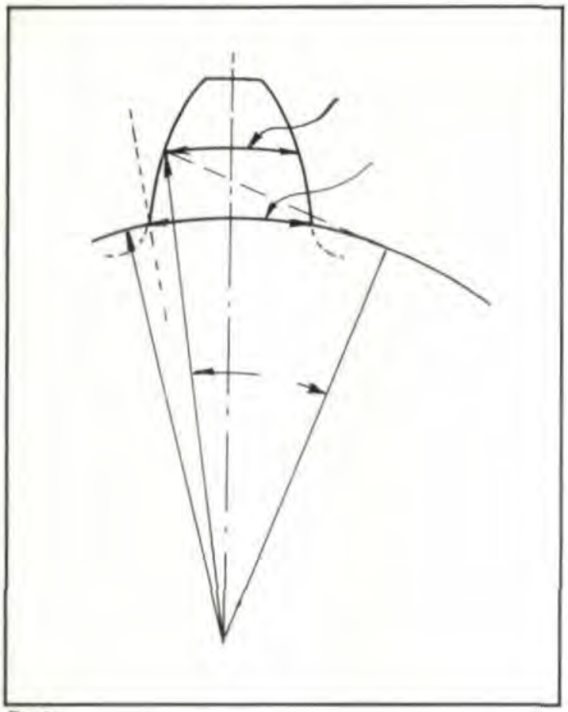
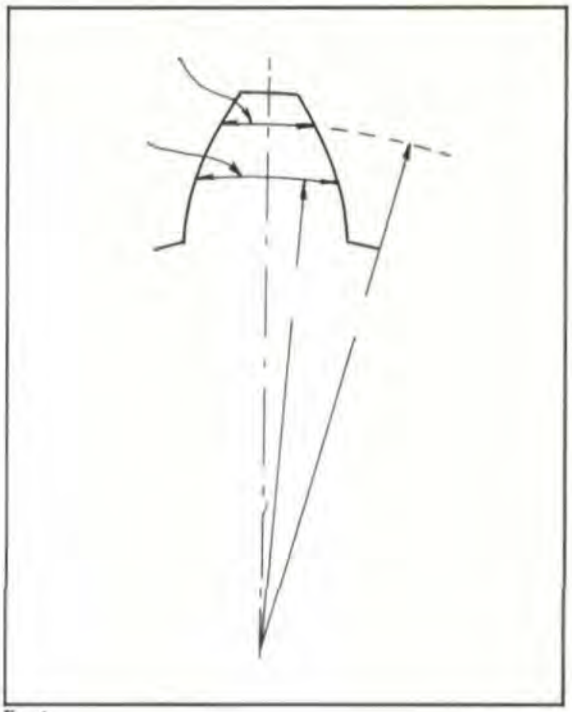


Eq. 3



Eq. 4

It is customary to use the circular dimensions for describing tooth thickness. The difference between the circular and chordal tooth thickness is very small, usually less than one-tenth of the backlash that will be provided. However, in some mathematical analyses, such as following the path of a cutter, chordal dimensions must be used to get the accuracy required.

Gear designers who work with "standard" gears usually have the tooth thickness at the pitch circle and then have to use two involutes to find the tooth thickness at another radius.

Given the arc tooth thickness and pressure angle in the plane of rotation of a helical gear at a given radius, to determine the tooth thickness at any other radius, use the following formulae.

$$\mathrm{Cos}\,\alpha_2 = \frac{R_1 \cdot \mathrm{Cos}\,\alpha_1}{R_2}$$

$$T_2 = 2 \cdot R_2 \left\{\frac{T_1}{2 \cdot R_1} + \mathrm{inv}\,\alpha_1 - \mathrm{inv}\,\alpha_2\right\} \qquad (4)$$

where

R_1 = Given radius
α_1 = Pressure angle at R_1
T_1 = Arc tooth thickness at R_1
R_2 = Radius of unknown tooth thickness
α_2 = Pressure angle at R_2
T_2 = Tooth thickness at R_2

The above formula assumes all the data is provided in the transverse plane (plane of rotation). Most helical gear data is available in the normal plane because the cutter is dimensioned in this manner. Such data as tooth thickness and pressure angle are usually in the normal plane.

The formula for the pitch diameter of a helical gear in the transverse plane is

$$PD = \frac{Z}{DP \cdot \mathrm{COS}\beta} \qquad (5)$$

where

Z = Number of teeth
DP = Diametral pitch (normal)
β = Helix angle (generating)

Converting the normal tooth thickness to the transverse plane requires the following formula:

$$T_t = \frac{T_n}{\mathrm{Cos}\,\beta} \qquad (6)$$

where

T_n = Normal tooth thickness
β = Helix angle
T_t = Transverse tooth thickness

Note that the helix angle must correspond to the diameter on the gear corresponding to the tooth thickness. The helix angle is different for different diameters of the tooth. The lead is constant for a gear and can be determined from a helix angle given at a given radius by the formula:

$$L = \frac{2 \cdot \pi \cdot R}{\mathrm{Tan}\,\beta} \qquad (7)$$

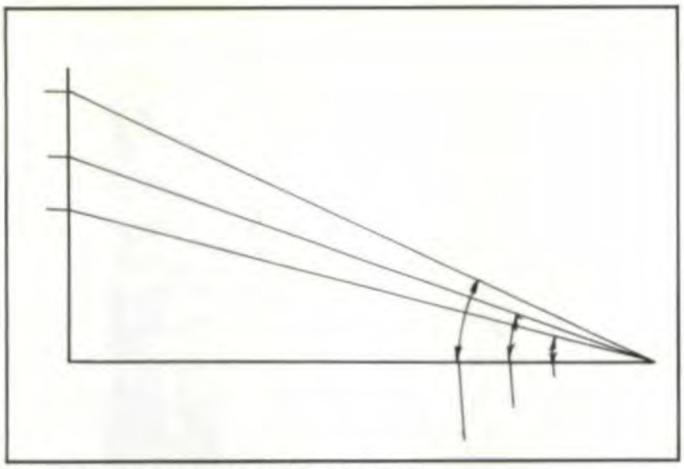


Fig. 6 – Diagram to illustrate helix angle variation.

where

$R =$ Radius of given helix angle
$\beta =$ Given helix angle
$L =$ Lead of the gear

Once the lead is known, the following formula may be used to find the helix angle at the radius corresponding to the given tooth thickness.

$$\beta = \text{Atan} \left\{ \frac{2 \cdot \pi \cdot R}{L} \right\} \tag{8}$$

where

$R =$ Radius of given tooth thickness
$L =$ Lead of the gear
$\beta =$ Helix angle for R

Fig. 6 is helpful in remembering the relationship between the lead and helix angle at different radii of the gear tooth. The lead is constant for a gear, but the circumference depends upon the radius being studied.

The study of the beam stress of the tooth and the form diameter is done in the normal plane. Doing these studies in the normal plane requires converting to a "virtual" gear. The normal plane of a helical gear is really an ellipse. The point of interest is the largest radius of curvature of the ellipse. This radius of curvature is used to construct a virtual gear.

The virtual gear is a simulated spur gear. Mathematically, the virtual gear is derived from the helical gear by the following relationships:

$$\text{Virtual pitch radius } (R_v) = \frac{\text{Pitch radius (helical gear)}}{\text{Cos}^2 \text{ (Helix angle)}} \tag{9}$$

$$\text{Virtual number of teeth } (Z_v) = \frac{\text{Teeth in helical gear (Z)}}{\text{Cos}^3 \text{ (Helix angle)}} \tag{10}$$

$$\text{Virtual base radius } (R_{bv}) = \text{Virtual pitch radius} \cdot \text{Cos } (PA_{normal}) \tag{11}$$

To obtain the outside diameter and root diameter, their dif-

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The given pressure angle will usually correspond to the pressure angle of the cutter. To convert this into the transverse plane the following formula is available.

$$\operatorname{Tan} \alpha_t = \frac{\operatorname{Tan} \alpha_n}{\operatorname{Cos} \beta_g} \qquad (12)$$

where

α_n = Normal pressure angle
β_g = Generating helix angle
α_t = Transverse pressure angle

Another calculation that is important in designing a set of gears is determining the center distance for two gears to be meshed together. Again, it is only necessary that the gears have the same base pitch for the involute action to be proper; however, there may be interference with the tips of teeth going below the form circle. The tips of the teeth may also bottom out in the root of the mating gear if the members of the pair were not designed together. If the gears are helical, they must have the same base helix angle. If they are external gears, the hand of the helixes must be opposite, assuming they turn on parallel shafts.

Given tooth proportions in the plane of rotation for two helical gears, to determine the center distance when they mesh tightly, use the following formulae.

$$\operatorname{inv} \alpha_2 = \frac{Z_1(T_1 + T_2) - 2 \cdot \pi \cdot R_1}{2 \cdot R_1 (Z_1 + Z_2)} + \operatorname{inv} \alpha_1$$

$$C_1 = R_1 + R_2 \qquad (13)$$

$$C_2 = C_1 \frac{\operatorname{Cos} \alpha_1}{\operatorname{Cos} \alpha_2}$$

where

R_1 = Given radius of first gear
R_2 = Given radius of second gear
Z_1 = Teeth in first gear
Z_2 = Teeth in second gear
α_1 = Pressure angle at R_1 and r_2
α_2 = Pressure angle at meshing position
T_1 = Arc tooth thickness at R_1
T_2 = Arc tooth thickness at R_2
C_1 = Center distance for α_1
C_2 = Center distance for α_2

The previous equations are useful for determining the center distance when run with a master gear. This is a tight mesh operating condition. If two gears are to be operated together, then the desired backlash may be added to the tooth thicknesses in the equation, and the corresponding center distance derived.

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ference from the pitch diameter is maintained in each plane.

Contact Ratio

If two mating gears are to provide smooth rotation, it is necessary that there be a continuing contact on the line of action. A second pair of teeth must mesh before the first set separate. This relationship is described by the "contact ratio". The contact ratio is the ratio of the length of active engagement of the gears on the line of action to the base pitch. For spur gears the contact ratio must exceed one and may exceed two for gears where quiet operation is needed.

The following figure and equations show how contact ratio is determined.

$$CR = \frac{\sqrt{R_{01}^2 - R_{b1}^2} + \sqrt{R_{02}^2 - R_{b2}^2} - C \cdot \mathrm{Sin}\,\alpha}{P \cdot \mathrm{Cos}\,\alpha} \qquad (14)$$

where

R_{o1} = Outside radius of gear 1
R_{o2} = Outside radius of gear 2
C = Center distance
CR = Contact ratio
R_{b1} = Base radius gear 1
R_{b2} = Base radius gear 2
α = Pressure angle
P = Circular pitch

With helical gears we have a second contact ratio to consider. The contact ratio expressed by the length of the zone of action divided by the base pitch is still used. The second, called face contact ratio, is a function of the helix angle and the width of the gear. For smooth action both contact ratios should exceed one. The formula for face contact ratio is

$$CR_{face} = \frac{F \cdot \mathrm{Tan}\,\beta}{P} \qquad (15)$$

where

F = Gear face width
β = Helix angle at pitch point
P = Circular pitch (transverse)
CR_{face} = Face contact ratio

Generation of Gear Teeth

While we discussed the forming of the involute surface of a gear tooth as identical to the curve of a string being unwound from a base cylinder, the following is descriptive of how a gear tooth is actually generated. There are two main types of cutters used to generate gears, hobs and shapers. First consider the "shaper". This cutter closely resembles a gear. One edge is sharp and, by gradually meshing with a blank as it moves back and forth axially, it will cut a mating gear. The blank should be the appropriate diameter, and the two centers must be externally geared together with the right ratio.

A second type of cutter is the "hob". Before discussing this cutter we must discuss the "basic rack". A basic rack is a seg-

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INVOLUTOMETRY . . .
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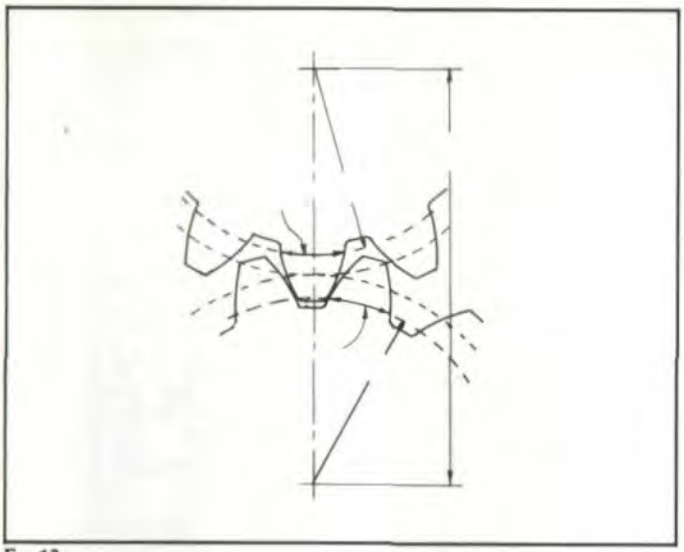
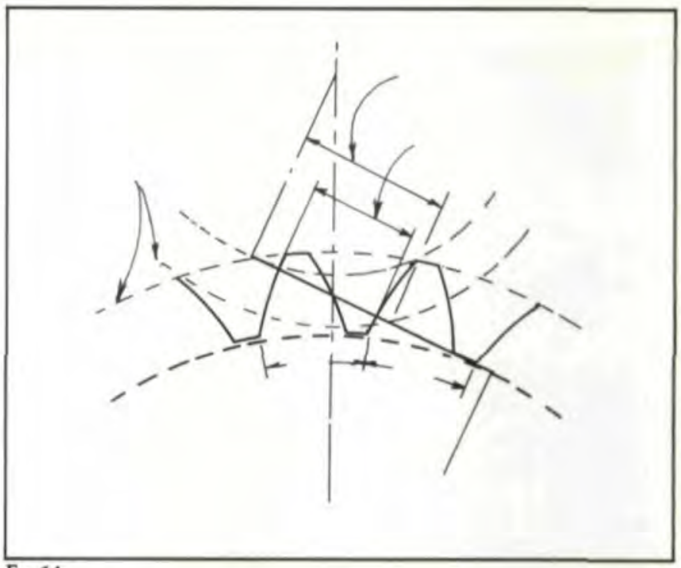


Eq. 13



Eq. 14

ment of a gear of infinite size, but with teeth the size of the gear we want to cut. As a result, the "involute" sides of the teeth are essentially straight lines. To generate a tooth, the basic rack moves in a straight line like a rack and pinion arrangement, with the cutting action being up and down axially to the gear. In reality, instead of having a very long rack, segments are arranged around a cylinder and offset to create a helix. With the axis of the cutter inclined, the teeth will contact the gear being cut parallel to the gear axis. By inclining the cutter at a different angle it can cut a helical gear. The cutting activity closely resembles a worm gear drive. Hobbing is the most popular and fastest method for cutting gears.

In our calculations it is necessary to determine the position of the basic rack with the gear being generated in order to establish the form diameter and root radius and determine

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whether the cutter will interfere with the outside diameter of the gear. Usually a gear tooth thickness is given, and the corresponding position of the basic rack is computed.

Given the tooth dimensions in the plane of rotation of a helical gear, to determine the position of a mating rack of different circular pitch and pressure angle (same base pitch), use the following formulae.

$$\text{Sin}\,\beta_b = \text{Sin}\,\beta_1 \cdot \text{Cos}\,\alpha_{n1}$$

$$\text{Sin}\,\beta_2 = \frac{\text{Sin}\,\beta_b}{\text{Cos}\,\alpha_{n2}} = \frac{\text{Sin}\,\beta_1 \cdot \text{Cos}\,\alpha_{n1}}{\text{Cos}\,\alpha_{n2}}$$

$$\text{Tan}\,\alpha_2 = \frac{\text{Tan}\,\alpha_{n2}}{\text{Cos}\,\beta_2} \qquad R_2 = \frac{R_b}{\text{Cos}\,\alpha_2}$$

$$X = R_2 - a + \frac{1}{2\,\text{Tan}\,\alpha_2} \cdot \left\{2 \cdot R_2 \left\{\frac{T_1}{2 \cdot R_1} + \text{inv}\,\alpha_1 - \text{inv}\,\alpha_2\right\} - \frac{\pi \cdot R_2}{Z}\right\} \qquad (16)$$

where

β_1 = Given helix angle at R_1
β_b = Base helix angle
α_{n2} = Pressure angle of rack
α_2 = Transverse pressure angle of rack
R_2 = Pitch radius with rack
a = Addendum of rack
Z = Number of teeth
P_{n1} = Normal pitch at R_1
β_2 = Helix angle for mating rack
α_{n1} = Normal pressure angle at R_1
α_1 = Transverse pressure angle at R_1
R_1 = Given pitch radius
R_b = Base radius
T_1 = Given tooth thickness at R_1
X = Dist. gear center to hob tip
P_{n2} = Normal pitch of rack

Base pitch must be equal ($P_{n1} \cdot \text{Cos}\,\alpha_1 = P_{n2} \cdot \text{Cos}\,\alpha_2$)

The position of the rack formula results in a figure for the root radius of the gear. The following formula will give the minimum root radius without undercut. By comparison, it can be determined if undercutting is taking place. Undercutting is undesirable. By taking the minumum root radius without undercut a new tooth thickness for the gear can be determined. By keeping one gear from undercutting, the mating gear should be checked to determine if it may be undercut as a result of shifting tooth thickness.

Given the dimensions of a hob, the helix angle and number of teeth in a helical gear, to determine the minimum root radius to avoid undercut, use the following formulae.

$$R = \frac{Z}{2 \cdot DP \cdot \text{Cos}\,\beta} \qquad \text{Tan}\,\alpha = \frac{\text{Tan}\,\alpha_{hb}}{\text{Cos}\,\beta}$$

$$R_{unc} = R \cdot \text{Cos}^2\,\alpha - r \cdot (1 - \text{Sin}\,\alpha) \qquad (17)$$

where

Z = Number of teeth in gear
α_{hb} = Pressure angle of hob
α = Pressure angle in plane of rotation
β = Helix angle
DP = Diametral pitch of hob
r = Radius of hob tooth tip
R_{unc} = Minimum root radius w/o undercut ■