Crowned Spur Gears: Optimal Geometry and Generation

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Abstract:

The authors have developed a method to synthesize the pinion crowned surface that provides a localized bearing contact and a favorable type of transmission error for misaligned gears. A method for generation of the pinion crowned surface by a surface of revolution (it slightly deviates from a regular cone surface) is proposed. Tooth Contact Analysis (TCA) programs for simulation of meshing and bearing contact for misaligned spur gears with the crowned pinion have been developed. A computer graphic program for the display of the pinion crowned tooth surface in 3-D space has also been developed.

Introduction

Involute spur gears are very sensitive to gear misalignment. Misalignment will cause the shift of the bearing contact toward the edge of the gear tooth surfaces and transmission errors that increase gear noise. Many efforts have been made to improve the bearing contact of misaligned spur gears by crowning the pinion tooth surface. Wildhaber[1] has proposed various methods of crowning that can be achieved in the process of gear generation. Maag engineers have used crowning for making longitudinal corrections (Fig. 1a); modifying involute tooth profile uniformly across the face width (Fig. 1b); combining these two functions in Fig. 1c and performing topological modification (Fig. 1d) that can provide any deviation of the crowned tooth surface from a regular involute surface.[2]

The main purpose of these methods for crowning is to improve the bearing contact of the misaligned gears, which addresses only half the problem. The transmission errors of misaligned spur gears are a main source of gear noise. According to the open literature, the influence of gear misalignment on transmission error has not been investigated. Maag's method of topological modification does not determine the relationship between the surface deviations and the transmission errors. Also, the optimal geometry for the pinion crowned surface has not been proposed.

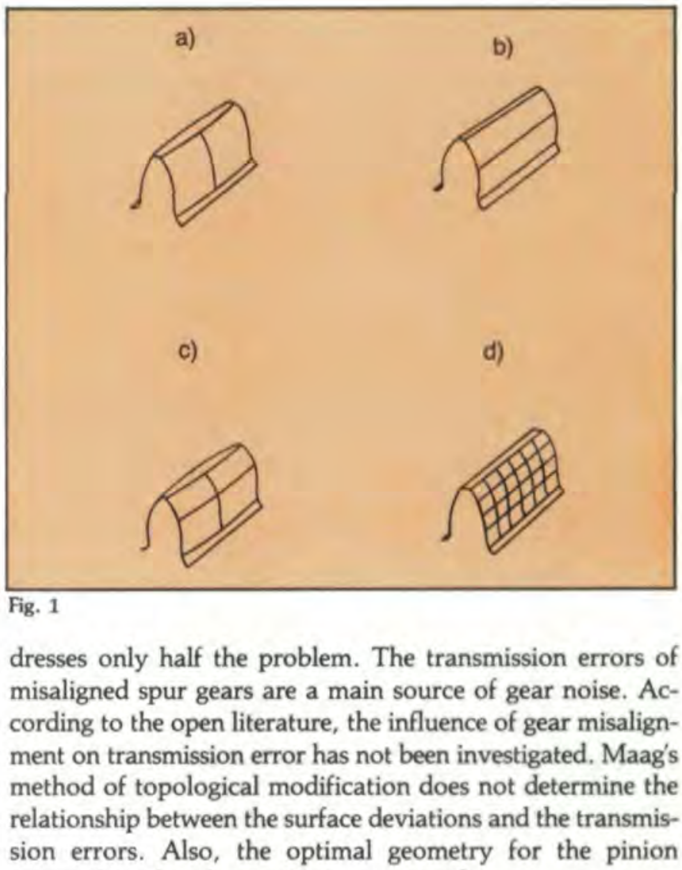

Fig. 1

The contents of this article cover the solutions to the

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following problems: optimal geometry of a pinion crowned tooth surface; new method of crowning that is based on application of a tool provided with a surface of revolution (the tool surface is slightly deviated from a cone surface); the development of TCA (Tooth Contact Analysis) programs for the determination of transmission errors for misaligned gears and their bearing contact and a computer graphic program for the display of the crowned surface and bearing contact in 3-D space. The first method of generation that provides the desired optimal pinion tooth geometry is based on the application of a computer controlled machine with five degrees of freedom. The second method for generation needs only a new tool shape and is based on application of the existing equipment.

The development of the optimal geometry of a pinion crowned gear tooth surface is based on the following considerations.

Misaligned spur gears with a pinion crowned tooth surface can provide transmission errors $\Delta\phi_2(\phi_1)$ of two types, shown in Figs. 2a and 2b, respectively. The transmission errors are determined with the equation as

$$\Delta\phi_2 = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \qquad (1)$$

Here: N_1 and N_2 are the numbers of gear teeth; ϕ_1 and ϕ_2 are the angles of gear rotation; $\phi_2(\phi_1)$ is the function that relates the angles of rotation of gears if the pinion is crowned

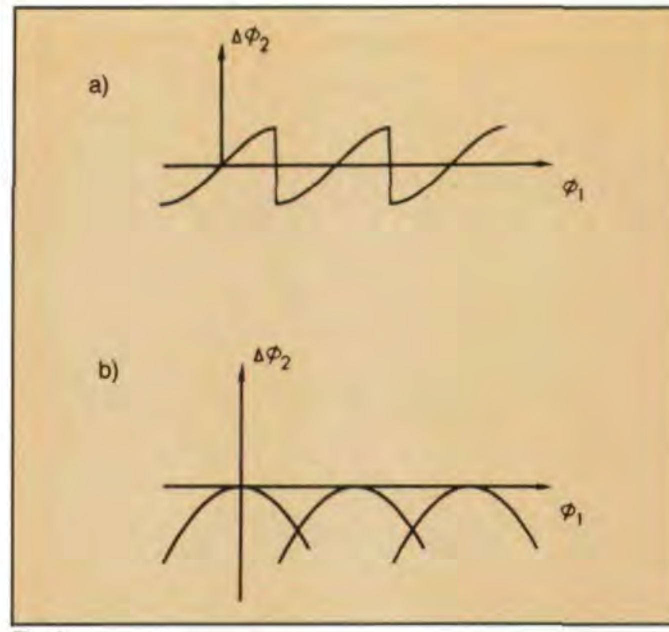

Fig. 2

and the gears are misaligned; $\phi_2^0 = \frac{N_1}{N_2}\phi_1$ is the theoretical relation between the angles of rotation of the gears in the ideal case where the gears are not crowned, not misaligned and the transmission errors do not exist. Type 1 transmission errors are not acceptable because the change of tooth meshing is accompanied by an interruption or interference of tooth surfaces. Type 2 transmission error is preferable if the level of error does not extend the prescribed limit.

At first glance crowning should be directed toward providing an exact involute shape in the middle cross section (Fig. 3). In reality, this type of crowning is not acceptable because the misaligned gears will transform rotation with transmission errors of type 1 (Fig. 2a), but not of type 2 (Fig. 2b). For this reason the authors have synthesized a specific pinion crowned tooth surface. Such a pinion, while in mesh with a gear that has a regular involute surface, is able to provide transformation of rotation with a parabolic type of transmission error function. This type of function of errors is synthesized for ideal gears that do not have any misalignment. Then, the tendency to provide parabolic transmission errors can be extended to misaligned gears and the discontinuance of meshing can also be avoided. Note that the proposed method of synthesis provides a shape in the middle cross section of the tooth that deviates in a certain way from the involute curve that is shown in Fig. 3. The longitudinal deviation from a straight line is not related to the transmission errors, but to the desired dimensions of the instantaneous contact ellipse for the gear tooth surfaces. The proposed pinion tooth surface can be generated by a plane chosen as the tool surface. The motions of the plane with respect to the pinion must be controlled by a computer, and the machine represents an automatic system with five degrees of freedom (first method for generation).

The second method of pinion crowning is based on application of a surface of revolution that slightly deviates from a regular tool conical surface (Fig. 4). Such a tool can be used as a grinding wheel or as a shaver. The motions of the tool

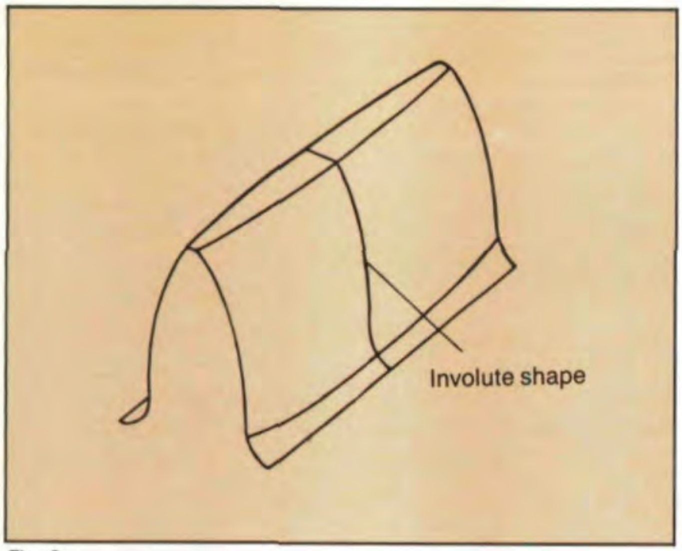

Fig. 3

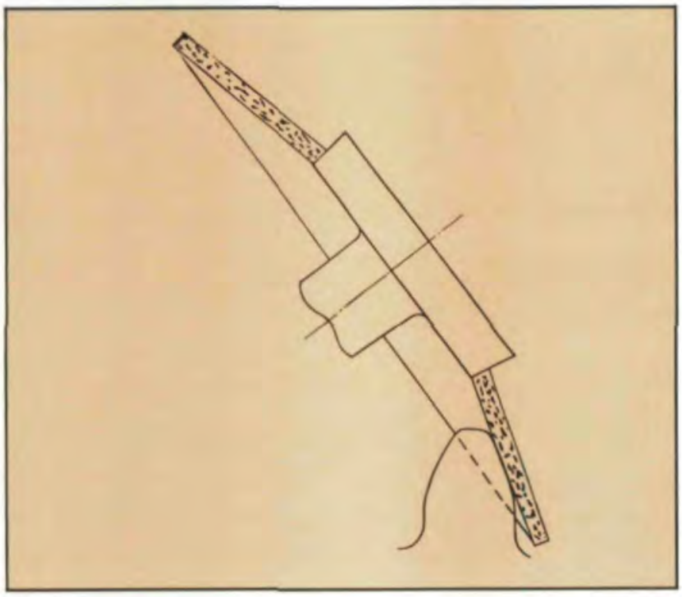

Fig. 4

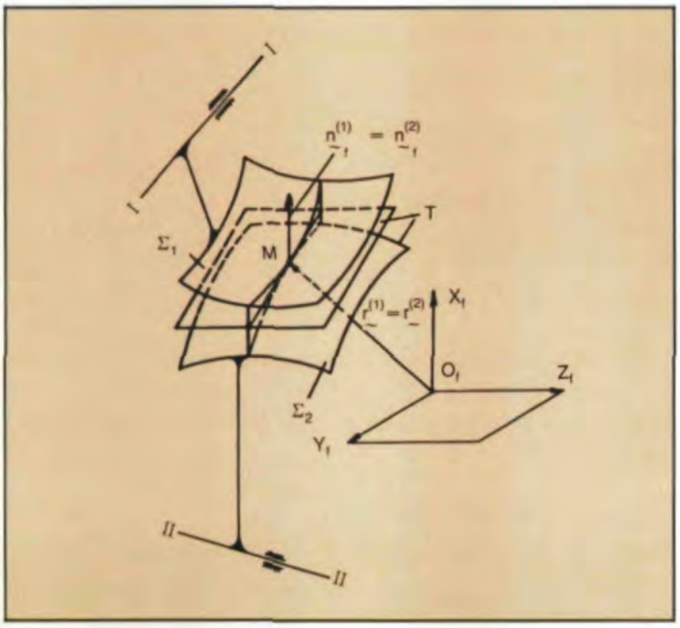

Fig. 5

and the gear being generated are related similarly to the motions of a rack-cutter and the gear. (See Section 3.) A tool with a regular conical surface can generate a pinion crowned surface whose middle cross section represents an involute curve (Fig. 3). However, this type of pinion crowned surface is not desirable because the misaligned gears provide Type 1 transmission errors (Fig. 2a). Therefore, a tool with *a surface of revolution instead of a conical surface is used.*

The evaluation of the bearing contact and transmission errors for the misaligned gears, as well as the investigation of the influence of errors of gear assembly, needs the application of TCA programs. The programs are based on the following algorithm:

(a) The contacting gear tooth surfaces are represented in a fixed coordinate system, S_f, that is rigidly connected to the gear housing (Fig. 5).

(b) The continuous tangency of gear tooth surfaces is provided if the position vectors and surface unit normals for the contacting surfaces coincide at the contact point at any instant. Then we are able to determine the path of contact on the gear tooth surfaces and the relations between the angles of rotation of the output and input gears. Knowing function $\phi_2(\phi_1)$ we can determine the deviations of this function from the prescribed linear function; i.e., the transmission errors.

(c) Due to the elasticity of the gear tooth surfaces, the surface contact is spread over an elliptical area. The dimensions

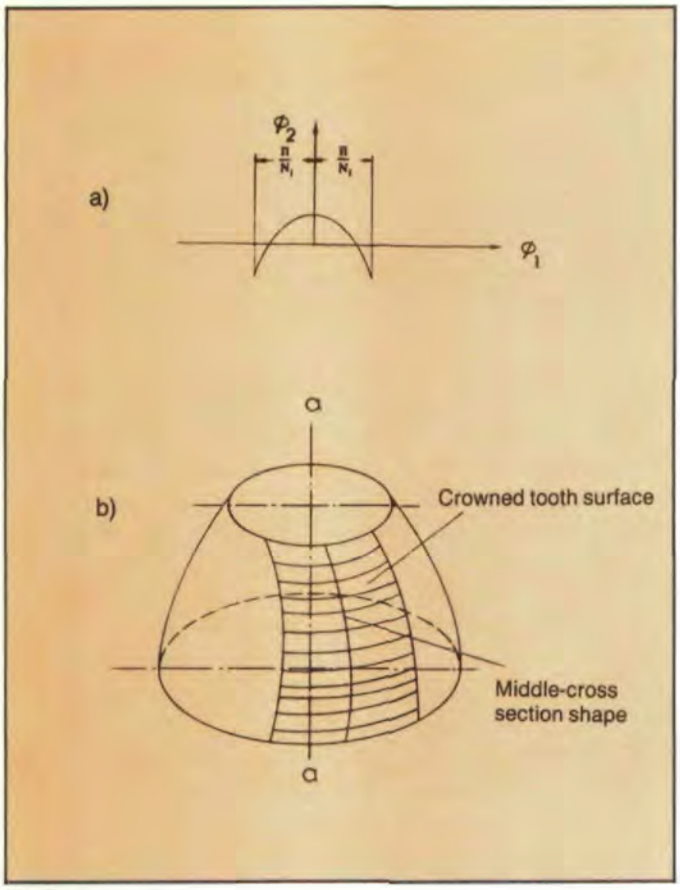

Fig. 6

and orientation of the instantaneous contact ellipse depend on the principal curvatures and the principal directions of the contacting tooth surfaces. The bearing contact is determined by the developed TCA program as the set of the contact ellipses that move over the contacting surfaces in the process of motion.

Synthesis of Pinion Crowned Tooth Surface

Consider that the gear is provided with a regular involute surface. The pinion will be provided with a crowned surface. The shape of this surface in the middle cross section is synthesized on the basis of the following considerations: Two shapes – a regular involute curve of gear 2 and the to-be-determined shape of the pinion tooth surface – are in mesh in the middle cross section. Although the gears are not misaligned, their shapes, being in mesh, must transform rotation with the function (Fig. 6a).

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + \Delta\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + b - a\phi_1^2 \quad (2)$$

Here:

$$\Delta\phi_2(\phi_1) = b - a\phi_1^2 \quad (3)$$

$\Delta\phi_2(\phi_1)$ is a parabolic function that must satisfy the following equation:

$$\int_{-\frac{\pi}{N_1}}^{\frac{\pi}{N_1}} (b - a\phi_1^2)d\phi_1 = 0 \quad (4)$$

Here $\frac{2\pi}{N_1}$ is the angular distance between two pinion neighboring teeth. Equation 4 states that the arithmetic average of the transmission error $\Delta\phi_2(\phi_1)$ over the interval $(-\frac{\pi}{N_1}, \frac{\pi}{N_1})$ is equal to zero.

After some transformation we obtain

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + d\left[1/3 - \left(\frac{N_1}{\pi}\right)^2 \phi_1^2\right] \quad (5)$$

The magnitude of d represents the level of transmission error. Using methods of synthesis of planar gears[3] we may determine the sought-for shape of the pinion middle cross section.

The longitudinal shape of the pinion crowned tooth surface may be determined from the requirements of the contact ellipse. The authors propose representing the pinion crowned tooth surface as a surface of revolution that can be generated by rotation about its axis a-a (Fig. 6b).

The advantage of the proposed geometry of the crowned pinion tooth surface is that the gears, while misaligned, have a parabolic type of transmission error, and the discontinuance of meshing can be avoided.

An example is given below to demonstrate the results that can be obtained through use of the proposed crowning method.

Example 1

Given: numbers of teeth: $N_1 = 20$, $N_2 = 40$; diametral pitch, $P = 10 \frac{1}{in}$; pressure angle $\Psi_c = 20°$. The pinion tooth surface has been designed as a crowned surface with a parabolic transmission error maximum d = 2 arc seconds in the aligned condition. The developed TCA program has been applied for the evaluation of transmission errors for the following misalignments:

(i) The change of the center distance is $\frac{\Delta c}{c} = 1\%$. The gear axes are not parallel, but crossed, and the screw angle is five arc minutes. The function of transmission errors caused by the misalignments mentioned above is of a parabolic type, and it is represented in Table 1. The maximal value of transmission errors is 1.2 arc seconds.

(ii) The gear axes are not parallel, but intersected, and form the angle $\alpha = 5$ arc minutes. The function of transmission errors is of a parabolic type, and the maximal value of transmission errors is 2.0 arc seconds (Table 2).

Generation of Pinion Crowned Tooth Surface by a Tool With a Surface of Revolution

Fig. 7 shows the installment of a tool with a regular cone surface. The cone surface is tangent to a plane which is the surface of a rack cutter. We can imagine that the two tools – a cone and a rack cutter – being rigidly connected, generate a crowned pinion surface and a regular gear involute surface, respectively. In the process of generation the rack cutter and the cone perform a translational motion, while the pinion and the gear rotate about their axes (Fig. 8). The rotation of the cone about its axis, c-c, is not related to other motions that have to be provided for the tooth surface generation. The angular velocity in the rotational motion of the cone depends on the desired velocity of cutting. The tool for the crowning of the pinion can be designed as a grinding wheel or as a shaver. The opposite sides of the pinion tooth are generated separately.

The described process of the crowning of the pinion by a regular cone provides an involute shape for the pinion tooth surface in its middle section. The crowned pinion and the involute gear, if they are not misaligned, can transform rotation without transmission errors, and their bearing contact can be localized. However, the misaligned gears will transform rotation with Type 1 transmission errors (Fig. 2a). To avoid the discontinuance of tooth surfaces that occurs at the change of teeth in meshing, a surface of revolution must be used instead of a cone surface. This surface slightly deviates from a regular cone surface and its application for crowning provides Type 2 transmission errors (Fig. 2b). Also,

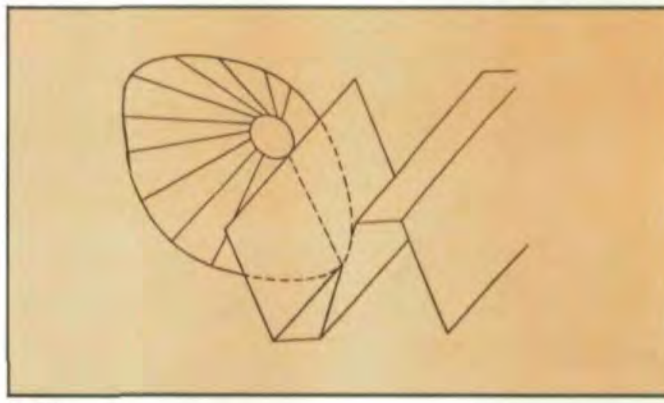

Fig. 7

Table 1
Function of Transmission Errors

ϕ_1 in degrees	−3	0	3	6	9	12	15
$\Delta\phi_2$ in arc seconds	−0.71	0.00	0.40	0.50	0.33	−0.09	−0.74

Table 2
Function of Transmission Errors

ϕ_1 in degrees	−10	−7	−4	−1	0	2	5	8
$\Delta\phi_2$ in arc seconds	−2.02	−0.92	−0.24	0.01	0.00	−0.16	−0.75	−1.78

knowing the topology of the pinion crowned surface that is generated by the surface of revolution, we can generate it by a plane. The conditions of gear meshing and their bearing contact have been simulated by the TCA program that has been developed by the authors. An example is shown below to demonstrate the concepts developed in this article.

Example 2

The input is the same as in Example 1. The pinion tooth surface is crowned by a surface of revolution with the following parameters (Fig. 9): $\Theta = 20°$, $R = 500''$. The misalignment of gears has been simulated and the transmission errors have been evaluated by the developed TCA program with the following results. The gear axes are not parallel, but crossed, and the screw angle is $\alpha = 10$ arc minutes. The change of the center distance is $\frac{\Delta c}{c} = 1\%$. The function of transmission error is of a parabolic type, and the maximal value, 0.35 arc seconds (Table 3).

The gear axes are intersected and form an angle, $\alpha = 10$ arc minutes. The function of transmission error is of a parabolic type and its maximal value is 0.34 arc seconds (Table 4).

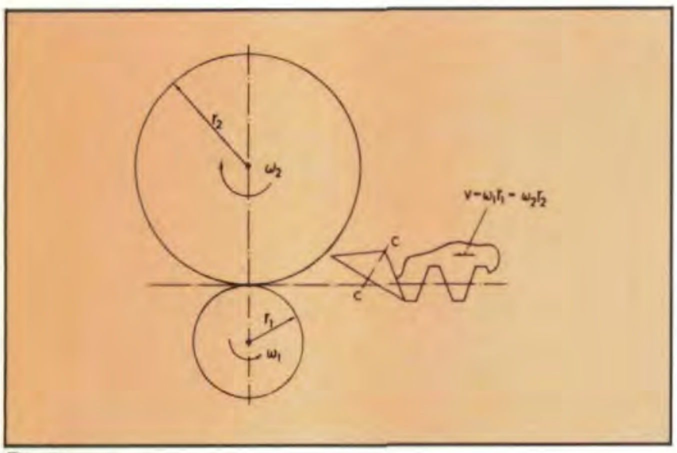

Fig. 8

Table 3
Function of Transmission Errors

ϕ_1 in degrees	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2$ in arc seconds	−0.26	−0.10	0.00	0.02	0.00	−0.02	−0.14	−0.33

Table 4
Function of Transmission Errors

ϕ_1 in degrees	−11	−8	−5	−2	0	1	4	7
$\Delta\phi_2'$ in arc seconds	−0.33	−0.15	−0.03	0.01	0.00	−0.02	−0.11	−0.20

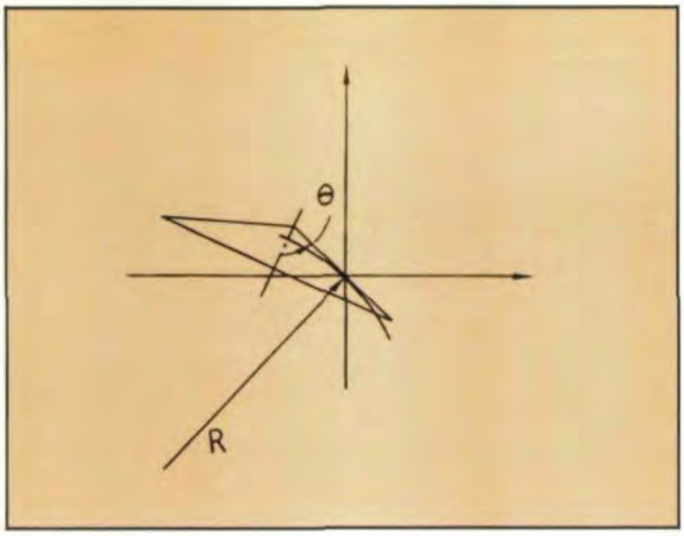

Fig. 9

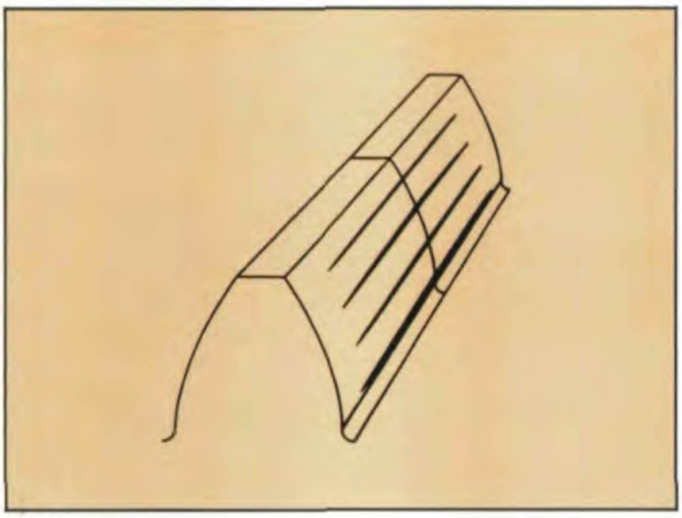
Fig. 10

Display of Analytical Results by Computer Graphics

A computer graphics program was developed to represent in 3-D space the pinion crowned tooth surface and the motion of the contact ellipse in the process of meshing.

The graphics program is based on the analytical solutions that have been obtained from the TCA program. It produces the intricate high resolution picture on a graphic terminal and laser printer. Fig. 10 shows the results of the computer graphic program that represents the pinion crowned surface and the location and orientation of the contact ellipses.

Conclusion

The authors developed:

- A method for the synthesis of a pinion crowned surface that provides a localized bearing contact and a limited level of transmission error of a parabolic type. This approach provides favorable conditions of meshing and contact for misaligned spur gears.
- A method for generation of a pinion crowned tooth surface by a surface of revolution that slightly deviates from a regular cone surface. This method can be applied for crowning by grinding and shaving.
- TCA programs to simulate the meshing and bearing contact of misaligned spur gears with the crowned pinion and to investigate the influence of misalignment on the transmission errors.
- A computer graphic program to display in 3-D space the pinion crowned tooth surface and the location and orientation of the contact ellipses.

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