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Approximating an Involute Tooth Profile

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On many occasions a reasonably approximate, but not exact, representation of an involute tooth profile is required. Applications include making drawings, especially at enlarged scale, and laser or EDM cutting of gears, molds, and dies used to produce gears. When numerical control (NC) techniques are to be used, a simple way to model an involute can make the NC programming task much easier.

Previously we had found that a second order polynomial (A*X*X + B*X + C) gave a very good fit to an involute. Out of curiosity, we thought to test how well a circular arc (a special case of a second order polynomial) fit an involute. A program, GEARFIT, was developed to do this, although the algorithm can be used without this specific program. GEARFIT does the following:

- Request data for gear parameters (e.g. pitch diameter and pressure angle) from the user.
- Generate N points on the involute between the minor and major diameters, using standard involute equations.
- For every combination of three points on the involute:
  - Find the circle which fits the three points,
  - Determine the distance from the center of the trial circle to every point on the involute,
  - Record the maximum error, relative to the radius of the arc fit to the current three points.
  - Select the circle which had the least maximum error.

The number of trial circles which GEARFIT generates is given by the expression:

NC = NP! / (3! * (NP - 3)!), where NP = number of points on involute.

This can be simplified to

NC = NP * (NP-1) * (NP-2) / 3!
NC = NP * (NP-1) * (NP-2) / 6

For 25 points on an involute, 2300 trial circles will be generated. For each trial circle, the distance to 25 points must be found to find the maximum error. If only 12 points are generated on the involute, the number of trial circles is reduced to 220. Running the program on an IBM PS/2 with a math coprocessor and measuring time by a wall clock, 18 seconds were required to solve the case with 25 points; 2 seconds were required for the 12-point case.

The best fit arc for 25 points had a maximum error of 0.0002; the 12-point case had a best fit error of 0.0003. These errors are typical for all gear configurations tried to date. This indicates the faster solution based on fewer points is close enough for most applications.

One test of GEARFIT used the following parameters (dimensions in inches):

Pitch diameter: 2.3125
Pressure angle: 25.0
Root diameter: 2.142 - 2.162
Outside diameter: 2.433 - 2.438
Arc tooth thickness: 0.0922 - 0.0952
Radius over one 0.1080 diameter pin: 1.2259 - 1.2289

The results from GEARFIT are presented in Table 1.

A CAD system was used to visually verify the results. The points on the in-

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Table 1
Results of GEARFIT

Input Data:

Pitch diameter:	2.3125
Pressure angle-degs.	25.0
Base diameter:	2.0958
Minor diameter:	2.1520
Major diameter:	2.4350
Tooth thickness:	0.0937
Pin Diameter:	0.1080
No. of Teeth:	37

Results:

Center of arc:	0.9525	0.4466
Radius:	0.4444	
Best fit maximum error:	0.0003	
Tooth thickness:	0.0940	
Radius over pin:	1.2276	
Points giving best fit:	2 12	23

Points on involute and error:

1	1.0758	0.0199	−0.0002
2	1.0817	0.0215	0.0000
3	1.0875	0.0232	0.0001
4	1.0934	0.0250	0.0002
5	1.0993	0.0270	0.0002
6	1.1051	0.0290	0.0002
7	1.1109	0.0312	0.0002
8	1.1168	0.0335	0.0002
9	1.1226	0.0359	0.0002
10	1.1284	0.0384	0.0001
11	1.1342	0.0411	0.0001
12	1.1400	0.0438	0.0000
13	1.1458	0.0466	−0.0001
14	1.1516	0.0495	−0.0001
15	1.1574	0.0525	−0.0002
16	1.1631	0.0556	−0.0002
17	1.1689	0.0588	−0.0002
18	1.1746	0.0620	−0.0002
19	1.1803	0.0654	−0.0002
20	1.1860	0.0688	−0.0002
21	1.1917	0.0723	−0.0002
22	1.1974	0.0760	−0.0001
23	1.2031	0.0797	0.0000
24	1.2087	0.0834	0.0001
25	1.2144	0.0873	0.0003

volute were entered. The approximating arc was generated and used to determine the center of the 0.1080 diameter pin. No discernable error could be found between the approximating arc and the involute points, except at extreme magnification. The distance from the gear center to the outside edge of the pin was determined by the CAD system to be 1.2277. This compares to 1.2276 predicted by GEARFIT; both values are well within the allowable limits specified for the radius over one pin.

In all cases tested, the error in using an arc to represent an involute has been less than +/− 0.0005 inch. This is well within acceptable limits for preforming operations or manufacturing by laser or EDM techniques. Using an arc to represent the involute allows the tangent points of blending root and tip radii to be quickly and accurately located. Most NC controls have circular interpolation, so the involute profile can be approximated by a single statement when the center, radius, and endpoints are known. ■